Aircraft Stability and Control

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Lecture-48
Perturbed Pitching Moment

You have seen how to understand what cm alpha dot? What is cmq? What is cl alpha dot? And you could very well appreciate the sign of cmq is negative and cm alpha dot is also will be negative because alpha dot is coming out because lag in down wash primarily. So actually the tail at time is seen more angle of attack. Because the down was had only had a time lesser than time t by lt by u1. So that also gives the nose down moment.

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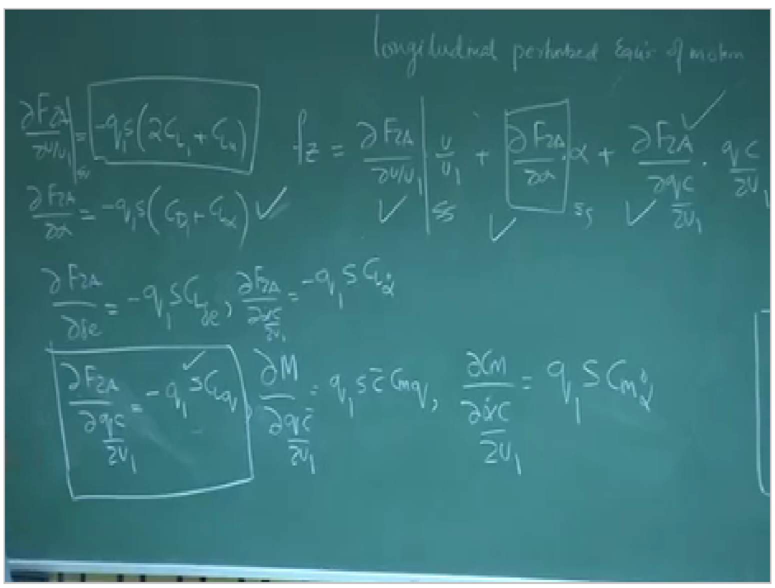

So cmq and, cm alpha dot both are negative. cm q and cm alpha dot are less than 0 ok. Now we comeback to this perturbed force as f z equation. We have seen fz by du by v1 have developed that expression and that was given as q1 s 2cl +cl u. We have done last lecture we have done this also at steady state this expression is this, this sub revising this dfz by dq here it will be what? Let us understand this.

This is the expression I have written very obvious for you know fz fza because of q will be what? That will be half rho v2 square s cl q into qc by 2u1 right. Only q right is a positive derivative this dc d cm by qc by 2u1 but this is the value so dfz a by dq c by 2u1 will be equal to nothing but q1 that's the evaluated as a steady state.

But this is remaining at q1. And then cl q ok q1 and cl q are of course that is S. And that is exactly what this expression. Very simple right similarly if we find out ok. Let me see this, this is done. Dfza by d alpha dot c by 2u1 so again fz only because of alpha dot holding others constant when we are trying to find out partial derivatives. So this will be 1\2 rho v square s cl alpha dot into alpha dot c by 2u1 right.

So d f z a by d alpha dots c by 2u1 will be nothing but q1 but this is evaluated as steady state is the ½ rho is steady state q1 into s into cl alpha dot right. So there is a slight mistake you could see that. We must understand this expression and this expression what is the mistake? You could see here please note down here as per as number is concert q1 is cl q fine. But what was the find out .that find out dfz a and I have been telling you that z axis is pointing downward and, lift is upward.

So these will this force will be towards lift direction because if q is positive force is toward upward direction. But which is opposite to z. So I have to put a minus sign here for cl q write coming back to alpha dot. What is happening y alpha dot is coming because the delay in downwash to come here? We have got what is the time. It is actually deceive time to more angle right.

So that will give the course positive upward but, z is downward so I have to put minus sign here. Please we careful about all the both of the derivatives will not make much of difference in stability for the most of the airplane. But we should very clear about our convention. Fz a bracket q means do not get mislead this is for q perturbation ok. That means helding other perturbation to 0 because we want to find out partial derivative ok.

