Aircraft Stability and Control
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Lecture – 42
Euler Angles

Good Morning. We are continuing on the series of equations on the motion. If you recall we derived this equations of motion.

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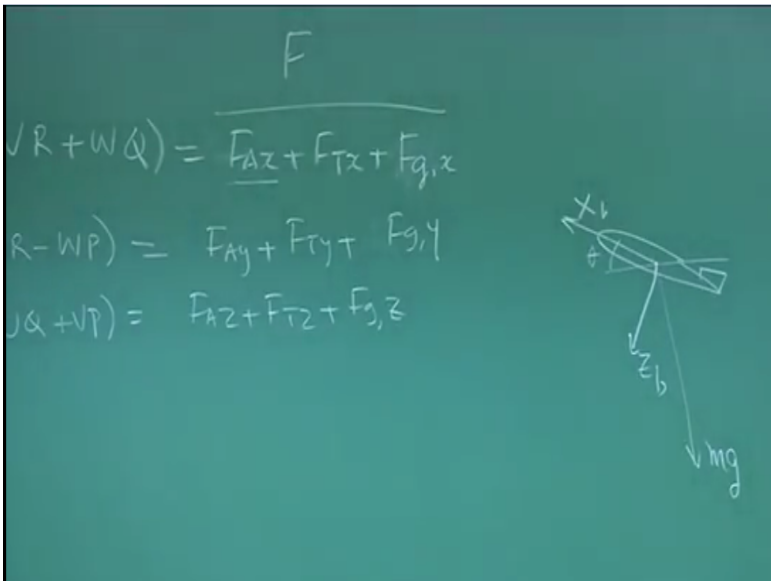

And here this force components, so to the force here what is the x component. So what is this X? X is the body axis X right. And we need to know what those forces are. This is aerodynamic force this is come because of the drag, because of lift. F T X is because of the propulsion or thrust and, the GX is the component of the weight of the gravity force okay. Similarly along Y, along Now the Question comes how do I define the orientation of the airplane?

Because I cannot use body fixed axis system. Because as the body rotates the body axis also rotates, so what is the way to define? There we try to define the orientation of body with respect to inertial frame.

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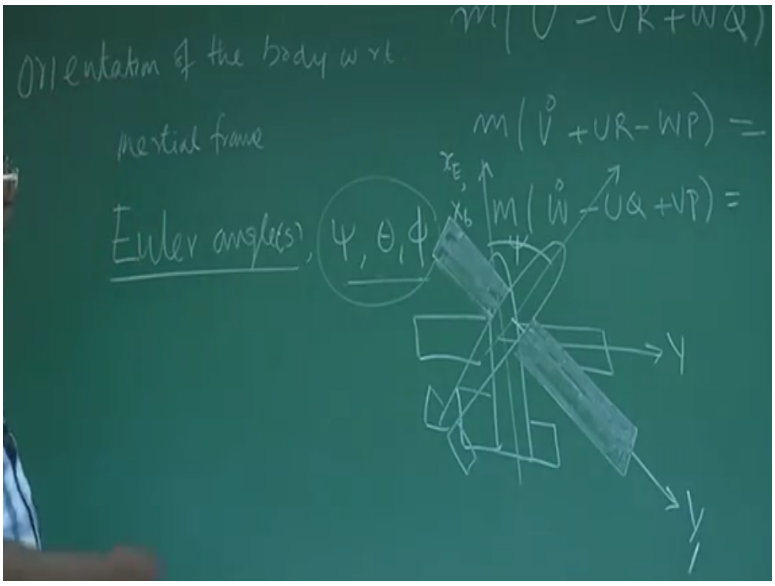

Orientation of the body with respect to inertial frame. We define Euler angle or Euler angles with chi, theta and phi. Please note that when you define Euler angle to locate a body in free space the order is very important. The order should be in terms of chi, theta and phi that is extremely important. For this part of the lecture of the course you take it as a statement, statement I am not going to deeply tell into it.

All these statements we are talking about our next course ten hour course on advanced aerodynamic stability. All ready we have covered so many things. But remember this Euler angle has to be in particular angle chi, theta and phi. Let us understand that chi means remember this is airplane and this is x as well as the x of the body have aligned at like this. And this is Y and, then Z is into the board.

