Aircraft Stability and Control
Prof. A. K. Ghosh
Department of Aerospace Engineering
Indian Institute of Technology-Kanpur

Lecture- 10
Longitudinal Control Continued

To revise before I start what we are telling.

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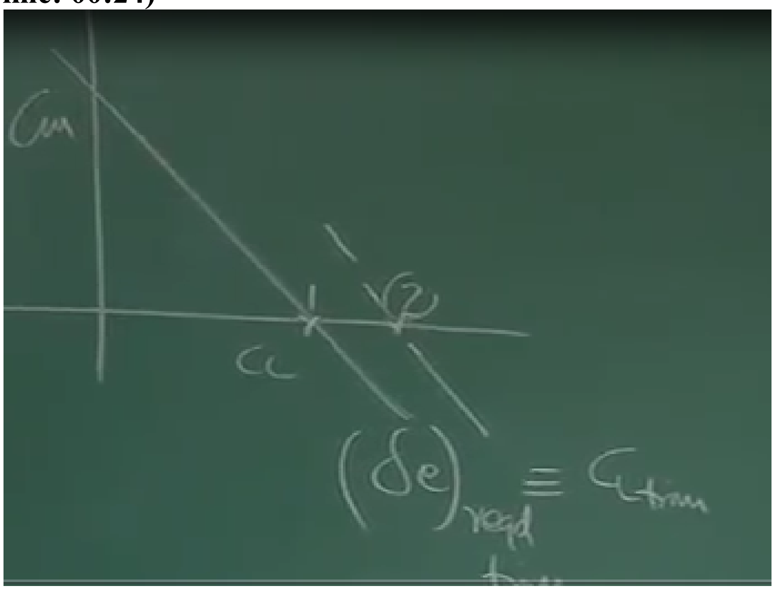

Suppose this is my CM versus CL I am flying here this 1 and to fly at 2 I want to know, how much elevator I should deflectrun so that, I can again trim at CL 2. So relationship between Delta E required some term we say Delta E trim and CL trim is what is a control problem for this flight mechanics what we are discussing, how do I formulate it?

Let us see, you know CL should be = CL Alpha into Alpha + CL Delta E into Delta.

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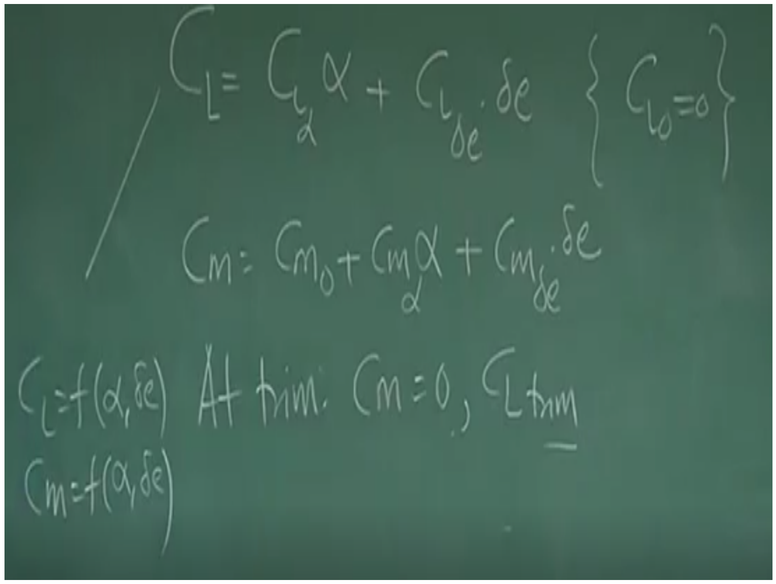

E, for simplicity I have put CL 0 = 0 you can introduce CL0 here and do the derivation, and most likely I will be give in that assignment. Then CM will be CM0 + CM Alpha into Alpha + CM Delta E into Delta E we are clear? Because here we have seen CL is function of Alpha and Delta E, we are not talking about any dynamic condition so, since CM has come because of CL CM also will be function of Alpha and Delta E, so CL have expanded in the linear way.

CL Alpha into Alpha + CL Delta E into Delta E, similarly I can that CM Alpha into Alpha CM Delta into Delta E, but CM0 we cannot neglect because CM 0 is very stringent condition to ensure that airplane has positive angle of attack trim, that is CM0 should be greater than zero. Now think once it has come to an equilibrium, what will be the value of CM at trim what happens at trim CM will be = 0 right? And the CL will be = CL trim, whatever I am looking for maybe I am looking for this.

So, that time CL = CL trim at 2 and CM has to be 0 otherwise it will not be trim, if I now do this introduce these things here what happens? let us see that if I introduce this then I get.

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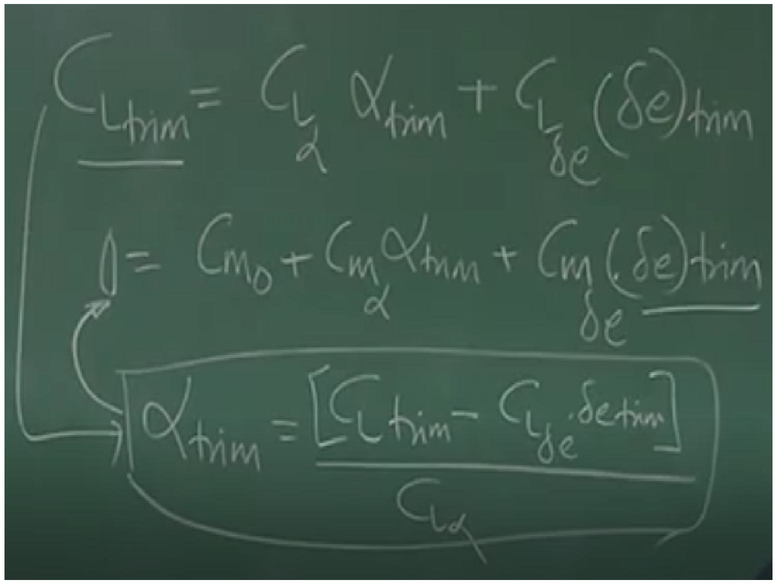

CL trim = CL Alpha into now Alpha becomes Alpha trim, CL Delta E into Delta E let's say trim what I am using which I physically mean that it is a Delta E required what will happens to this equation, since it is at trim so CM has to be 0. So 0 = CM 0 + CM Alpha into Alpha trim + CM Delta E into Delta E trim, what is our aim let's not get lost in all these situations. What is our aim? We want to find out what is the relationship between Delta E trim and CL trim so, that I can often know if I want to trim at this particular CL trim what is the Delta E trim, or Delta E required right? so to do this Algebraic adjustments.

