

Indian Institute of Technology Kanpur

National Programme on Technology Enhanced Learning (NPTEL)

**Course Title
Engineering Thermodynamics**

**Lecture – 22
Second Law of Thermodynamics: Carnot Cycle and Efficiency**

**by
Prof. D. P. Mishra
Department of Aerospace Engineering**

Let us start this lecture with a thought process from Peter Theodore Landsberg.

(Refer Slide Time: 00:21)

Lecture 22

It has been suggested that thermodynamic irreversibility is due to cosmological expansion.

Peter Theodore Landsberg

Who says that it has been suggested that thermodynamic irreversibility is due to cosmological expansion is it really, so or not you know is a questionable thing because we do not know that whether that is the cause or some other things, so in the last lecture we basically discussing about irreversibility and we have seen that irreversibility is inevitable however one can minimize their reversibility to improve the thermal efficiency and our efficiency of a cycle or a system and other things. So what we will do today we will be basically looking at the converse cycle.

(Refer Slide Time: 01:16)

The Carnot Cycle

According to 2nd law, $\eta_{TH} \leq 100\%$ for all heat engines

Q: *What is the most efficient cycle we can have?*

To have max η_{TH} , processes of HE must be reversible.

Such reversible cycle is known as **Carnot Cycle** by French military engineer "*Nicolas Sadi Carnot*" in 1824.

The Carnot cycle consists of 2 reversible isothermal and adiabatic processes.

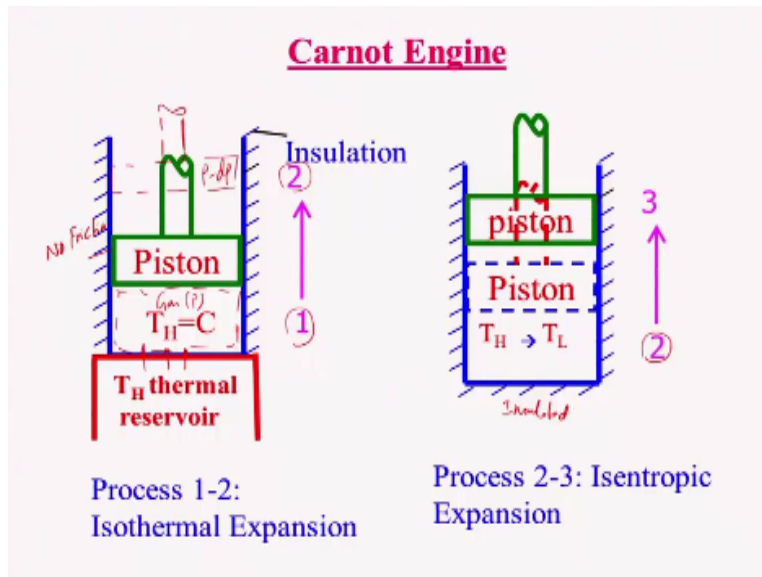
Let us consider a HE operating between T_H and T_L

So according to the second law of thermodynamics the thermal efficiency cannot really equal to the 100% but you know it can be it must be less than the 100% efficiency kind of things right so therefore if you look at that it is always will be less than you know less than 100% for all heat engines whatever, so this it cannot really equal to 100%, so what is the most efficient cycle we can have because we know we are interested to have a heat engine and heat engine is cycling operating device.

So we always should know that what is the most efficient you know heat engine we can have and says that we can achieve the maximum thermal efficiency and it is possible only if the processes involved in the heat engine should be reversible then only heat will pass so these ideas is basically used by Nicholas Sadi Carnot in 1824 who is considered to be the father of thermodynamics and you know and he proposed a cycle thermodynamic cycle which is known as the corner cycle and this Carnot cycle consists of two reversible processes

One is isothermal reversible process other is adiabatic reversible process right and of course if you look at these are the two processes but however to complete the cycle you know you should have two reversible isothermal processes and two adiabatic processes that means total four processes will be there, so let us now consider a heat engine which is being operated between the two temperature source and sink and the source temperature is T_H and we are saying high temperature and the sink temperature will be lower we are calling it as a T_L .

(Refer Slide Time: 03:52)



So let us consider a engine like a piston cylinder arrangement here as I have shown here and these are all insulated and there is no friction between the piston and the cylinder surface again hypothetical you know like in real situation it would not be and except the lower portion which is in contact with a thermal reservoir and some heat will be transferred to this gas you know if it is a gas here and we are having a system and this if you look at like we are as making yourself that the gradient is very small that is a one we are saying.

So that it will be reversible and then what will happen when the heat being transferred to this gas then the pressure also will go up, so that piston will move but that again the pressure whatever is acting let us say if it is pressure here and then here the pressure will be acting $p - dp$ and this is very small, so that the of course to keep this gas constant temperature, so that the piston will move up or there will be expansion it will go from state 1 to the state 2 you know keeping the temperature of the gas constant.