Chi means right wing going like this. This is the chi rotation okay this is chi, chi rotation means what is happening now? As a rotate by like this, this wing and this becomes your new Y1 direction. Correct body fixed. Remember it is like this, and now it becomes like this. If I make it this will be more exotic for your understanding. This is second orientation this orientation is after giving chi rotation. Now what is happened this y becomes y1, this is the body fixed axis.

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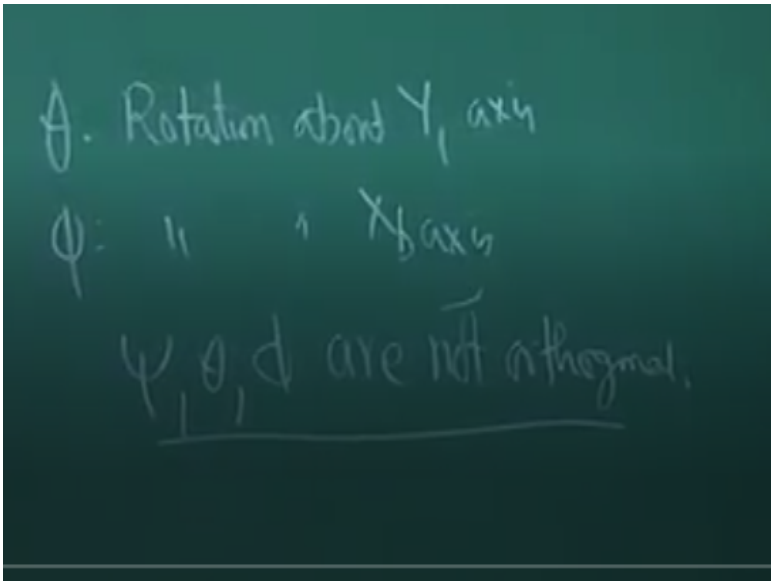

What is theta? Theta is rotation about y1 axis. So chi and theta is about y1 axis. And once the y1 action is given, now phi will be rotation about this x axis. So phi will be rotation about Xb axis. Is this part clear? Let me demonstrate this. Let us understand the physically what are the Euler angle. Assume that this is the airplane and this is the X axis of the airplane and, this is the inertia of the plane and, I rotate it by chi. the moment I do the chi Y axis will become like this.

Theta is what rotation about this axis? Now this is x body axis. So what is phi? About this axis clear I repeat again. This is rotation about z axis. So this your new Y axis. Rotation about Y axis is theta and new Y axis rotation about phi okay. This is the order of chi, theta, phi and you need to understand chi, theta, phi are not orthogonal okay. This is extremely important because we have seen that rotations are like this.

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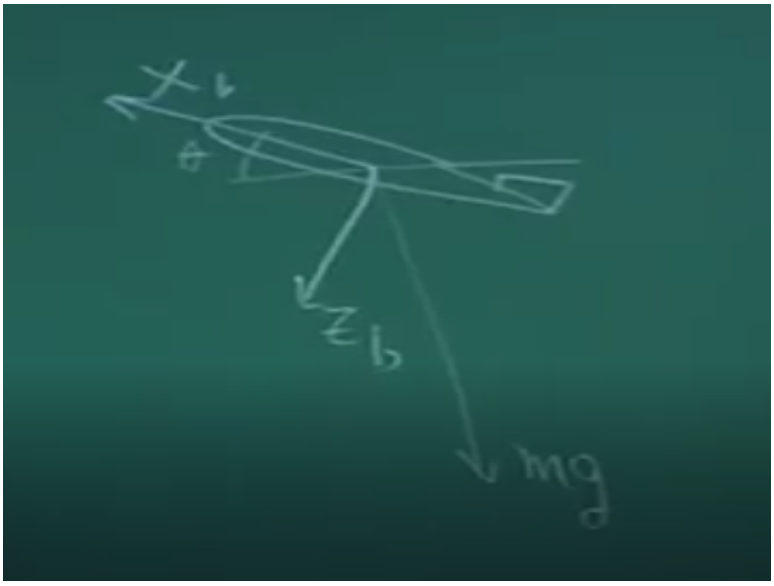

Why do we need chi, theta, and phi? Because I need to find the orientation of the airplane so that I can get angle of attack extra, extra okay. Orientation is important right. If I need to get the forces I need to know what is the orientation so that I can get the component of force. We need to know the orientation of airplane because we need to find out the forces. Along the local body fixed axis system okay. Let say for example if need to find out.