What we will do is I will write the step I can write Alpha trim is = CL trim - CL Delta E into Delta E trim, divided by CL Alpha isn't it from this equation I can write this Alpha trim = CL trim - CL Delta into Delta E trim divided by CL Alpha, so this Alpha trim expression I will be plug in to this equation, that is CM0 + CM Alpha into Alpha trim instead of Alpha trim I will put this expression, CM Delta E into Delta E trim.

Now, you could see that Alpha trim is vanished Delta E trim and CL trim will be in this so if you do this, manipulation I leave it to you to derive this small relationship, I repeat again I will be putting Alpha trim here in this expression, and then try to find out relationship between Delta E trim and CL trim, so that eliminating Alpha trim from using these two equations, and then you get some term like which I am sure you will be able to do it, write the final expression.

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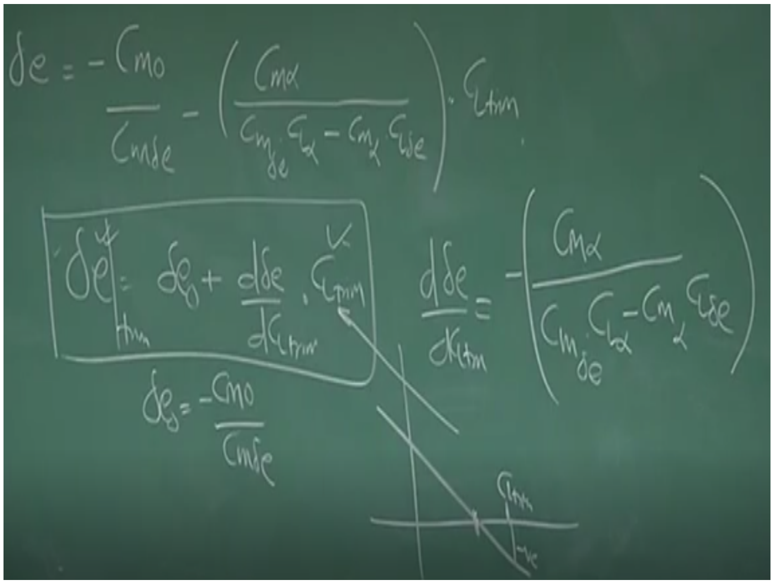

And that will be Delta E = - CM0 by CM Delta E - CM Alpha by CM Delta E into CL Alpha - CM Alpha into CL Delta E into CL trim let me check, - CM0 by CM Delta E - CM Alpha by CM so I can write this Delta E = Delta E0 + D Delta E by DCL trim into CL trim where, D Delta E by DCL trim is nothing, but - CM Alpha by CM Delta E CL Alpha - CM Alpha CL Delta E right. What I have done, this whole what is Delta E0 from this expression very simple CM0 by CM Delta E.

So I have put a neat diagram neat I have developed and I am presenting a neat expression, where Delta = Delta E0 + Delta E D Delta E by DCL trim into CL trim what is the meaning of this? This I called again trim, because this is required if I know Delta E0 value how do I know. I need to know CM0 which I know and I know CM Delta E I know so I know Delta E 0 I need to know D Delta E by DCL trim so CM Alpha I know airplane CM Delta E I know CL Alpha I know so, I know Delta E so this term also I know okay.

So whatever CL trim I want I put it here this numbers are fixed for low speed. So I can get what is the Delta E trim required is this clear? that was our purpose, I want to know what for a given CL2 how much Delta E I should deflect, or it is again if I take you back to that discussion I flying here if you want to fly here this will give a negative pitching moment, so I need to generate a positive pitching moment by putting the elevator up, how much elevator up?

That will be decided by what is the CL trim here, and what are these values so I know how much elevator up I have to do as simple as that okay. So that is why we say this is controlled solution for elevator and we call elevator control okay. Now we will go little more insight into it, so that we get little bit of insight of what are we doing, we will do some approximation.

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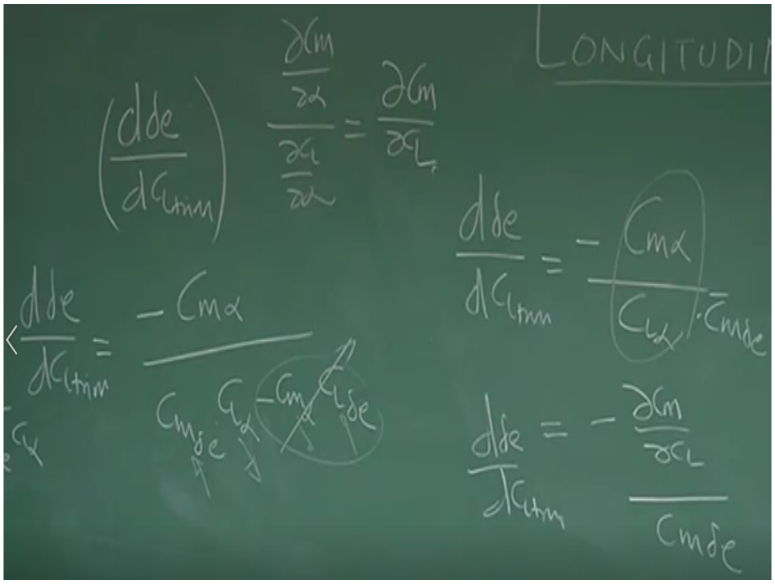

We will now go little bit of discussion for this term what is this? D Delta E by DCL trim, this is given by - CM Alpha by CM Delta E CL Alpha - CM Alpha into CL Delta E. Typically you know CL Alpha value will be around 5 between 5 and 6, CM Delta will be around 1 CM Alpha would be around -1, this is also - 1 CL Delta is little less point 5 point 7, so I can do a simple approximation for this specific purpose I say this gentleman right. They are this term is less compared to the first term, so I can write D Delta E by DCL trim approximately = - CM Alpha by CM Delta E into CL Alpha.

