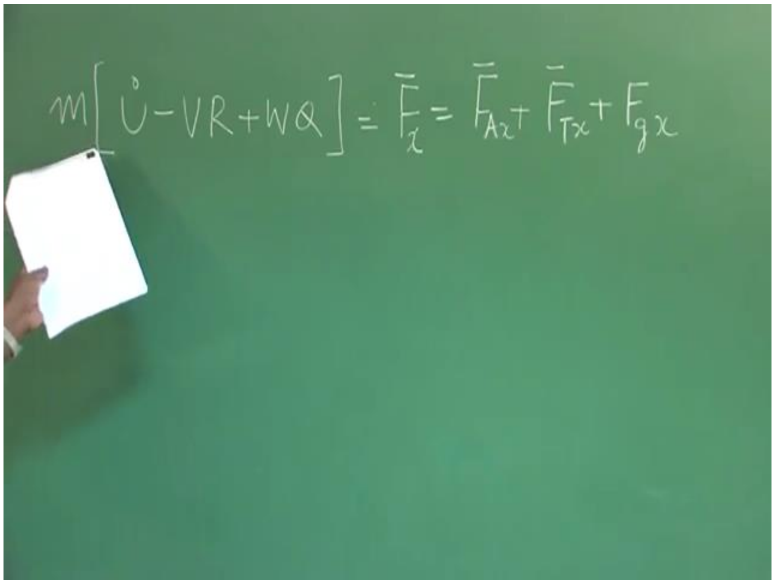
Aircraft Dynamic Stability & Design of Stability Augmentation System
Professor A.K. Ghosh
Department of Aerospace Engineering
Indian Institute of Technology Kanpur
Module 2
Lecture No 11
Euler Angles

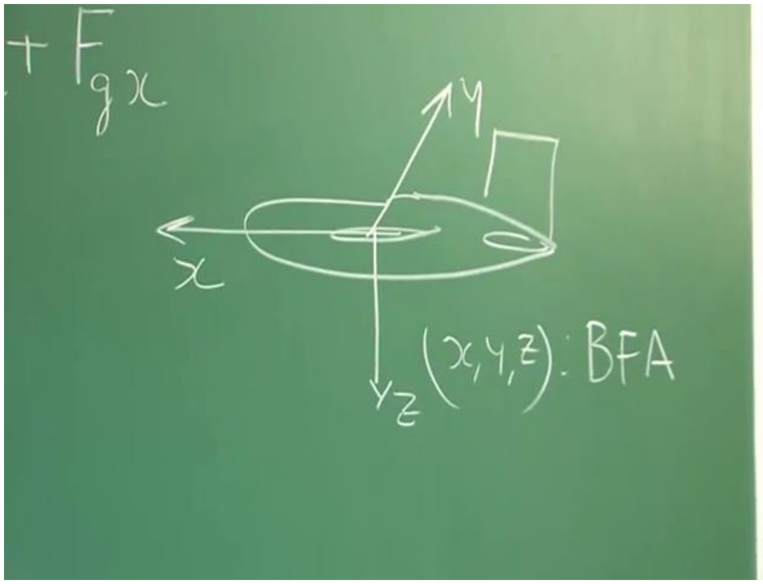
Good morning friends. So far we have developed equations of motion and we realised that there are distinct advantages in working in the body frame. We all know that if I am using Newton's laws of motion, then I have to work with reference to inertial frame of reference. We have found out a way to equivalently work in body frame applying appropriate additional corrections or terms so that we still can work in body frame. But we do not violate the condition that Newton's law of motion is valid in reference to inertial frame.

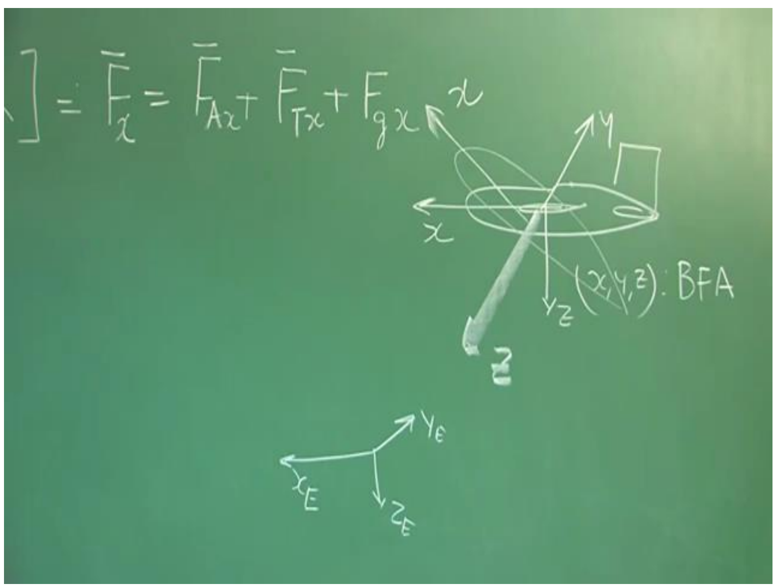
(Refer Slide Time: 01:48)



And doing that, we found out an equation. Just I will write 2 of them, M U dot - VR + WQ equal to force, external impressed force and that is in the X direction and this is composed of aerodynamic force in next direction, propulsive force in X direction + gravitational force in X direction. This I am writing along X direction. What is this X? Please understand.

