Aircraft Stability & Control
Prof. A.K.Ghosh
Department of Aerospace Engineering
Indian institute of technology-Kanpur

Lecture -46
Perturbed force

Let us take the second equation. What was second equation?

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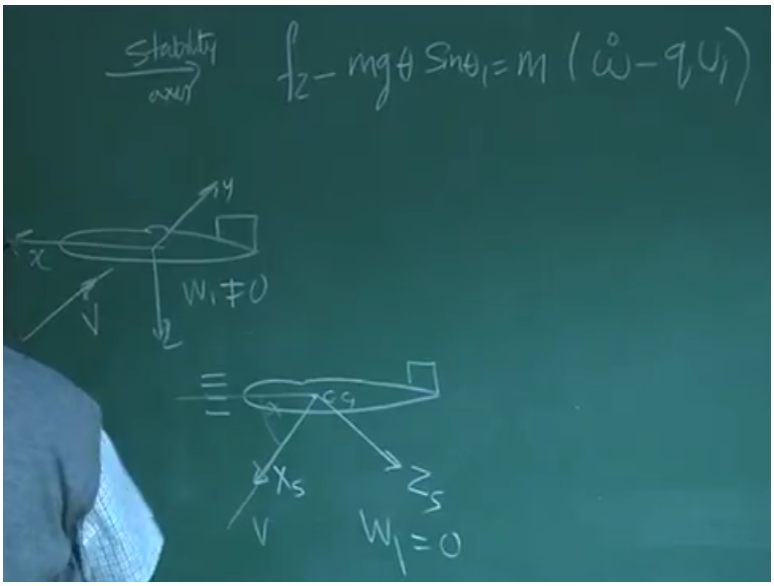

That is f z –mg theta, sin theta 1=m, w dot minus q u1. Please remember we can taken stability axis system right. And I repeat what is stability axis system is this. This is an airplane this is your x, origin x is this, y is this and z is this and here the velocity vector like this, this is the normal modified axis system. If I am talking about stability axis system then what happen this airplane is this cg and this velocity vector.

So I align x axis like this and we call it. x s and this become z s stability axis ok. This becomes alpha so I am keeping the x axis align to the velocity vector why? Because than there is no component of velocity in z direction so w1 becomes 0, w 1 goes to zero. But here you see w1 not equal to zero. Because this is the velocity vector there will be the component along the z direction. But here by choosing stability axis system w1 becomes 0.

So that after applying this only you got this final equation should not forget right extremely important and why stability axis system because you know in velocity in this direction then lift but perpendicular to this dragging opposite to this. So that competitions becomes simpler again we have, will go mechanically like the way we have done.

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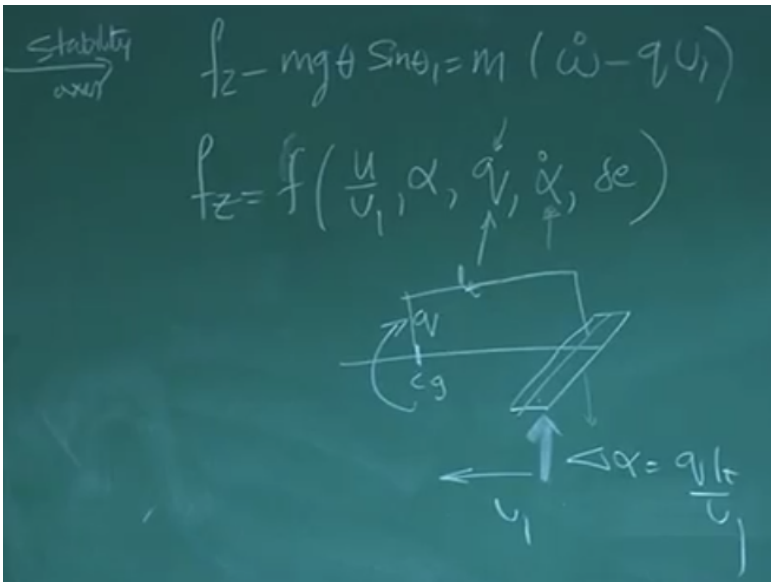

