Jet and Rocket propulsion
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Lecture - 9

So, welcome to this lecture on rocket propulsion. Before we proceed, let us first recap what we have discussed so far.

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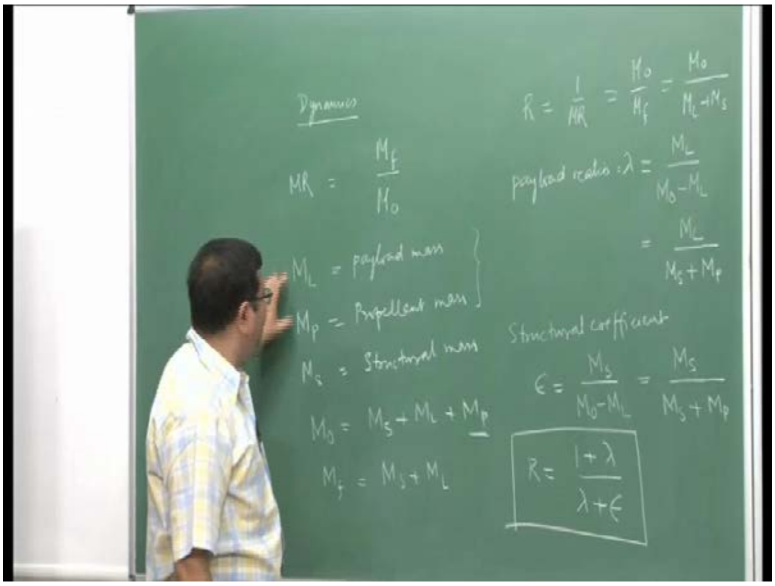

We have been discussing the vehicle dynamics for the last few lectures. So far, we have been focusing on single-stage rockets. And we have defined some parameters. One of such parameter is the mass ratio, which is given by the final mass by the initial mass. Then we have defined various type of masses; that is M L is the payload mass; then M P is the propellant mass; M s is the structural mass, which the structural mass includes everything except these two; except the payload mass and propellant mass, everything else is included in the structural mass. Then the total mass at the beginning is the sum of all these. And at the burn out, all the propellant is supposed to have been used up. Therefore, what remains is only the structure and payload. So, the final mass is nothing but the sum of structural mass and payload mass given by M S plus M L. We have seen this last time.

Then, we had defined another parameter R, which is inverse of mass ratio. Therefore, R is equal to M naught by M f. Now, if we combine these two, we get R equal M naught upon M L plus M S. We had done it last time. After that, last time we defined some more parameters. One of the parameter was payload ratio, which is designated by lambda. And, payload ratio is defined as the ratio of the payload mass divided by all the initial mass except that of the payload. So, it is M L upon M naught minus M L.

Now, again coming back to this expression, if we subtract the payload mass from the initial mass, what remain is the structural mass and the propellant mass. Therefore, lambda is equal to M L upon M S plus M P. We defined lambda in the last class. Similarly, we defined another parameter, which is structural coefficient designated by epsilon. And, we had defined epsilon as again the structural mass divided by all other mass, except the pay load. We defined it like this. Therefore, this again comes out to be equal to M S upon M S plus M P. We have done… We have defined these two parameters in the last class. After that, we had combined these three definitions R, lambda and epsilon; and, we had shown that R is equal to 1 plus lambda upon lambda plus epsilon. This is what we showed in the last lecture. So, we have done it in the last lecture. Now, let us proceed from here.

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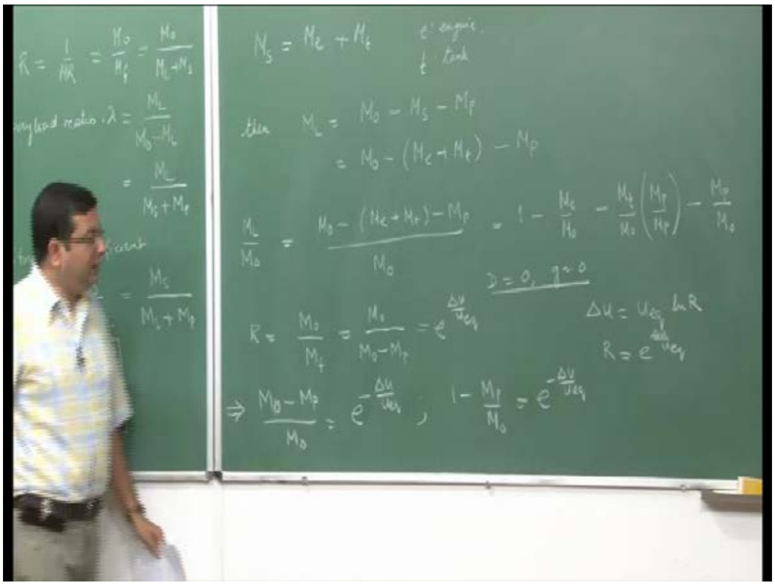