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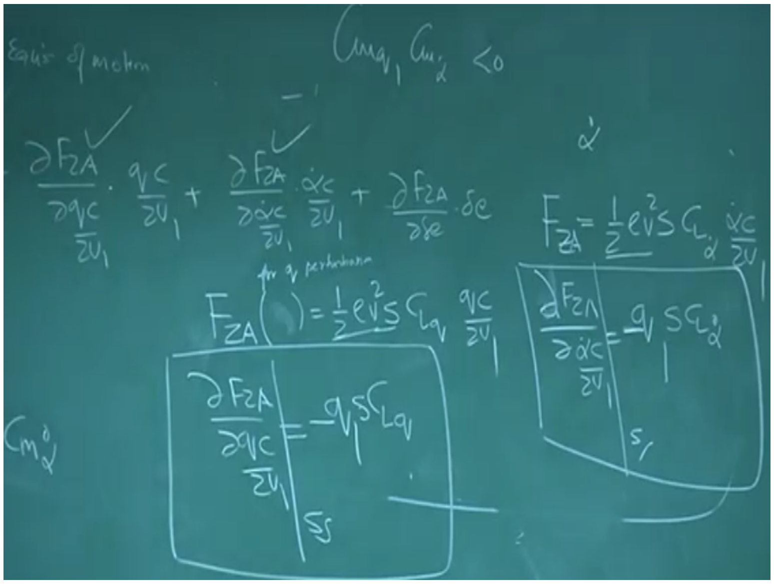

For this time it is very important please developing the habit of asking yourself whether you are giving correct sign or not ok. So this is over now I can easily expand fz using this expressions correct. Let us see what happens

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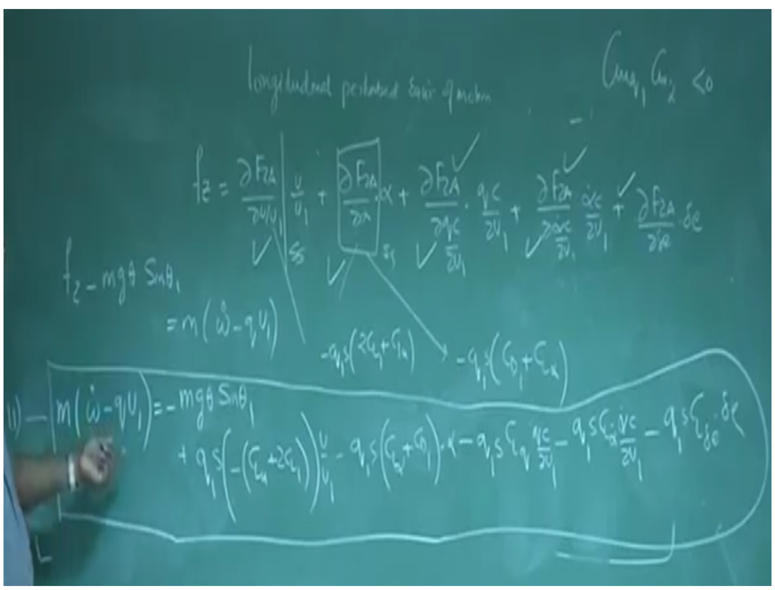

If I do that you know the equation of motion is fz – mg theta sin theta 1 equal to m w dot- q v 1 right. We have to substitute fz using this expression and using the value of expression for all this term. For example for dfz by du1 will put minus q1s and 2cl 1 + CLU right. Dfz by d alpha will put –q1 s + cl alpha like that. For dfz by dq was cl also like qc by 2v1.

If I do that then I get equation m w dot minus q v1 is equal to minus mg theta sin theta 1 ok.so, + q1s into minus cl u + 2 cl 1 ok into u by u1. Similarly minus q1s into cl alpha + c d 1 into alpha. We are substituting the values similarly minus q1 is cl alpha very good. Then minus q1s cl q into qc by 2u1. So will be minus q1 s cl alpha dot into alpha dot c by 2u1 and that minus q1 s cl delta e into delta e. For that because my second equation.

But we need not to get disturbed by this equation. I am sure and understand now you are here about each and every time of this expression. W is perturbed w dot q is perturbed pitch rate. Theta is the perturbed pitch is angle. U1 is the steady state velocity. Cl you know, cl1 you know, cl1 is lift equal to weight from cl because we have assured to have equilibrium. We are giving the disturbances.

We are giving small disturbance more precise about cruise ok. For this expression is going to this is fantastic one equation you have got through fx equal to another you got fx is basically force on the x direction and fz is plunging in this direction. Now what is left is the moment equation ok. We will finish the moment part now. So will same we are trying to develop equation of motion perturbed equation of motion.

