**Aircraft Stability and Control**

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**Lecture – 41**
**Vector in a Rotating frame**

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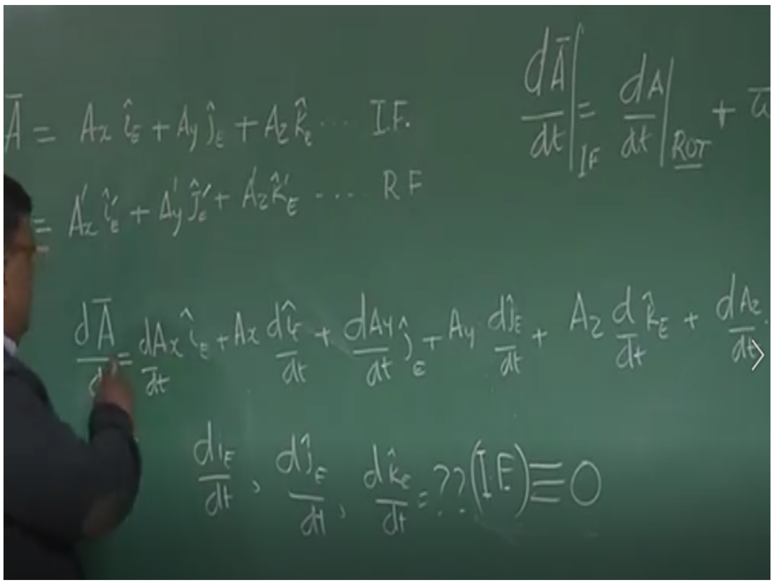

So now Let us try to understand this, we will be deriving this, many of you might have already done it yourself, but I will prefer that we will go back to this derivation and that will help in, I F means. When I write IF the initial frame and ROT means rotational frame. For our problem, which we are solving writing equation of motion, this rotating frame is the body fixed axis, which is also rotating with the body, with the body fixed. Initial frame means it is earth fixed initial frame, okay.

So the definition is that initial frame I should try to understand that unit vectors i, j, k are not changing, the magnitude is one, direction is not changing it's fixed, but for rotating frame the i, j, k as the body rotates they also rotate. So the direction is changing, all the magnitude is not changing, its unit. But the direction is changing so this vector is changing okay. That is how it doesn't remain initial frame of motion. So how that is to be developed, and let us see.

Let me define a vector t. I can define this vector both in initial frame and rotating frame because both are heavy axis system. So for e in inertial frame I write as a x i e + ay je +  a z k e okay. So Let us dot, dot, dot, this is define inertial frame. I can define this vector frame I define this in rotating frame. So in rotating frame the scaleur component of a will be different now.

So this is I said a x prime e I put the prime right, the moment I put prime is this is referred to rotating frame + ay prime je + a z prime k prime. Please understand I should not slip this prime. Once I said prime means this is defined in rotating frame r f rotating frame. Clear okay.

Now Let us take this a. Let us talk about inertial frame, I am doing derivative in inertial frame, so da/dt equal to in general what it will be, if I write mechanically the vector definition. So this will be d x by dt into I e + a x + d I e by dt. Similarly second term will be da y by dt into j cap earth frame. So I must write e earth + ay into d j earth by dt + similarly a z d by dt of k + da z by dt into k. Let me repeat dx/dt, i vector a s this is find dy/dt je y d find z d k e by dt + da z / dt I all did in first now what is important.

What is value of dI/dt and d je / dt and d k prime/ dt what is the value of that because you are now computing this as the inertial frame we should not forget this is in the inertial frame. What we do in the inertial frame it earth fixed and the unit vectors along this directions y z directions are not changing in direction and in magnitude. What is magnitude vector and it is the fixed its not rotating we assumed that earth as non-rotating platform for our purpose.

So these derivatives since they are not changing this is equal to 0 in inertial frame. Correct, any doubt?