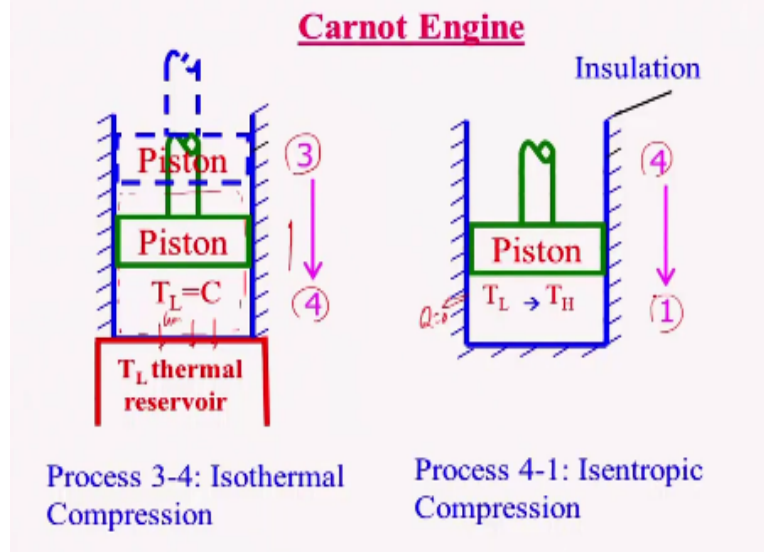
So that we call it as I basically the isothermal expansion kind of things right the piston will be moving over here right, so this process you know we can call it as a basically reversible isothermal process and if I will you know bring it I can bring this system from the state 2 the state 1 and if I do in a very slow manner and there is no irreversibility therefore it can comeback.

Let us say that after this what will happen this thermal reservoir which was here earlier being removed and this is being insulated earlier this portion cylinder is insulated all are insulated now

only lower person also insulated and the piston will move from state 2 state 3 expansion will be taking place as a result the temperature you know which was at a higher temperature which was equal to the thermal reservoir temperature and that you know drops down to T_L is a lower temperature right.

And as a result there will be some isentropic expansion because adiabatic and there is no reversibility in this, so therefore this expansion we call it as a basically isentropic expansion process right and if you look at like what we have to do.

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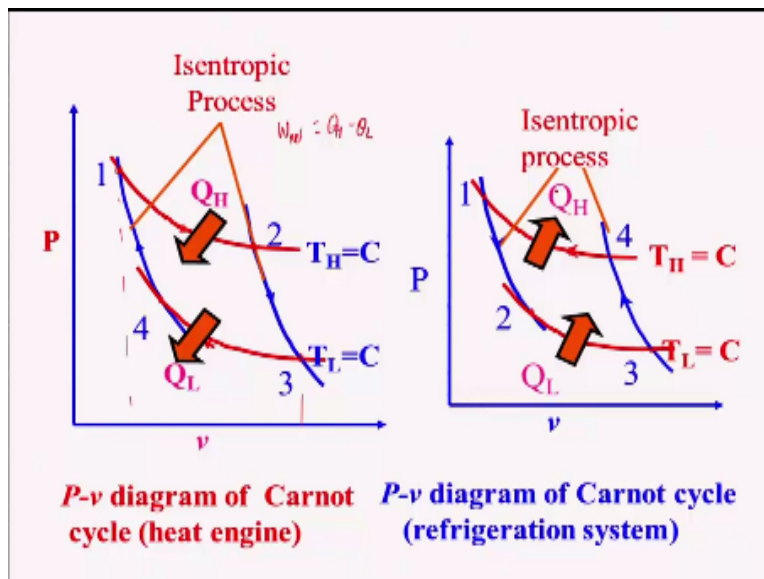
That we will have to now compress this these things as that what will happen again will remove this what you call insulation from the top of the cylinder and then bring this sorry the bottom of the cylinder will be brought in contact with the thermal reservoir, so that you know when it is compressing that means it is coming from state 2 the state 4 slower manner that you know temperature will go up but however some heat transfer will be taking place to this from this gas you know this is your gas right.

This is of my system to start with and then you know like it will be from state 2 to 3, so that the temperature will be remaining constant and this process is basically isothermal compression right and you keep in mind if I want to go back from 4 to 1 and that is possible that is possible you know like I can go back, so therefore it is a reversible isothermal process and now this again the thermal reservoir is being removed and then bottom of the cylinder is insulated and the piston

will move from state 4 to the state 1 you know says that what you call temperature will be increased from T_L that is the lower temperature to the higher temperature.

And there is no you know heat transfer is taking place because heat transfer is 0 here and it is a reversible process therefore this process is basically isentropic compression process that means we started with from the state 1 right and we have written back to the state 1 again undergoing the if you look at isothermal expansion adiabatic expansion then followed by isothermal compression and isentropic compression process that means the total 4 processes 2 isothermal what you call compression and expansion to isentropic compression and expansion right basically you know that together isentropic compression and isotropic expansion at 2 processes right.

