Indian Institute of Technology Kanpur

National Programme on Technology Enhanced Learning (NPTEL)

Course Title
Engineering Thermodynamics

Lecture – 23
Second Law of Thermodynamics: Clausius Inequality

by
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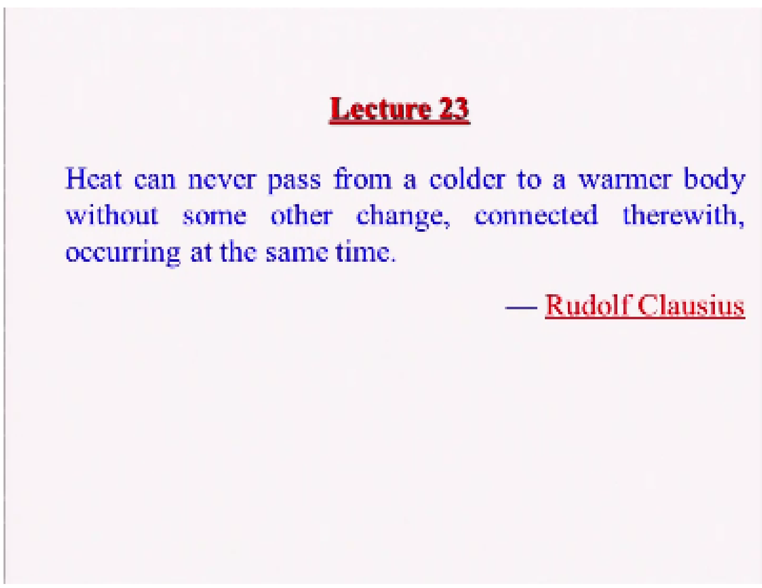

Let us start this lecture a thought process from Rudolf Clausius he says heat can never pass from a colder to a warmer body without some other change connected here with occurring at the same time. And of course that we have seen as a part of what you call Clausius statement it is impossible to transfer the heat from the cold region to the hot region without any external aid. And we have basically discuss about various aspects of second law of thermodynamics including Kelvin Planck statement, Clausius statement and then we moved into the Carnot cycle analysis.

And we also found out that the thermal efficiency of a Carnot engine is equal to 1-T_L / T_H and which will give the maximum efficiency whenever an engine is operated between a thermal reservoir and rather a source and a sink. So we looked at basically two corollaries that is a Carnot corollary 1 and Carnot corollary 2. And we will be now taking up an example to see how you can apply the Carnot corollary and solve a problem.

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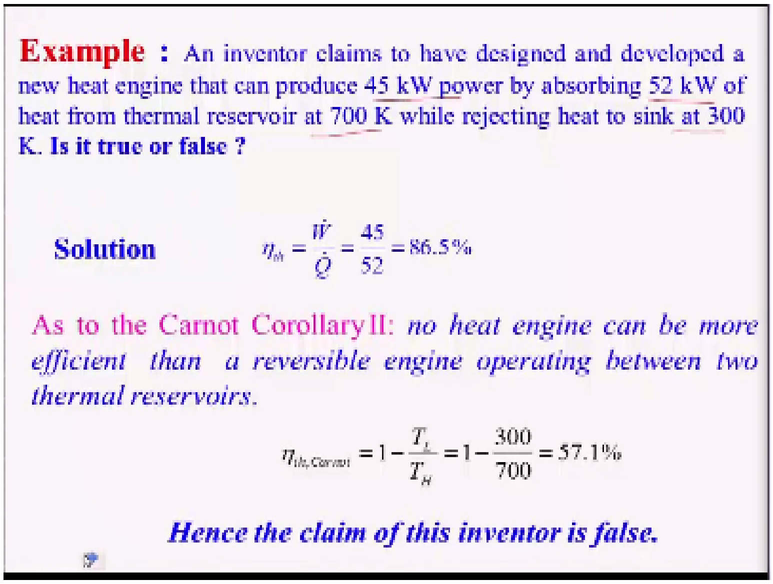

So let us take an example, so this example generally you know it is related to again you know claim being made by an inventor. And which is things one has to clarify and one has to invoke the second of thermodynamics, the example is like this then in winter claims to have design and develop a new heat engine that can produce 45 kilowatt of power by observing 52 kilowatt of heat from a thermal reservoir at 700 Kelvin while rejecting heat to the sink at 300 Kelvin is it true or false that you need to find it out.

If you look at this problem the question arises whether you know first law of thermodynamic being, you know satisfied or not right and then we need to look at also whether the second law of thermodynamics is being satisfied or not. If you look at like it is basically what you call absorbing 52 kilowatt of power from a source from a thermal reservoir at 700 Kelvin and it produces 45 kilowatt of power that means it is whatever heat being observed is, you know some portion is going to seek and 45 kilowatt of the power is being produced and sink is that 300 K.

That means it looks to be, it may be true. Let us look at whether it is true or not, so for that what we will have to we will have to find out the thermal efficiency, thermal efficiency is nothing but W./Q. and the work produced by this heat engine is 45 and heat observed from the source is 52 the efficiency is 86.5% quite a bit. And now what you will have to do, you will have to basically invoke the Carnot corollary 2 which says no heat engine can be more efficient than a reversible engine operating between two thermal reservoir right.

