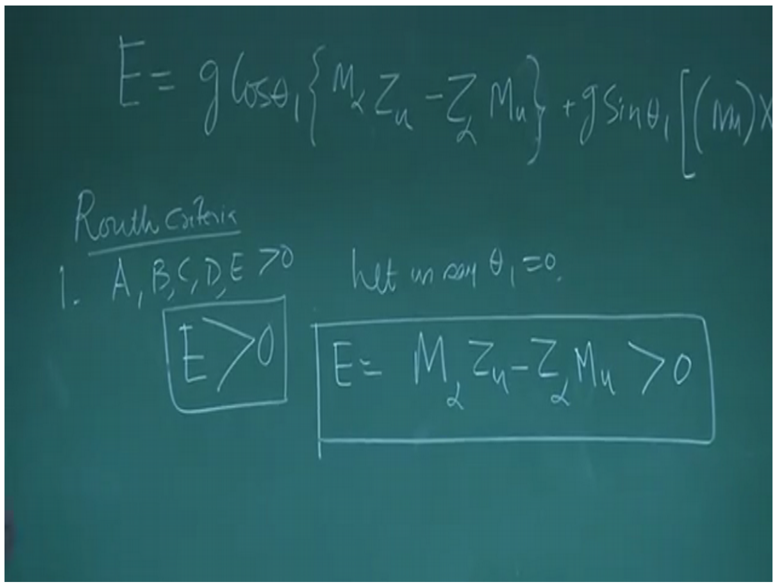
Aircraft Stability and Control
Prof. A.K. Ghosh
Department of Aerospace Engineering
Indian Institute of Technology-Kanpur

Lecture-50
Dynamic Stability

(Refer Slide Time: 00:20)

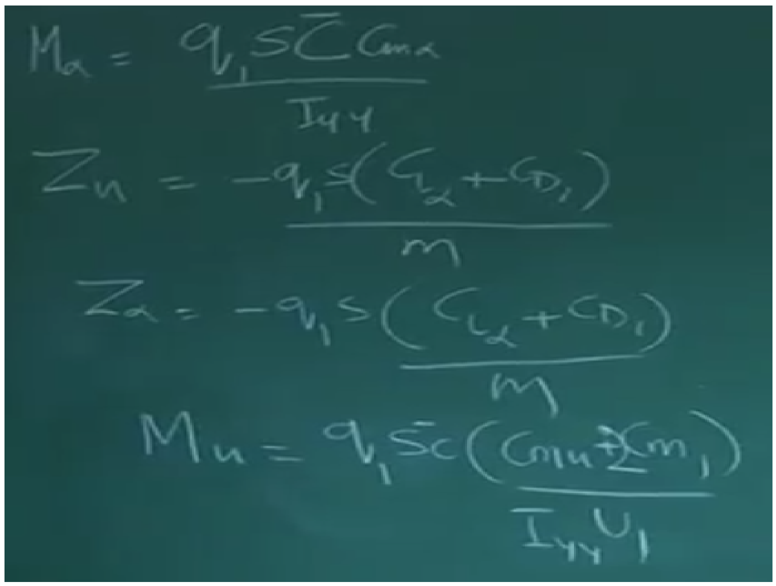


In Routh criteria, we can apply to check whether these things are stable or not. One of the conditions is that all the coefficients A, B, C, D, E should be greater than 0. There are other conditions also, we will come. First we take A, B, C, D, E greater than 0, that means when you expand this matrix and put dynamic values, coefficients etc., etc., then you will see what is the value of A, B, C, D, E, one thing we have to ensure that if in the dynamic stable, then all the signs of A, B, C, D, E should be greater than 0.

One of those cases I will take, what do you mean what additional information we get when you say E greater than 0, okay. That is the case now we will be studying, okay. We will talk about little bit of Laplace transform and little bit of oscillatory area in our subsequent Mann Ki Baat session but we have to complete this part. So I am just for time being we will take that Routh criteria, one of the condition is A, B, C, D, E greater than 0 and we are trying to look for the significance of this sort of huge, huge expression, what is the meaning of this expression, how best we can take care of it.