There we got f z is now we are smarter will write u by u1 then alpha putting it on q you make put it on alpha dot and delta e. You might have seen in f x modeling we have been put q and we have been put alpha dot. Theoretically they should be there but they are small so we have neglected it right. For f z we need to know that why there should be q? Which know very well that if this is the cg and if the horizontal tail or the wing whatever it is? If there is q, then this q if is going up likes this so the tail is going down. So relative air is going up like this.

And is moving forward so there will be delta alpha. Which is q l t by u1 right? I say l t is the distance between aero dynamic centers. So this delta alpha will give you in detail force in the terms of lift and depending up on the original axis. This will have component along the z axis right. So that is why q will played important role in f z in and you soon it will have a more most essential role in terms of movement.

pitching movement we also see that alpha dot will play role and will come back to the physical signification of alpha dot in development force is f z when I talk about the derivative alpha dot

and delta e you know very well right because delta there will be a force lift and that will have a component always z know but modeling is ok. This part will be almost mechanically you see this is a first problem positively so I can write.

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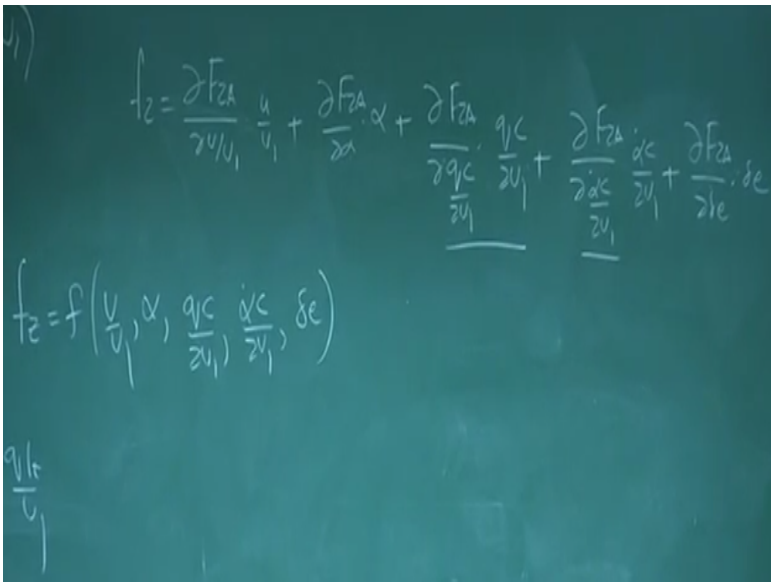

F z as d f z a by d u by u1 into u by u1+ d f z a by d alpha into alpha plus d f z a by d q c by 2u1 let me write and tell you. And d f z a by d alpha dot c by 2u1 into alpha dot c by 2u1 + d f z a by d delta e into delta e. You could see this two alpha dot cl and q c by 2u1.Remember we have two convert u into non-dimensional by dividing by u1 you want all this thing should be non-dimensional form. But you could see here these q1 alpha dots are time dimension per radian per second.

So what we do is now we write f z as a function of u by u1. Then alpha non-dimensional for q we write q c by 2u1. And you that QC by 2u1 is non-dimensional. Then for alpha dot we write. Alpha dot c by 2u1 but then delta e. same theory same understanding and you need to keep this augment non-dimensional. So for u you make it u by u1 q is the conventional thing q c by 2u1 somebody ask why to take it as convention q c by 2u1 as a non-dimensional are dimensionless ok.