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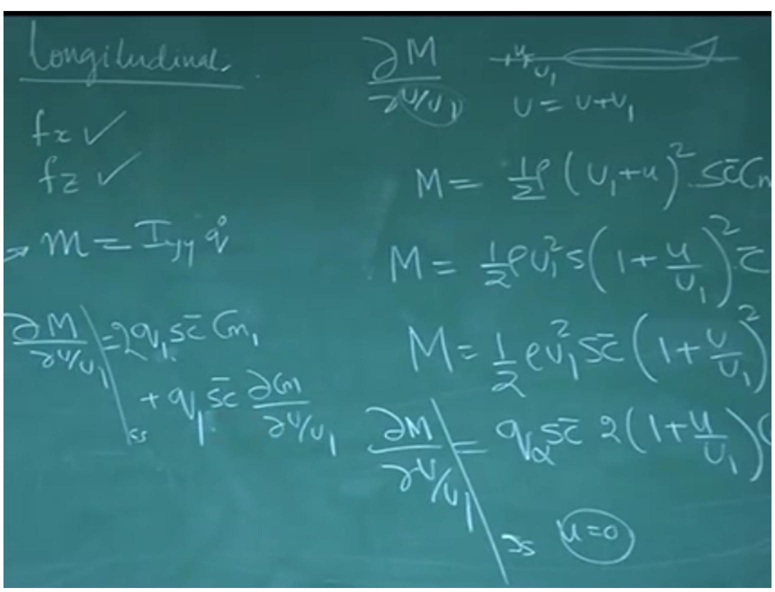

For longitudinal case well we have already seen the equation along the x axis has been completed, along z axis has been completed along x is this and along z is plunging. Now what is remaining is pitching. Because you know we are talking about longitudinal perturbation. That is perturbation is man aircraft will do like this and the passage will go up and down and the velocity along in also is reduced.

We are not talking about lateral motion, because we are talking about longitudinal case. We are assuming that aircraft will be in vertical plane and that is possible only when we give small disturbances ok. So Last equation was pitching moment n is equal to I y y q dot ok. And we are trying to find out how this moment can be modeled. Right how efficiently z was modeled.

And in terms of motion and, control variable. Delta e means control variable. So similarly you do model pitching moment perturbed pitching moment with in terms of motion variable and control variable very simple.

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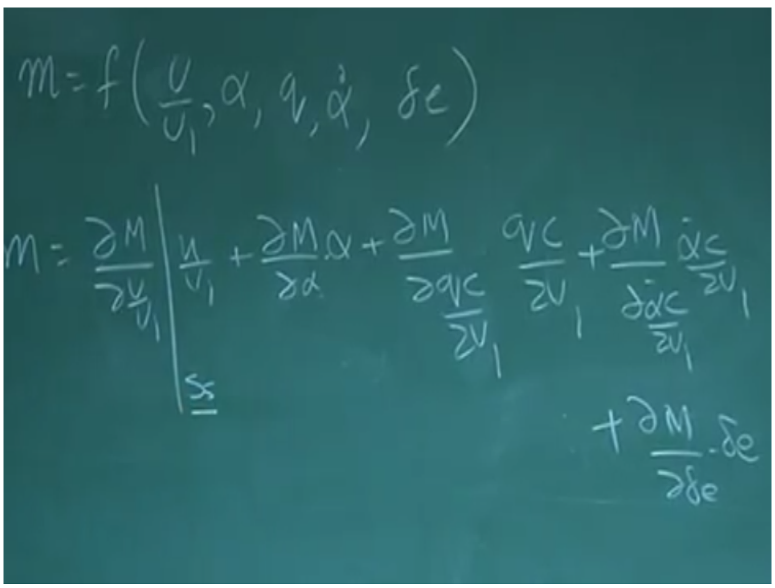

Now you are expert m in write function of alpha and or u by u1 alpha u alpha dot delta e ok. What is next so mu writes us dm by du by u1 into u by u1 + dm by d alpha into alpha + dm by dqc by 2u1 into qc by 2u1 then dm by d alpha dot c by 2u1 into alpha dot c by 2u1plus dm by d delta e into delta e ok. These are the equation for this expression or oscillating pitching moment which is the perturbed of pitching moment and assumption is pitching moment well depending on it up to perturbed u alpha q dot alpha dot delta e.

Small disturbance condition is ok. I repeatedly say for high performance airplane I have been changing delta e dot that would be alpha q. So much can happen right but we are taking simple thing to understand the basic principles and basic techniques. And how do I use those to get basic parameters that are more parameters important for us. So let us understand we have to find out this dm by u by u1 it should be at steady state ok. Let us first find out this dm by du by u1.