So we are taking a case where E greater than 0 means what. E greater than 0 let's say theta 1 = 0 okay. And we are not considering thrust part, which we not considering you know it can be easily done in similar way. Theta 1 is 0 then I have E = m alpha z u - z alpha m u greater than 0. We need to understand if theta 1 is 0 this is 1 and g is always positive, so e greater than 0 means m alpha z u - z alpha m u greater than 0 correct.

(Refer Slide Time: 02:28)

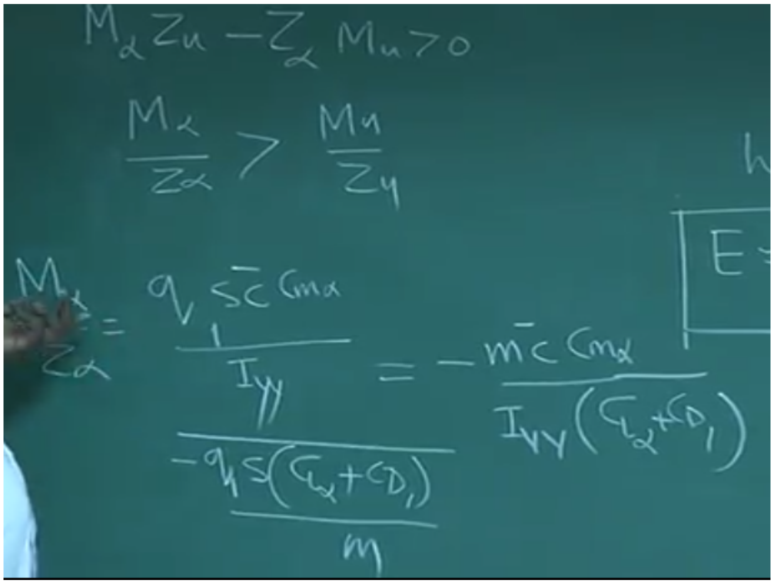


So this is the condition. Question is what does this tell us? Let's go back what was m alpha. M alpha was defined as q 1 s c bar c m alpha by I y y and then second term is z u, what is this z u expression is - q 1 is C L alpha + c d 1 by m and what is your z alpha z alpha is - q 1 s C L alpha + c d 1 by m and what is m u. mu is said to be q 1 s c bar cmu + c m 1 into c m 1 I y y u 1, right.

So, what is the condition E should be greater than 0 because you know that for Routh criteria you take it for granted at this point that all the coefficient A, B, C, D, E for that equation a s four + b s cube + c s square + d s + e = 0, all the coefficient will be greater than 0. So, we are taking a case where E is greater than 0 that is. I am seeing what does E greater than 0 means to us ok. So now from there we are studying a case where we are saying theta 1 will be 0. What is theta 1? Theta 1 is the theta at the time of cruise okay.

Now again we will see the expression what the m alpha z u, z alpha is and m u which we have already derived and now we will put this here and see what does this finally mean to us okay. So that is lot of involved, it involved lot of code word, so don't get upset.

(Refer Slide Time: 04:24)