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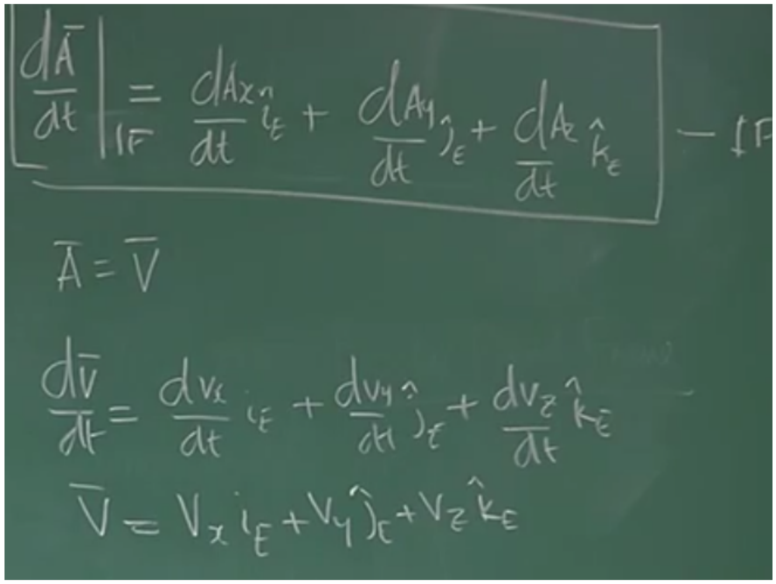


So this is 0. What will happen the da / dt was what? Now da / dt let me write here. So that I can use it appropriately. So da / dt in initial frame is as simple as d x / dt i e + da y/ dt je + da z / dt k e as simple as that. This is derivative vector in the inertial frame, it is nothing great. You have been all doing in class 9th, 10th. I am talking about every problem have solved in inertial frame. So what is that if I say a is v then what it is mean the d v / dt is d v x is component into I e + d v y / dt je + d v z / dt and k e but the v as component.

V as component and v x and I e + v y je + v z k e so what that v as we use it and v x in the component of v in the inertial frame x component v x and v y and the v z component is z so what will be the acceleration. That is because it is the initial frame. So along x acceleration will be d v x/ dt along y the d v y / dt along z will be d v z/ dt correct it is will be very, very clear now and the initial frames no problem. Now let us see derivative in rotating frame, because we want to work in rotating frame, okay.

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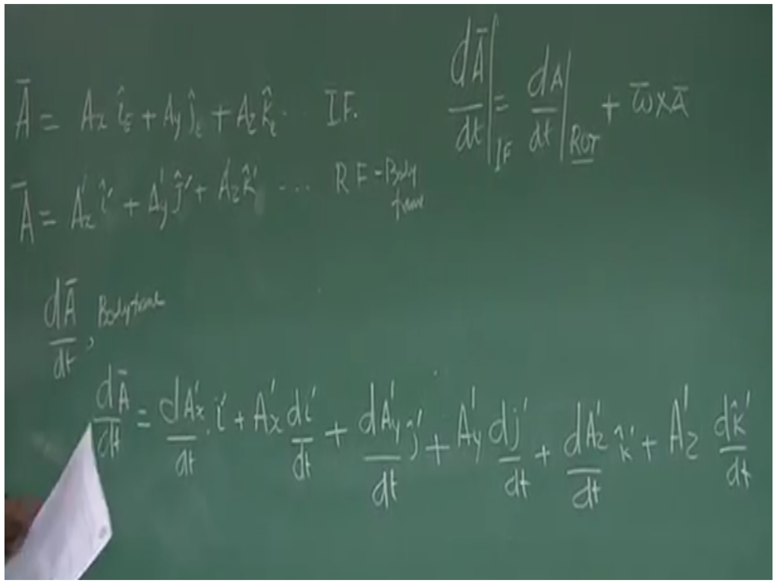


Let me start from the beginning, so that there is no confusion. Before I do that please understand that a is defined as well as initial frame c m a vector a is same as well as we defined in putting the rotating frame we doesn’t change whatever frame we will put the vector is same ok. So I know what in the getting frame . the rotating frame is our case is the our body frame is example of rotating frame because the as well as because the body rotating frame as well as body fixed frame it also rotate. Let us see how do the different the da by dt using the body frame.