(Refer Slide Time: 10:14)



So if we put that thing in a PV diagram right, so if you look at we are here state 1 and this is your reversible isothermal expansion is taking place while keeping the temperature constant which is same as that of here what you call source or the high temperature thermal reservoir as a result

some heat being taken from the thermal reservoir that is q_h and after this then there will be the system will be undergoing a process what you call the isentropic process like from station 2 to station 3 of course the temperature is changing but there is no heat transfer taking place between the system and surrounding right.

Now if I want to if you look at the work done this process will be what you know this process 1 to 2 and 3 this is my work done if I look at this is my work done is not it some work being what you call done because by the system is expansion, so what is being done by the gas on its surrounding right now if I want is a look it is a reversible process instead of here in this process can I not go back here if I will go back here what will happen this work you know is what you call will be nullified and similarly available.

So therefore we need to take this what you call you know from station 3 to station 4 that is your isothermal what you call compression process in which I will have to you know the system will be giving certain amount of heat to the sink that is Q_L and after this there will be what you call Isentropic again compression will be taking place from state 4 to state 1.

So this is the work you know like if you look at it is given some work input will be given to the vertical by the surrounding to the system because work is done on the system right not by the system on the system so this is the amount of work which you need to supply that is 1, 4 and three this amount of work will have to supply and one two and three this work of being produced by the engine.

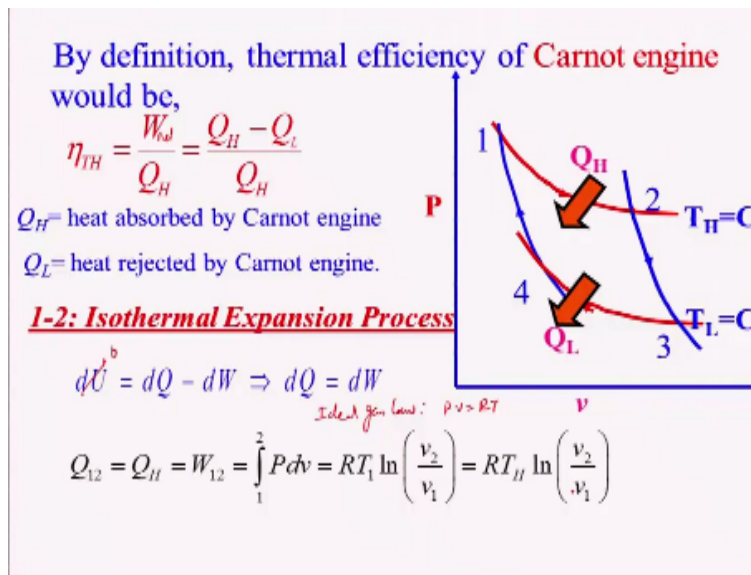
So therefore what will be the net work done here net work done if you look at we know that net work done will be $Q_H - Q_L$ am I right because heat is coming in and then is being rejected and that is the work, so this is the engine why you know like this is the engine like what we call is basically Carnot heat engine because this is a hypothetical engine right it is not that it has been produced you know but it is a hypothetical engine which will give you a benchmark to check how good you know it is the other engine with reference to the Carnot engine.

Now I can think of a process in a reverse way right reverse way in the sense like instead of you know starting with the station one what I will do I will just expand right from station 1 to state 2 right and then it will be again I will go from state 2 to the state 3 so that I will take some amount

of heat from the sink that is the low temperature thermal reserve Q_L and then I can move from state three to the state one through again isentropic compression process right.

And from the state 4 to state 1 I can go or to call with the isothermal compression process while taking you know gear ejecting the amount of feed to the high thermal deserve that we call it a source that is Q_L so this cycle you know is known as the vertical reverse Carnot cycle and this will be representing a reverse are you know as Carnot refrigeration system right one can think of so if you look at these are the process and what we will be doing now.

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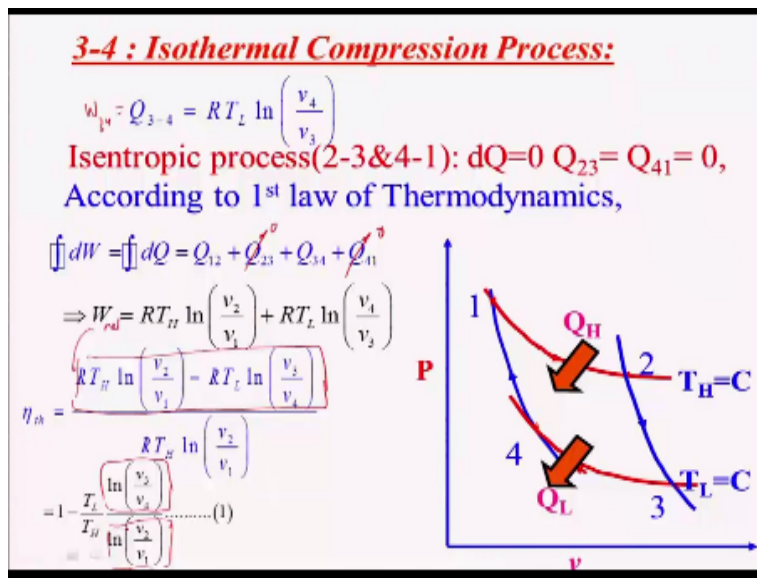
We will be applying this what you call first law of thermodynamics and finding out the efficiencies and you know thermal efficiency what it would be for a Carnot engine, so by the definition you know like we know the thermal efficiency is basically what you call w that is the net you know this is the net what that means work is produced by the engine and also work is being supplied to the engine net engine it is nothing but $Q_H - Q_L / Q_H$.

