Aircraft Stability and Control
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Lecture- 36
Point Mass Equation of Motion

After hectic section, now the time has come and when we should relax and try to see what we have been discussing all through. So what I decided today? Let me go through quickly whatever we have done so for.

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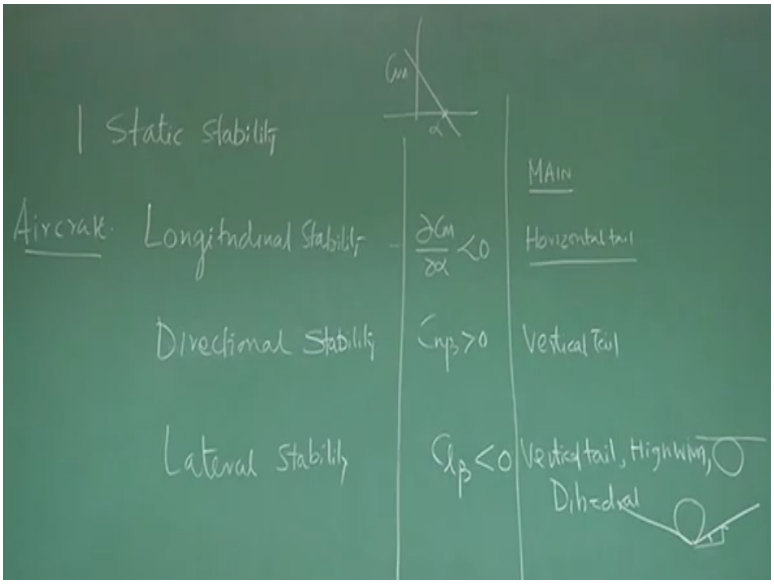

If you see we started with definition of static stability, now when you are discussing about static stability, we have very well understood that it is the initial tendency of the body to come back to equilibrium while it is disturb from the equilibrium right. So far aircraft we were discussing about longitudinal stability, we have discussed about lateral stability, we have discussed about or let me write this here the sequence we have done directional first directional stability then we have lateral Stability.

All are static stability right. Let me write here stability that makes things look better. Now ask our self a question what was the condition for longitudinal static stability. We know that from CM vs Alpha graph we know that if the aircraft has to be stable at Alpha trim which is here CM is 0 then the condition was DCM by D Alpha less than 0. Similarly for the directional stability

we realized if an airplane is going straight like this and because the cross wing Beta introduced, so the airplane should have initial tendency to nullify that, so it will turn the airplane are generating Yawing moment positive.

So it said C and Beta greater than 0.And for lateral stability what you understood was the airplane is going like this because, of some disturbance it has bang like this, so if it is laterally statically stable if you generate a moment which will it produce a negative disturbance is like this should generate negative CL, negative rolling moment right. And how that mechanism was generated. Are how this mechanism works. It is as the airplane bangs it starts side slipping.

And when it side slips when the vertical tail family that gives the rolling moments, of course you can get it from the wing also little bit. So the condition was for a positive Beta it should generate negative rolling moment. That is if there is a bank like this it induces positive Beta, and if it has to come back to it generate negative CL because CL is positive right wing going down okay. So we say CL Beta should be less than 0, and who are the main contributors towards these conditions.

If you see first CM Alpha it is the horizontal tail main contributor. Okay. Please understand the QR is main contributor. Then for CN Beta it was vertical tail, and for CL Beta also we have seen it is a vertical tail and also we have seen High wing okay, and also we have seen if we have given some Dihedral to the wing. High wing you understand high wing is this wing is here and dihedral you understand that I am giving some angle like these and this angle is just a Gamma or Dihedral angle.

So you understand what are the conditions, you also understand who are the main contributors? when I say DCM by D Alpha less than 0, the main contributor for stability or static stability Horizontal tail wing also contribute towards stability, if the aero dynamic center of the wing is the head of CG of the airplane it is De stabilizing, if is aircraft CG you know it is stabilizing similarly for fuselage is here is mostly De stabilizing and CN Beta also gets from fuselage.