The body frame is this d/ dt a bar it will be equal to see here will be da x prime by dt into I prime. What is the I prime? I prime is the unit vector in the body frame oh this is should not be e please correct it this is wrong it is not a at frame ok it is a body frame I repeat here. Please see I written a as I like this a initial of the e was there so I am using that because this is between the of the rotating frame in the body frame.

Sorry for this confusion don’t worry better some time write some time wrong and that stress is your mind I know you learn better so do a connection when I am defining a vector in body frame the what in the frame there are a scalar components and the unit vectors are different now ok. The unit vectors is define as the I prime and j prime and k prime write the components a x prime and the ay prime and the a z prime correct.

Now I am differentiating so i get d x  and  dt I prime + a x prime d I prime / dt initial you know that d I prime / dt will not be 0 because what we frame and the body rotates frame rotates so the direction of unit rotates so derive frame the dt prime not be 0 vector is a different vector if be the different direction. Ok. What is the next step, next will be.

da y prime / dt into j prime + y prime into d j prime / dt the similarly the third term will be da z prime / dt k prime + a z prime d k prime / dt all terms are here will be correct this I e je k e in the art fixen the I prime j prime k prime in inertial vector in the body frame the initial I wrote the  I e je k e I swill be  wrong absolutely ok.

A vector are different ok see you know that now I cannot d I prime / dt r and d j prime d z prime d k prime  is equal to 0 because the derivation is changing difference is changing  the question is I can I find out this vector ok.

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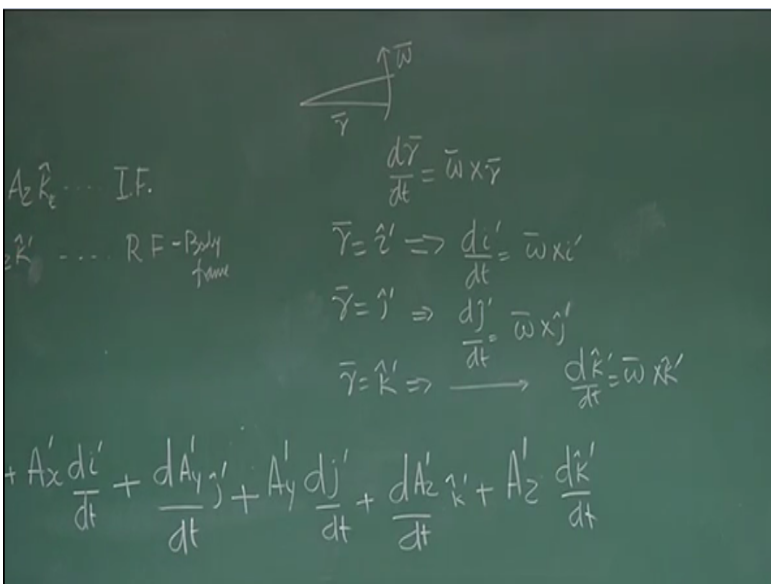

This Is the circular motion this is the r vector.This will be rotating with the omega. Then we know that d r / dt is omega cross r very simple is it but it is  the vector. What is will on omega city of the  rotated  vector ok so now for r by write I prime ok this is initiator in the body frame of  reference then what will be happen is in then d I prime/ dt equal to omega cross I prime it so simple  so r if I write I prime it inertial unit vector in the body frame of the reference  then what will happen it will be  simply d I prime/ dt is equal to omega cross I prime so simple.

If I put r bar equal to j prime is impale  d j prime / dt is equal to omega cross j prime similarly  I put r is equal to k prime is it be d k prime / dt is equal to omega cross k prime so simple way of find out. The d I prime and dt prime  d j prime / dt and d k prime / dt ok now what is solution?