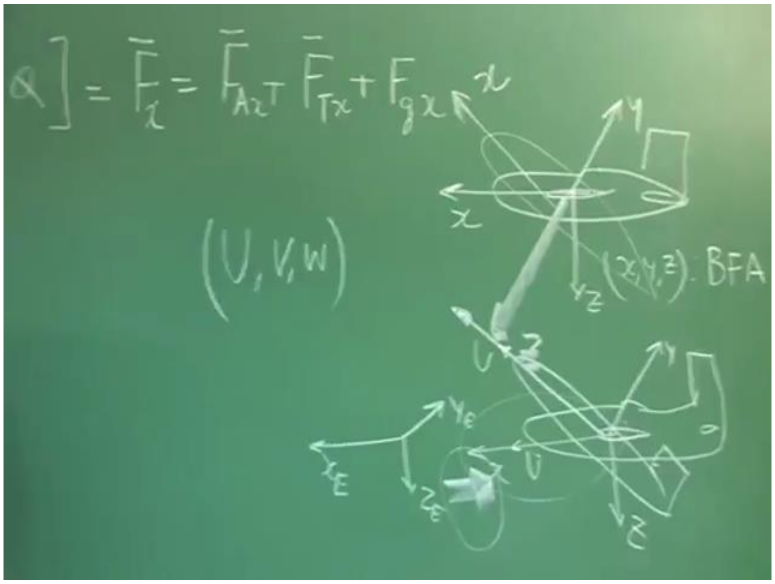
(Refer Slide Time: 02:50)





This is the body of the aircraft and X, Y, Z, this XYZ, they are body fixed axis system. What is the implication of that? That although inertial frame of reference, if I write XE, YE and ZE, in inertial frame of reference, the direction is fixed. However, when I am operating in body fixed axis system, as the body rotates, let us say the body is rotating like this, then X becomes this. X changes its direction. Z becomes this. So there is change in the direction of X, Y, Z in a body fixed axis system.

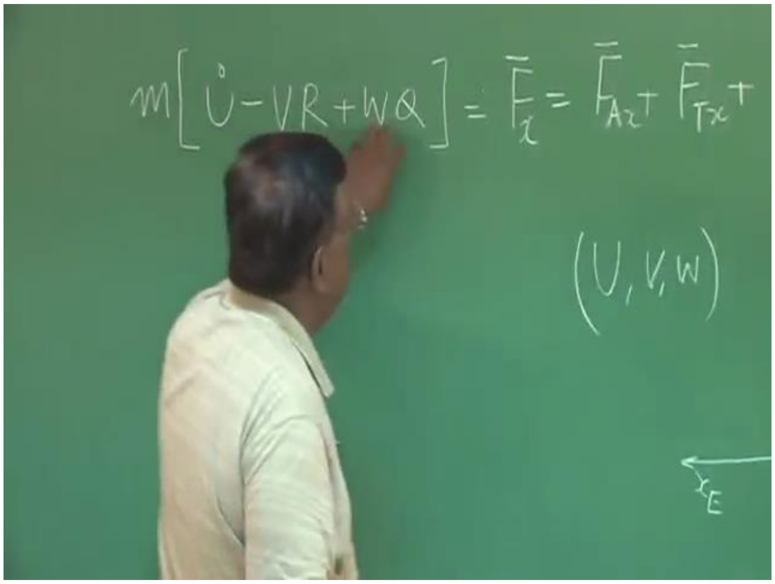
(Refer Slide Time: 04:20)



And when I write this equation, what does this mean? This means U dot, what is this U dot? U dot is, in body fixed axis system, U, V, W are the components of total velocity which is measured with respect to inertial frame but resolved along body X, Y, Z axis and since body X, Y, Z axis is changing its orientation, so naturally U, V, W also go on changing. Just to give an example, suppose this is the airplane and this is X, this is Y, this is Z and let us say relative wind is coming like this, relative air velocity.

Then you could see that forces along X direction is primarily the drag or the thrust. But if we do not change this and the body changes its orientation, then earlier U was this one. Now, U will be along X direction, local X. This so this will be U.

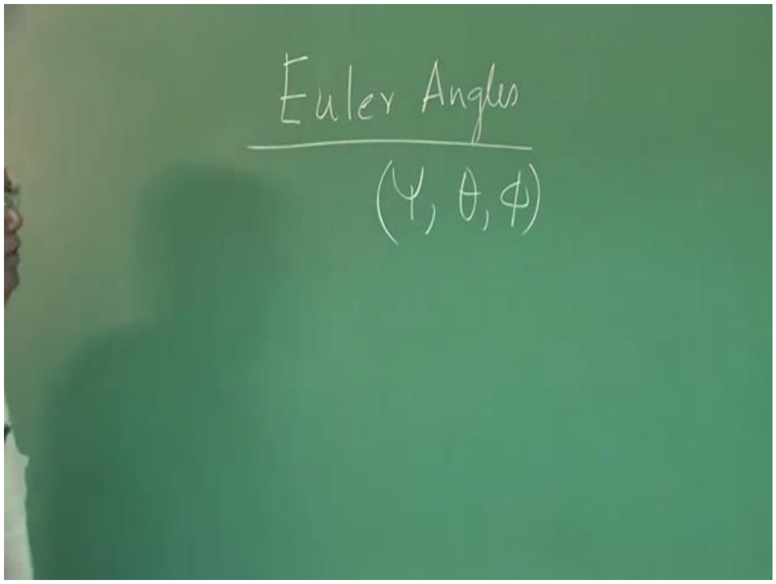
(Refer Slide Time: 4:22)