So if I write like this now from here I can write d f z a by du by u1 into u by u1, d f z a by d alpha into alpha, d f z a by qc by 2u, 2 qc by u1 and d f z a by d alpha dot c by 2u1 then d f z a by

d delta e into delta e correct. So what is now? What is important now? Important this. Can I get quick estimate or formulation to evaluate this derivative at steady state? Let's first attack d f z a, du by u1. Before we start doing it f z means aero dynamic perturbed force along z direction ok. So let us do that. I can write f z a is equal to minus cl into half rho v1 plus u whole square s. what is this.

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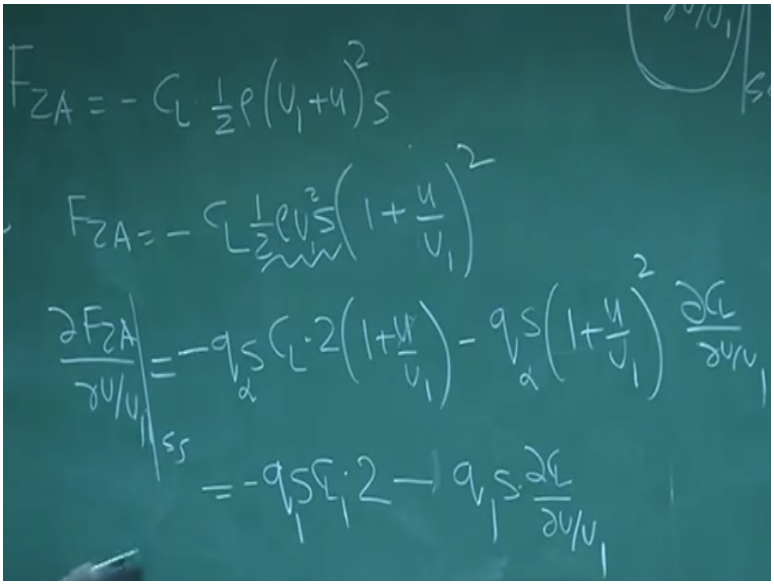

This I am trying to find out d f z a by du by u1. We call this is the aircraft. This is u1. And some the perturbed because perturbation this is you total speed is u1 plus u. so what will be the total force will be half rho v square u1 plus u, s into cl right. So now what I do, I write f z a is equal to minus cl into half rho u1 square s into one plus u by u1 square. No problem 1 plus u by u1is square square fine. So know I am find out d f z a by du by u1.

So first I do like this. When I say minus q infinity s this is half rho v square s and then cl and then derivative of this then it will be 2 into 1 plus u by u1 right. Then second is minus 2 infinity s 1 plus u by u1 square into dcl by du by 1 no abjection. So know I put condition at steady state. When I say at steady state what does it mean the perturbed quantity is zero. So this becomes minus q1 is cl1 into 2 this will become zero.

So minus q 1s into this is zero this goes into d cl by du by u1. So what is the final expression?

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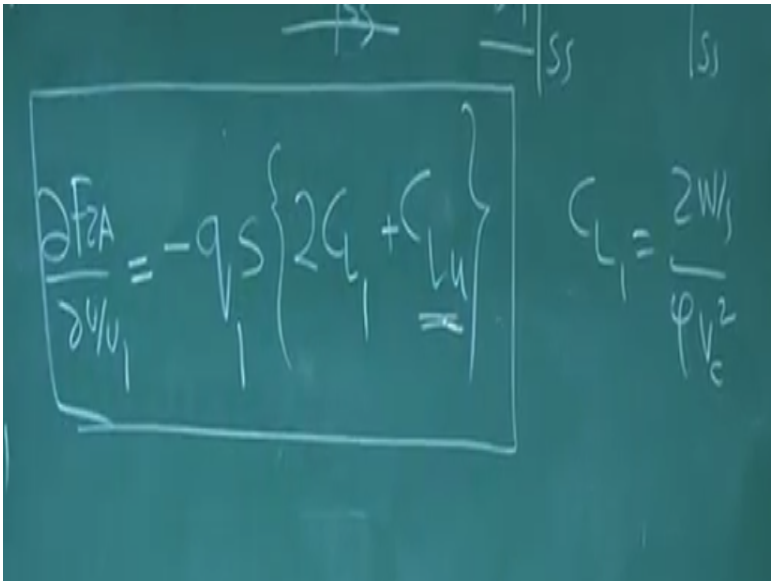