Suppose, you know this engine and then this is the actual engine or its efficiency is 86.5 and if you say that Carnot heat engine is being operated between the temperature 700 Kelvin that is the source and the sink is 300 Kelvin then what would be the efficiency would be efficient should be equal to $1-T_L/T_H$ and the T_L is 300 Kelvin right and the T_H the temperature of the source is 7oo Kelvin and you are getting the efficiency of 57.1% that means a reversible engine is giving 57.1% and the inventor is claiming is efficiency to be 86.5 therefore it is not really possible right.

And hence the claim of this inventor is false. So by, you know using a very simple way one can really find out whether it is true or not.

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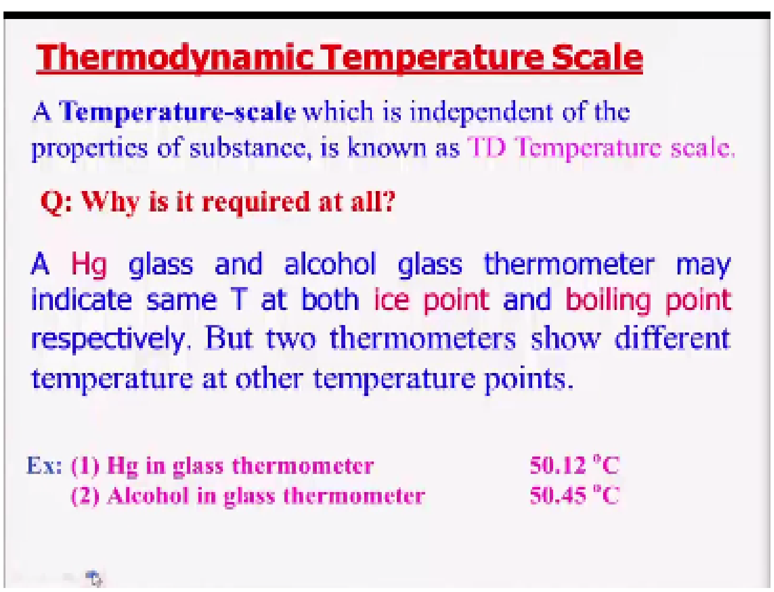

So let us now see how we can apply you know thermodynamic apply the second law of thermodynamics for a temperature scale which we have already discussed earlier let us devise a temperature scale which would not be dependent on the properties of the substance that we have already defined as a thermodynamic temperature scale, if you recall that is the thermodynamic temperature scale which would not be dependent on the properties of the substance like you know what we use.

Let us say in a mercury and glass thermometer, so if you look at the temperature what it will measure it will be dependent on the expansion of the, what you call mercury. But if I use alcohol then you know that expansion will be different, so it will be dependent on the, what you call substance the thermometric fluid rather, but however we need to devise a temperature scale which will be not dependent on the properties of substance.

So why is it required we know because we have already know that whenever this mercury glass and alcohol glass thermometer will be used even though, you know it will be calibrated at the ice point and boiling point, but however it will give you different, you know what you call temperature at the at other points, you make elevator the same ice point and boiling point and assign those values, but however if you go deviate from this ice point and boiling point it may give you different what you call values.

For example like mercury in glass thermometer will give you 50.12°C whereas the alcohol in glass thermometer will give 50.45°C, of course you may say look it is not much, but in accuracy wise it is not good. So therefore it indicates that what you call this temperature scale, you know is basically dependent on the properties of the thermometric fluid right, because mercury and alcohol are two different and their properties are different therefore.

So therefore we need to have a scale which is independent of the what you call properties of the thermometric fluid or the substance.

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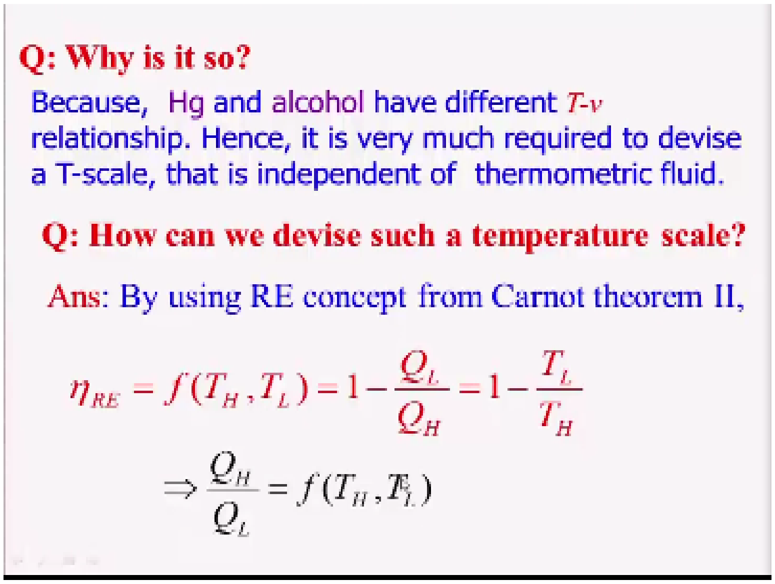

So for that of course we know that alcohol have different T-V relations it is very much required to devise temperature scale. And how can we devise such a temperature scale for that what we will do by using the reversible engine concept from the Carnot theorem 2 that we have seen that the efficiency of a reversible engine or what we call Carnot engine is basically 1 - Q_L / Q_H which is equal to 1minus T_L/T_H. And we say that Q_H/ T_L is basically a function of T_H and T_L temperature both the source and the sink temperature it is function.