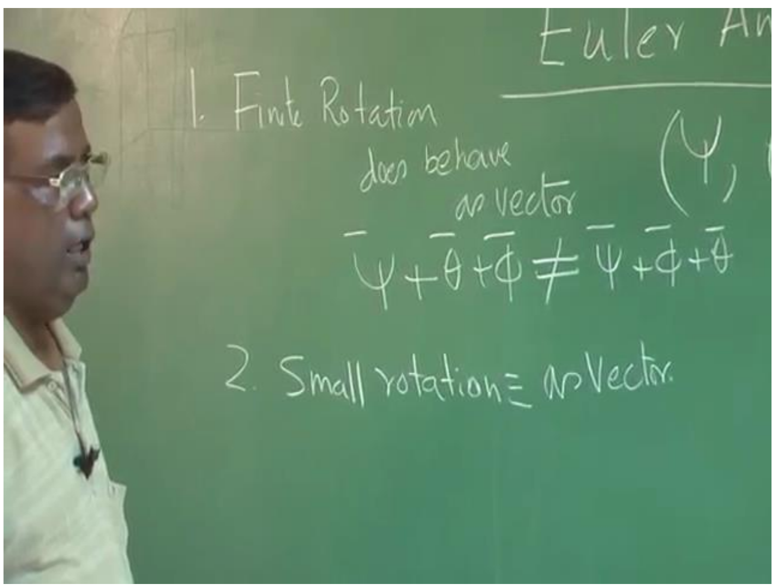


So if I were to write, M U dot, all this equation, then FX or F aerodynamic X will be the component of this force along this direction. So it will change. How it will change? It will change depending upon what is the orientation of the body fixed, X, Y, Z axis. So I am using the word, orientation. So we need to know how to define the orientation of the airplane in space. I cannot measure the orientation with respect to body fixed axis because as the body rotates, axis also rotates.

Why the axis rotates? Because it is a body fixed axis. So I cannot measure the orientation with respect to the body fixed axis. So what is the way out? Way out is, we measure the orientation of this body with respect to inertial frame. Right? So that is the way we handle the orientation of the axis or we try to locate the airplane with respect to inertial frame.

(Refer Slide Time: 06:12)

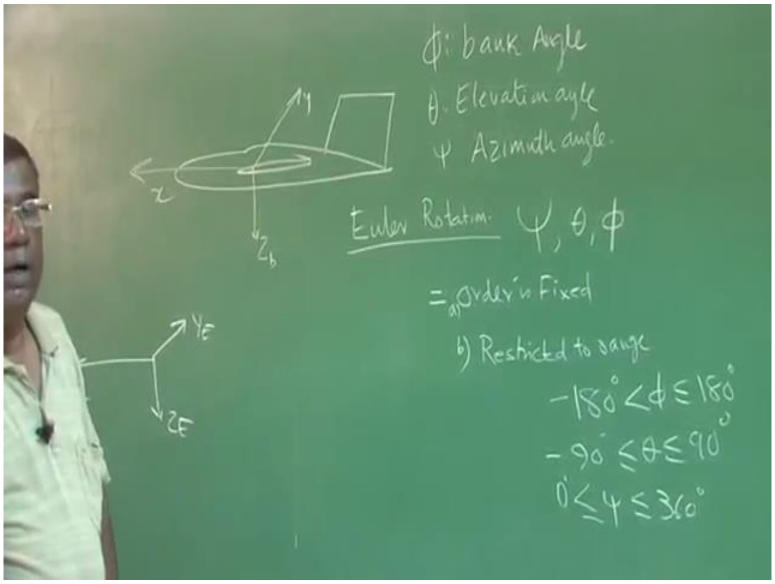




And now to understand that, we define something called Euler angle and we need to know what is the meaning of Euler angle? Typically you will find, Euler angle has seen rotations, sigh, theta and Phi. And we will see what this Psi, Theta and Phi means? Before we go into Euler angles, 2 things we need to understand.

One is, this finite rotation does not behave as vector. That is, Psi + Theta + Phi may not be equal to Psi + Phi + Theta or any other combination. So, finite rotation does not behave as a vector. However small rotation, they do behave as vector. Okay? So these are the 2 things that we should keep in the back of our mind. And, it is very easy to say this that this is indeed true. Now, with this understanding, we should go for defining Euler angle.

(Refer Slide Time: 08:26)



If I take this as an aircraft and this year X Earth or inertial frame, Y Earth, Z Earth and this is X body, Y body, Z body, then the rotation is, the Euler rotation, one is, order is important. Order is Psi, Theta and Phi. Second thing, how do I define Psi? Suppose this is the airplane. If this is the Z axis, let us say Z of Earth, then Psi is the rotation about that axis. Now what will happen? As I rotate Psi about Z axis, now the body fixed axis is changed. Earlier, this was X. Now, X is also tilted like this and Y came like this.