Which is also again destabilizing, but vertical tail next is stabilizing right? wing also if you give a sweep it will have stabilizing effect, if I summarize this I write CM Alpha, because of wing CM Alpha because of fuselage CM Alpha, because of Horizontal tail if you see this 3 derivatives 1 thing I take guarantee, that this will be less than 0, this will always stabilizing.

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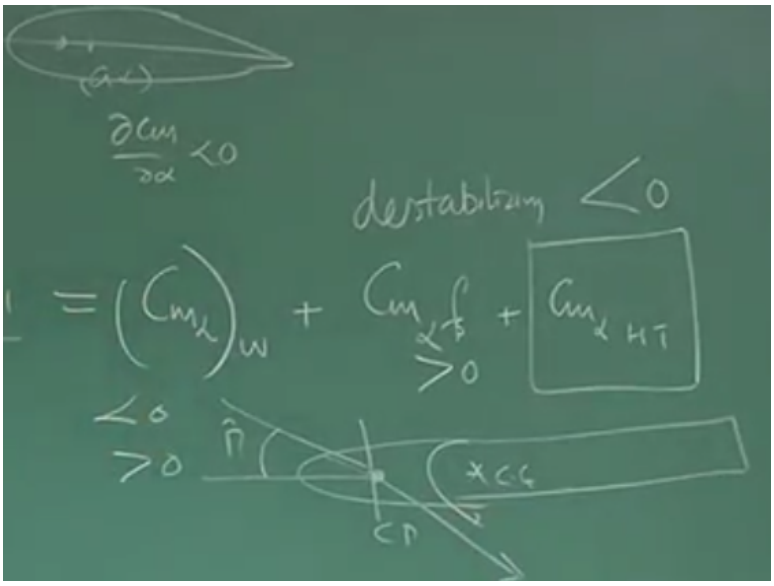

As well as fuselage is concerned, you can very well see fuselage typical shape is like this and its center pressure will be somewhere here, and CG will be some were here, so it is always destabilizing, or I can write CM Alpha greater than 0, and for wing we know if AC of the wing is behind, CG of the airplane then it is then DCM by D Alpha is less than 0, or wing contribution is stabilizing if AC of awing is a head of CG it is destabilizing.

So it could be either less than 0, it could be either greater than 0, depending upon where is the AC location and relation to the center of gravity of the airplane. No issue about this okay.

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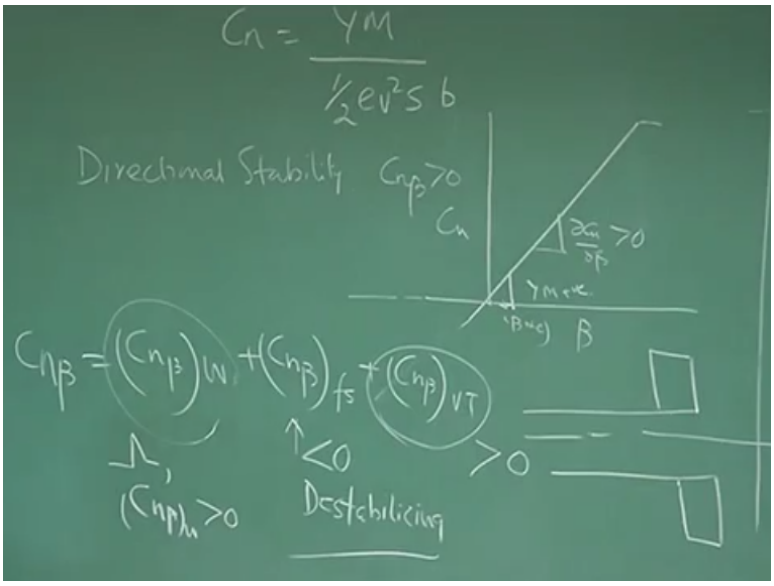

Now coming back to the vertical tail or CN Beta what was CN Beta what was CN. CN was a yawing moment non dimensional by half row V square S into B B is the Bank. And what we have seen for directional stability was directional stability CN Beta should be greater than 0.If I plot CN verses Beta slope will be like this, and this is DCN by D Beta should be greater than 0. You could see from here Beta = 0 is the trim, because we want to fly at Beta = 0 almost all the time except 1 or 2 occasions.