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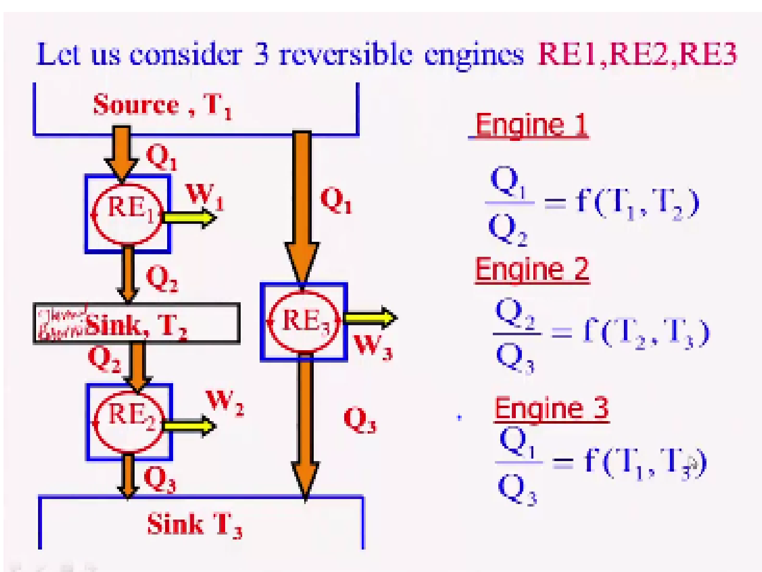

So let us consider the three reversible engine that is reversible engine 1 2 & 3 which is, you know let us say the reversible engine is observed what you call Q_1 amount of heat and rejects Q_2 amount of weight to the sink 2 right, and T_1 is greater than T_2. And it does the W1 amount of work right, and let us say that whatever heat is coming from what we call the sink 2 the sink 2 then that will be observed by another reversible engine and it rejects the Q_3 amount of heat to the sink T_3 that if you look at so far the reversible engine two is concerned this is this sink is not a sink this is basically a source for the reversible engine right.

And it is a thermal reservoir I mean you call it scenes or I should call rather it is a thermal reservoir right. So it is basically, you know thermal reservoir right, thermal which can act as a source as can a sink depending on the word let us say that another engine that is a third engine which observed the same amount of Q_1 from the source that is T_1 which is at the T_1 and it rejects Q_3 amount of heat to the sink 3 and it produces W_3 amount of work.

Now if we look at for the engine 1 we know that Q_1 Q_2 is a function of T_1 and T_2 if it is reversible engine that we have already seen. And similarly for the engine 2 the Q_2 and Q_3 ratio is a function of T_2 and T_3, T_2 is what you call the thermal reservoir 2 temperature which will be acting as a source for the reversible engine 2 while it will be acting as a sink for the first reversible engine. So similarly for the engine 3 we can say that the key one Q_1/Q_3 is basically the function of T_1 and T_3 right.

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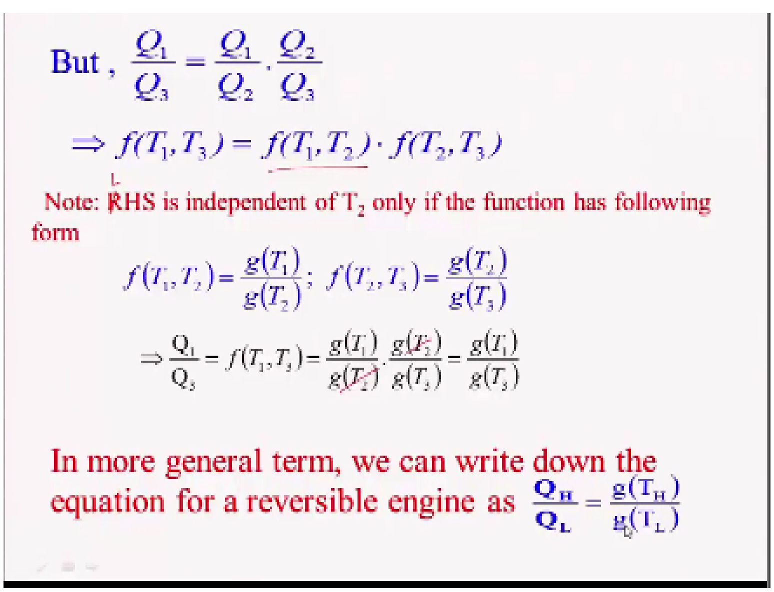

Now if you say that like $Q_1/Q_3 = Q_2 = Q_1/Q_2 \times Q_2/Q_3$ it is possible only if you know that we can say from this that it is basically Q_1 by Q_3 is a function of T_1 T_3 and Q_1 by Q_2 is a function of T_1 by T_2 and Q_2 by Q_3 is a function of T_2 by T_3. So if you look at the this portion that is if you look at this the if you look at LHS you know this is your LHS is the independent of the temperature T_1 and you know it is a LHS is a function of T_1 T_3 and the LHS is for basically independent of temperature T_2.