First of all, let us look back at the structural mass. Structural mass as we have said is everything except the propellant mass and the payload mass. So, now, if we look at a practical system, a rocket essentially is the payload and the engine and tanks – fuel tanks. Therefore, if we take out the propellant and the payload, what remains actually is the engine and tank. Therefore, the structural mass is mass of the engine plus mass of the tank; where, the subscript e stands for engine and subscript t stands for the tank. Therefore, the structural mass is just the sum of the engine mass and the tank mass. Then the payload mass now can be written in terms of this. What we can see from this expression is that, the payload mass is equal to initial mass minus the structural mass minus the propellant mass. So, payload mass is equal to M naught minus M S minus M P. And, we can put M S equal to M e plus M t like we have done here. Therefore, the payload mass is equal to M naught – M e plus M t minus M P. This gives us an expression for the payload mass.

Now, what we are interested in is finding out the payload ratio; not the payload ratio, the payload fraction; that is, this is called payload fraction – M L by M naught. So, this is equal to then… First of all, what we do is we take this expression for M L and then we divide it by the initial mass M naught. Then after this division, what we will see is that, this term is equal to 1 minus… We write M e by M naught as the engine mass ((Refer Slide Time: 07:08)) fraction. Similarly, we have the tank mass fraction. This thing let us multiply and divide by the propellant mass; and then the propellant mass fraction upon M naught. So, the payload mass fraction we can write like this. We have just expanded this.

Now, let us look at the expression for R, which we have defined here. R is nothing but M dot by M f. And, M f is equal to the initial mass minus the propellant mass according to our definition. We can write it like this. Hold this equation for the time being. Let us go back now to the expression we had derived for the velocity increment. We had… What we have done is we have derived expression for various cases. We have considered first for all no lift for most of the cases and they have… We have considered no gravity, no drag, etcetera. For the time being, let us look at the simplest case. What was the simplest case? When we had no drag and no gravity. Now, if I look at the velocity increment for this, then we have proved that, this is equal to delta u – u equivalent ln R. This we have proved in the previous classes; this equivalent velocity R; this ratio as defined here; and u equivalent is… delta u is the velocity increment.

Now, from this, then what we can do is if we integrate this, we can get an expression for R in terms of delta u and u equivalent... So, that will be equal to e to the power delta u by u equivalent; u equivalent, not u v. So, integrating this, we can get the expression for R as a function of velocity increment and equivalent velocity. So, now, let us put it back here as equal to – R equal to delta u by u equivalent. Then now, what we can do is first let us invert this. So, we can write M naught minus M p divided by M naught is equal to e to the power minus delta u by u equivalent. The left-hand side of this expression can be written as 1 minus M p upon M naught. This is the left-hand side of this expression is equal to then e to the power minus delta u by u equivalent. So, we are just simplifying this expression. Now, what? By doing this, what we can do is… What we can get is an expression for M p by M naught; that is, the propellant mass fraction, which is a very important design parameter.

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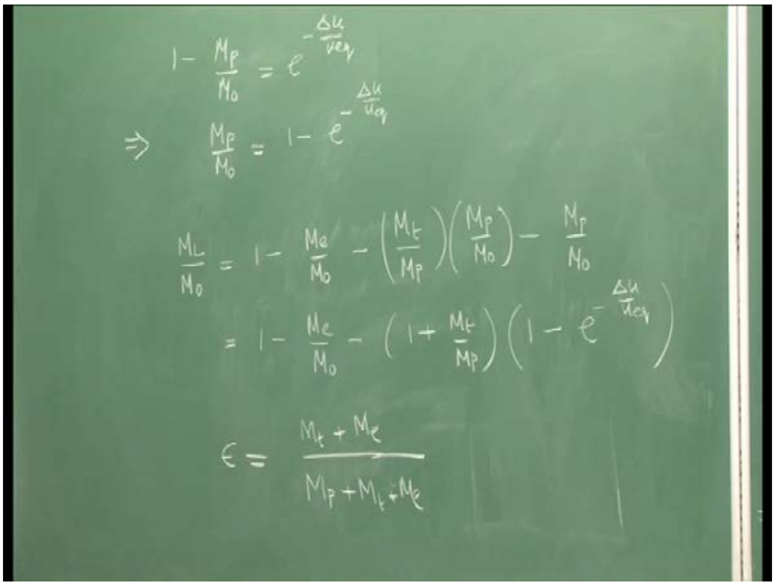