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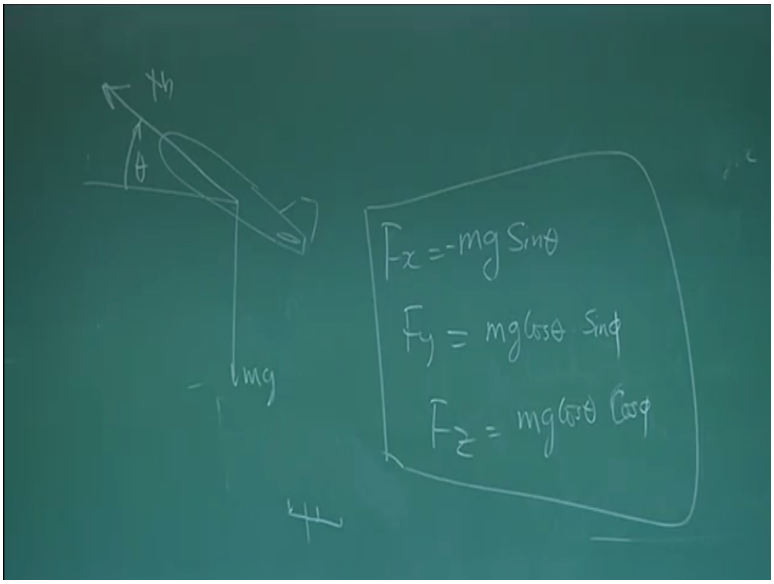

What is the component of gravity force along local body Xb and you know this is theta and this is nothing but mg. Then I can write FX = mg - sin theta. Similarly you can see for the given configuration and bank angle and I accept you to derive that yourself okay. And fz=mg cost theta cost phi okay.

Please do yourself by giving appropriate rotation. And now you know if phi is 0 there will be no FY. If phi 0 means there will be no Fz. Therefore fz will not depend upon bank angle FY. I need to explain this understanding to you so you can understand what is the final form. Do not forget we are developing this equation motion for the stability analysis**.**

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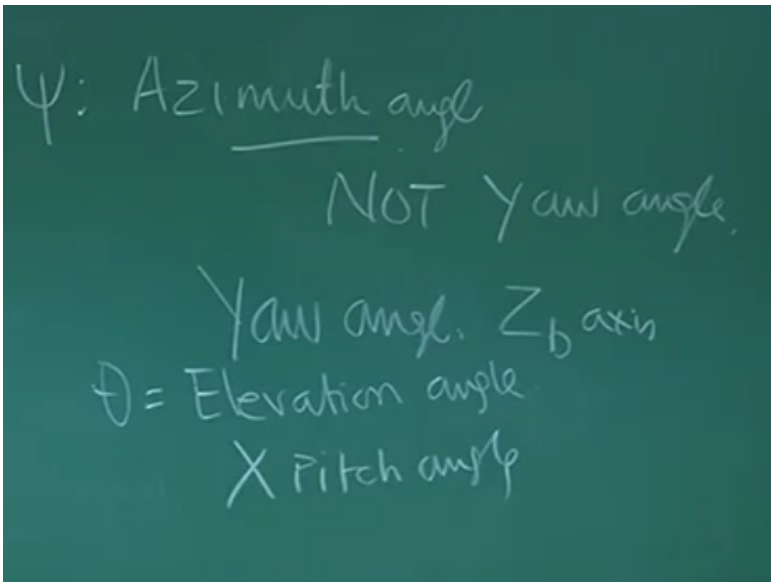

Please remember chi is called azimuth angle and not yaw angle. Strictly speaking Right. Because yaw angle actually talking about yaw angle will be about Zb axis. What about azimuth angle? Z about inertial angle. There are some certain things you should understand theta Euler theta is also called as elevation angle but not pitch angle and I am talking about Euler chi theta. Why? What is the pitch angle? Pitch angle is the rotation about body yb. What is theta? Theta where same Rotation about y1 axis.