So what does it tell you so, what we are getting an expression is, D Delta E by DCL trim is = - CM Alpha by CL Alpha into CM Delta E, now could you say CM Alpha by CL Alpha is what? CM Alpha by CL Alpha is DCM by D Alpha divided by DCL by D Alpha, if everything is linear it is DCM by DCL.

So, I can write here this = - DCM by DCL by CM Delta E, this is D Delta E by DCL trim how do I add value, to this from designers perspective, see here 1 thing I know that is CM Delta is very high very large, this value will go down in absolute sense what DCM by DCL, remember which I have derived DCM by DCL as minus static margin right.

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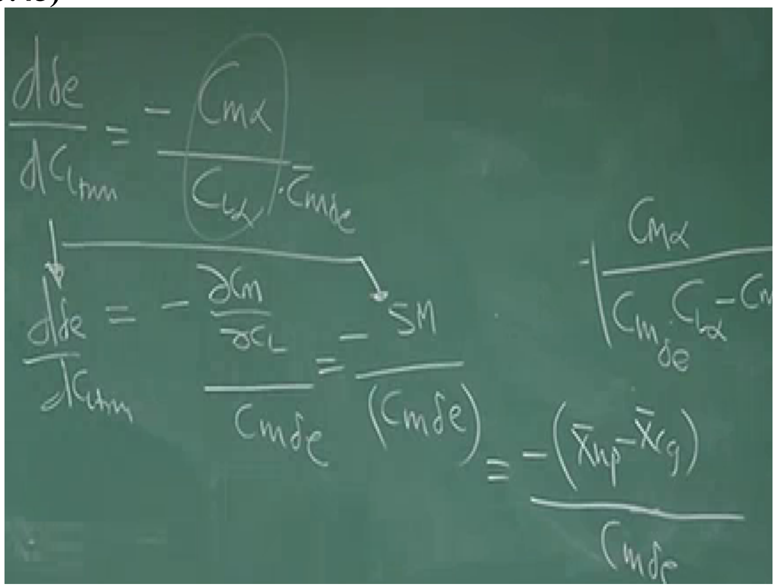

Which is nothing but - XNP fixed neutral point neutral point fixed - XCG bar so divided by CM Delta E okay. So now you could see that, if static margin is high then D Delta E by DCL trim will be what? If XNP and XCG are coincident then, D Delta by DCL trim will become 0, is a neutral stability point, but if this stability margin is large and large D Delta E by DCL trim.

Also will increase that when you need larger Delta E to change the same CL trim from one point to another point. The gradient will be too high, the pilot will have a lot of difficulty to trim from 1 aircraft trim to another aircraft trim so, and this gives you the connection the handling qualities for the pilot through static margin okay.

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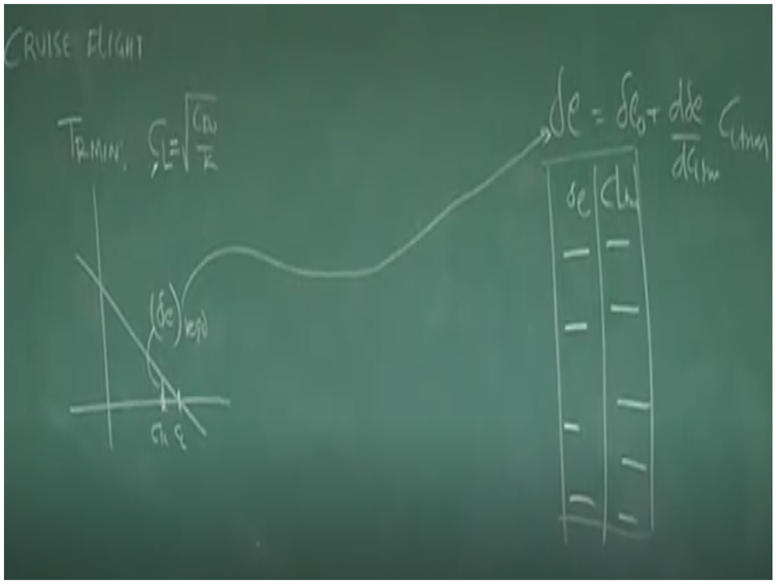

This is one understanding. Second 1 once, I write Delta E = Delta E 0 + D Delta E by DCL trim into CL trim. I am knowing that for CL trim for this CL trim, this Delta E required for this CL trim this Delta E required like this you know, so pilot will develop a field from the Delta E, how much Delta he required to give for the CL trim or he thinks in terms of he gets the feel in terms of speed, so your giving Delta E and speed that's sort of mapping you’re doing for a pilot okay.

This is the one way of developing a field for the pilot but that’s not a very good way of developing a field, that is why the reverse will control the aircraft, we tried to give the field through the stick force, that is where some little we do will talk about stick free stability just I stop here, but please remember you can give a pilot field through Delta E CL trim or V trim combination, that if you are add this speed put that much of Delta E, so it develops a field okay.

However, another way of giving a field for reversible control is that give through the stick force gradient or through the stick force right, which we will be talking sometime later. So after doing all this thing what is our learning so many expressions we have written so, what is the learning? Let us just summarize in few minutes as I have been telling you once for all you should do this derivation, the very straight forward derivation however.

You should understand what is their real importance and in terms of designer how you are going to use it. Let us say this a cruise flight okay what CL will be flying at a cruise? say if you say I want to cruise for thrust required minimum that means you know that I have to fly such that CL = CD0 by K right. This already knows in performance so, there is a particular CL okay.

Suppose you were flying the machine at some CL which was not for thrust required minimum, now you will find for thrust required minimum I have to fly at a CL here let's say. So let me the higher speed but CL is given by CT0 by okay, so immediately you see that this relationship between Delta E = Delta E 0 + D Delta E by DCL trim and into CL trim will tell you if I now want to fly at this CL trim how much Delta E required?

That comes from here okay; similarly if you want to go for a climb you need a particular CL so how much Delta E to have that CL, again this expression will tell you clear. So that is the importance of this term okay, I will be solving problem based on this which will make things much clear to you; but as I have told you that I will have a mann ki baat session where we will be taking up some problems also.