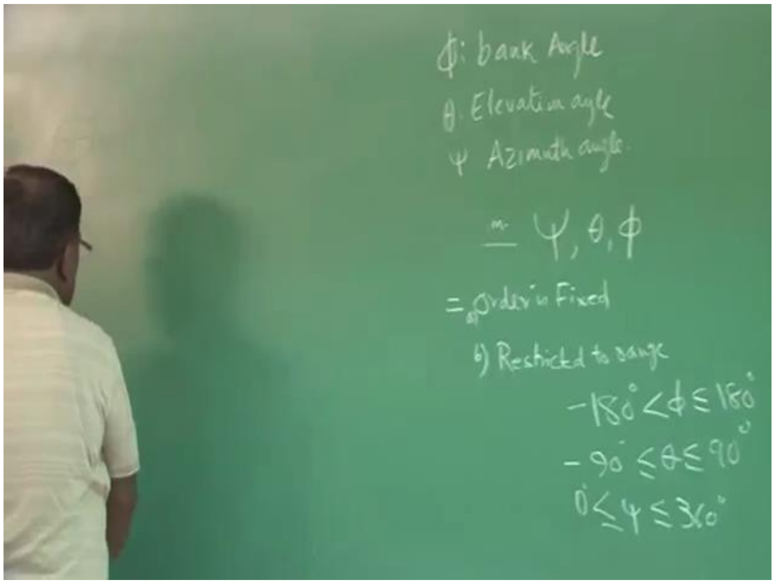
So Theta will be with respect to rotation about this new Y axis, Theta. And the now you see, the X axis also changes. So Phi will be about this axis to stop okay. So that is to be kept in mind and this has been cleared in my last module also, the course on static and dynamic stability. So please remember this that Psi, Theta, Phi has a particular order. The order is fixed and second thing which is also important, I am just brushing the earlier understanding, second thing you should be clear that the standard convention is restricted to a range - 180 degree less than Phi less than equal to 180 degree.

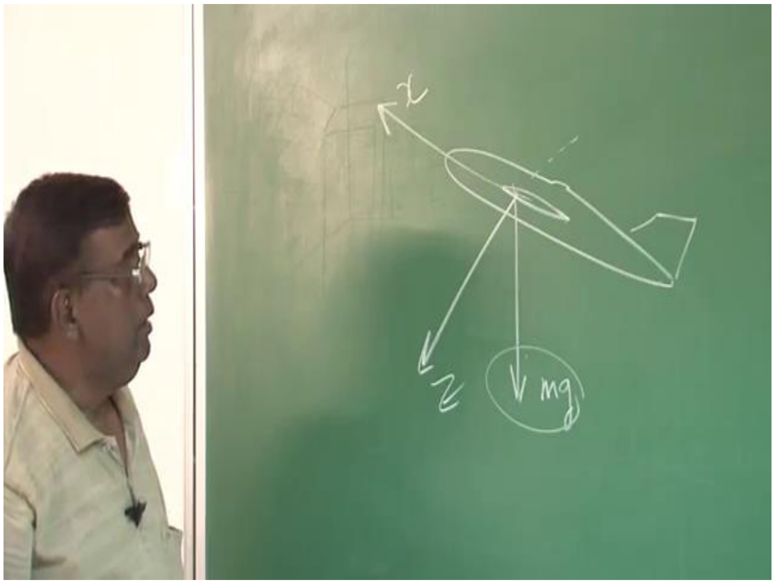
Then - 90 less than or equal to Theta less than or equal to 90 degree. And 0 degree less than or equal to Psi less than or equal to 360 degree. These restrictions, you should remember when you are trying to develop flight dynamic model using Psi, Theta, Phi or Euler angle concept. And there is a special name given for this Psi, Theta, Phi where Phi we call as bank angle. Then Theta is called as elevation angle and Psi is called as azimuth angle.

Please understand one thing. Loosely we often tell Phi as bank angle, Theta is which angle and Psi as yaw angle which is not strictly correct. Why? See the point. What is Theta? Theta is not about body axis? Theta is not about the final orientation axis. Theta is about the intermediate axis. What? Suppose this is the orientation am starting with. I am giving a Psi. Now this becomes Y. Earlier, Y was like this. Now this becomes Y.

So Theta is above this Y axis. Similarly Phi. Phi will be about this axis. So they are different as compared to the concept of roll, pitch and yaw angle. I will strictly recommend you to read a few pages from flight dynamics book by Warren Phillips, wonderfully it has been written. I will try to give some examples in the forum so that you get better ideas. This is mathematics part of it. Why we are doing all this? Let us understand. That is more important.

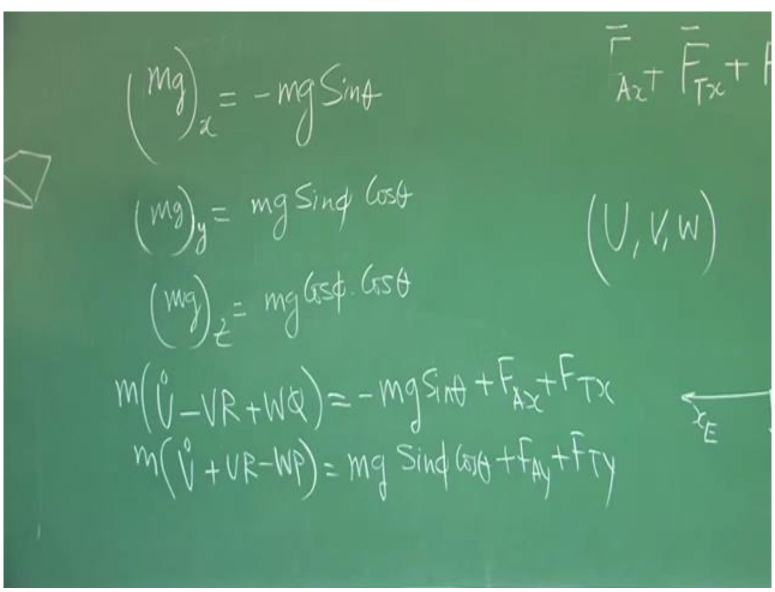
(Refer Slide Time: 12:20)





We have just seen that if I take to configurations like this, this is the X body axis, Z body axis and Y. Now the gravity force MG will always act downwards which will be in direction of the inertial Z axis. But as the orientation is changing, this component along X, Y, Z will also change depending upon what is the angle, elevation angle, what is the azimuth angle, what is the bank angle? So if I want to resolve this component of it along X, Y and Z, I need to know how this MG is being resolved using Phi, Theta and Psi.