During landing or some time we want to reduce speed by increasing the drag, it go like this but take it for understanding that we want to fly at Beta 0. No module I have to fly like this always want to fly like this. So from here you could like this if by disturbance Beta positive is introduced it will automatically generate a yawing moment positive and what is yawing moment positive? It is the right wing going back. So this will make the Beta to come back to Beta = 0.

So it will have initial tendency to come back to 0. So this is the understanding of the directional stability. And again you should know the directional stability contributors are CN Beta wing, then CN Beta fuselage and CN Beta vertical tail right? So once I have seen this vertical tail, if I put vertical tail I can guarantee that CN Beta will became greater than 0 that is if I put a vertical tail on a fuselage whether this way or whether I put the vertical tail like this, that both the case CN Beta will be greater than 0.

There 2 different configurations okay. Sometime we put the vertical tail up sometime we put vertical tail down. Whatever this combination is CN Beta will be always greater than 0 rights. then you have CN Beta fuselage based on this understanding, if there is a Beta then you could see here, if there is the positive Beta, this will give force in this direction and that will give a moment which will take left wing backward.

That is if I am moving like this, force is here that will take the left wing towards back, then the CN Beta is less than 0 so, this fuselage CN Beta is less than 0. That is destabilizing because for directional stability, I want CN Beta should be greater than 0 for wing we have seen, if I introduce sweep angle then CN Beta because of wing will also greater than 0. It is stabilizing and this is the interesting part many times we will be fine suddenly there was a hype that the vertical tail less airplane Right.

Type airplane without a vertical tail because there are some configurations, which are for military purpose, you do not want to key vertical surface because, that increases the radar cross section area. Radar knows it can be trace by radar. So better to not have a vertical tail so that you reducing the radar cross sectional area RCS or radar cross sectional surface. So people will tell oh it is great job you are doing there the vertical tail, so directionally it will not be stable. Because directional stability is given by vertical tail primarily.

But you could see that, if you give a sweep you can get it from wing also so nothing very big claim you can get CM Beta positive.

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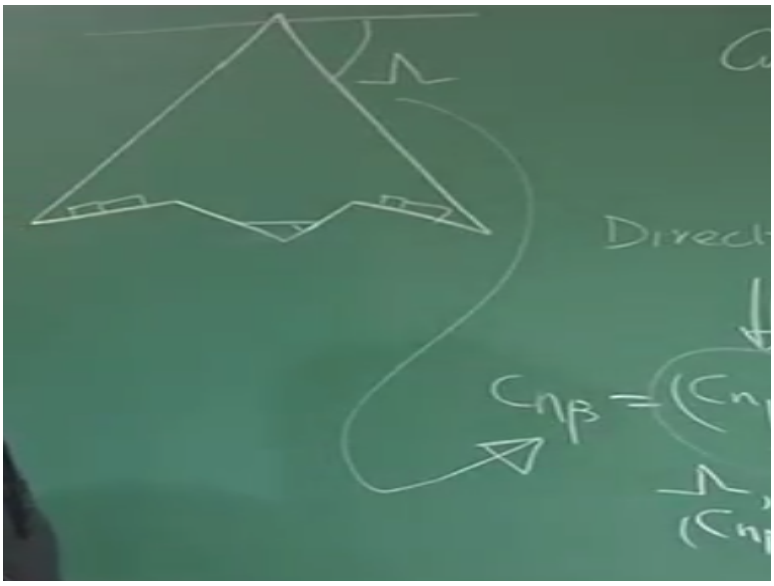

You can stabilize directionally by making the wing having a sweep that is why vertical tail less airplane whatever we might have seen, stealth aircraft B 2 bomber type, we have no vertical tail there are control source is here, here also something is there right. It's does not know the vertical tail. But it has the large sweep and this sweep we are demonstrated to the expression this sweep gives CN Beta, greater than 0, So because of this sweep you are having directional stability in term static stability is concerned.