And that will give me the thermal efficiency and as I told so the Q_H is a heat absorbed by the Carnot engine and Q_L is a heat rejected by the Carnot engine and if you look at this isothermal expansion process between the station 1 and 2 in this case you know the temperature is remaining constant that for dt is 0 and what is happening here that means if I apply this first law

of thermodynamics that is $du = DQ - DW$ you keep in mind that we are using a control mass system right kind of things rather we are using a control mass system.

So then for this process is isothermal so therefore this will be 0 right is not, temperature is not changing so therefore the DU into change in internal energy will be 0 so then that will be $DQ = DW$ and for this process nothing but DQ is what Q_{12} and which is nothing but the amount of heat which is observed from the article source and that is equal to the work done one to two and we know there is a PDB work so that will be PDV when I integrate between station 1 to 2 write and you can assume the ideal gas law right ideal gas law we know that $p v = RT$ right V is the specific volume right and instead of P I can put this RT by V here and then integrate that I will get $RT \ln(V_2/V_1)$ and T_1 is nothing but your T_H right because this is the T_H right so that is the work done is equal to $RT_H \ln(v_2/v_1)$.

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So let us consider the isothermal compression process that is what you call 3, 2, 4 right this is basically we are considering 3, 2, 4 here and that is again same way I can write down that Q_{34} is equal to $RT_L \ln(V_3/V_4)$ and which is equal to basically w_{34} right and for isentropic passes for both the what you call system two to three and four to one right four to one the DQ is zero so therefore $Q_{23} = Q_{41} = 0$.

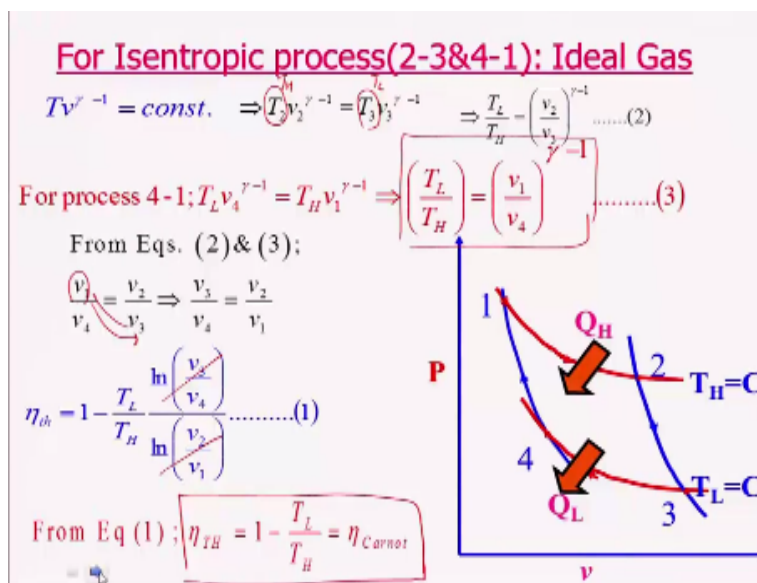
So according to the first law of thermodynamics then we can write down the cycle integral of work is cycle interval q is nothing but $Q_{12} + Q_{23} + Q_{34} + Q_{41}$ Emily you go this way you

know 1-2, 2-3, 3-4, 4-1 and we know that this is basically if you look at $Q_{23} = 0$ and this 410 right so then the work done is total work done or the network done right this is your network done right is nothing but $R T_H \ln(V_2/V_1)$.

We have already derived this and the Q_{34} is nothing but $R T_L \ln v_4 / v_3$ right, so if we will apply this that we know thermal efficiency is nothing but your w_{net} / Q_H and what we will do we will just put this values you know net this is the work net here I am like you know this is nothing but that and divide by the Q_H which is nothing but $R T_H \ln(V_2/V_1)$ and we can if you look at this divided this and this term will be you know is it divided then it will be one and then i can write down the efficiency thermal efficiency is basically is $1 - T_L/T_H$.

Of course RR will cancel it out into $\ln(v_3/v_4) / \ln(v_2/v_1)$ so this is the thermal efficiency for the Carnot engine right now what we will do, we will try to look at this how we can relate this term you know this $\ln v_3/v_4$ and this term and see that how we can you know relate this thing and see whether we can simplify this expression for the thermal efficiency.

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So we know that for an isentropic process right we will be considering the process isentropic between station what you call two and three this Isentropic process am I right and similarly 4 and one these are isentropic process right and we know that for Isentropic process $T V^{\gamma-1} = \text{constant}$ for an ideal gas we have already assume ideal gas in that, so then I can write down between station 2 and 3 right station 2 & 3.