So that your understanding is not only cleared is crystal cleared okay. We have been talking about stability and control of an airplane and by now you must be tired we have written so many expressions in my opinion at this juncture, we should try to solve some problem, which are directly relevant to a designing of an airplane. Once we solve those problems you will get better feel for this whatever expression we have written whatever equations.

We have derived so what will be my approach is we will start from the beginning what we have been talking about, and at an appropriate time we will try to solve a numerical right, like this we will create a work space for you where you can yourself work for a problem and you know how to apply whatever we have been talking so far in a real design scenario okay so. We will little slow.

Because I could understand that writing so many expression black board, and some time you will forget that we are talking about an airplane many times I used to get a feel that I am doing a some sort of a mathematical or numerical course, where I need to know how to differentiate how to find maxima how to find minima, So it’s right time we should come back and try to see what we have done, and how can you use this for a real air craft. Let us go back what is our primary aim our aim is to design an airplane.

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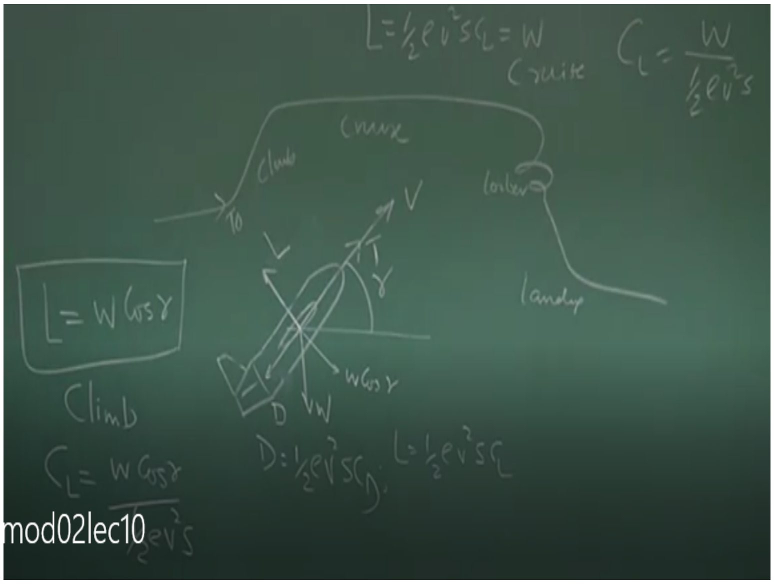

We should have take off climb, so it take off then climb, cruise may be lighter, landing and there could be maneuver as well right. Let us see here when I try to climb what is happening? If I again go back little bit if this is the airplane okay, and it is going like this is you know by now this is flight path angle.

And if I am talking about the study climb then I know that, if this is a CL this is CD this is the weight and somewhere there will be thrust, what I know for a study climb, study claim and accelerated claim that is it will follow rectilinear motion the center of gravity will follow this flight velocity, assuming that angle of an attack is very small. Most important thing center of gravity will follow the rectilinear path and that is possible only when you could see that if this is CL.

I mean it has a lift I can multiply by similarly for drag, we can write drag which is half row v square into CD so for a completion I right drag = half row V square is CD and lift = half row V square SCL. Coming back to the steady climb when this airplane is going to be like this, what is expected what I should maintain I should maintain that, if I write it here L for your otherwise you will get confused ,see L should be able to balance W COS gamma right.

So L should be = W COS gamma, what does it tell you? If I now switch over from climb to cruise in cruise I know L should be = half row V square SCL and that should be = W, what is the difference in climb and in cruise, from trimming the airplane point of view what is the difference you know that from here if I say CL, CL will be W COS gamma by half row V square S and here CL will be = W by half row V square S. So, which CL is more definitely COS gamma is a positive number.

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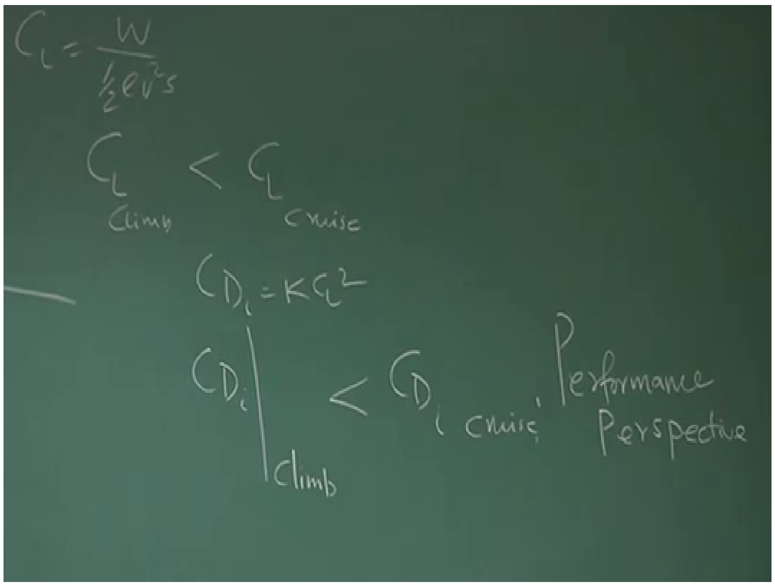

If it is climbing so this CL climb is less than CL cruise what is the implication of this? In a performance we have seen it means the induce drag CDI which is KCL square that means induce drag CDI for a climb is less than CDI for a cruise, that is a drag coefficient okay. CDI for a climb is less than CDI for a cruise this is from my performance prospective no issue right.

But from stability and control point of view what does it mean? From stability point of view you could see that the CL required to trim a climb is less than the CL required to climb at cruise, what is the implication in terms of control let us understand this.