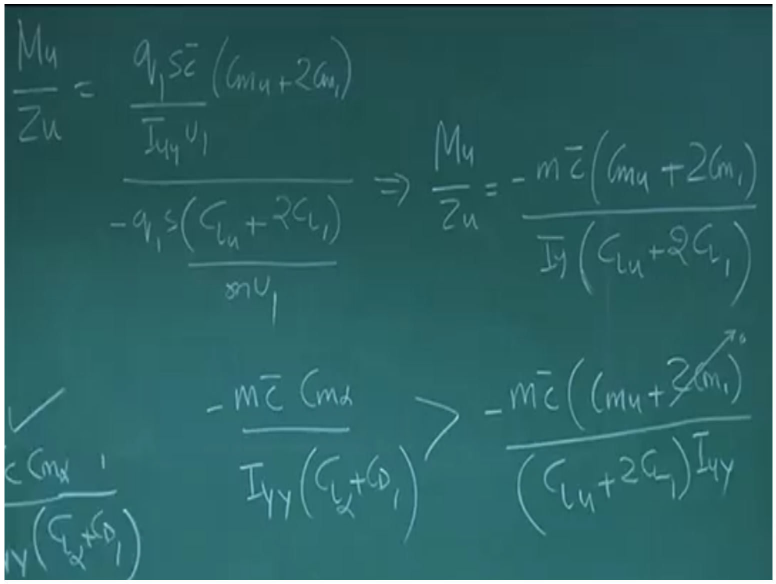


So e greater than 0 means we will see that m alpha z u - z alpha m u greater than 0 that means m alpha by z alpha greater than m u by z u. We are seeing that we are trying to understand what additional information relating to stability we are getting through this coefficient. We know for dynamic stability all this A, B, C, D, E should be greater than 0 so you are studying a case for E greater than 0. e greater than 0 means this expression greater than 0.

Which tells m alpha by z alpha greater than m u by z u and now I substitute m alpha z alpha m u z u in this expression and then what will happen you see. M alpha by z alpha will be nothing but let me write q 1 s c c m alpha by I y y that is y that is m alpha q 1 s c c m by I y y q I by z alpha - q 1 s c 1 alpha + c d 1 by m right.

Q 1 s c 1 alpha + c d 1 by m. if I divide this i get this expression as - m c bar c m alpha by I y y c l alpha + c d 1 ok. So what is the final equation m alpha by z alpha is this ok. Similarly we find out what is the second term m u by z u.

(Refer Slide Time: 06:25)



so m u by z u when I am doing I know the expression of m u so I put q 1 s c bar by I y y q 1 into cmu + 2 c m 1 is divided by - q 1 s clu + 2cl1 by m u 1. You could see that we have to substitute the expression for z u and if z u here. It is wrongly written so let me correct it and you also correct it should be. Z u should be - q 1 s clu + 2cl1 divided by m u 1 ok.

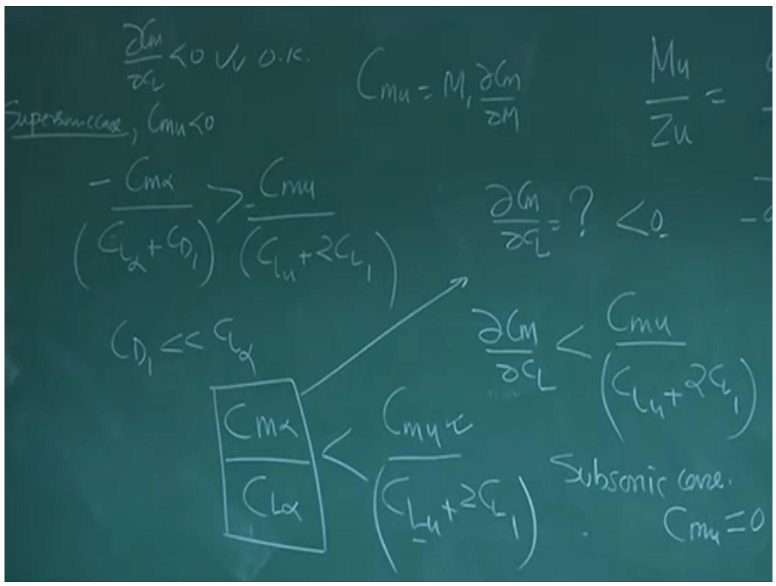
So that was wrongly written please correct it so here it will be q 1 s clu + 2cl1 by m u 1. So once I do that I get an expression is m u by z u = - m c bar cmu + 2 c m 1 divided by I y clu + 2cl1 please understand you should do it yourself otherwise there is possible chance to mixed up you know.

Writing this expression not a simple thing is a real challenge you should get de focused ok. So what we are doing. We are trying to extract maximum information from this relationship which has come because of the condition through routh criteria that all the co efficient with that equation A, B, C, D, E should be greater than 0. So let’s moving what happen what information when condition e greater than 0.