So nothing to hype about it, there is a vertical tail so how we will get a directional stability okay Right. so this was the discussion we had a lot of expression are there so, what I have decided in 1 of the forum I will also pasting those derivations step by step, so that you can do your home work you can re visit those expressions, and derive yourself right. Sometime extremely difficult to continue deriving expression here in a this sort of a mode of interaction. But I do not know understand how much you getting into it right.

Where to stress and where not to stress especially on derivations part so 1 of the blog you will find all the derivatives will be given right? And you can ask question based on those steps okay. This we have done and why we have done all these things that is a question first of all final aim is to also check whether the airplane, whatever we are discussing about dynamically stable are not. That is why we started with static stability, now we will have swiping to us dynamic

stability, and while will go to this process will try to develop equation of motion okay. So what will be the philosophy.

You have equation of motion and then give preservation and see how the displacement is responding. If it is damping then you say it is dynamically stable, if it is going on increasing you can say it is dynamically unstable. So what are the steps required here?

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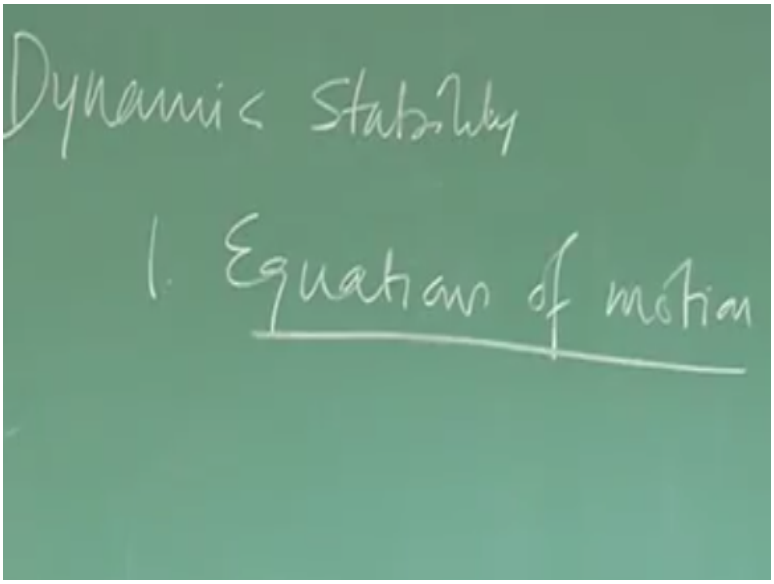

If I want to go for dynamic stability study what I will do first of all we think of equations of motions. Just to give 1 example what is the meaning of equation of motion for flight vehicle. I will take a simple example and try to share with you or create a road map for the final 6 degree fuel of freedom equation or motion. Equation of motion. So what will do let us take a simple example right.

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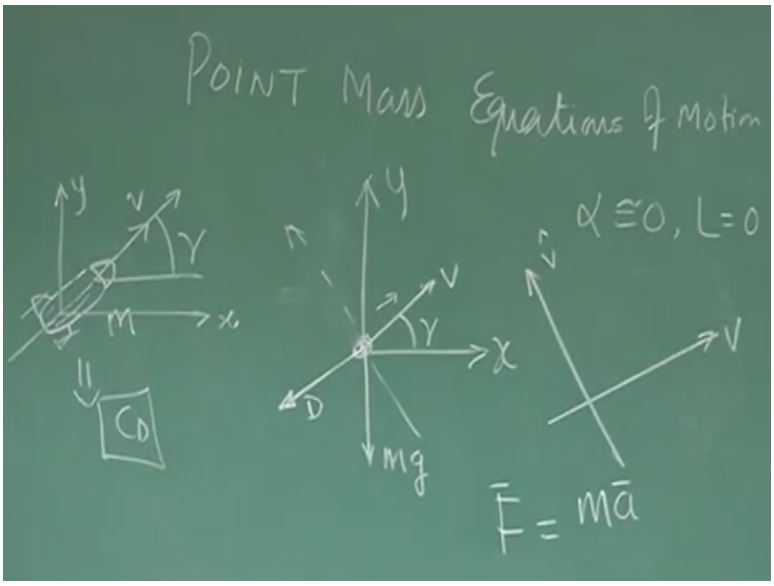

let us say let us say projectile of mass M. is fired at an angle Gamma at right path angle velocity vector, and we want to write the equation of motion that means, we want to know how its position is changing. That is and how is its velocity changing speed changing what I do we fix some X Y let us say, I write it X and Y it is the co-ordinal system I am thinking of let us say simple plane air ,That is assuming that the projectile is fixed in the vertical plane.