So, what we have shown here; there is that, 1 minus M p by M naught is equal to e to the power minus delta u by u equivalent. Therefore, we can write M p by M naught, which is the propellant mass fraction, is equal to 1 minus e to the power minus delta u by u equivalent. Now, what is the significance of this expression? This expression is what the designer will be using. Let us look at this equation. M naught is the initial mass. First, we have to leave this initial mass and we have to attend certain velocity at the end of the burn out. So, that delta u is dictated by the vehicle mission dynamics. So, the mission director will specify how much delta u will be required at the end of burning of propellant. Then you have already chosen a propellant. So, you have chosen a propellant; u equivalent is fixed, because that is the function of I s p – specify impulse. So, once you have chosen the u equivalent, delta u is specified by the mission requirement; then from this equation, first of all, we know the propellant mass fraction. And, since M naught is the initial mass, then we know how much propellant we need to carry for achieving this mission. So, this is a very important equation that tells us how much propellant needs to be carried by the vehicle to achieve a particular delta u and a particular fuel, which will be dictating the u equivalent.

Now, first, once we have got this, let us come back to this expression for the payload mass fraction. So, we had shown it in that board that, the payload mass fraction is given as 1 minus M e by M naught minus… Now, this term here we will simply little more. So, let me put it as M t by M p and M p by M naught; and then minus M p by M naught. Now, this M p by M naught we have derived here. So, between these two, if I take M p by M naught common, then this equation can be written as 1 minus M e by M naught minus 1 plus M t by M p times 1 minus e to the power minus u by u equivalent. We get this expression. Now, this expression then tells us that… See the engine mass again is something that is fixed. Time mass is something that is fixed.

Now, this propellant mass we are estimating from here. Delta u by u equivalent is not known. So, we can now tell that, for the given propellant that we are carrying and the initial mass, how much payload we can carry. So, once again, the mission goal is to carry this payload and give it that increment in velocity delta u. From this, we can estimate are we able to carry that payload or not. Or, in other words, what we can do is if M L is given, M naught is given, and these are given, we can find out how much propellant is required or if we need to change the propellant. Is the u equivalent good enough to give us that velocity increment or do we need to improve on the design of this or make the tanks lighter? Various things essentially can come up. All the mission and design requirement will come up by looking at these equations. Therefore, these equations are very critical.

Now, I have said at the beginning that, the tank mass and the engine mass are the part of the cell mass; they are included in the structural mass. So, now, if I use that and define about structural coefficient again, epsilon; then by that definition, M t plus M e, which is equal to the structural mass M s divided by the propellant mass plus the structural mass will come in here. So, this is the structural coefficient defined in terms of the tank and engine mass as well. So, now, let us look at this expression. What we will do is we will take this equation and then put it back along with this and this. We will combine all three equations. Once we combine all three equations, we will get expression for the structural coefficient in terms of other masses and the mission requirement. Now, this is the mission requirement – this ratio.

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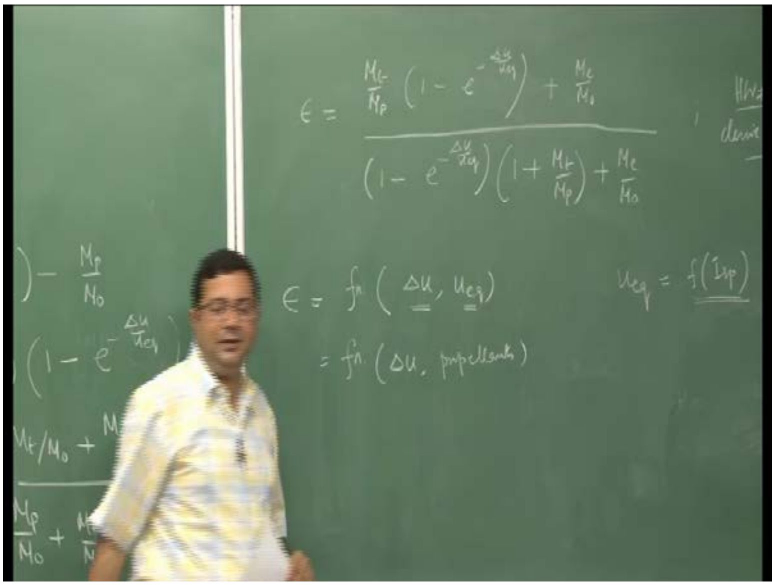

So, this is equal to M t by M p 1 minus e to the power minus delta u by u equivalent plus M e by M naught. As we can see here, this plus this essentially comes from this term here, that is, M e plus M t by M p. So, this is essentially nothing but this divided by M naught. So, we are… Essentially, what we are doing is we are dividing both the numerator and denominator by M naught. That is what we are doing. And then M t by M naught can be written as M t by M p, M p by M naught like we have done here. And, M p by M naught we have already expanded in terms of this. So, this is what we are doing. So, if I write the expression here; continue with this expression, this is equal to the 1 plus M t upon M p plus M e by M naught.