So from there we have come here now we know this expression one is m alpha and z alpha we know this and m u by z u is this so now we will write like this that - m c bar c m alpha by I y y c l alpha + c d 1. What is this, this is basically are m alpha by z alpha see m c bar c m alpha by I y y this is simply greater than m u by z u with this expression that is - m c bar cmu + 2 c m 1 divided by clu + 2cl1 into I y y.

clu + 2cl1 I y y m c. now how do simplify this you know this c m 1 is 0 this is at trim this value is 0 or trim. Or trim in cruise this value is 0 so. Now further if I manipulate this expression. we get interesting result and all this efforts were for that if I further simplify this.

(Refer Slide Time: 10:01)

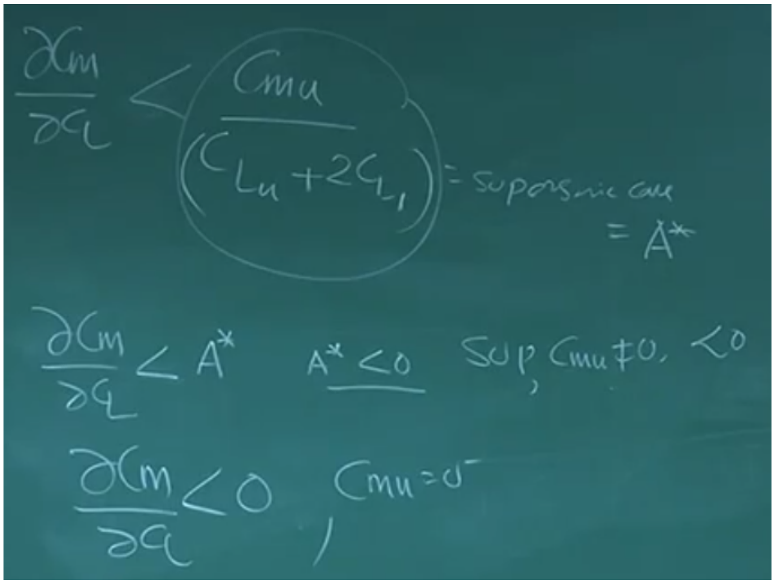


I will get - c m alpha by c l alpha + c d 1 is greater than cmu by - cmu by clu + 2cl1 you could see here this m m get canceled c c gets cancelled I y y get cancelled c m 1 is 0 so this is nothing but m u, cmu ok. So that is what is prevailing here c m 1 is of course is 0 now if I say c d 1 is less compared to c l alpha is very true c d 1 could be 0.025 c l alpha around 5.5 so it will be good approximation and then what we will get which is as c m alpha by c l alpha less than cmu by clu + 2cl1 you could see this sign has changed to less than greater than - - sign by multiply - sign in both sides naturally this becomes less.

What is the meaning for c m alpha by c l alpha let us understand. What is ratio c m alpha by c l alpha this is nothing but dcm by dcl right. And what is this, this is you know - static margin so what we are getting from here the dcm by dcl should be less than cmu by clu + 2cl1 for dynamic stability the condition is dcm by dcl should be less than cmu by c l 1 + 2cl1.

Now for subsonic case, you know for subsonic case this cmu = 0 so naturally we have this condition dcm by dcl less than 0 we are ok with this right. But for supersonic case, cmu will be negative right. You know cmu is negative. Subsonic case cmu is less than 0 that you know because cmu is nothing but m 1 into dcm by d m and what happens as the aircraft goes subsonic increase the speed the center pressure anatomic center of tail goes backward so these tail moment have increases that gives that nose down moment and give tunnel effect right.