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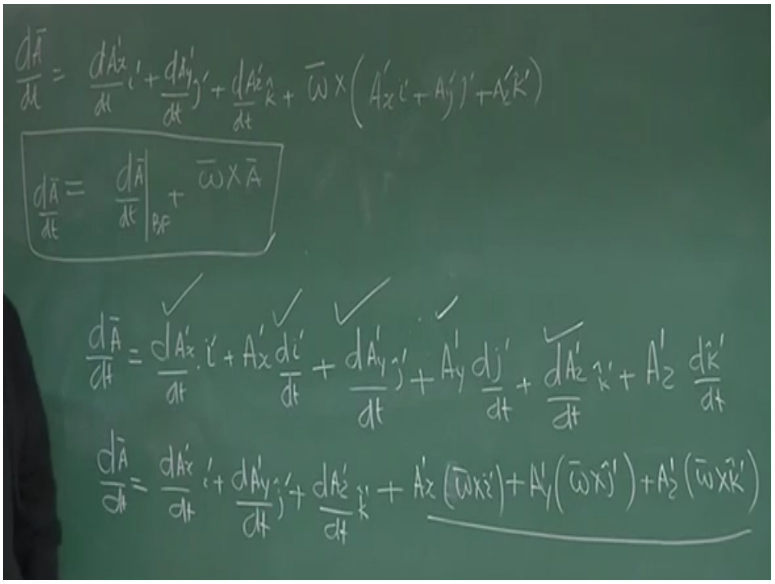

Is in it put it here so da/ dt is equal to da x/ dt so I write it  da x/ dt I prime  da prime y and I do some rearrangement so j prime similarly d z prime / dt k prime and what is left this of taken here this of taken here this of taken here what is left this is a x prime d I prime/ dt and d I prime / dt is omega cross so this is omega cross I prime similarly this of taken here for this ay prime the d j prime and d j is  what the  omega cross j  and this is omega cross j prime. similarly for z I will get a z prime to omega  cross k prime last the this term .

This term I can easily write  the complete term so that you understand better in the this part also and that will understand.

what will doing and the what will getting  da / dt is equal to da x prime by dt ineo I prime + da y prime by dt into j prime + dAz prime by dt into k prime + for this I can like this omega cross Ax prime i prime + Ay into j prime + Az k prime right dismally  I just the common so this is what is da prime by dt and day prime by dt daz prime by dt .

What it is? It is don't worry about rotation. Rotation will be taken this will be gentleman. You see what is you're your component of the vector in x direction, Local x direction, what you x direction, find that simply what you by find that so you are operating in the body frame now ok. Because x prime, y prime, z prime is the exceptive the body axis right. So this is alright.

I write da by dt in the body frame and very simply you know + omega cross A this is equal to your dA by dt. You may have a question what you take the omega carbon so in this cash when I route this you see Ax prime, Ay prime, AZ prime ok. So what you have understand to this very simple now.

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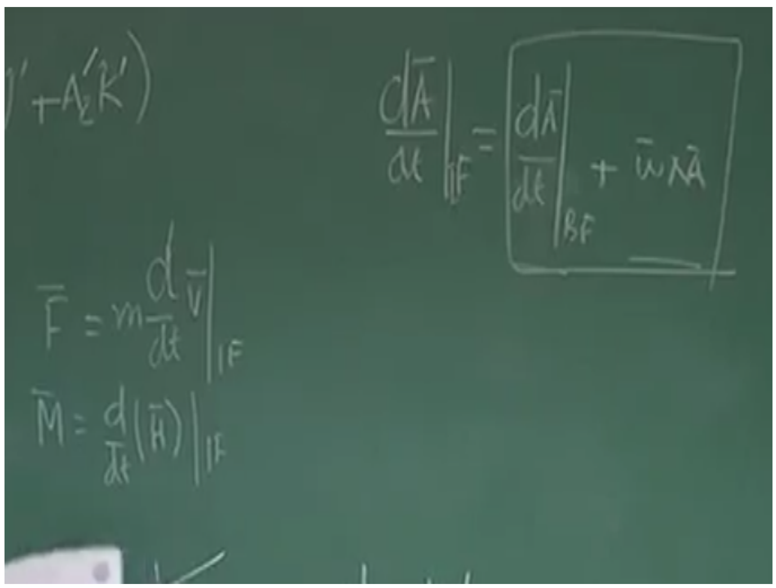

That da by dt this is got to initial frame is equality I can work in body frame then body frame of add to omega cross to A. so in our problem we have vector differentiation is demanded if is add in initial frame what weather differentiation what was d by dt of v, m dived by dt .