(Refer Slide Time: 14:55)



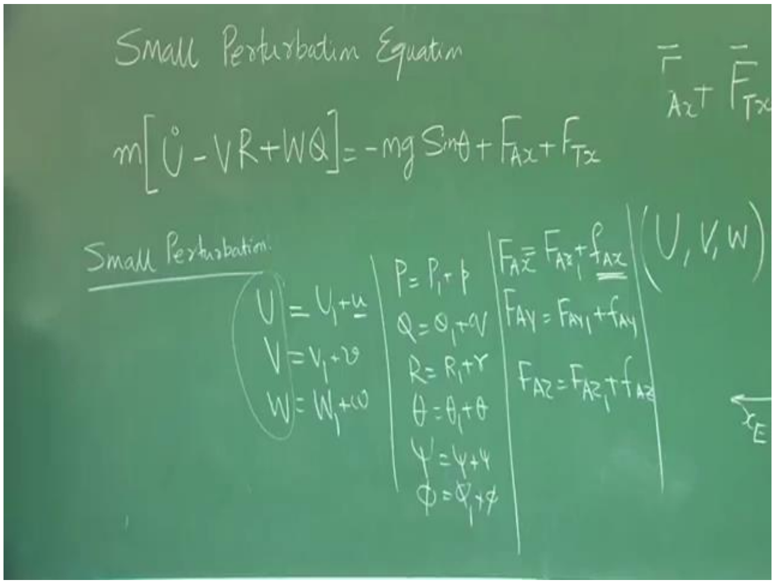
And you have seen that we can easily write it as MG X component is equal to - MG sin Theta. Similarly MG Y component, I can write it as MG Sin Phi Cos Theta and MG Z component, I can write it as MG Cos Phi into Cos Theta. So when I write this equation, M U dot - VR + WQ equal to FGX, for this FGX, I will write - MG Sin Theta, then remaining F, aerodynamic force F + propulsive force, F.

Similarly for other equation also I can write, MV dot - or + UR - WP equal to MG Sin Phi cos Theta + FKY + FTY. And other, you can easily write using this term. But before we go further, please understand, I have not, we have not developed a perturbed equation. Equation of motion.

We have developed a general equation of motion. But when I was giving you an example of mass spring damper system, and, like MX double dot + KCX dot + KX equal to F of T and there the X was measured with reference to the point of equilibrium. So it is like a short or a small perturbed equation of motion. So we also need to develop a small perturbation equation using this basic equation.

That is our aim. We are only trying to understand how these components can be resolved using orientation using the environmental condition that is P, density, aerodynamic characteristics, propulsive characteristics and we like to see if I give a small perturbation, how those forces also change. That is why we are developing this.

(Refer Slide Time: 21:35)



So we will be starting with small perturbation. We will just take one by one equation and try to understand how it is developed. Once we understand how to develop, then it becomes very mechanical for you to do it. So I will give one example. Let me take MU dot - VR + WQ equal to - MG Sin Theta + FAX + FTX. Please understand. What are all these equations talking about?

What are U, V, W? U, V, W are the components of the total velocity measured with respect to inertial frame but resolved along local X, Y, Z or body fixed X, Y, Z direction. Similarly, Q is the pitch rate, R is the yaw rate. These are the forces along X direction. Now once I do small perturbation meaning thereby, it is a small perturbation. I am very clear that once I introduce the perturbation at equilibrium, the perturbation should be so small that it is not going to change the linearity of the whole dynamics.

In a sense, if it is the aerodynamic forces which we will be using assuming a linear model, so by changing the condition about the equilibrium, maybe a cruise in this case, the aerodynamic shall still remain linear. And under that small perturbation concept, we can use that if I am flying at a steady-state condition, U1, next is the cruise. We are introducing disturbances about cruise. Then it is cruising.

I give a small perturbation and see what is happening to the machine dynamically? Whether it is coming back once the disturbance is withdrawn. What is it doing? So I can write, a small perturbation is, if this is the steady-state condition and this is the perturbation introduced, small U, then total velocity, I can write as the sum of this.