(Refer Slide Time: 13:21)

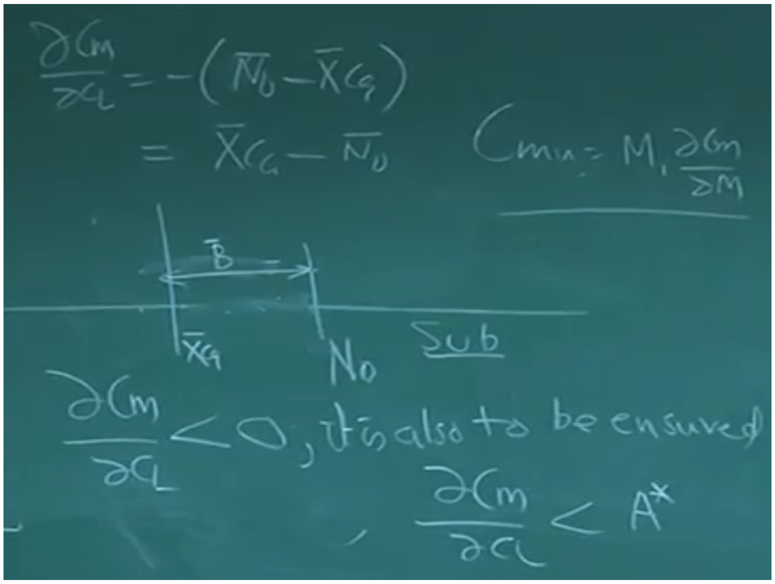


So this is negative so dcm by dcl again negative what is the difference in the first case when cmu was 0 dcm by dcl should be negative its ok statically stable. Second case also dcm by d cl less than 0 so this is negative again dcm by dcl less than 0 so there is no violation in the static stability however what is the different let us see that.

So we see here the dcm by dcl = or should be less than cmu by clu + 2cl1. We know cmu is negative for supersonic case so dcm by dcl less than 0 is fine but what is let's say this number for supersonic case is A star I can get the value of cmu and c l 1 so this is negative that you know so this will be indeed less than 0.

dcm by dcl ok it = A star well A star is less than 0 fine, this supersonic case for The subsonic case same it is given d cm by dcl is less than a star and less than 0 because cm equal to 0 here cmu not equal to 0 in fact the less than 0. What is dcm by dcl. dcm by dcl is that you understand let me erase this part.

(Refer Slide Time: 14.48)



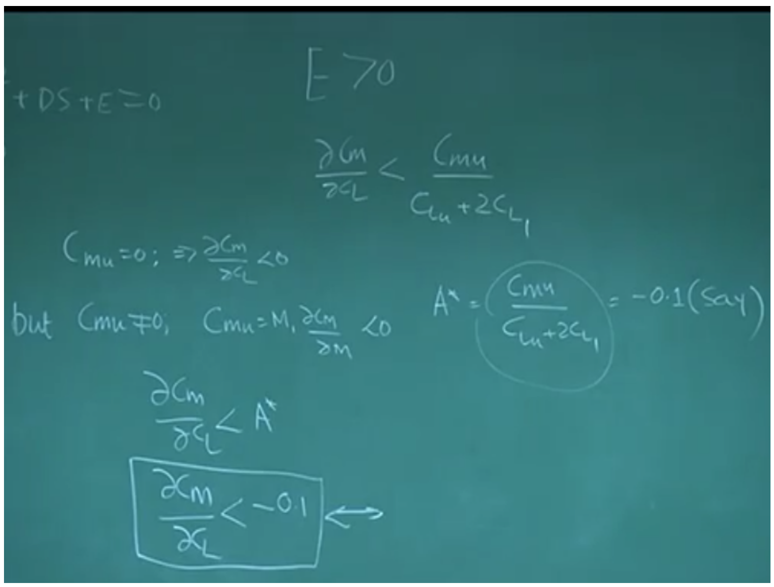
So what is dcm by d c l we have see dcm by dcl is nothing but - starting margin that is - x or - neutral point - x c g bar or we can write as x c g bar - n not bar. N not is the neutral point. For subsonic case what is just simply telling is that if this is the n not ok. Aircraft will be subsonic right from there what I am saying getting is dcm by dcl should be 0.