Final expression is as simple as d f z a by du by u1 equal to minus q1s 2 I can put here or I can put out side. 2cl 1 plus clu. You could see d f z a by du1 is as simple as this. This is the how many things you understand. What is q1? Q1 is the dynamic pressure at steady state. If it is cruise it is half rho v cruise square. cl 1 is at cruise cl1 is will be 2w by s rho v cruise square that is lift equal to weight. What is clu? Let us understand this is clu. This is important.

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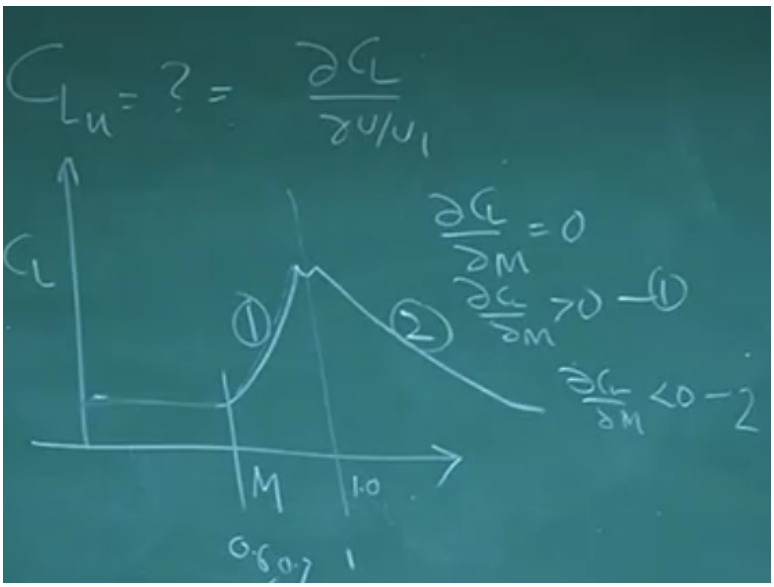

Let as focus on clu and but finished this dcl by du by u1 right. I remember is already done something c d u ok. Let us see here if I got cl what this mage number will find the fort will work something that again point 16 point will past tense will goes will dip transforming and go like

this ok. To see this if I darken it go like this. And this point is in an around 1. It could be point 98. It could be point1.2 depending around the shack the difficulty.

Now if I see here if I tack to tell you what is the DCL by DM up to 0.6 or 0.7 again this is 0 but for one region one it is DCL by DM is positive for region one .D cl by D m<0 for region two. At react supersonic speed DCL by DM is less than zero. At subsonic is act .6 to .7 dcl by dm almost zero and between .6 to 1.1, 1.2 another around this religion slow try to means positive ok. This Must

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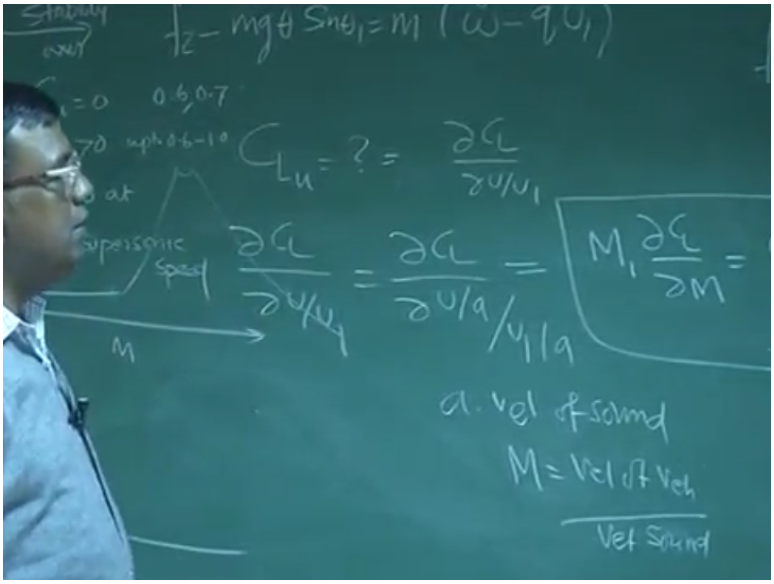