This example is just to illustrate you; how to start thinking when I want to write equation of motion typically, what this is called we will see we are actually trying to develop point mass equation of motion. When I say point mass that means I can equivalently write this, in this fashion right? Has if the whole mass is concentrated at the CG of the projectile, is it impress assumption this is stable right. We will not stable then we will not able to solve the way we are doing it.

And now if I write it like this, I know what are the forces acting on this projectile, we know that there will be drag on the fin drag on the body drag we everywhere fin pressure drag pressure drag will come So, we are expert now we know the total drag I know how to compute, and from there I can find out what is the CD drag coefficient, let us say that is known so, this the velocity direction the drag will be in the opposite direction.

Where will be the lift? Lift will be perpendicular to the velocity vector, and there will be w8 here MG correct. now make further assumption will say angle of attack Alpha is almost 0 all we are neglecting the lift, which is in true case for this sort of a shell when they are fired the angle of attack is very very small point 1 degree point 2 degree be that is the angle at which it is in equilibrium.

All are symmetric everything symmetric right? we have realize that so, we are neglecting this lift so, lift = 0, now we want to write the equation of motion so, what I do I write I choose 2 axis 1 along V and 1 perpendicular to V right. let me write the motion along V, and 1 perpendicular to V I can as well write the whole comp1nt along Y and X, but I am just giving you exercise, we are solving an exercise.

Where I am writing equation of motion along V and perpendicular to V. So, what is the step I should follow, I know I should be very clear that have to follow F = MA force = mass into acceleration And I should know that this are all in inertial frame now going to write the equation of motion along v direction.

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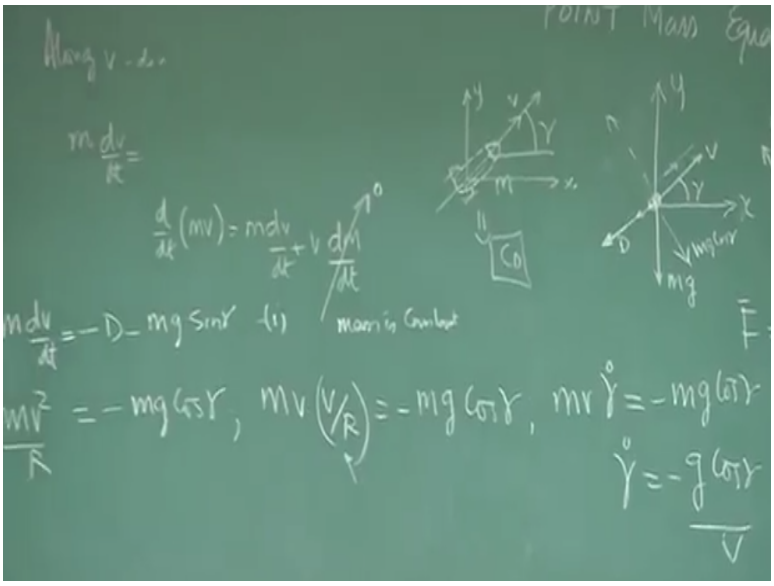

So, that wills what. MDV by DT = the net force why this MV by DT because actually should be DV by DT of MV this is MDV by DT + V DM by DT, but we are assuming mass is constant we are assuming mass is constant this assumption will not be strictly true if it is aircraft. Because in

aircraft the fuel consumption will be there this assumption will not be true for rocket having solid propellant, because has time goes the propellant mass gets consumed so, the w8 reduces this will be true strictly for a shell, Like a buffo shell right.