That means x could be anywhere as long as it is in this side correct, but for supersonic case what is dcm by dcl when it takes care of dynamic stability also it is not just less than 0 it is also to be ensured that dcm by dcl is less than A star where A star is negative which is given by cmu by clu + 2cl1 so that means just putting c g here ahead of n not will not be sufficient we have to ensure that x c g is such that n x c g - n not this separation now will be governed by this should be less than A star that is this distance ok if I said B would be subsonic case this B should be less than A star ok. This part clear.

dcm by dcl should be less than A star here we know that A star is negative this is positive this is positive and this can be neglect very small. This is positive this is negative so this values less than 0 this is fine. So that is what you should be very very clear. But for dynamic stability supersonic case how cmu which is nothing but cmu nothing but m1 dcm by d m plays an important role through the phenomenon called tuck under this speaking supersonic.

A c of the tail goes backward so tail momentum increases and additional nose down moment comes which is known as tuck under effect ok. This is clear. So cmu plays important role which restrict the stability margin to ensure it is also dynamically stable and this will be derived from the condition that E is greater than 0 ok. Which is condition for dynamic stability through Routh criteria ok?

(Refer Slide Time: 17:48)



So we try to revisit this e greater than 0 this condition which is came from the characteristic equation A s four + b s cube + c s square + d s + e = 0 and you know from last criteria one of the conditions satisfies all this co efficient greater than 0 and there are other condition that will come and by using this condition one case E greater than 0 what does it mean and we got the relationship wonderful relationship.

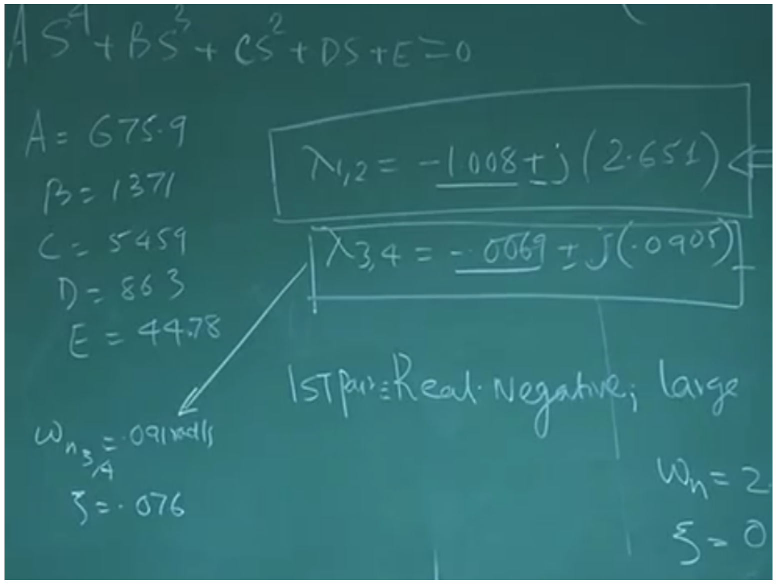
dcm by dcl less than cmu by clu + 2cl1 ok and you know from this relationship it realize that if cmu = 0 this implies simply dcm by dcl less than 0 which is very familiar with the slope of dcm by dcl at equilibrium simply as a 0.

But high speed cmu is not equal to 0 then what, then at high speed cmu you know is nothing but m 1 dcm by d m and which is less than 0 then this condition says dcm by dcl should be less than A star but what is A star. A star is nothing but cmu by clu + 2cl1 typically let us say this value ok let say - point 1. cmu is negative ok if this is - .1 for sake this is an example telling to understand this that means the condition is just not dcm by dcl less than 0 the condition is dcm by dcl.

Yes indeed less than 0 but should be less than point 1 that is the modified condition on undertaking dynamic stability also into account so the location neutral point and c g because this is extremely important it has to satisfy this condition as well. Which includes the condition that dcm by dcl is less than 0 but dcm by dcl is less than this value says for example we have taken - . 1? This should be understood very carefully ok.

Why this is important because we have taking care of fact that I know as speed increases a c of the tail goes backward and the tuck under phenomenon ok. And then this is one another way to be friendly with big big equation is we see that what finally this expressions are meant for us. Let's take one example let's say for the flying jet airplane all those derivative were computed and after solving we found that value of A, B, C, D, E which are dependent upon the geometric and aerodynamic coefficient or derivatives.