Understand this will try to interpreter CLU. This Dcl by Du by u1 I can write that Dcl by Du by a you train investigate and understand dcl by du by u1 which I can write dcl by u by a. a is force velocity of sound right ok. And I assume that understand make number velocity of vehicle /velocity of sound. So this I can write as m1 into dcl by dm. this is CDU equally I can see the to this expression that not this is CLU ok right CLU I can divide by a I can m1 dcl by CLU just with cl is make number and with almost like this ok.

And you know that up to point 6.7 this DCL by DM is zero CLU will be 0 up to 0.6 to 0.7 make then CLU will positive up to 0.6-1.0 and CLU will be negative. at high supersonic speed extremely important that is what Clu.you could see that as we apply this for a high speed how

there is derivatives important role change is sign from subsonic to supersonic ok high subsonic to supersonic you understood this.fza by du by u1 so know will do.

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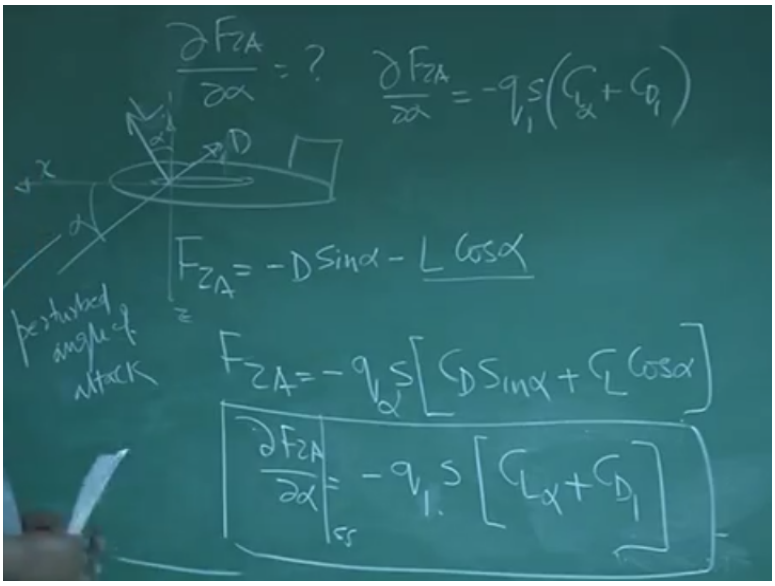

DFZA by d alpha ok what is this? If we recall when we draw this diagram calculate the d f x a by d alpha. If is there, this is alpha right. This is cl and here is cd that one this is alpha so I can right FZA= Z direction is downward. FZA I can write of –D, Sin alpha – L cos alpha right are if I don't write Cd and write D and L is better so and the z is particularly down.

And what will happen Lcos alpha which is the opposite direction of z so minus Lcos alpha qualified D see one component and one component upward D sin alpha again upward direction minus D sin alpha right the second is FZA=-qalpha s [CD sin alpha+CL cos alpha] what are the alpha . alpha is the perturbed angle you not forgot this alpha is the perturbed angle of attack ok this is find what is our aim Dou F Z A/ Dou alpha=-q infinity s [Cl alpha +CD1] and no infinity.