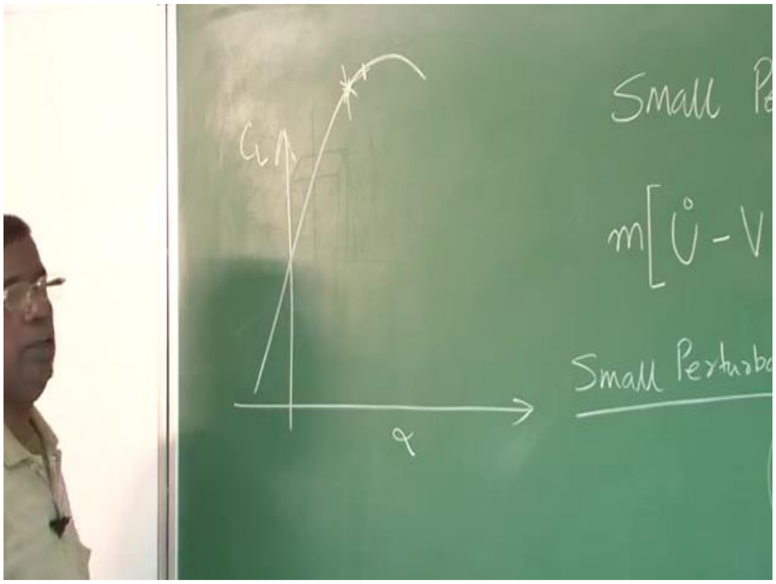
That is the advantage of small perturbation under the domain of linear approximation. Similarly if I say V, I write V1 + small V. W equal to W1 + small W. Similarly you could see, I write P is equal to P1 + small P, Q is equal to Q1 + small Q, R is equal to R1 + small R, Theta is equal to Theta 1 + small Theta. Psi is equal to Psi 1 + small Psi, Phi is equal to Phi 1 + small Phi.

What are these 1 designated variables? They are the condition at steady-state. So suppose I am flying at cruise, level cruise like this. Okay? That steady-state you understand that P1, Q1, R1, all are 0 at steady-state if I am doing a level cruise. Because there are no roll, there are no pitch and there are no yaw at steady-state if my equilibrium state is a normal cruise level, unaccelerated cruise. So these approximations, I will be using.

Second thing, on aerodynamics part, we will say, FAX, on the left-hand side, I hope you understood, after introducing the perturbation, what is the total velocity, U, V, W in their respective axis, X, Y, Z? So FAX again I will write as FAX1 + FAX. So this is a perturbed or dynamic force along X direction and FAX1 is the steady-state.

Similarly FAY, I will write as FAY1 + FAY. FAZ as FAZ1 + FAZ. Clear? So what are FAX, FAY, FAZ? They are the perturbed aerodynamic forces that is introduced because of the perturbation given to the airplane at cruise. Clear: these are total. This is composed of FAX, steady-state value and perturbed the value. Naturally you could see that again everywhere you are using the concept of, advantage which you have by assuming it to be a linear, small perturbation, a result of small perturbation. Here is word of caution, you must understand.

(Refer Slide Time: 22:04)

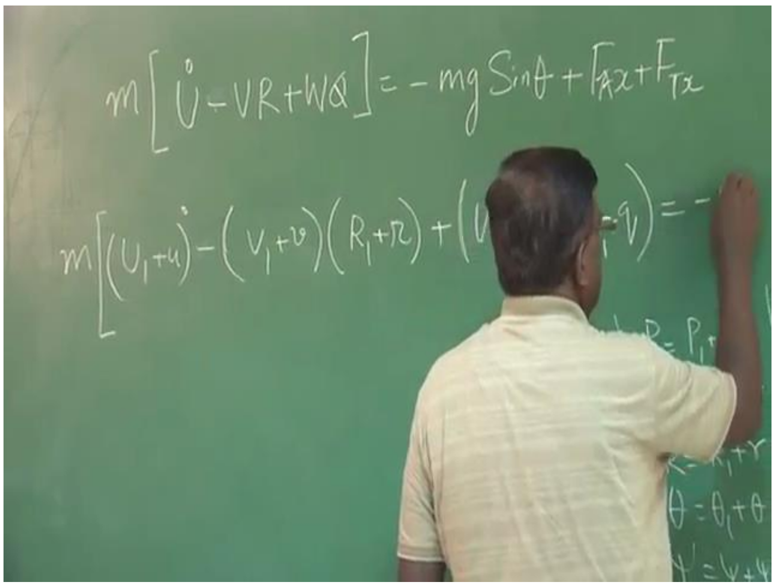


Suppose I am flying such that CL versus alpha, follow this trail and you are flying somewhere here. And now if you give a small perturbation, it may be possible that are you going into a non-linear domain. So there strictly, this will not be valid. But there are ways to handle it. We will try to make it (())(22:16) linear. But once you will be careful, once you are applying this, do not apply this mechanically.

Especially, today is the era of high angle of attack, high performance aircraft. So mostly you have to be very very careful how best these assumptions are true when you are applying for dynamic stability analysis.

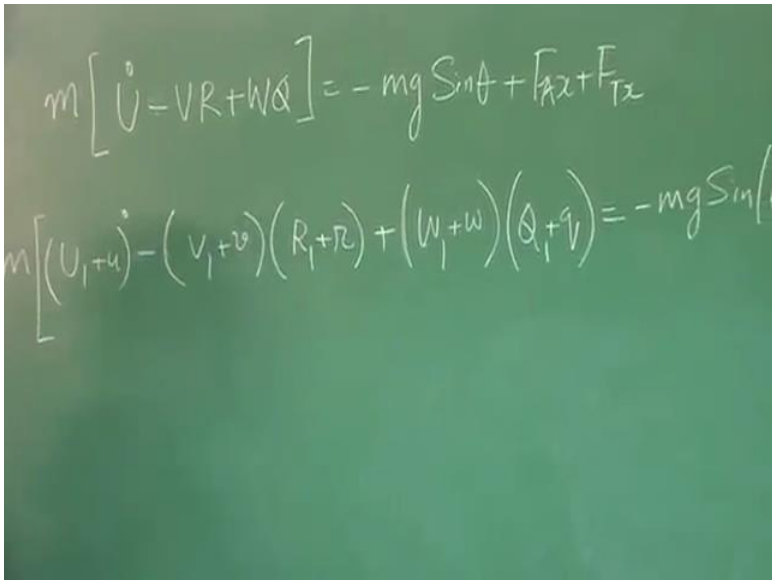
Yes, good morning friends. Let us solve an example of how to develop perturbed equation of motion.