You know that moment will be or I write moment will be equal to1/2 rho u1+u because we are trying to find out for perturbed u. So what is the concern the airplane was moving it u1 and we got perturbation which is u and total velocity u + u1. We are assuming again here everything is linear. So this ½ rho v square s into cm of course there should be another length term so c bar.

So ½ rho v2 is c bar represent cm. so now we are to find out dm by du by u1 you know it is technically symbol of that ½ rho u1 square is and 1 + u by u1 square. And here I write c by here and cm for this I again write and better format SC bar into 1 + u by u1 square into cm. What I am trying to find out dm by du by u1. So What I do dm by du by u1 nothing first time I am doing.

This is q infinite s c bar and this is 2 1 + u by u1 into cm + q infinite s c bar into DCM by du by u1 into 1plus u by u1 square. First I am taking derivate is u by u1 the first step the two as come here and then second is cm. know what is the understanding this should be evaluated as steady state at steady state you know u is 0. Perturbed u is 0 at steady state.

So what do I get I get dm by du by u1 as q infinity s 2 will be here c bar into cm + q infinite SC bar into dcm by du by u1 ok. But says I am evaluating at steady state not only I put equal to 0 but also I should know this dm or q s infinity q1 dynamic at steady state cm 1 that is steady state. And this becomes one and, this is the steady state fine. What is cm for an airplane cm in steady state is how much? It will be 0.

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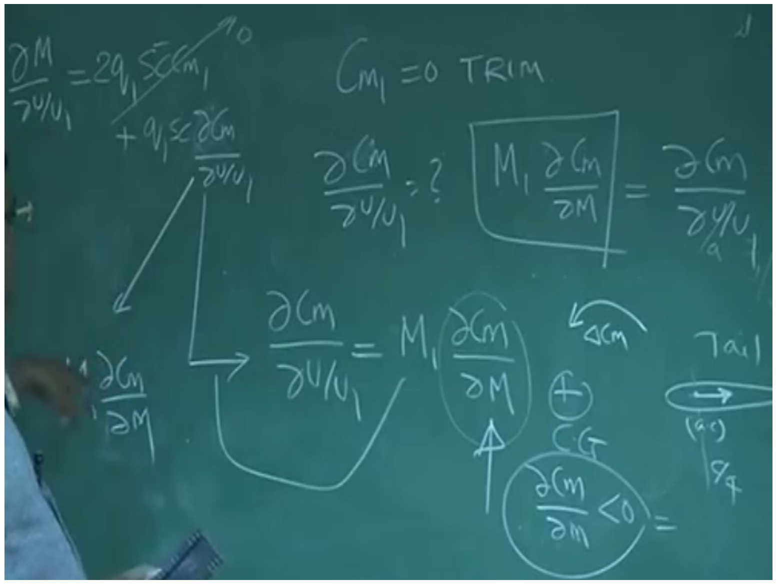

Cm 1zero that is trim ok. Then what is dm by du by u1 or DCM ok this is important DCM by because. This man goes at steady state cm 1 is 0. For cruise that is phase about we giving all disturbances so DCM by du1 I came write this as M 1 into d cm by dm you know how. You are all expect now we have done it earlier also I have divided here by a divided here by a either speed of sound so this become d cm by dm and u1 by m1 which comes here right.

So now understand d cm by du by u1 is nothing but m 1 d cm by dm and this is very very important parameter. This is extremely important parameter what is d cm by dm? remember this is the tail right, a c of the tail is somewhere here this is c by 4. What is subsonic speed right as will go supersonic high speed this a c goes backward. This d cm by dm is extremely important and I try to explain that you know low speed the s c of the tail, this is tail as soon the c g somewhere here on the aircraft.

S c of the tail is as increase the tail of supersonic a c of the wing or the s c of the tail move backward it tries to approach around 50 percent of the chord. So what is happening as I am using the mark number this 18 is going on increasing because a c of the tail is moving backward right. So the distance from c g l t is increasing that increase in l t will give additional moment delta cm pitch down moment. And what will happen accelerate if I don't model understand this phenomenon as there to oscillator supersonic there will be pitching moment airplane will try to go like this.

This called tuck under phenomenon. Lot of accident happens when you are not able to understand what this phenomenon tuck is under this clear. So d cm by dm is extremely important you could see that as I am increasing the speed to supersonic this additional pitching moment d cm is stunning because the a c of the tail is moving backward and this d cm by dm is negative it gives pitch down moment and hence tuck under phenomenon.