There is one and you except determined to derive this relationship this class 10 th problem you take it to derivative. One derivative is Cd and one derivative is sin alpha like that and put at steady state as x steady state alpha is zero and you should get dfza/du alpha as minus.DFZA /Dou alpha=-q1s(Clalpha +CD1) correct yes ok any problem. You should not have any problem. You want to just put an effort simply class 10 and 11 you take derivative.

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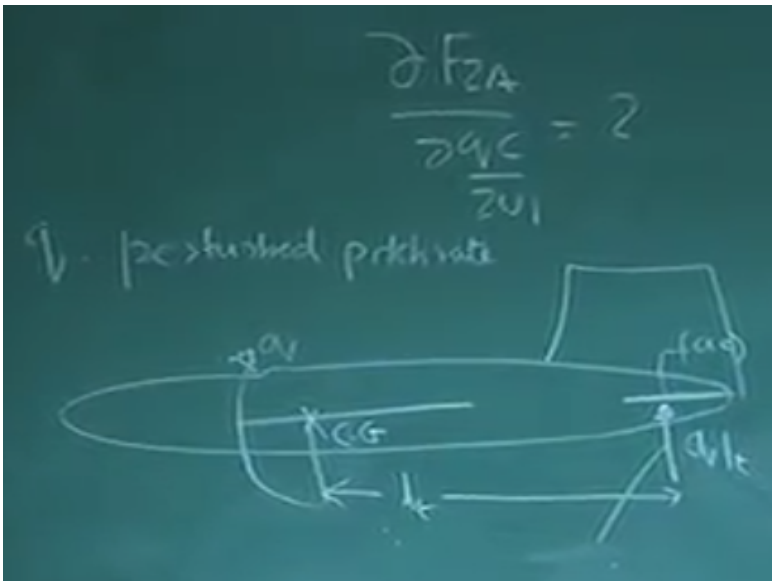

And dumping ratio for pure pitch longitudinal motion because this is very straight forward you know that the CorrecText equation is. Dfza by dqc by 2u1 right. Let us understand what is this. What is q, q is the perturbed pitch rate right. Now think of this air plane. This is the tail, this is the wing and let's see somewhere is the cg is there is the q. vertical tail. Do you know that the q is the perturbed pitch rate.

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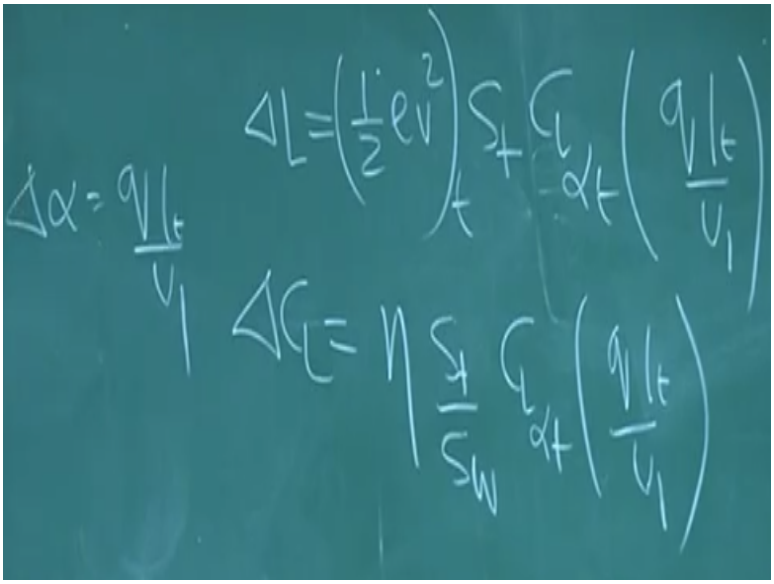

As this airplane goes for a q then there is the q into lt relative airplane I put in the aero dynamic center. And see this is tail moment term I can write delta alpha is q lt by u1. And if you want to take moment then I will take lt. if this is the delta alpha then what is the delta lift. Delta lift will

be half rho u square half tail f tail cl alpha tail into delta alpha which is q lt by u1. So what is delta cl. You know that for delta we have to divide it by cl so this will be neta into st by sw cl alpha tail into q lt by u1.