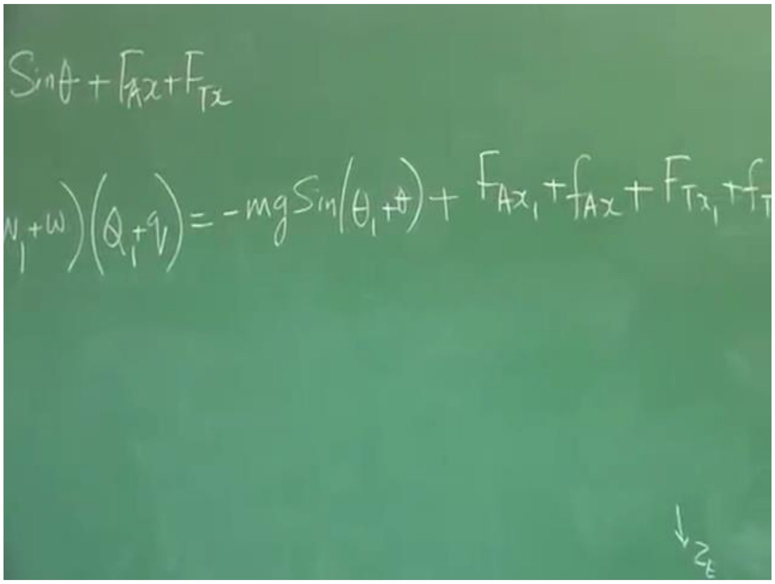
(Refer Slide Time: 24:25)



So I will take one equation. That is MU dot - VR + WQ is equal to - MG Sin Theta + FAX + FTX. Now I will introduce small perturbation. What do I do? I write MU dot is, U I write as U1 + u dot +, for V, I write V1 + small V, for R, I will write R1 + small R. For W, I will write W1 + small W. Then for Q, I will write Q1 + small Q. And on the right-hand side, I will write MG Sin Theta 1 + small Theta. For FAX, I will write FAX1 + FAX and for that, FTX1 + FTX.

(Refer Slide Time: 25:00)

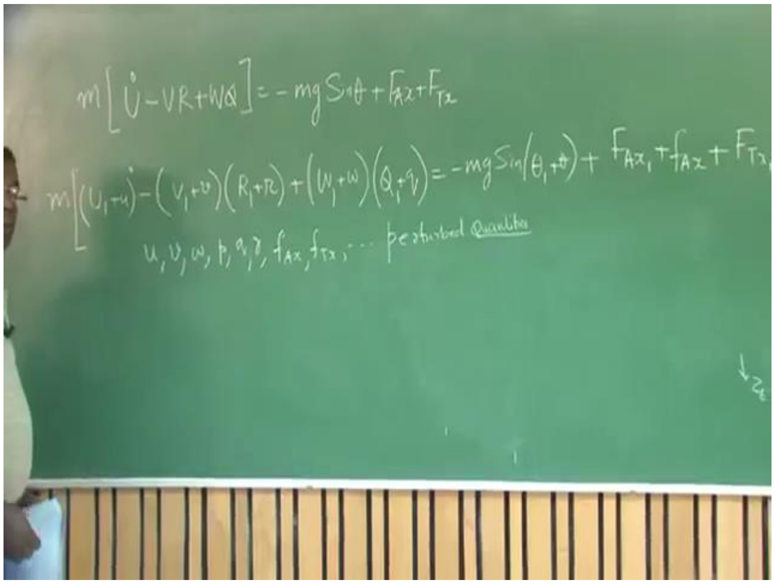




Please see what we have done. I have taken one equation from the equations of motion and as we realised that when we did mass spring damper system, we wrote the equations of motion in perturbed quantities. Right? Because we measure that X about a displacement, about equilibrium. We are also trying to develop the perturbed equation of motion and we want to see what is happening if this airplane is disturbed about its equilibrium, about its steady-state. U1, V1, R1, wherever 1 is there, they represent the steady-state condition.

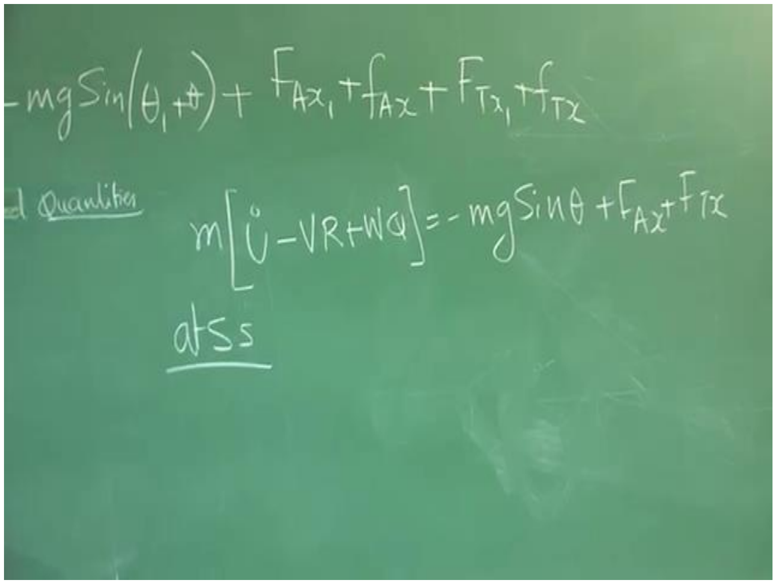
So what I have done? For U, I have U1 + U dot. For V, V1 + V into R is R1 + R. Similarly other terms.