I can write down T_2, V_2 power to the $\gamma - 1 = t_3 v_3^{\gamma - 1}$ and then you know we can rewrite that because the T_2 is nothing but your, so if you look at T_2 is nothing but your what T_2 is t_H right and t_3 is T_L so I can get this thing that means basically T_L / T_H I will take this the you know denominator and then that will be equal to v_2 / v_3 power to the $\gamma - 1$ right and in the similar way we can also further derive the similar expression for process four and one.

And that is nothing but $T_L / T_H = V_1 / V_4$ power to the $\gamma - 1$ so this expression you know and if you look at this equation 1 2 & 3 what you will see you will see that both our T_L / T_H the left hand side both the equation that means that we can write down $v_1 / v_4 = v_2 / v_3$ what we will do we will basically rewrite that expression we can take this here and bring this over to here i can write down the v_3 / v_4 is nothing but equal to v_2 / v_1 and then we can say that from here that this can be cancelled out you know this is can be cancelled because both are same be $v_3 / v_4 = v_2 / v_1$. So therefore i can say that the thermal efficiency of the Carnot engine is $= 1 - T_L / T_H$ right.

This is a very simple you know formula what we got that is basically it says you know the maximum efficiency what a engine can achieve that is you know stipulated by the Carnot engine. So and a what it says it does not depend on through it what it is it will be dependent on the temperature of the source and the scene the thermal efficiency will be not dependent on the what you know matter you are having and what it is it will be only depend on the temperature of the source and the sink.

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η_{Carnot} is the max. efficiency, a HE can attain.

Example: A GT engine is operated between $T_{\text{H}}=1200 \text{ K}$ ^{Source} and $T_{\text{L}}=300 \text{ K}$ ^{Sink}.

$$\eta_{\text{TH}} = \eta_{\text{Carnot}} = 1 - (300/1200) = 75\%.$$

No HE can be more efficient than the Carnot engine operating between the same two thermal reservoirs.

“Carnot Theorem 1”

All reversible engines operating between the same two thermal reservoirs have the same thermal efficiency.

“Carnot Theorem 2”

So that you should keep in mind because we will be using this concept what you call very often to derive something so therefore the Carnot engine is basically then can have the maximum efficiency what a heat engine can attend because it is a reversible engine you know like UV ray and always irreversibility will be there in the system. So therefore I mean a one cannot really go beyond a reversible engine so that is why the Carnot engine is having the maximum thermal efficiency. So we will take an example like a gas turbine engine let us operate it between temperature of 1200 Kelvin and temperature of 300 Kelvin that means.

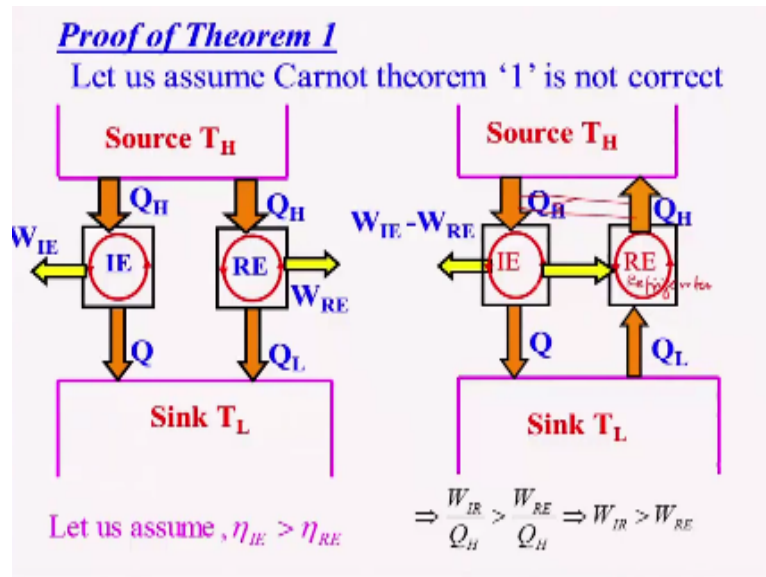
You know if you look at this is your what you call source temperature right from which it is taking the heat and this is the sink temperature right. So which is the heat being rejected by the engine and what will be it is efficiency then is efficiency will be basically $1 - 300 / 1200$ under that will give you 75% efficient a could achieve in real sense this kind of order of efficiency for a heat engine it will be great.

So from this one can really say that no heat engine can be more efficient than the Carnot engine operating between the same to thermal deserver that means one is source other is sink if you know engine eight engine is being operated right and that engine can never we have a higher efficiency than that of the Carnot engine which is operated between the same temperature you know sources and sink.

That means you know so therefore that is the Carnot theorem one and there is another also theorem which is little differently told all reversible engine operating between the same two

thermal reservoirs have the same thermal efficiency right so this is known as Carnot theorem 2. So we will look at you know whether it is a true or not by using the deduction method. So we will do that now.