That was first external and we should be initial frame we got equal of course equal sum m dot is 0. Simply you know for M we should be d by dt(h) all the moment of initial should frame for this string there is should be innocent frame all equality you are in the body frame and add this ok. So that is we work in body frame and the same time would not violate the condition for neutrals that we should be inexceptive inexpansion  ok for now this understand thing will complete.

First part of derivation of equation of motion ok. So now will be use in this to complete the derivation of equation of motion.

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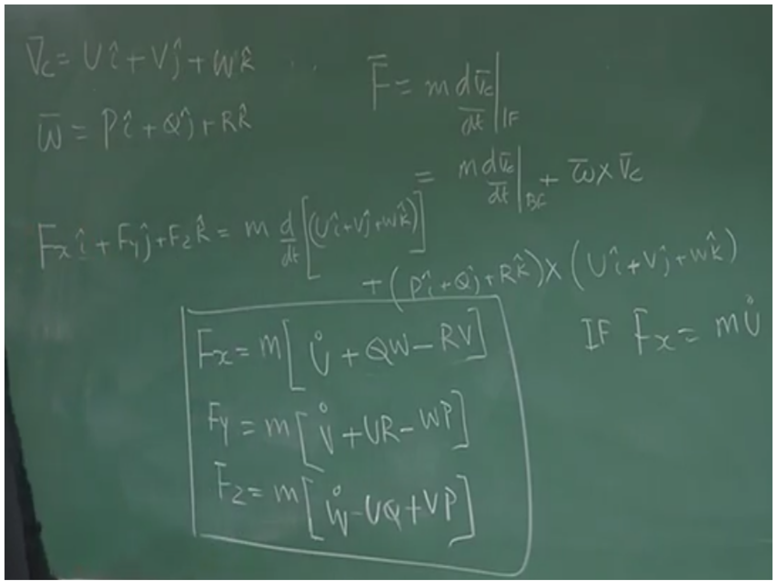


What we wanted was F equal to m dvc by dt in initial frame and now we know that. That is equivalences in m dvc by dt in body frame or rotating frame and comma omega cross vc for this equivalent  to working in body frame right. It is the initial frame we are not expecting in this. What is VC please understand  in body frame then + vj in the body frame wk in body frame we working in body frame please understand this right.

And what is omega before we come to omega what is the meaning of u v w it is the x component of the vc which is by to the expect in initial frame but the X component it always local body direction body fix axis I direction so learning for v and w. Now it is omega equal to Pi + Qj + Rk adapt pqr. PQR the component of omega and omega of expect to which from initial frame. But PQR the component of that omega into is alpine to local xyz body fix axis codeines all axis system.

So now if I do this but I get fx i + fy j + f z k will be equal to this m d by dt of this is ui + vj + wk this + omega Pi + Qj + Rk omega cross v c. V c again U i + V j + Wk. It is very simple form. You find lets axis side you can easily right from here by equating in x components and y components fx as m u dot p + qw minus rv. fy as m v dot + u r minus w p ok. And fz as m w dot minus uq + vp. This is beauty your now allow to work in the body frame.

C from here if you want the initial frame suppose we want the initial frame fx is simply m u dot this term will log come if the initial frame this term are coming because of this omega cross a right because of a try to correct for not to compounding in the relational frame by omega cross a that vector of a cross what about we are angler movement from c. I repeat here what are u v w.