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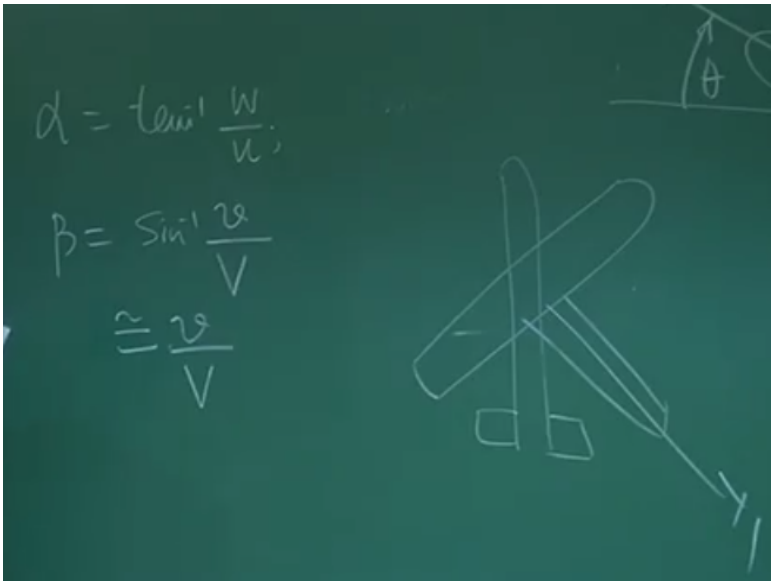

That is Plane was like this after this one angular chi rotation. Then the plane Y1 is this theta rotation is about y1 axis Euler theta rotation. But we talk about pitch angle it is the final. Whatever orientation is there y v axis about that is theta. Is something loose the use People often commit mistake and I thought I will tell you we are not going deep into this part. Understanding So that you can take advance dynamic stability 10 hour course. Where we will go 1by 1 into this thing.

It is also important to know what the angle of attack is**.** Alpha was tan inverse w by u. We can make out vertical and horizontal. And then beta we are using sin inverse lateral versus total velocity. Which often find for small angle it will be like this okay. To complete this set of equation of motion.

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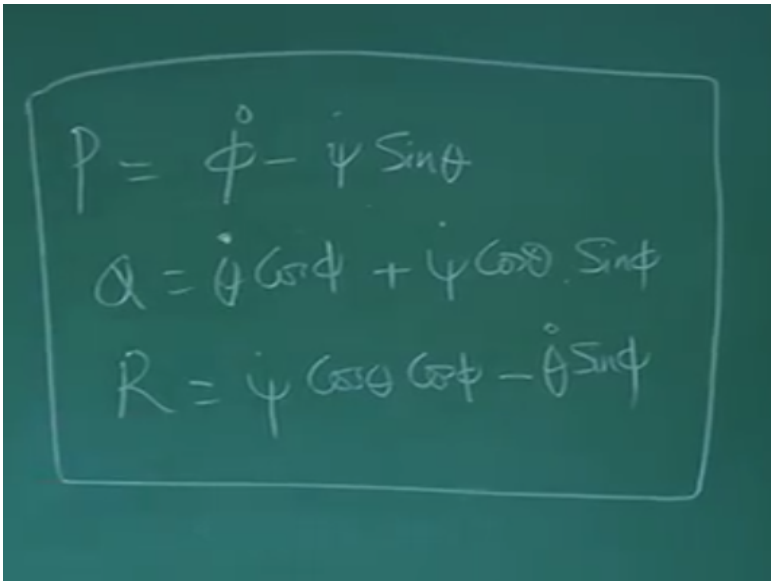

We need navigation equation also. That phi dot –chi dot son theta Q = theta dot+ chi cos theta and sin phi and R = xos theta cos phi - theta dot sin phi. To complete the whole equation of motion we also need to develop relationship between chi, phi dot, Q and etc. And the equations are like this. We are not deriving this but we are making physical interpretation of this. What is p Q R? P Q R are the component of the velocity of airplane fixed to the body.