(Refer Slide Time: 28:24)



So let us you know assume that Carnot theorem is not correct right that we will do that means what that means basically what you call there is a heat engine right which is having the higher efficiency than the Carnot right which is operated both are operated between the two temperature. So you know source and sink so let us say there is an irreversible engine that means you know most of the engines whatever we use is irreversible engine.

Right and it is taking certain amount of heat that is Q_H from a source at temperature T_H and it is rejecting certain amount of heat Q_L to the sink which is at T_L and producing certain amount of work that is W_{IE} see this is basically irreversible engine and let us consider another of course the reversible which is taking the same amount of feed that of the irreversible engine from the same source and it is of course rejecting Q_L amount of heat to the sink.

In case of irreversibility is Q okay it is not same and it is producing the work done that is the reversible work done and let us assume what we will be doing the efficiency of the irreversible engine is greater than the reversible engine. Right we are assuming that means we are saying that car not theorem is not correct so therefore the efficiency of the irreversible engine should be greater than the reversible engine.

What is the meaning is that basically by the efficiency we know that work what you call irreversible work done by the irreversible nation divided by the amount of heat which is observed on the source is greater than the what you call the thermal efficiency of the reversible engine that is nothing but the work done by the reversible engine / the Q_H and Q_H is being same therefore we can say that the work done by the reversible engine will be greater than the work done by the reversible engine.

Right this is you know we can say from this assumption and also from this deduction. Right now what we will do we will just you know because the work done by the reverse irreversible indicator than the reversible let us say that this work will be given to the reversible engine as it is a reversible engine it can operate other way around. Other way around in the sense it will be taking Q_L amount of heat from the sink right and giving Q_H amount of the source and this is the basically refrigerator.

Right this reversible engine is not religion is are free right can I not do that with a reversible engine I can operate on the reverse way yes or no right possible. So that what will happen I joined together that means whatever the heat is given by this ray what you call a refrigeration system Q_H instead of giving here I will divert this key ways to this place you know engine. So that it will go and look at the whole system right and instead of giving this cook you and then it will go to the Q_L . So that what will be happening that means this will be right so this Q_H and how much work will be when we join together this one and make one system to operate.
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Let both IE & RE operate in a single cycle.

$W_{net} = W_{IE} - W_{RE} = (Q_H - Q) - (Q_H - Q_L)$, then $W_{net} = Q_L - Q$

$Q_L - Q$ is being absorbed from T_L and performs an equivalent amount of work.

⇒ Such a device violates the Kelvin-Planck statement.

Hence, $\eta_{IR} \leq \eta_{RE} \Rightarrow \eta_{RE} \geq \eta_{IR}$

⇒ **Carnot theorem 1 is correct.**

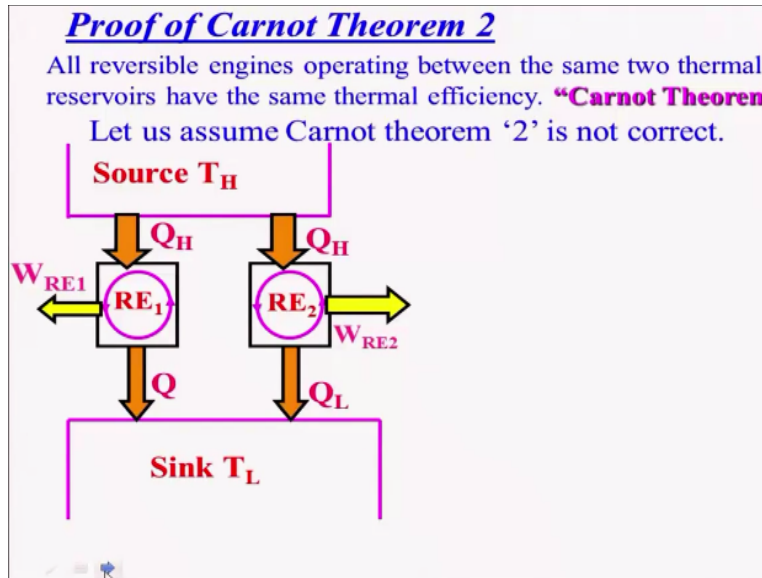
Sink $T_L < T_H$

That you know when the both irreversible engine and the reverse reversible engine that is basically refrigerants operate in a single cycle what it is saying it will be basically taking the $Q_L - Q$ amount of feed from the sink right and it will be producing this amount of work done that is $W_I - W_E$ right. So it violates what it violates the Kelvin plank statement yes or no that means it is taking basically you know certain amount of heat $Q_L - Q$ and that is the network which is nothing but $W_I - W_{RE}$. it has been produced.

So therefore the as i told that this combined system now is violating the Kelvin plank statement right so therefore that assumption what we are made is not correct. That means that irreversible engine efficiency can be less than equal to the reversible engine. Right in other words the reversible engine efficiency will be greater than equal to the irreversible that is another way of looking at because that is not correct therefore this will be may be possible.