Only if function as the following form I mean that is only possible right that if I say that this function that is F_1 T the f this function is basically of is equal to g/g is another function of temperature T_1 and is also a ratio of the temperature T_1 and T_2 and f_2 the function f which is a function of T to these basically gT_2 2 / gT_3 now if I say then only we can say that because this if it is there then I can it will cancel it out so that we can say that Q_1 by Q_3 is function of T_1 by T_3 right two functions are there.

And so if you look at if you take a you know of course it can be any function you can say in the more general term we can write down that equation for this reversible engine is basically Q_H by Ql is equal to the g which is a function of the and the function of PL right.

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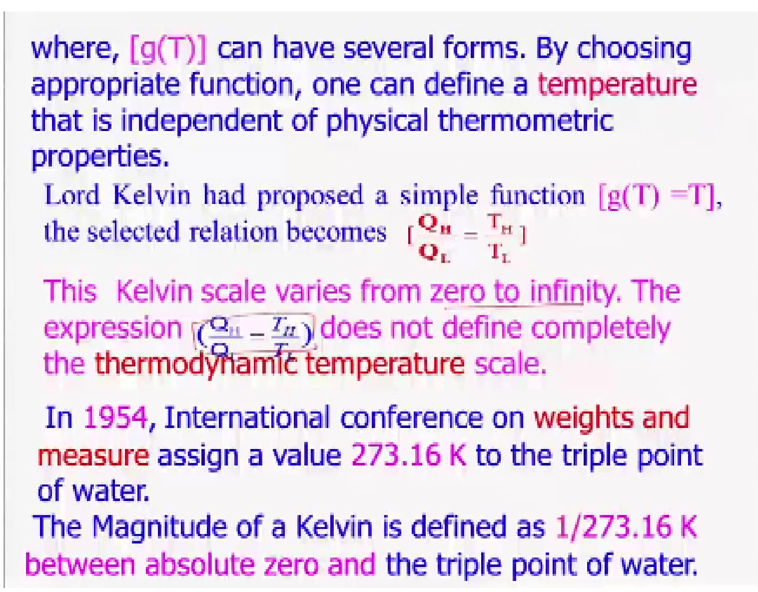

And where gT can have several forms right you can think of any forms but by choosing an appropriate function one can define a temperature and which is can be considered as independent of the physical thermometric properties right and the if you look at Lord Kelvin he had proposed a simple function he says that gT_H that is the function is equal to the temperature then he says I am then the selected what you call the relations that Q_H by Q_l will be equal to P_H by T_L you may see that you know people have used a complex function and they have arrived at similar relationship.

So therefore this is the scale which we call it as basically Kelvin scale which varies from 0 to the $\propto$ right and then expression Q_H by Q_l is does not really on protocol depend on the properties of the thermal when deployed and it can be you know even though it is a relationship even though this relationship it does not really define the thermodynamic scale completely but it gives a relationship so in 1954 the international conference on weights and the major assigned value 270 3.16 to the triple point of water right.

As a result like you can get one point here then after that you can find out that magnitude of Kelvin is basically defined as the one by 270 3.16 Kelvin between absolute zero and triple point of water triple point of water you know like 0.01^0 C right so therefore he has defined that you know one over 70 and that 0 is considered to be there Kelvin a absolute zero so by this way one can use the basically the second law of thermodynamics to define a thermodynamic temperature scale which will be independent of the thermometric flow so let us take an example.

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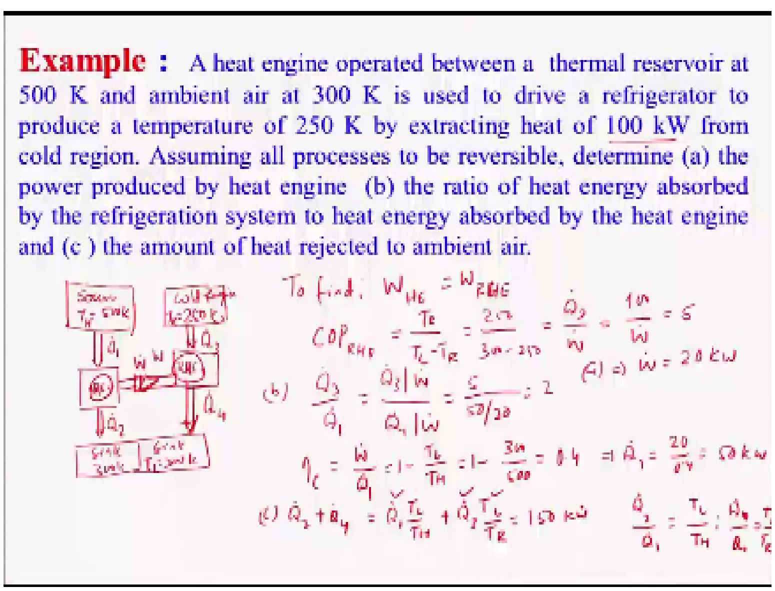

Like whether this relationship can be used for solving some other problem we have talked about a temperature scale that is of course to you know to define a scale Kelvin scale basically defined from that and then you can use it but here we will be applying that for solving a problem let us say a heat engine operated between thermal reserve at 500 Kelvin and ambient air at 300 Kelvin is used to drive a refrigerator to produce a temperature of 250 Kelvin by extracting heat of 100 kilo watt from the cold region right.