(Refer Slide Time: 21:18)

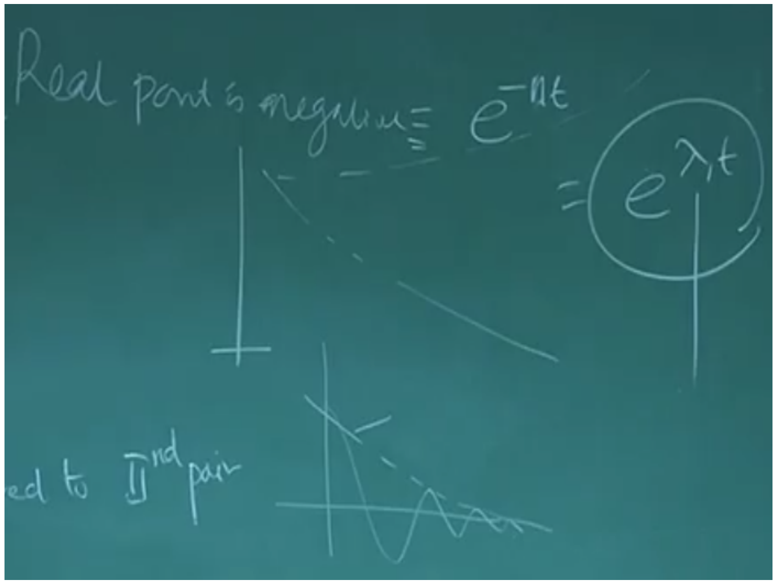


Lets us see the A value, A is 675.9, B is 1371 and c is 5459 and D is 86.3 and E is 44.78 and let us see the all efficient unit. At this point let us talk about the signs and all. You could see A, B, C, D, E all are greater than 0. One of the conditions for Routh criteria for its stability is satisfied ok. Now If I can solve this equation, and I can find out the roots so numerical methods which is possible. Let's see that has been done and the roots are lambda 1, 2 roots are - 1.008 + j as complex 2.651 and another pair of roots 3, 4 which is - 0.0069plus - j, j small j .0905.

Typically if I most case of jet airplane if you get the co efficient that have solve this equation will find it will get segregate that into one pair here one pair here so two second order system behavior then the beauty of longitudinal dynamics find in most of the airplane response or its response will have two types of excitation one thing is that they are having the complex parameter in oscillatory response ok for stable system.

So oscillatory response that is why this is complex here or here but what is the difference you see in first pair you can find real root. First pair ok this typically we can see in most of the airplane. First pair in real negative and it is very large compared to the second pair. Large compared to second pair. You could see here this is - 1.008 here - .0069.

(Refer Slide Time: 24:01)



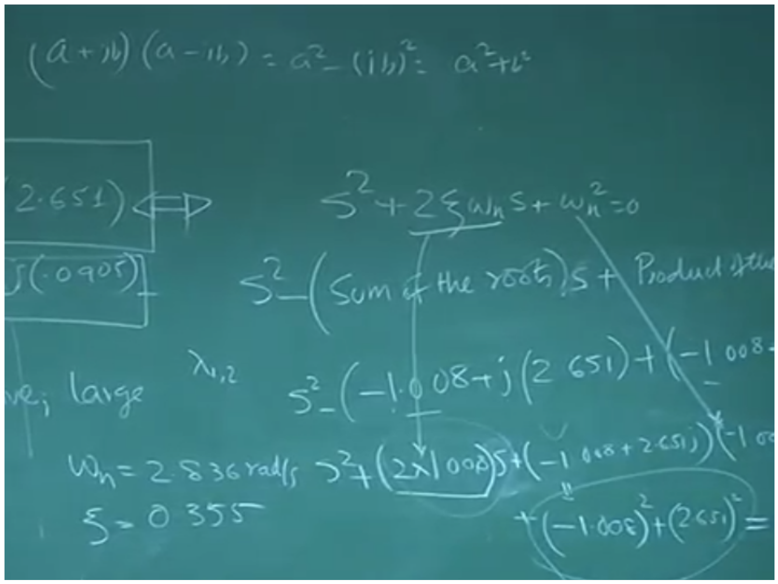
What does this real root does, real value of this root does you know that this L is the real part is negative that means it means it difficulty form of E to the power - t n t right. It will put envelop for decaying the amplitude will go on decaying. If the real root is positive then it will go on growing in root will be go on going in because this is typically e to the power lamda t. If lamda 1 is negative the real part is negative which is here both are negative both are trying to cut down the amplitude and it is oscillatory also so if you see the root typically root like.