Right and so if you can look at that you know like one can also argue well by saying that you know like it will be reversible engine always will be greater than the irreversible engine that means the thermal efficiency of the reversible engine will be greater than the thermal efficiency of the river and therefore cannot theorem is correct.

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So let us look at the proof of the Carnot theorem 2 that is what we you know we know that according to the Carnot theorem 2 all reversible engine operating between the same two thermal reservoirs have the same thermal efficiency. Right what we will do now we will say that let us consider a reversible engine 1 right which is observing certain amount of heat that is Q_H from the source and rejecting certain amount of heat Q_2 the sink while producing the vertical certain amount of work that is W_{RE1} by the reversible in one and there is another reversible engine to which is taking the same amount of heat from the same source as that of the reversible engine one.

Right which is observing certain amount of heat that is q_h from the source and rejecting certain amount of heat q_2 the sink while producing the vertical certain amount of work that is where one by the reversible in one and there is another reversible engine to which is taking the same amount of heat from the same source as that of the reversible engine one but however it is rejecting amount and it is producing different amount of work there is w_{re} to write that is the we are considering and we are saying that let us assume this corner theorem is not correct.

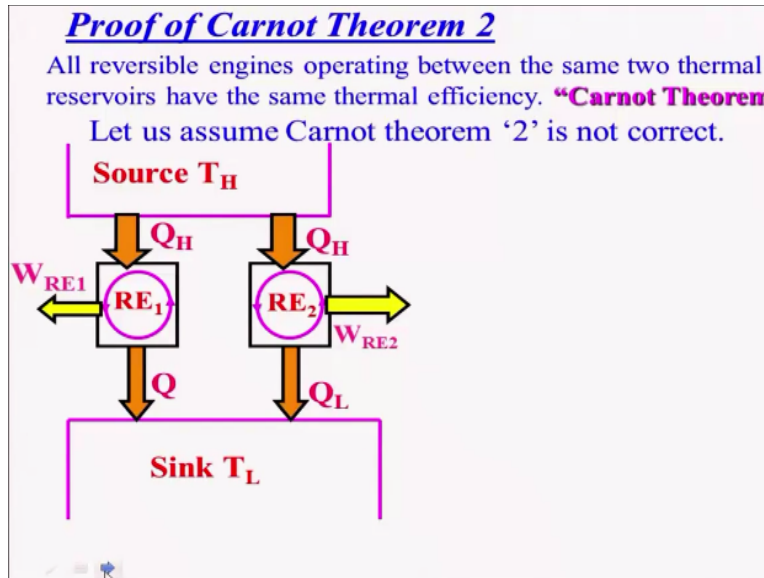
What is the meaning that means that means the let us assume that the thermal efficiency of the reversible engine right is greater than the thermal efficiency of the reversible engine were assuming that then we know the thermal efficiency of the reversible basically the ratio of the reversible you know work done by the engine one and the amount of heat being observed from

the source which is greater than the work done by the reversible engine 2 /qh that means basically were 1 is greater than were 2.

And let us what you call operate this the reverse engine ton a reverse cycle that means it will be operated as an affiliate system right so what will happen it will be taking this'll amount from the sink and you are giving some amount of work input that is w_{re} and this refrigeration system you know which is the earlier it was reverse reversible engine now it is acting as a refuge system so it will be giving the amount of heat to the source so what we will do we will basically again combined it this q whatever it is going to the source i can divert this thing to the what you call this reversible engine.

One instead of taking this h_e from the source the reversible engine one will take from the reversible engine 2 and similarly you are rejecting this amount of it i can give you know this heat to the water column q_l instead of to the sink or will be giving this Q amount of weight it which will be to this versal engine sir this refrigeration system so this we can say that the reversible engine one and the refrigeration reversible system.

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Basically will be the new cyclic devices then what will happen like again if you look at it is absorbing the key well minus Q amount of heat from the sink and doing the certain amount of work done that is w_{re} were to that means whatever heat being taken from this sink is being done by this you know combined cycle and which violates basically what you call the second law of thermodynamics rights therefore we call that you know basically the thermal efficiency of the reversible engine one cannot be greater than thermal efficiency of the reversible engine 2.

Because both are operating at the same source and sink that means it can be other way around that what we call reversible engine tools efficiency thermal efficiency is greater than equal to the reversible engine thermal efficiency one right so by the similar arguments we can also prove that that this is also not correct that is the thermal efficiency of reversible engine two cannot be greater than the thermal efficiency of reversible engine one.

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The $(Q_L - Q)$ absorbed from sink, T_L converted into equivalent amount of work which violates 2nd law of

$\Rightarrow \eta_{RE_1} > \eta_{RE_2}$ is not correct.