The u v w are the air directive speed resolve in to hole body fixed access coordinate in to what is v. v is the v c is the velocity of the vector which respective initial frame ok got it that convergences all this written in the local body quaintest system like we wrote a vector both in

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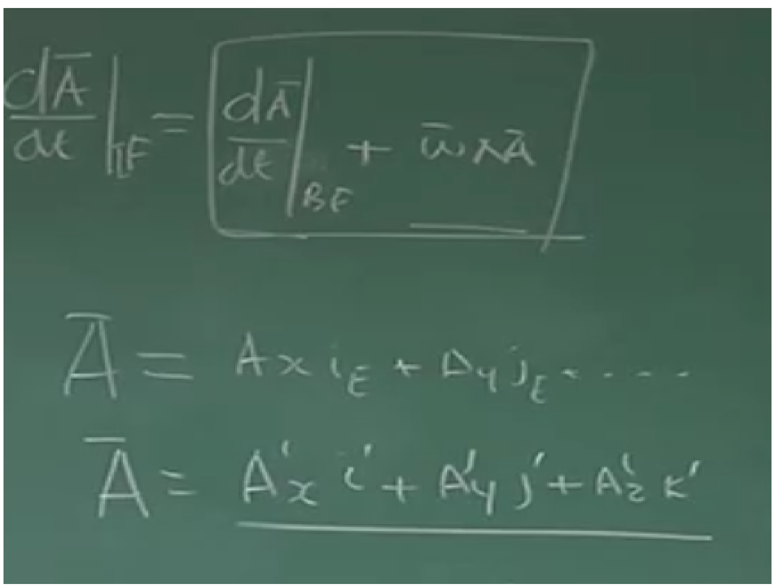

What we did a wrote a in a x I + ay je just like that similarly like that a x prime I + ay prime y prime + a z prime. So this is what will derive write think that vector wish you as prime in the verity exceptive the local frame now you could see the censer I the aircraft I can easily get the u v w because now it a body directive the aerodynamic force is will be easily completely because now the u v w we getting the alternative right the airspeed and the especial what is fx and fy and f z we are discuss about that the fx. fy, and f z.

Let as complete the movement equation part ok the another equation is waiting for as to be handle let as handle that, let as handle complete the movement equation and be following the same help d u d v assume frame da dt was a rotating frame and body frame of is so omega cross.

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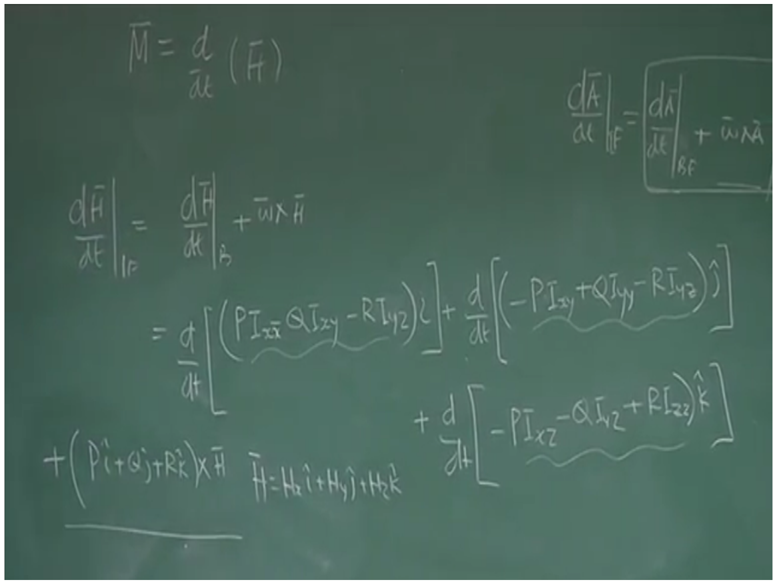

So we know need to calculate m as d/dt(h) angular movement of vector .so I lose this dh / dt in initial frame will be equal to I find dh / dt . and dh/ dt in the body frame + omega cross h and you know h vector as to be expressed in body frame you know the component of h a h so I specify hd / dt or of it me write p I x minuses q I x y minuses r y z and this is now I that is in the body frame .