Where they are given initial kinetic energy so, they are no acting propellant sitting inside so, mass will be remain constant so, that is how I can take this has MDV by DT so, this is the acceleration generated who is crossing this acceleration.

That is by some force what the force is. force is a vertical of V direction so, force is - drag and - MG SIN Gamma I could see from a this is Gamma so, this from per MG SIN Gamma this is 1 now, what about when I write the equation along direction perpendicular to V so, what is the equation will be so, here what are the forces acting perpendicular to V is - MG COS Gamma
And what is this crossing this is crossing you could understand semicircular motion.

This is V and this is 10sion so, this 10sion is perpendicular to V so, what sort of acceleration you get. You get centripetal acceleration this 10sion which is perpendicular to V will generate a centripetal acceleration, and you know centripetal acceleration is MV square upon R okay. Now we see I can further write this as MV into V by R is = - MG COS of Gamma what is the R here.

MV square R R is a radius right? Because this force whatever is there MG COS Gamma right? It will try to turn the traject so, the will be a radius of turn So, now what is V by R V by R is at what rate this velocity is turning right? I see the angle of attack is 0 so, whatever the rate angle of attack is turning but assumption it is the same rate at which your axis of the projectile is also turning.

So, we call it now MV into Gamma dot that is angular speed with which the velocity vector is turning is = - MG COS Gamma so, you get Gamma dot = - G COS Gamma by V.

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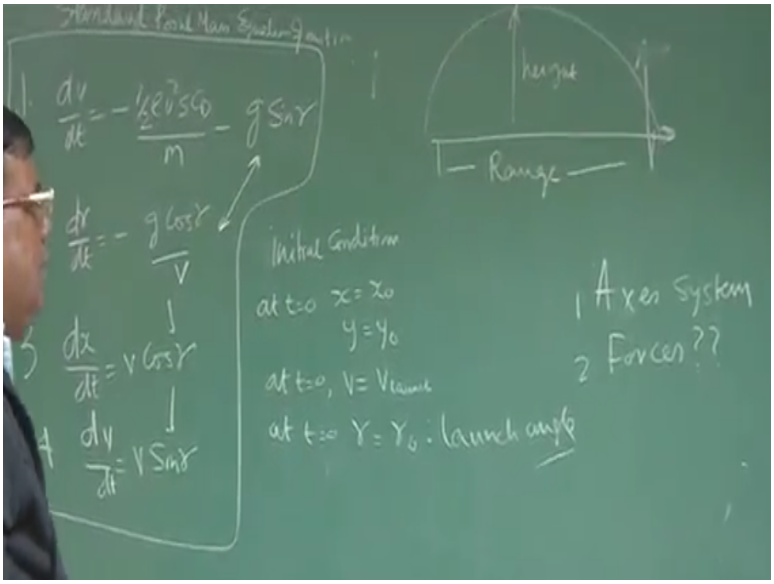

So, what are the equation of motion we got when we wrote it like this, 1 is DV by DT is = - half ROW V square SCD by M - G SIN Gamma have divided this by M all through, And our second equation I got is D Gamma by DT is = - G COS Gamma by V what is our aim. We want to find this coordinate of this projectile, in terms X and Y axis right? but they assuming that is it planet trajectory so, we have to add third equation that is DX by DT is V COS Gamma and 4th equation is DY by DT = V SIN Gamma that clear?

Because this is V and this angle is Gamma so, this is V COS Gamma and this is V SIN Gamma what is V COS Gamma. It is the rate at which the projector distance is changing along X, that is V COS Gamma and V SIN Gamma is a rate at which the distance of the CG changing along the Y direction okay.

So, now I could see that these are the 4 very standard point mass equation of motion, how to solve it. let us see first of all we hand if is that set of differential equations first order differential equation, also you see that equation number 2 and equation number 1 this Gamma is present Gamma is here Gamma is here in here also Gamma is here here Gamma is here so, you could see V is here V is here so, you could see V is here so, you could see that there coupled differential equation first out of differential equation right.