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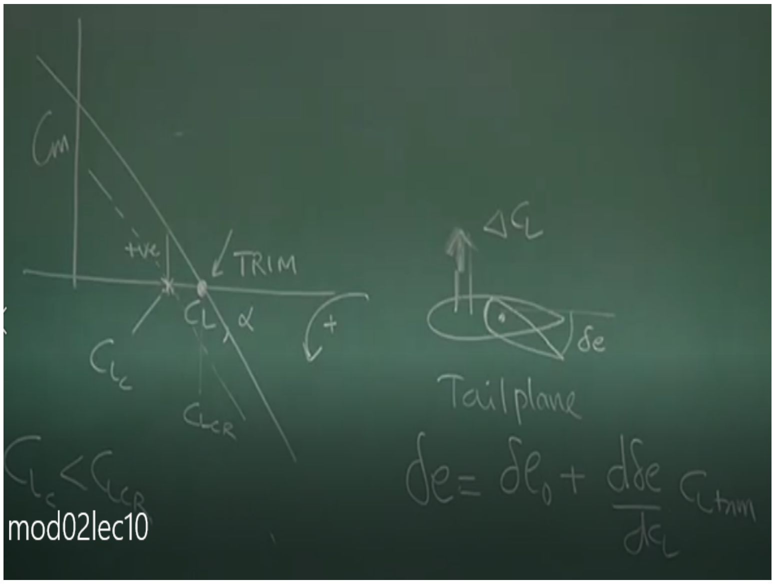

Then we take you back to this graph CM versus CL or Alpha whatever you are comfortable, and this is the CL and this is the trim point right or the equilibrium point trim a climb it is CL I see C this is for climb and for cruise I right CL CR.

So, if I am trimming the airplane for a cruise and if this is the point CL cruise then for climb I have to have a CL which should be less than CL cruise is it clear? Because CL climb if I write CLC is nothing but W by half of V square S into COS gamma, and we have seen CLC is less than CL cruise. So if I am trying to climb at CLC you could see that if I have to trim the plane if the CM vs CL graph.

If I try to trim the airplane at CL cruise graph it will automatically generate a positive pitching movement because this is statistically stable, but we want to trim the airplane here that is I want this graph to look like this what I have to do I have to nullify this positive movement so, how can I nullify this positive movement? We know it can be done through elevator this is the elevator, now how the much elevator to be defected that will depend upon.

How much positive moment has to be nullified by generating a negative moment. So how do you put this negative moment using elevator if this is the tail plane what you have to do negative movement means you have to put the elevator down, as you put the elevator down, there is additional Delta CL and which will abort a CG will give you negative movement, so this Delta E whatever you have deflected should be sufficient enough to generate a negative movement who's is magnitude will be = this much of positive movement.

So that is the controlled point okay. This is clear now the question is how do I know what is that elevator required? And we know that for that we have divided a devised a relationship which is Delta E 0 + D Delta E by DCL into CL trim.

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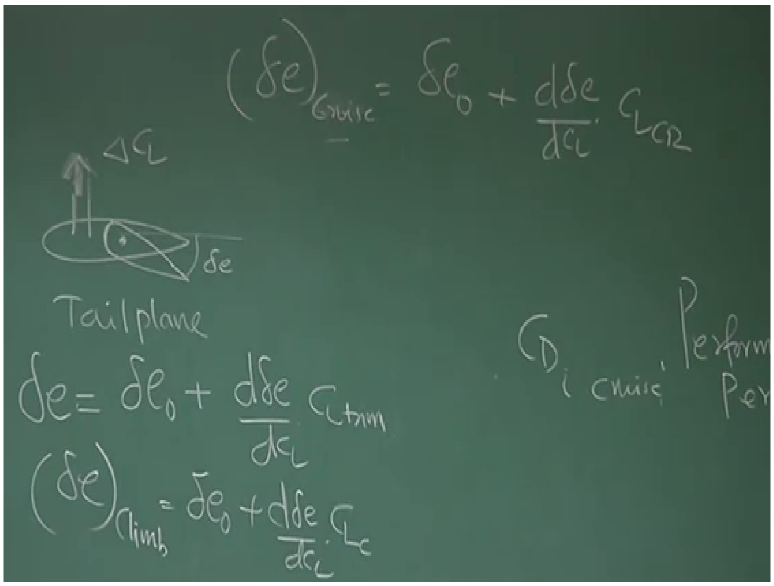

So if I want to know what is the Delta E required for climb I should replace so I write Delta E during climb will be = Delta E0 + D + Delta E by DCL into CL climb CLC.

If I want to know how much Delta E is required for a cruise so, I will right Delta E is required for cruise Delta will be = Delta E 0 + D Delta E by DCL into CL cruise that will tell you know how much Delta E is required. This part is cleared okay. Now next question may come on what is Delta E 0 and what is Delta D by Delta CL and we have also elaborately explain that and just for revision because, we will be solving problems soon we need to recapitulate whatever we have done before we ensure that we are lost into equations.

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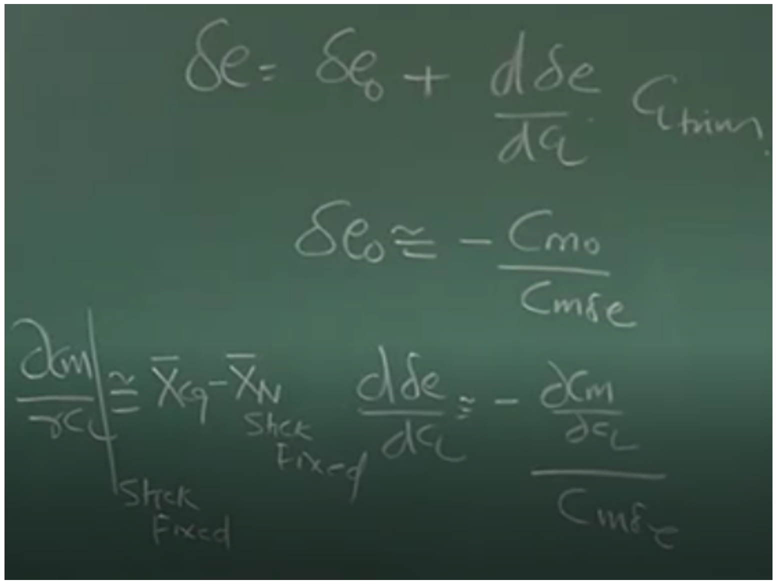
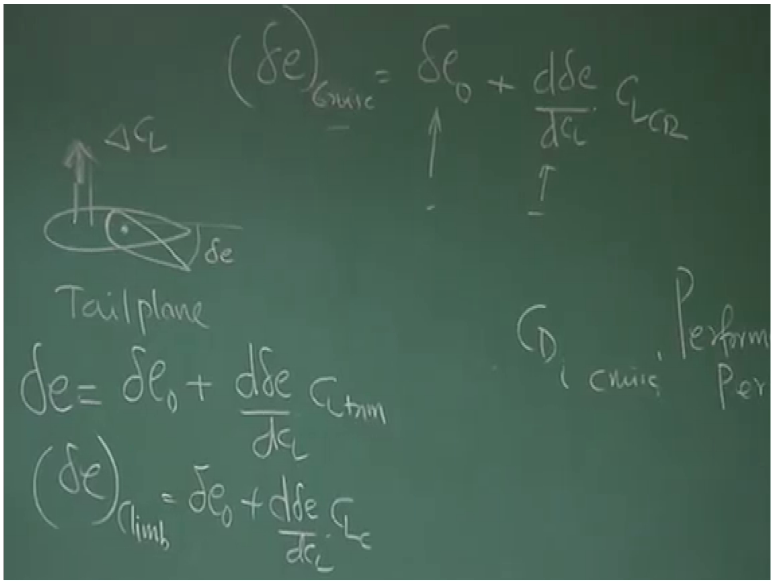