See in this heat engine is basically produced giving some walk it is producing some work and that work is being utilized to run the refrigerator and assuming the all the processes to be reversible and we need to determine the power produced by a heat engine right and the ratio of heat energy observed by the refrigerant system to the heat energy observed by the heat engine and we will have to also find out the amount of heat rejected to ambient air so let us say that that is a source that is T is equal to 500 Kelvin and it is observing some amount heat.

Let us say key one and there is a reversible engine which is producing amount of work w dot and it is rejecting certain amount of it to the sink that is your sink, sink is which at what temperature that is a 300 Kelvin and of course this sink you can say the same thing right and there is a refrigerator right I say that R_E, R_E or I can say this is basically he reverse heat engine or the refrigerator and this region is the cold region which is at 250 Kelvin and what it does it observe certain amount of it.

Let us say this is Q_3 and it is rejecting certain amount of heat that is Q dot for right what we will have to find out to find and this work is being supplied right if you look at this work is being supplied what is that that is w that means you can say that this amount of work is being I think this amount of it being supply now what is it we found out to find basically we need to find the power produced w right and that is WHE is equal to w R_E, R_E is the RH_E let me put it that means reverse heat engine which is nothing but a refrigerator right.

And how we will find out that how we will find it out that we know that coefficient of performance of the refrigerator what is that SI of P of refrigerator if I say simply are H_E or I can say R is equal to what is that what is that if I say this is T_L this is T_L and this is T_H right this I can say T_R right can I say that what will be coefficient of performance of the refrigerator that is nothing but your T_RT_L - T_R yes or no we have already seen the coefficient of what you call reversible refrigerator right.

Are reverse heat engine whatever you call so that is equal to what you call T_R is 250 this is 300 this is 250 is equal to what is equal to what that is Q_3 / W yes or no what is Q_3 here what is Q_3 Q_3 is 100 kilo watt is not this is your Q_3 so that is 100 / w dot so W that would be what and what is these values this one 250 /350 it will be nothing but your 5 this is equal to 5.

And what is these values this one 250/350 it will be nothing but your 5 this is equal to 5 so $\dot{W}$ that would be what 20 how much it will be w that will be 20 kilo watt, so I mean we got now the a so if you look at this is your a we got now we need to find out b, what is b, b is the ratio of the heat absorbed by the refrigeration system are much heat being observed it is 3 divided by the heat observed by the heat engine what it would be what it is it is it is $\dot{Q}_1$ yes or no right.

So how we will find out that I can write down this $Q_3/\dot{W}$ yes or no similarly I can write down $\dot{Q}_1/W_1\dot{W}$ so if you look at $\dot{Q}_1$ is it we know this one Q·1W we know or not we really do not know but we know this $\dot{Q}_3$ we know and $\dot{W}$ that we know that is nothing but your word COP of reverse engine reverse heat engine or the refrigerator right, and how I can get this $\dot{Q}_1$W how, basically that is nothing but your Carnot cycle efficiency.

What is that Carnot cycle efficiency cannot I am just writing c is equal to by definition is yes or no that is equal to 1 minus P_L/P_H that will be equal to what 1-3 under / 500 will be equal to what 0.4 can I say so therefore the Q_1 will be equal to leave at 20 right Q_1 will be equal to 20/0.4 is equal to 50 kilowatt yes or no, so now we will have to find out basically the what you call if we know this Q_1 then I can substitute over here that is $\dot{Q}/\dot{Q}_3$/W is 5 and this is 50 / 20 that will be getting what will be this will be 2 yes or no.

This will be 2 so we need to now find out that is C, $\dot{Q}_2$ that is the amount of heat rejected amount of heat rejected will be $\dot{Q}_2+\dot{Q}_4$ that is the amount of heat rejected to the ambient air is it not $\dot{Q}_2$ and $\dot{Q}_4$ so how we will get this $\dot{Q}_2$ we know that $\dot{Q}_2/\dot{Q}_1$ is equal to T_L/T_H yes or no right. Similarly we can also find out $\dot{Q}_4/\dot{Q}_3$ is equal to T_4 is T_l divided by Tr from this you can substitute these values you can get $\dot{Q}$ is basically $\dot{Q}_1T_L/T_H+\dot{Q}_3\ T_L$/Tr.

If you substitute these values you will get basically because you know that Q·1 is known so T_L and T_H is known right $\dot{Q}_3$ is known or not $\dot{Q}_3$ is given to you 100 kilowatt so T_L you know Tr you know all those things you know he will substitute this value you will get something 150 kilo watt, right. So now you know how you can apply what you call this whatever we have derived the relationship.

That is basically Q from the what we call thermodynamic temperature scale we have learned that Q· or the heat ratio is equal to the temperature ratio whenever the engine is operated between the source temperature and the sink temperature right, reversible it should be reversible that you should keep in mind. So now what we will see how we can up you know take this use this second law of thermodynamics for other purposes.