So, you to solve that we need to have initial condition initial conditions, and let set the initial condition if I want to see this 3 equation I need to write at T = 0 X = lets X 0 and Y = Y 0 So, depends this is decided by where from your firing, where from your launching right? So, there That reference will have some coordinate, then also here we see in this at = 0 what is V? V = V 0 or V launch right. And also you know this Gamma at = 0 or Gamma = Gamma 0.

On that is called launch angle right. at what angle I have launching 10 degree twenty degree forty 5 degree fifty 5 degrees to get different types of values of X, this value of X value where it is falling like this this is typically called has a range right.

And whatever in the Y direction happening all's X of forces and around the Y direction know this are in h8 of the traject so, we have a maximum h8 we are a maximum range. I am only talking about plane at rejections, in actual practice we will find the 3 dimensional so, there be a drift also I will have a distance cover along the lateral side so, those will be adding in 6 dots in point mass ,this is fairly simplified way to write the equation of equations of motions.

And you realize that this are first order differential equation coupled first order differential equation, and you need to solve them by numerical methods and you know that there will be there will be so many methods to solve this equations, if possible will loading 1 or 2 software or core that it can u use it but any way you can go mat lab and solve this equation using stranded routine.

This lecture is given to you make you understand simple mass module, when I am developing what are the things I need to know first, Know first is what is the axis system I am talking about then what are the forces. In all directions and that is where the fixed of concept into mind. Then how do I do that all the today I am not talk about stick job just a warm up know what we are heading towards.

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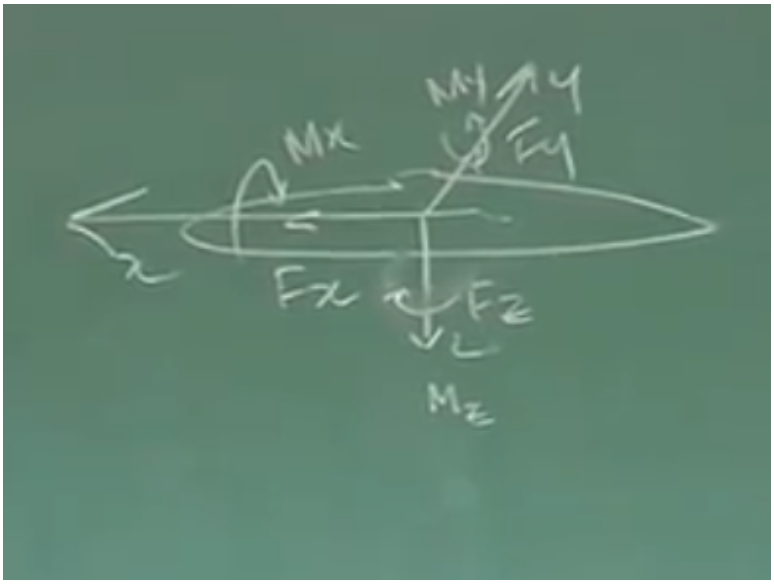

We are familiar this is the axis system is through this is X , this is Y this is Z so, I will have force along X direction I call it FX force along Y FY force along Z FZ , then moment along X MX moment about Y MY moment about Z MZ right. We know the convention that MX is positive, when right wing going down which is rolling moment, MY is positive moves it pitching up and MZ is positive when Right wing is going back I need to know how to calculate this FX, FY, FZ and MX, MY and MZ.

So, that I can write the equation of motion here only drag force were required because it has a simplified assumption point mass So, I need to know what is the drag coefficient value right. To find the drag but by miss 6 of I need to know so many co efficient so, that I can model FX then FZ FY MZ MY MX all 6 that how its became little involved. Not complicated it gets involved. It is question of matter of putting time nothing you don't required any extra IQ for all these things.

This has been done 50 70 years back I mean hundred years back, do not get nerves unnecessarily yes you may not like this Expressions does not matter may not like there are useful this neem tooth paste I never liked, but they are very useful. Okay so, please be clear this need clean attention it is effort do not unnecessarily say it is complicated absolutely today youngsters should not find this complicated at all okay.