So this envelop will be governed by the real negative root and where ever this root is last negative they will decay fast compared to the smaller one here so I could see here there are two more excited where one is decaying very fast another is taking long time taking long period. This is long period and this is short period mode short period mode. Both are being like a second order system both this mode right.

Now come back to the airplane what happen if the airplane is moving like this we give this disturbance. In one way it is get excited is that is like this and call that two equilibrium other is other could be goes like doing like this and finally come back. Second one this one is nothing but nothing but longer period this real root is smaller negative. First one this one is short period is this one. So we will characterize this lamda 1 lamda 2 at short period root and this as long period root or we also says phugoid roots.

We will discuss about short period mode and phugoid mode in detail but just see in the beautiful equation also use ugly root expression in formatives. Now how do I handle what information more I should extract from this, if it is second order system? What I look for is natural frequency I look for damping ratio. Let us see if I can used this equation short period natural frequency and long period natural frequency and damping ratio let's do that.

(Refer Slide Time: 27:01)



If I take this since I have seen this is at typical second order response so I write this as characteristic equation as s square + 2 get a omega n s + omega n square = 0 you know this is typically second order characteristic equation in terms of omega n and zeta and so I construct this equation from this root this form. So I write s square - sum of the roots into s + product of the root equal to 0.

So if I take this first equation first root for lamba 1, 2 this equation becomes s square - sum of the roots will be what - 1.008 + j 2.6 This + - is 2.651 ok this is one root + again - + - 1.008 + - now j 2.651 ok this becomes sum of the roots. First root is I miss that the - sign here + - complex sphere complex sphere so what we note is - 1.008 + j 2.651. Second root is - 1.008 - 2.651 into sum of the root into s + product of the root you know how to find out product of the root.

So this will be - 1.008 + j 2.651into - 1.008 - j 2.651 and that equal to 0 if you do like this you will get this equation in this forms square - sum of the root + product of the root when we compare it with this equation and you will get s square - this and this get added so this becomes + this 2 into 1.008 then + product of the root so that will be - 1.008 + 2.651 j ok into - 1.008 - 2.651 j which should be equal to 0. This is nothing but a + b into a - b so you will get equation where 2 into these into s will be there put s here let's erase this. S +

So I know I compare this with this equation s square + 2 zeta omega n for 2 zeta omega n I will write this equal to this compare this and omega n square I compare with this product should be typically will be - 1 point A + b into a - b so a square - b square and j is there become a square + b square little bit typically where not mistaken this value will be - 1.008 whole square + 2.651 square so a + b into a.

A + j b a - j b equal to a square - j b whole square equal to a square + b square so you can see that this will be this product will be a square that is - 1.008 + b square is 2.651 this square this will be equal to 0 so I compare omega n square equal to this term and 2 zeta omega n equal to this term and by solving that I will get omega n equal to 2.836 radian per second and zeta I will get around 0.355 this is clear how to do it.

You could see I could use this equation and find out the roots similarly for this also I can find out in similar way and you are supposed to do it and find that omega n 3,4 will be equal to 0.091 radian per second and zeta equal to 0.076.

And you could see that gets for first one is very high compared to second one that is why second one is the phugoid mode long period mode it goes on doing like this. Goes on converting kinetic potential energy among each other and then it process and that equilibrium. First one is simply like this that's why zeta is 0.355 compared to 0.076 same thing deflected in omega n this is much larger in short period then omega is for phugoid this you must do one exercise yourself and this is very straight forward thing.

We will solve one or two example for this so that it was handy this lecture was meant to be give an idea how to handle this big big equation and find some value and number here today I will end in next class I will start from here another example so that you will get familiar to what we have been.

Thank you.