This is extremely important we must value at this when we are trying to do dynamic stability especially in supersonic moment right ok. So what we have seen now, we have seen dm by d u1 is nothing but I must find out dm by du by u1 and we have seen this equal to 2 q1 s c bar cm 1 + q1 s c into d cm by du by u1 and which is nothing but is m1 into d cm by dm this is nothing but m1 into dm c by dm and this is negative supersonic speed that is.

The derivative by dm by du by u1 and you know the trim is at cruise then cm 1 is definitely 0. This is one time evaluated ok. Know we are prevailed You please understand this derivation then all lectures will have all the derivative neat and clean and then put the equation and get the nice equation. At this point every derivative note down this is the meaning of this, this is the physical significance of this that is more important don't get lost into this big expressions.

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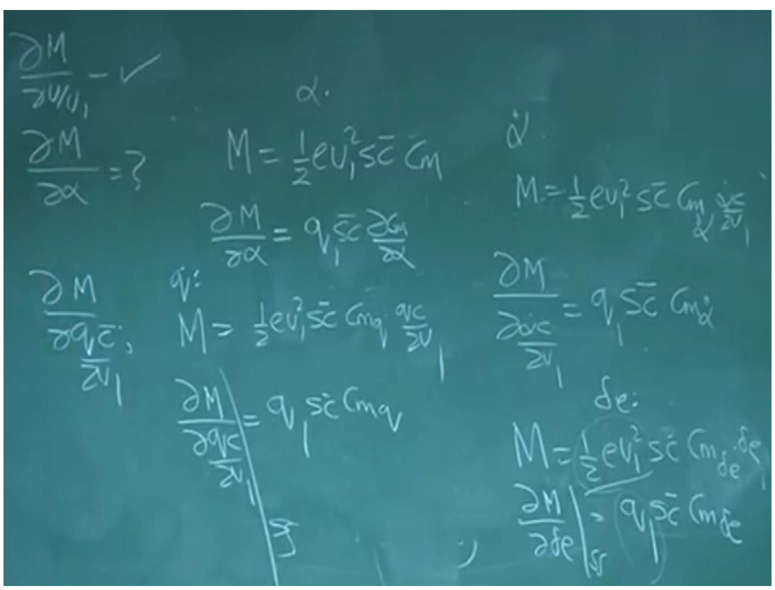

So we have seen dm by du by u1 you have evaluated so dm by d alpha will be what. Very simple you know that how to do it m is half rho u1 square s c bar into cm so we are talking about alpha derivative right.

So dm by d alpha derivative will be q1 s c bar d cm by d alpha right. So, that also you get now we want to find out dm by d qc bar by q1 very simple this is q derivatives ok. Second will be half rho v1 square s c bar into cm q into qc by 2u1. So dm by d qc by 2u1 will be equal to q1 because of evaluating at steady state so this is cm q.

Similarly alpha dot derivative m I will write as half rho u1 square s c bar into cm alpha dot into alpha dot c by 2u1. So dm by d alpha dot c b y 2u1 will be equal to q1 s c bar cm alpha dot. Similarly controlled derivative if I want to find out delta derivative elevator so I will write m is equal to half rho u1 square s c bar cm delta e into delta e right.

So dm by d delta e at steady state will be q1 this dynamic steady state q1 s c bar cm delta e. q1 is the dynamic pressure at steady state one means at steady state. So it is very simple we have all the expressions with you dm by d u1 have here dm by d alpha dm b y d q dm by d c q by d u1 we have dm by d alpha dot c b y 2u1 dm by d delta e and you also know how to estimate cm alpha dot you know how to estimate cm delta e.