This is first term and the second term will be d/ dt of what is this turn this is minuses p I x y + q I y minuses r I y z this sis the elevating body geo correction so the third term will be d/ dt minuses p I x z minuses q I y z + r I z k prime it will be right r I y z so many exclusion and derives be care full that why I reap it

Once life time is should like this do this and it should be where this can avoid the really want to good go to dynamic generation rotation by we want double spirit p I x for the I x + the I y write that I y y and I z write I z z so third is minuses p I x z minuses q I y z and r I z z fantastic term this is the first term omega cross h + you know p I + q j + r k omega and x also you know which as this expressions .

write so h and the expression of h and q is here and I know this is the and it simply derivative and expand it do not forget this term important h it will be write which as h h x I + h y j  + h  z k prime and this is nothing but h s  the nothing but  h y and this is h z we can put It here  l this is and to this operation and then we will get Sixth equation will see what are the equations once you do this part already

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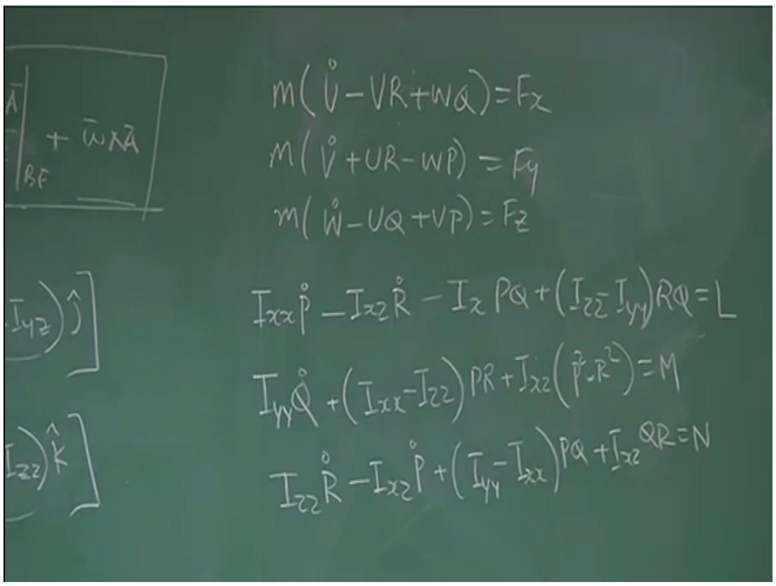


To be seen m u dot minuses u r + w q is equal to fx and you know m v dot + u r minuses w p is equal to fy then m w dot minuses u q + v p is equal to f z if you do this operation and do the I x x p dot minuses I x  z r dot so without the fxfy and f z  right here from once set of equation and need to do this cooperation we will got equation like this I x z r dot minuses x p q + I z z minuses I y y into r q  the component of  movement.

Because remember this is data change of will be the movement term and that is equal to movement ok and the cosign the data   the angular movement term and this m that the component miss equal to l + m j + m k what is the l. the l is about the x axes the rolling movement .This the rolling movement we go detail about it that percent you know m is understand  the pitching movement about y axes that the c j y here and n is whole movement ok .

Which is about z axes so If I take l m n and compare there competent will get this starter the equation this l put it l movement when we will get I y y q dot of be write because every possibility it I z z p r + x z of p square minuses r square ok is equal to the pitching movement and the third one is I z z r dot minuses I x z p dot + I y y minuses I x x and p q + I x z q r is equal to neural movement .

Finally we got 1, 2, 3, 4, 5, 6 and six of z of equations and you know the three for translator motion and the three are angular the motion you can see is rolling about x axes q is pitch rate about pitching about the y axes and are the z axes so we are got the we need to know what are the fxfy and f z what are this l m n how to the very man in aircraft is either in touch in impassion that also external condition and both will able to just is are stable and analyses before the doing that the missed class.

what I do the align start from here explain the physical mean of term and you know that completing f I fx f z are the movement we need to angle of that the size so angle between the angle that the vector and the code line of the access airplane that is extremely important to know we find some equations and I can see how the module access intention of the airplane because the access will be the body was access to the how to define the oration so that you can see how do I module the access oration of the airplane.

Because the access is fixed the body so the body was access to define the ordination so that ordination we have to do which respect to rational frame ok and there is catching the and you need to understand and settled deprave that the expectations reservation you know but vector by small let of change the vector to the other vector so there is will them complexity so over roll will be to go slowly and clear for this doubts so that we can want to implement the this your master of this equations right. Thank you very much friends.