These are local X Y Z direction. I repeat again P Q R are the component angle of the velocity of airplane fixed to the body right. They resolve along local X Y Z direction. And this we can derive little bit of more of mathematics rotation of axis system we can easily find this we can refer any book. Any way our next course will be only talking about these things. I am just keeping all this derivation.

Because our aim will be only to focus on dynamic stability pact with most derived u dot v dot w dot p dot q dot r dot equation. And among that in this course will be mostly doing the longitudinal equation of motion mostly perturbed equation. As you recall what we decided we develop the equation of motion and reduce perturbation and you see that how the perturbed quantities are varying. If they are coming back to 0 these are dynamically stable okay.

For the approach will be playing around the equation to develop feel.

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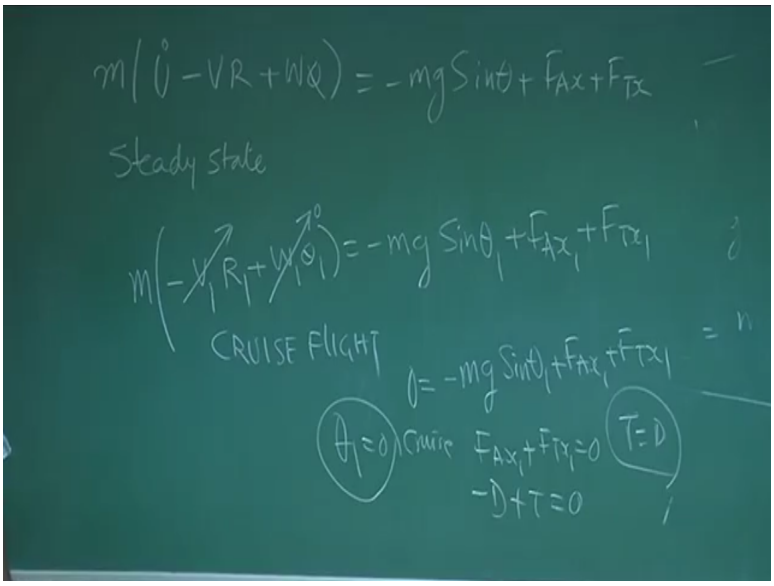

Let us m u dot – vr + wq =- mg sin theta +fax + f t x. When you talk about steady state equation of motion. If I am talking about steady state what will happen this will end up in u dot becomes 0 then – v1 R1 +w 1 o\theta 1 = - mg sin theta + f a x 1+ftx 1. Similarly I can write for other equations. If I see it here. For example if it is a cruise flight let say cruise what will happen in cruise what is the value of v1 or r1 this goes to 0 this goes to 0 there are no q1 no pitch rate.

So what I will get? 0 = -mg sin theta1 + Fax1 + FTX1. Now see here if theta 1 is 0 which is eventually true for cruise. Then what you get Fax1 and ftx1 = 0. That is nothing but –t = 0. This is nothing but t = D. You could see that from this equation I get simplifying the equation. Similarly you could see for the equations and try to remember what does this help? Now here if theta 1 is not 0 what we are getting let us see this.

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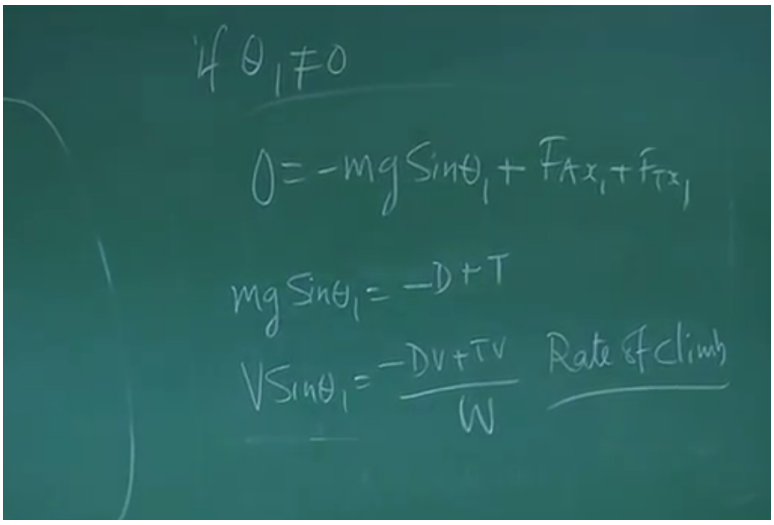