So you know Delta E = Delta E 0 + D Delta E by DCL into CL trim and Delta E 0 was approximately shown to the - CM 0 by CM Delta E and D Delta E by DCL was - DCM by DCL by CM Delta E. I am putting a approximation because we have neglected few terms okay. So what is the message for an airplane Delta E 0 is known because airplane will have a fixed CM0 and CM Delta E, if it is low speed airplane.

If it is a high speed airplane this value may change with incorporating MAC number effect. Similarly, D Delta by DCL how do I know this is DCM by DCL okay. DCM by DCL is what? It has a direct relationship with static margin what is static margin? It is the distance between neutral point and CG you remember DCM by DCL approximately I can write as XCG - XN stick fixed, and this is stick fixed. Why are we talking about stick fixed because it tells you absolute value of Delta E required right okay.

So DCM by DCL is known for a given configuration XCG will be known, and neutral point also will be known so this is known CM Delta E and CM0 also known, so I actually know these numbers this number, this number, these are known to us and we have to just plug in the CL cruise or CL whatever trim.



You are using right. So once this part is clear I as a student would like to know CM0 as revision and what is CM Delta E, before I really become champion of using this expression so, what I will do? We will now go back what is CM0, what do you understand by CM0 and what is the meaning of CM Delta E in a little exhausted manner, because that will help us in solving problems. Please remember whatever so far I am discussing here I am not talking about a maneuvering flight okay, correct.

For a maneuvering flight, we have seen that you have to put additional elevator deflection which will be again revisiting when we solving the problem, but at this point for this whatever I am discussing we are not talking about any maneuvering flight okay?

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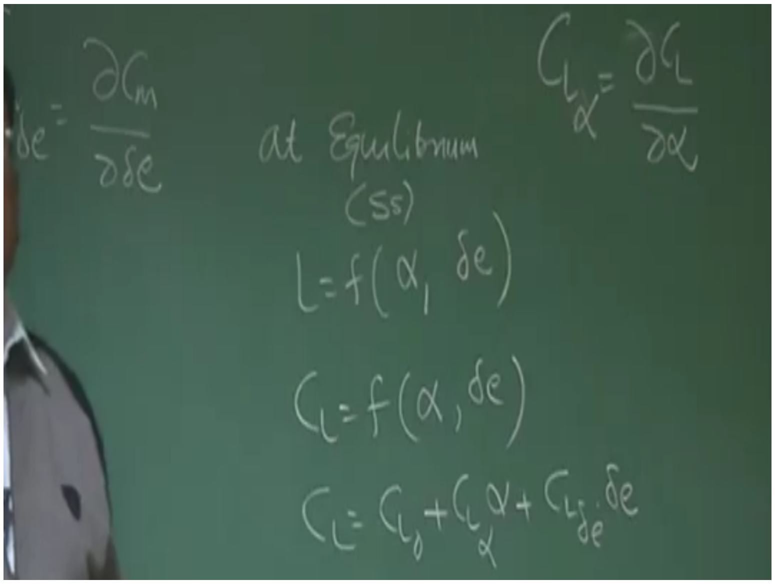

Now, Let us see what this CM0 and what is CM Delta E, and allow me to put something like CL Delta E what are these parameters or what are these aerodynamic derivatives. For example what is the meaning of CM Delta E.

Let us understand this is DCM by D Delta E partial derivative what does it mean? How do I physically generate feel for this you know at equilibrium I am talking about steady state, let’s say the airplane at equilibrium to cruise right. And you know that lift force will be function of Alpha and the Delta E because, otherwise lift also function of how do you’re doing the pitch rate right. how the rate of change angle of attack is changing.

But we are concentrating at case where is equilibrium it is just like we having some Alpha E and some Delta E some Alpha and Delta E Alpha is angle of attack so that is how we write CL also the function of Alpha and Delta E, so CL we wrote as CL0 + CL Alpha into Alpha + CL Delta E into Delta E, we are exploiting the fact that CL is linear function of Alpha and Delta E so that, we can write it like this. So here also if I ask you without the question CL Alpha.

CL Alpha is DCL by D Alpha is the partial derivative, what does it mean it means what is the change in CL for unit change in Alpha keeping other things constant right. That is important okay. Delta E also constant other thing is speed etcetera is constant so that is the meaning of CL Alpha or the partial derivative now similarly what the meaning of CM Delta E?

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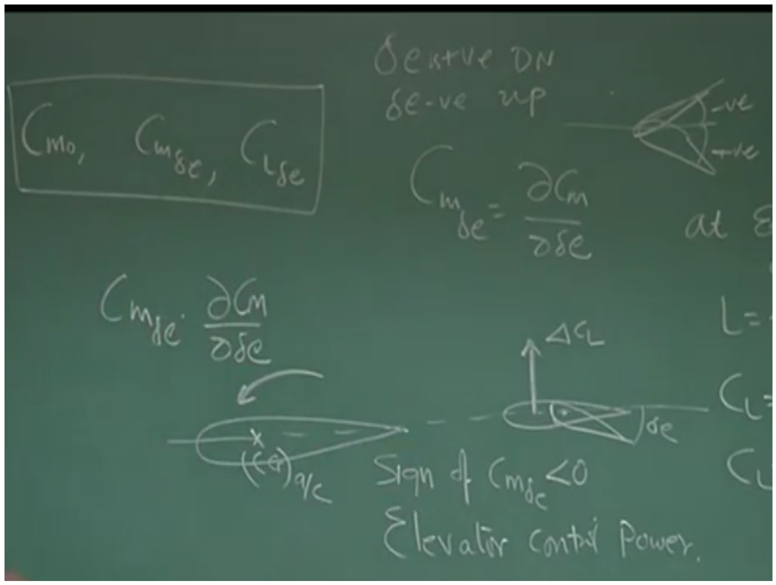

Let us understand that again this is partial derivatives DCM by D Delta E okay. What is the physics behind this let's say this is the wing this is the tail okay and here is the CG of the airplane now if I give a Delta E deflection as you recall as for the convention Delta E is positive up Delta is positive down sorry as per convention, Delta E is positive down and Delta E negative up that is, if I am putting the elevator up like this is negative Delta E and if I putting it down like this.