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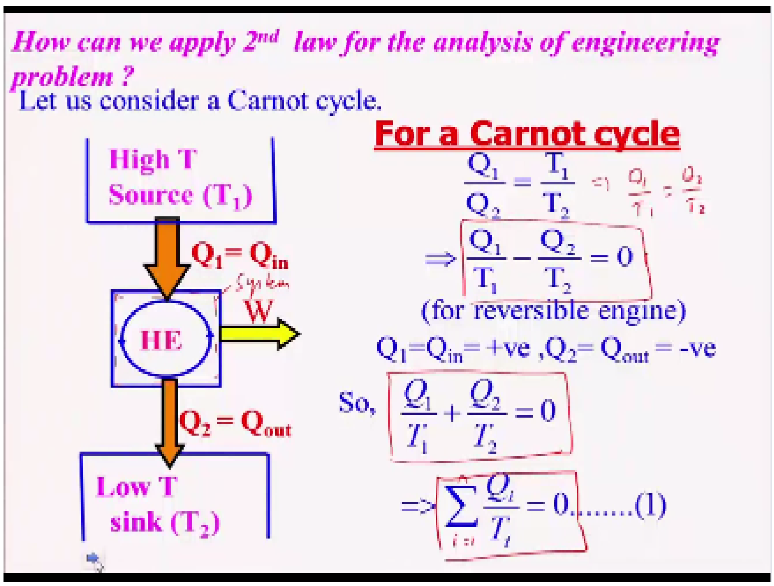

And we have you know we will have to apply the second law of thermodynamics for analysis of engineering problems and this whatever we have done is basically cyclically operating device not that all the cycling operatic device will be handling but we will be handling also the what you call various cases for that let us consider the corner cycle right, and which will be in this case the heating in observe the Q_1 amount of heat, right and it rejects Q_2 amount of heat to the sink which is at temperature T_2 and it does the W amount of work because is the reversible engine right.

So for Carnot cycle we know that from this the $Q_1/Q_2=T_1/T_2$ and from this what will we can write down we can write down basically here we can say $Q_1/T_1= Q_2/T_2$ and I can take this out that is nothing but $Q_1/T_1-Q_2/T_2=0$ this is one relationship which we can clear. So and of course this is mean for the reversible engine only right.

So in this case if you look at if I take this heat is entering into the heat engine right, if I take this my system right, it is entering what it would be it will be positive and if it is going out it will be negative right from the system if I take this is my system right, so then we can write down this expression as you know $Q_1/T_2+Q_2/T_2=0$ and this is of course between the two what you come to thermal reserve one is of course the source other is a sink.

But there might be several deserve worse right one can think of so we can write down this expression that what we have derived for the reversible engine which is operated between two thermal reservoirs it can be n number of you know thermal reservoir we can say that engine can operate so therefore I can write down i is equal to 1 to n right, and then this I can write down I mean from here I can write down that Q_i/T_i some of the heat and then you know heat and temperature ratio is equal to 0.

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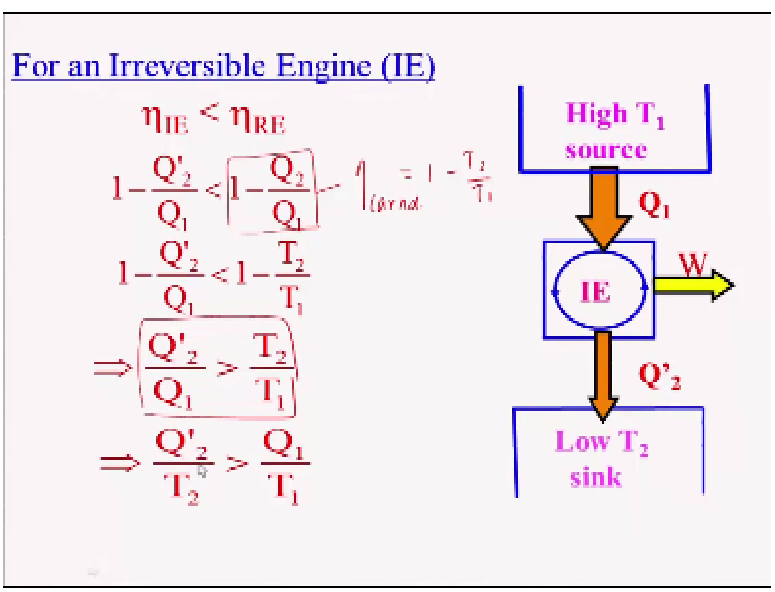

So let us look at because all the time we need not to what you call use the heat engine we will have now considering irreversible heating irreversibility. In earlier we have considered the reversible heat engine and again it is taking the this irreversible heat engine is taking the heat Q1 from the source which is at temperature T1and it is rejecting the Q′2 amount of heat to the sink and it produces double amount of work.

So for the an irreversible engine we know that the according to the cognate corollary right, that is the thermal efficiency of the irreversible engine always will be less than the reversible heat engine, so that means if you look at the irreversible engine efficiency will be what $1-Q'_2/Q_1$ and less than the $1-Q_2/Q_1$ right. And we know for the reversible heat engine the what will be the efficiency, efficiency will be $1-T_2/T_1$ right, yes or no from the current because the reversible engine is your Colonel this is corresponding to what you call Carnot efficiency right.

Carnot efficiency is not equal to, is equal to $1-T_2/T_1$ so therefore I can write down can I substitute here in this case no I cannot because that is it this one is mean for the irreversible heat engines therefore I cannot really apply this thing. So then I can write down here you know if you look at so this is less than that so therefore I can write down here that is Q_2'/Q_1 is greater than T_2 by T_1 that is being written here okay. So I can write down that you know like Q_2'/T_2 is greater than Q_1/T_1.