That means, $\eta_{RE_2} \geq \eta_{RE_1}$

By similar argument,

we can also prove that

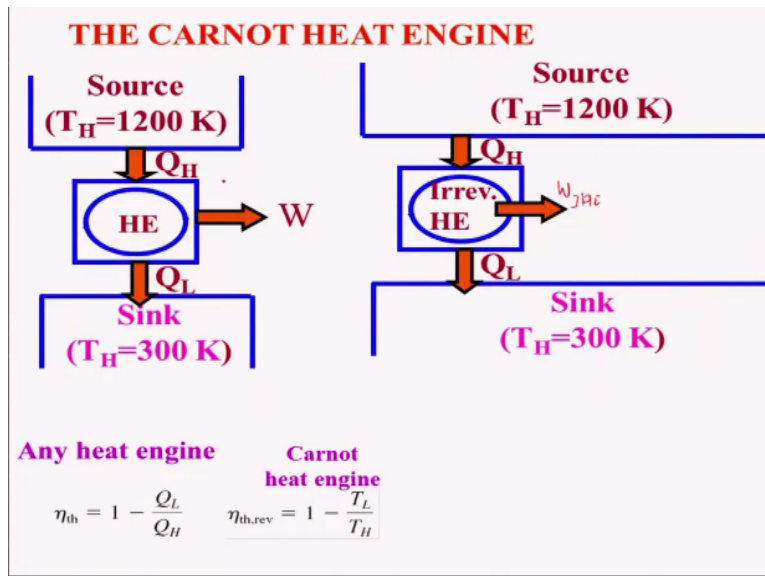
$\eta_{RE_2} > \eta_{RE_1}$ is not correct.

So by the similar argument it can be proved therefore we can say that thermal efficiency of reversible engine two is equal to the thermal efficiency of river one which are operated between the same source and sink .

So as I told you I mean this what you call theorems and also the Carnot engine indicates that that thermal efficiency does not depend on the type of working fluid rather it depends only on the temperature of the reserve between which it operates so let us look at the Carnot engine further I am like you know let us say that the source it is in the car heat engine which is observing q_h amount of it and then it is rejecting q_l amount of heat to the sink and it is producing ascertain amount of work so any heat engine you know the efficiency will be basically $1 - Q_L / q_h$ right okay .

This is you know for any heat engine it need not to be reversible right but forte Carnot engine which is a reversible engine we have seen the thermal efficiency is basically $1 - T_L / T_H$ both okay so if I consider there is a same source right and that is a what you call a reversible engine right which is taking a certain amount of heat q_h and it is giving the same amount of q_l and it will be producing some amount of work done right let us say this is w I r.i.h let me write down irreversibility engines .

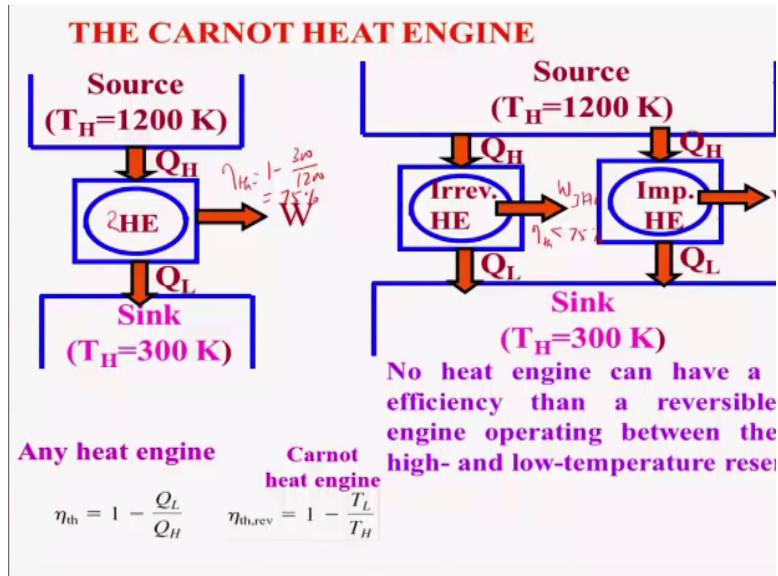
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If you look at the efficiency here what it would be this thermal efficiency in this case if it is a you know reversible engine if I say this is reversible engine what will be thermal efficiency something 1 minus what we call 300 divided by 1200 is nothing but your 75 percentage right isn't it thermo 11 seventy percent now these efficiency for the irreversible engine what it will be will it be less than seventy-five percent or greater than thermal 75% it will be definitely less than seventy-five percent because it is irreversible right it cannot be greater than or equal to the 75 percent okay.

Let us consider that there is a person who claims that there is an engine which does the same thing observed the same q_H amount from the same source and Q_L and engine that which claims that that efficiency is you know thermal efficiency something eighty-five percent if it is eighty-five percent that means it is impossible to have any heat engine which will be have higher efficiency than that of a Carnot engine which is reversible engine so therefore no heat engine can have higher efficiency than reversible heat engine operating between the same high or low thermal reservoirs.

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So to just to summarize that that means the thermal efficiency is same as that of a reversible engine right of any heat engines will be same and if it isothermal efficient engine for irreversible heat engine will be less than that and of course it is not possible to have a heat engine which will be higher than the you know Carnot thermal efficiency right so with this will stopover and will take few examples in the next lecture and then discuss about that okay about how to apply under things you.

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