And this is positive Delta E this is a matter of convention, okay. So know in order to understand what is CM Delta E which is a partial derivative and by definition you understand, the change in the CM that is the pitching moment coefficient per unit elevator deflection, so we give first a positive elevator deflection let's say this is Delta E, so what will happen? It will generate additional lift Delta CL.

So that will give a moment about CG and that can be non-dimensionalize and we can get the value of CM Delta E, but one thing is clear at this point for a positive Delta E the moment will be negative. So sign of CM Delta E is negative okay. And this is also called if we recall called elevator control power. Naturally if CM Delta is strong so it will have a very command over pitching the airplane giving nose down or nose up moment.

So it is a control power and it is called elevator control power because this moment is generated by elevator okay. So that is the physical meaning of CM Delta E. So CL Delta E also very clear from here that, how much change in CL because of elevator deflection right. So these are the two terms we should know before we try to solve a problem.

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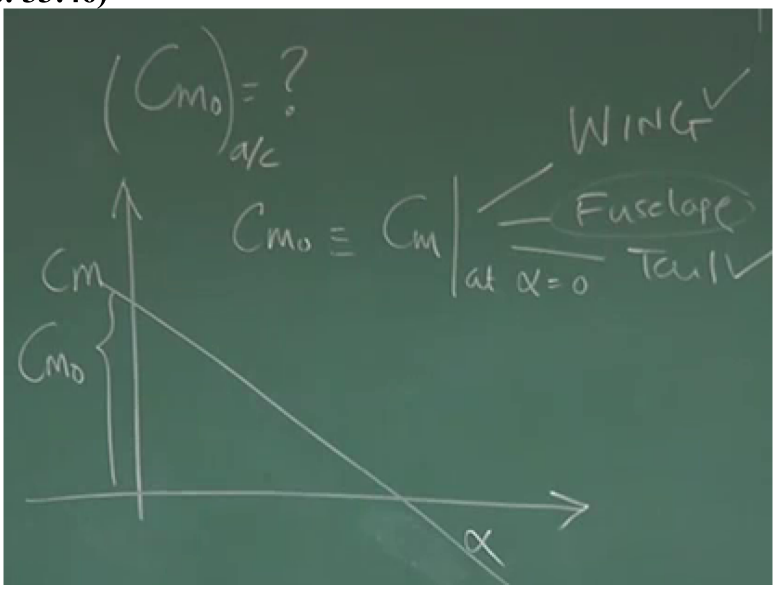

So next will try to know what is CM0 what is CM0? this is CM0 when I write it means CM0 of the aircraft, total aircraft what is happening you see what is in our mind when you talk about CM 0 we have seen if this is CM versus CL, and this is CL or Alpha whatever suits you so this part is CM0 right. Let's for simplicity I assume everything symmetric so let's me put versus Alpha right so what is CM0 then? CM0 is CM at Alpha =0.

So with this CM0 contribution should come from wing it could come from fuselage it could come from tail there could be other auxiliary surfaces also, but we will be more talking about wing contribution and tail contribution, and assume that fuselage contribution to CM0 is negligible.

Are there are empirical way to find out which will be doing showing you when you solve some numerical problem what is the message here why do I want CM0 positive? we want CM0 positive because, I want to trim the airplane at positive angle of a attack. If CM0 was negative I still can have in trim here that is statically stable but, this is at trim negative angle of attack. So I do not want to do that I want to have positive angle of attack so that is why we need CM0 but then CM0.

I should generate automatically from the aircraft by adding the contribution of wing fuselage and tail right? So what is the contribution of wing? We have already done we just write an expression, to give you better inside into it.

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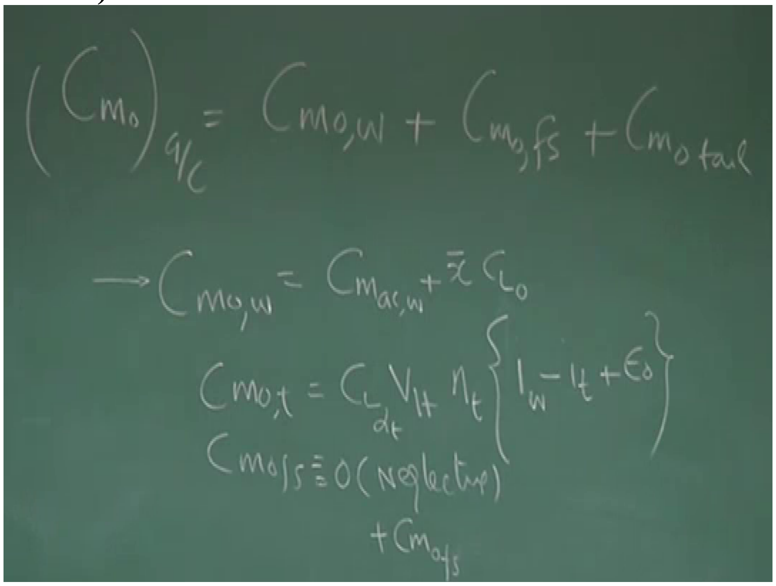

CM0 of aircraft is nothing but CM0 due to wing + CM0 due to fuselage + CM0 due to tail right. We have already done this and we derived expression CM0 due to wing. I can write it as CMAC wing + X bar into CL 0 similar to CM0 tail I can write as CL Alpha tail into VH Neeta T into IW - IT + Epsilon 0. We have already seen those derivation and we are saying CM0 fuselage we are neglecting now or we can put some value called CM0 FS right which we do not know at this stage how to compute but we will solve numericals to tell you how we can compute that, let us come back to very important here.