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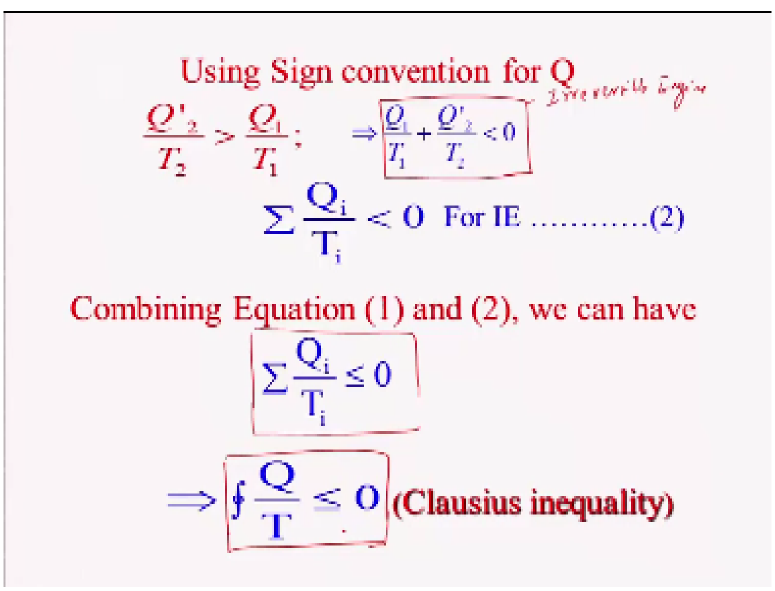

So using the sign convention what we have done we have already discussed right in this case Q_1 is basically being observed and Q_2 dash is given to the sink that means it is going out of the out of the engine irreversible engine. So therefore I can you know take this negative and therefore I can write down that that $Q_1/T_1 + Q_2'/T_2$ is less than zero, so this is mean for what for irreversible engine and if it is I at number of engines irreversible engine I can write down summation of QI' / T_I less than zero right.

So if you combine this equation that is the reversible engine which is equal to basically summation of ki y x TI is equal to 0 that is reversible and irreversible engine will be less than equal to 0, if I just combine this thing I can write down this expression that is summation of this heat interaction basically a queue I it is the you know engines t r is less than equal to 0 and that is nothing but your Clausius inequality this is your Clausius inequality what if state Clausius inequality?

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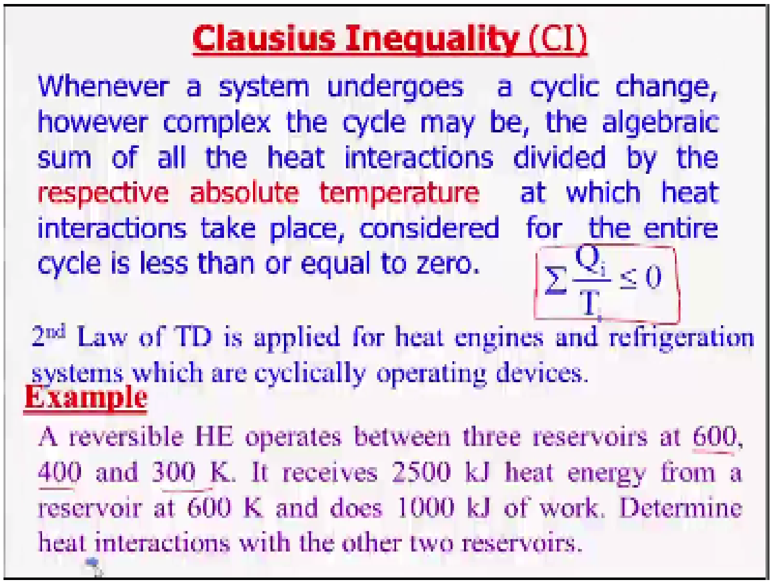

States that whenever a system undergoes a cyclic change however the complex cycle it maybe the algebraic sum of all the heat interaction / the respective absolute temperature at which heat interaction take place consider for the entire cycle is less than equal to 0 that is the Clausius inequality I mean which just now we have derived from taking very simple concept of the vertical Carnot engine that is a reversible engine and we have also considered the irreversible engine and done that and that is nothing but you're mathematically one can write down the queue

I divided by T are some of over all the this thing is less than equal to 0 if this is basically mathematical expression of the classiest inequality.

And keep in mind that this is the basically the second law of thermodynamics which can be applied not only for the heat engine for the also for the refrigeration system and which must be cyclically operated that means it is applied for the cyclically right and let us consider an example a reversible heat engine operates between3d servers that is at 600 Kelvin400 Kelvin and 300 Kelvin it receives two thousand five hundred kilo Joule of heat energy from the thermal reserve at six hundred and does the thousand kilojoules of walk and we need to determine heat interaction with other two reserve that is 400 Kelvin and 300 Kelvin that is the other two receivers we need to you know. So question arises how to go about it.
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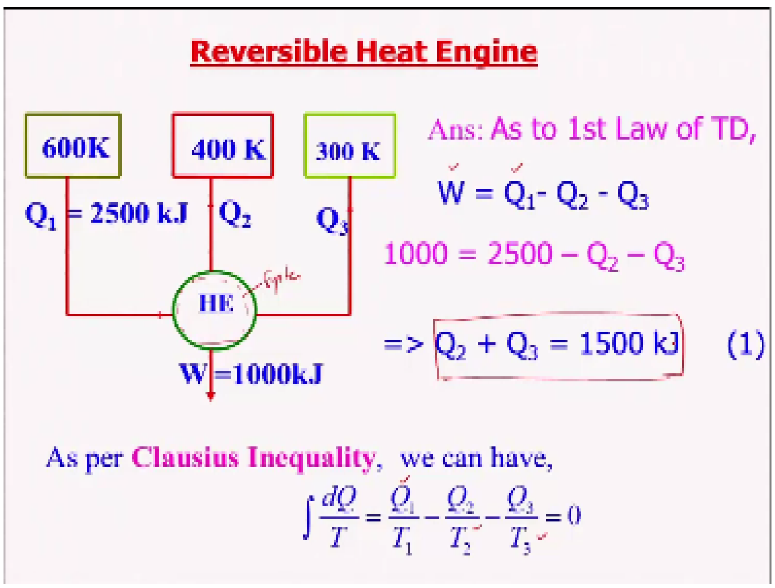