(Refer Slide Time: 25:35)



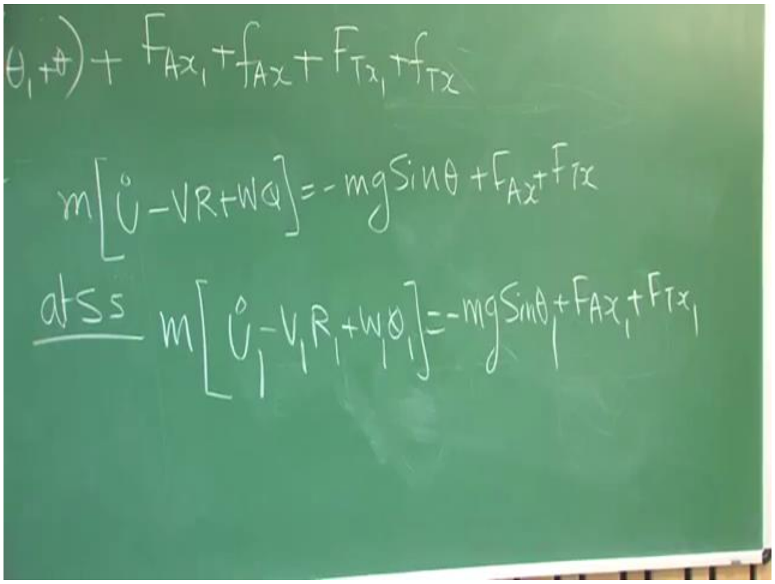
And we know, all this U, V, W, P, Q, R, FAX, FTX, etc are perturbed quantities. And our focus is to check how these variables are changing. Once we give the disturbance, once we the disturbance and based on this response, we can comment whether the aircraft is dynamically stable or not. And using this, we try to find out the criteria to be given to a designer so that he can ensure that the aircraft is dynamically stable.

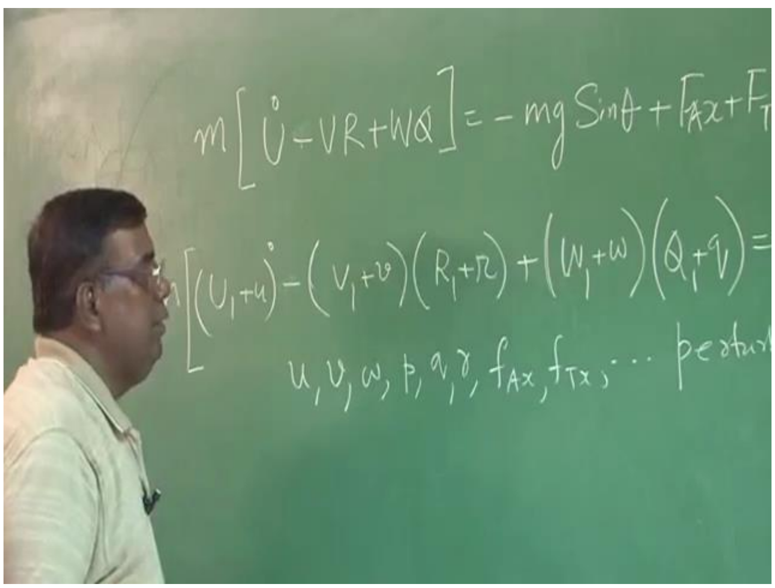
(Refer Slide Time: 26:23)



Now here you see, once I am writing this equation like this, we also know that M U dot – VR + WQ is equal to - MG Sin Theta + FAX + FTX. So we know that at steady-state, what is our steady-state for this study? It is the cruise, level cruise, level unaccelerated cruise.

(Refer Slide Time: 27:33)





At steady-state that means in your notation I can write U1 dot - V1R1 + W1Q1 will be automatically put to MG Sin Theta 1 + FAX1 + FTX1. So from here you could see that I could easily use that equation with this equation and try to simplify this. Also we need to know what is the expansion of Sin A + B. I have to just do this and then I can get a neater equation. So today what we have seen?

We have seen that what is the meaning of Euler angle and I will be giving you some examples in the forum. We understood the rotation of Euler angle. We also tried to understand how should we think in terms of developing perturbed equations of motion by giving a disturbance about steady-state and we write this equation because finally we want perturbed equations of motion to analyse the dynamic stability of the airplane and our focus will be monitoring how these quantities are changing.

Once we do that, we will be able to develop equations of motion at our own will. And my next venture will be on how to mathematically pass it. Please understand, we are talking about small perturbation and then we also introduce the concept of product of 2 perturbed quantities can be easily neglected or the perturbed quantities or perturbed disturbances are so small, we call them small perturbations and that must satisfy that the product of 2 perturbed quantities are negligible.

We will all use these concepts to develop the equation in a pure mathematical form which we will be using. So that I am ending it here. I do not want to pump in a lot many mathematics. We will be going in a smaller module. You absorb it, again go back and then go for next step. Thank you very much.