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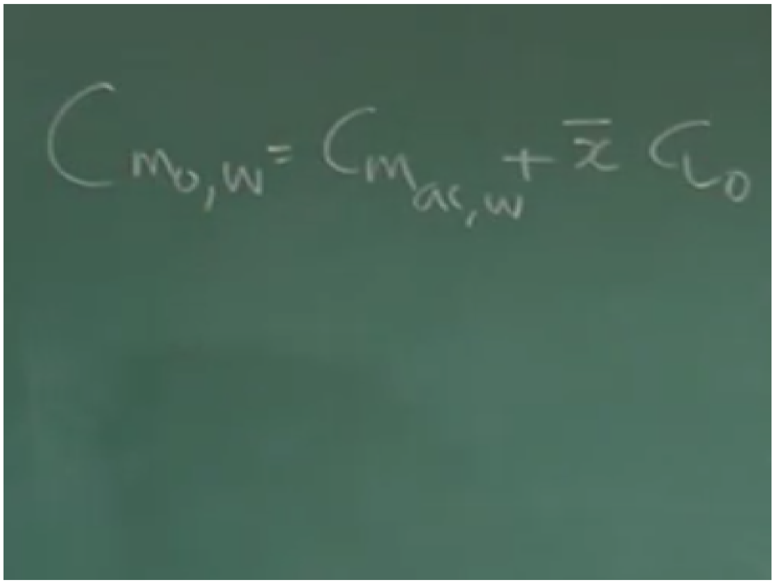

What is CMAC wing? so we have seen CM0 wing is CMAC wing + X bar into CL0, let us understand each term here, please do not forget what is our aim?

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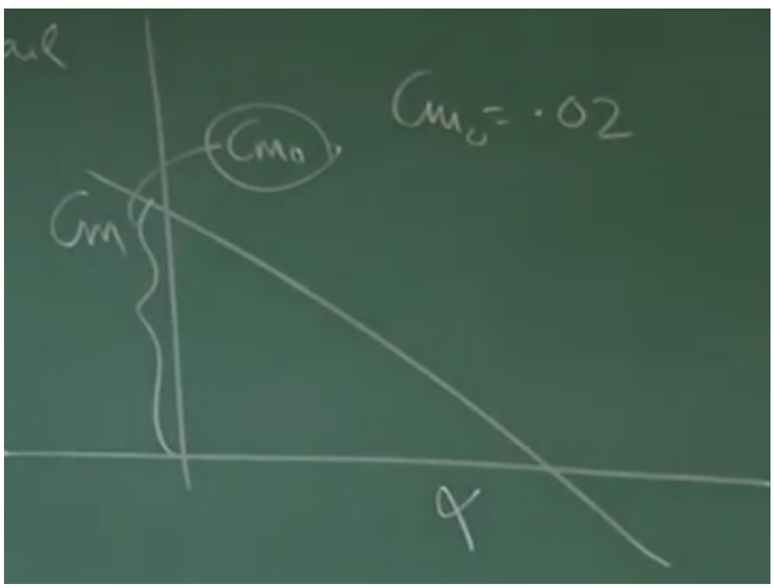

Our aim is to ensure that CM versus Alpha or CL graph, if its Alpha this is CM0 I need to have this CM0 desired, for example if I need CM0 = point 0 2 then, CM 0 because of wing because of fuselage because of tail so totally give me the value 0.02 okay that is important. So know what we are seeing what is CMAC wing? Because we are trying to find out what is the contribution of wing towards CM0 of the whole aircraft right?

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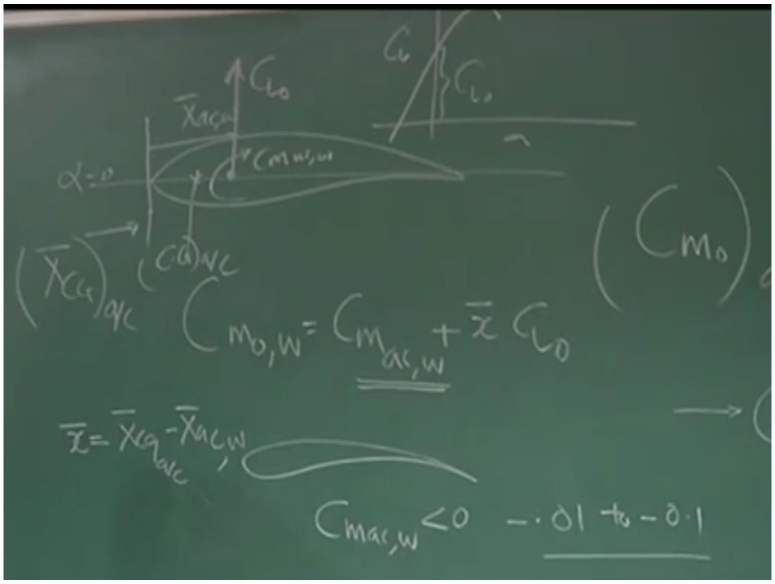

So now you know that we are very clear now that, big for a cambered aerofoil CMAC wing less than zero which is negative typically - point 0 1 to - point 1. We will find numbers like this, let us see what are these CMAC wing and X bar CL0 let us draw a wing, and let's say this is XAC wing bar and let's say CG of the whole aircraft, please understand so I give this nomenclature XCG aircraft. This is for the aircraft do not forget this people it creates confusion within themselves.

So now we know that this cambered aerofoil will have CMAC wing which is negative though by SIN conventional this is the conventional SIN positive SIN. So what is happening CM0 wing this is about CG actually right? That will be what you have to transfer CMAC wing here so it is here and then at Alpha = 0 you could see that there will be a lift here, which is CL0 is a coefficient part remember CL versus Alpha this is CL this is Alpha for a cambered aero foil at Alpha = 0 there is a CL which is called CL0 this CL0 also give you a nose down moment about CG, do you see this.

So if I write X bar as XCG aircraft bar - XAC wing you could see that CM0 wing about CG is CM0 CMAC wing + X bar CL0, and if AC of the wing is as tough of CG of the airplane, so this contribution also will give a negative pitching moment. So, total CM0 wing because of cambered aerofoil because of AC of the wing is behind CG of the airplane this will contribute to negative value. Thank you very much.