So let us say that is the heat engine this is your heat engine basically you know I can take this is my best system I can say right heat be observed q1 to 2500 you will and some heat being rejected to 400 Kelvin and this is again rejects q3 to the 300kelvin thermal reserve right and it does the thousand kilo Joule of cortical walk and keep in mind that this is a reversible heat engine right.

So how will go about it will basically first we will have to look at the first law of thermodynamics right, so according to fast law what is says W is equal to Q1 - Q_2 - Q_3 right. So if you look at the this is given w is given and Q_1 is given and Q_2 and Q_3 are not given so you cannot really proceed now right you can say look this is just an energy balance it is not like then what we'll have to do you will have to apply now second law of thermodynamics right and of course for this we will have to use the classiest inequality that is dQ by T over the cycle is equal to the Q_1 divided by T_1 right minus Q_2 divided by T_2 - Q_3/3 right yes or no is equal to 0 because this is a reversible eating it.

And in this case this Q_1 is given and T_1 of course P_2 is given and T_3 is given but Q_2 and Q_3 is not are not known but however you know you can get an expression from here and we have already derived an expression from first law of thermodynamics that is Q_2 plus Q_3 is equal to 15 hundred kilo Joule. So if we just solve this two equation we can get that.

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$$\Rightarrow \frac{2500}{600} - \frac{Q_2}{400} - \frac{Q_3}{300} = 0$$

$$\Rightarrow 3Q_2 + 4Q_3 = \frac{2500}{600} \times 1200 = 5000 \text{............(2)}$$

$$3Q_2 + 3Q_3 = 4500 \text{..............................(3)}$$

(Q_3 = 500 kJ)

By solving Equation (2) and (3), we can have

Q_3 =500 kJ; Q_2 =1000 kJ

How do we apply 2nd Law of Thermodynamics for non-cyclic process?

Let us see that is we are getting 2500 that is q1 divided by 6 and 8 - Q_2 / 400 - Q 300 and they equal to 0 why here it is minus because heat is going from the engine to the sink, so therefore Q 2 we have taken minus and so also the Q_3 the sign convention and that is your what you call equation 2 we are getting 3 Q_2 plus for Q_3 is equal to five thousand and if you already we have you know derived this thing that is from the first law of thermodynamic if we just minus here you know this one so what you will cancel this will cancel it out.

So we will get 7 q $_3$ is equal to 500 right and from that Q_3 is equal to from this what will say you will get Q_3 is equal to 500 kilo Joule and from the other expression you can get from the equation am like a from here any one of them that Q_2 is equal to thousand Kelvin. So that till now what we have seen we have seen that how you can apply the second law of thermodynamics for the cyclic process right but not all the processes should be cyclic in nature there will some processes which will be non-cyclic in nature right.

Now question arises how to go about it, if you remember that we are derived the what you call the processes first law of thermodynamics for the cyclic process for first and then we apply it for the non cyclic process what do we do there at that time could you recall how do you do we have done that already for the first law of thermodynamics a cyclic integral of the heat will be proportional to cycle integral of the work and then we say that it is equal to the JJ is there your joules constant right and then from that we have derived expression that is de is equal to dQ minus DW that is the first law of thermodynamics for non cyclic process yes or no right.

So how did you we arrived when that will be following the similar procedure right we will be following the similar procedure to do that, so I think we will stop over here but let us recall that how what we did we basically looked at the how to apply the first law of thermodynamics for acyclic process right and we derived the classiest inequality relationship right, we say that whenever a what you call reversible engine whenever the heat engine will be interacting with the particle system and is whenever it will be interacting with the thermal reservoirs right source and see however the complex cycle it may be the sum of the heat interaction and the respective thermal reservoir ratio will be equal to less than equal to 0 right.

So that is the thing what we have seen so if you look at we have seen whenever system undergoes a cyclic change however the complex cycle may be the algebraic sum of all heat interaction / the respective absolute temperature at which heat interaction take place consider for the entire cycle is less than equal to 0that is a Clausius inequality and before that of course we have seen that how one can devise the thermodynamic temperature scale by using the second law of thermodynamics. So in the next lecture what we will be discussing basically how we will apply the second law of thermodynamics for the non cyclic process thank you very much.

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Manoj Shrivastava
Padam Shukla
Sanjay Mishra
Shubham Rawat
Shikha Gupta
K. K. Mishra
Aradhana Singh
Sweta
Ashutosh Gairola
Dilip Katiyar
Sharwan
Hari Ram
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