Suppose if theta 1 not = 0 what happens here the 0 = - mg sin theta 1+ Fax1 +FTX1ok. For now I write this as mg sin theta 1= - D + T and th rust then I write V sin theta 1= -dv + TV by w from this I can get all the equation. Rate of climb versus power. We are getting all those equation by simplifying it. Like that you can go and checking that thing before I mentioning things we go for a dynamic stability analysis okay. Note the equation and try to develop.

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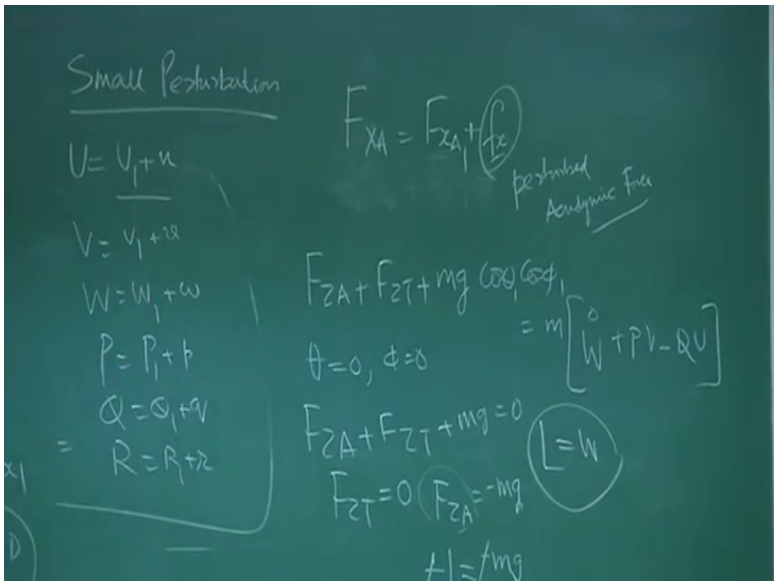

This another equation F z a + F z t -mg cost theta cost phi =m w not + p v- Qu. Theta =0 and phi =0 to more precise I should use an notation 1 steady state. Then what will happen fza +Fzt + mg =0. What is the value of fzt? Fzt means component of thrust along z direction is 0 then we have fza =- mg then you get a relation lift = mg. so you get this popular equation lift = weight. This is the way I thought I will give you the example so give the feel.

This part of the lecture was almost a discussion I just prepare you for dynamic stability analysis. Do not forget. What is the approach will be when I am going for interaction we will talk about small perturbation. Your understanding is that I can linearly add that we have the perturbed velocity to the steady state quantity to get total quantity. Then you will get the total quantity U= u1+u. So this linearity is allowed also V=v1+v and W= w1+w. so what are the u1, v1 and ww1 they are the quantities at the steady state.

That is the value of v1, w1 is at cruise flight like then P=p1+p, Q=q1+q and R=r1+r. that is talking about small perturbed that is neglected amount of perturbed quantity will be neglected we can left it from the analysis this also we have. This is one and second thing you must understand the aerodynamic force fxa I will write as FXA1+ fx similarly for fz it is small fz. What is the fx? What is it is the perturbed aerodynamic force.

And this will see that fx will be function of alpha, perturbed alpha, perturbed pitch ray perturbed delta e like that. And then you see how beautifully we are using the same concept of Euler series and linearity try to get those derivatives and completely estimate the model and functionality of perturbed forces in terms of perturbed motion variables okay. Then you will see how do we handle and get the characteristics equation and get the roots and negative and defines meaning positive different meaning dynamic stability of an airplane. Thank you very much.