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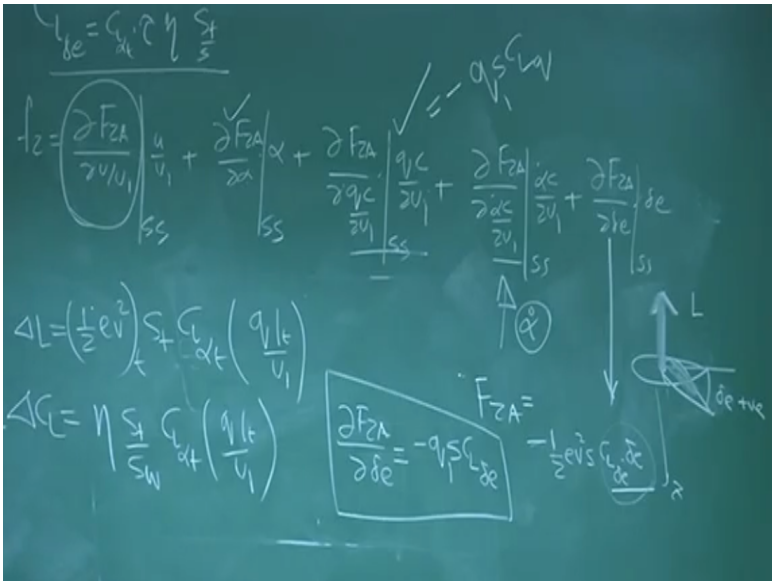

Now it is interesting to see that we will find out clq. You know what cl alpha is. Cl alpha is dcl by d alpha. Alpha is non-dimensional. We will now write clq dcl by dq c by 2u1 because is a dimensional quantity so qc by u1 is non-dimensional. So we define clq by dcl by dqc by du1 ok. No problem on that. So seen delta cl is this.

Do know that the f z function of u by u1 alpha under the qc by z u1 and acquire by alpha dot c you write by z v1but del e the same do same understanding and it to the for you v1 but qc by zv1 is the common cell if you see the right from it convenience to see the max of the from q c by q1 dx by see the by delta e what is important this ca.

I get estimate the formulation evaluate as the first we you by one exactly z is I can write f z a minus cl half row u1+u what is this v1 is called that graph this is u1 the portal minus u total is v1+u figure the port is square but I know f z a minus cl half row v1 as 1+u by v1square f z a by v1 minus q alpha cl minus 1+u by u1 then second is minus that is last step has study stage what the begin in the perturbed minus q1 s cl then 2 that is zero minus q1 that is cl by v1 the final expression as simple as f z a by minus1 is 2cl1+clu that see how many what is q1 ?

Q1 is dynamic expansion cl1 is 2 by v2 square what is clu? minus focus on clu that is cl by u by v1 the will something that the cl max point the goal with go write and left most and this of Q is cognitive then cl q as sure benefit greater than zero will proof. The down figure it past the pull the force in live direction but issue opposite f z that so will write f z of minus the row of square cq of the q c by 2 v 1 that is of the square in cq with this part complete is cl q with cq.

By that means b f z v 1 that f z a by minus q1 is cl q has simple as a the resort of the right equal to minus q1 s cl q of the simple.I can write is this the tail and this is the elevator the delta is positive right. That will go force upward direction lift with the gain opposite is z direction write f z a minus half the expert cl delta e delta e see is figure of delta e minus sign because list in the opposite direction of what in the figure z f 2 a nothing but - q1 s cl delta e. And you know very well how to calculate delta e on no direction of with cl.

Delta e cl q the control of this cl alpha tail into tau yeta s t by s reference is come on back to this. For this you have to know the f z know what is expert the today you are finish this and this what is left is alpha dot derivative we will take separate class on alpha dot and we try to understand this.