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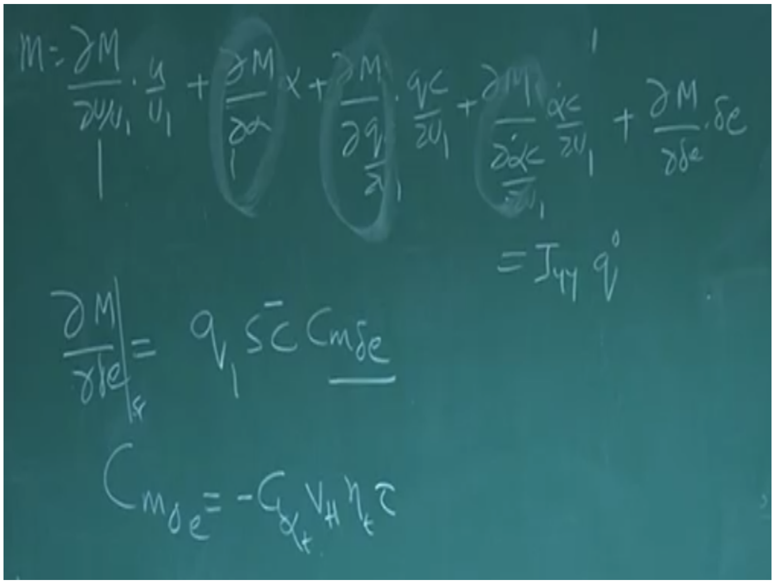

We have written m is equal to dm by du by u1 into u by u1 + dm by d alpha into alpha + dm by d qc by 2u1 into qc by 2u1 + dm by d alpha dot c by 2u1 into alpha dot c by 2u1 + dm by d delta e into delta e ok. We have developed expressions for all this derivatives for example if I take dm by d delta e which is nothing but q1 s c bar cm delta e and we know the expression for cm delta e we might have seen in initial lecture.

We will have the expression for cm delta e dm by delta e is q1 s c bar cm delta e and we have already done cm delta e is nothing but minus c l alpha tail into v h neta t into tau so what is the message. Even the configuration of the airplane even the fight region the altitude the speed region can easily find out these derivatives which are evaluated at steady state. What I need information.

What is that dynamic pressure at which the airplane at trim at a c of the aircraft and cm delta e what I want c l alpha tail I know v h volume ratio I know neta t may be .91 and tau value I know. So all these value for example for cm alpha dot also I have shown you expression I can find cm alpha dot find cm q all the things I can find out not sake of time saving all the expressions values of this derivatives will be known ok right.

At once I know that I know this is equal to I y y q dot right m equal to I y y q dot. So I can easily write equation q dot in the form

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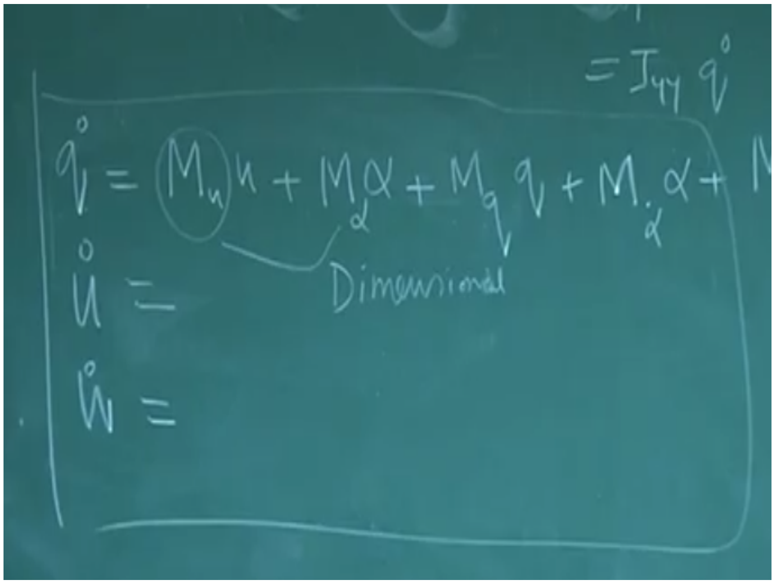

q dot equal to m u into u + m alpha into alpha + m q into q + m alpha dot into alpha dot + m delta e into delta e. Like the way we infer the x direction for z direction also we will write like that and then once .We found the equation we will be actually solving them to get what exactly we are looking for ok.

What we are try to do is we have developed q dot we have already developed q dot we already developed w dot in terms of dimensional derivatives in terms of m u m alpha dimensional derivative. I have shown similar thing for other cases dimensional derivatives for example m alpha is dimensional but cm alpha is non-dimensional. Once I write the three equation like this then I will know how to solve this equation for different control input delta e and see the response and check the airplane is dynamically stable or not.

So that will be our next task so far we struggled we worked very hard to develop this expression and simply splitting it out. What are these expressions what are the partial derivatives and what are the important points you should understand the designer right. So next class we will summarize them we synthesis them and get unit equation how to solve it to get stability characteristic at this point I must tell you if you have time just go through Laplace transform right ok. That will help. Thank you.