Now, I have given you two home works so far; this is the third home work. Derive this expression. So, now, coming back to this equation or this expression, what do we see? We see that, the structural coefficient depends on what are the parameters. If we consider a particular mission – the tank mass and the engine mass surface because we have chosen that; then the structural coefficient essentially will be dictated by this term – initial mass is also dictated fixed. Therefore, what we see is that, the structural coefficient is a function of delta u and u equivalent. So, structural coefficient depends on how much velocity increment we want and what is the equivalent exhaust velocity. On the other hand, we have shown that, the equivalent velocity is a function of I s p. That we have shown before; that equivalent velocity is a function of I s p – specific impulse; and, specific impulse depends on thrust per unit mass fuel flow rate. So, is a function of the fuel that we are considering or the propellant that we are choosing. Therefore, this u equivalent – equivalent velocity then is a function of delta u and propellants. So, the structural coefficient also depends on the mission requirement and the propellant chosen to achieve this mission. So, this expression gives us that functional dependence. Let us now look at a practical case and see particularly how the structural coefficient depends on these two parameters.

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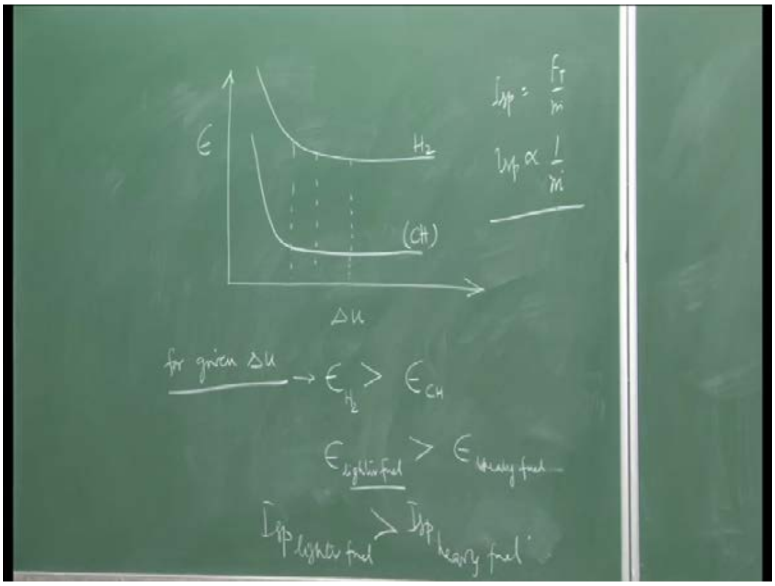

For that what I will do is I will plot the variation of structural coefficient as a function of delta u for two different fuels or two different propellants. Particularly, we will consider fuel as the… The major propellant is the fuel. So, let me plot structural coefficient along the y-axis and the velocity increment along the x-axis. Let us consider two fuels: one is a hydrogen fuel. If I plot the variation of delta u for hydrogen fuel, it is like this. It varies like this. This is for the hydrogen fuel. If I plot the same for a hydrocarbon fuel, there is a steep drop and then it is like this. This is a hydrocarbon fuel. So, this is what has been seen experimentally that, the structural coefficient variation with respect to the delta u, that is, the velocity variation. Or, in other words, if you vary delta u, how structural coefficient is going to vary for different fuels; that is what we have shown here.

Now, let us see that, what we see from this plot. You see that, for a given delta u, you look at anywhere, any location here. For given delta u, we see that, the structural coefficient is greater for hydrogen as the fuel compared to hydrocarbon as the fuel. So, for any given velocity increment, the required structural coefficient is going to be greater for hydrogen than hydrocarbon fuel. Now, hydrogen as we know is much lighter than hydrocarbon. So, essentially, what is this graph is showing is that, for a given velocity increment, the structural coefficient for a lighter fuel is greater than that for a heavier fuel. So, this graph shows that, the structural coefficient for a lighter fuel is going to be higher than that of the heavier fuel.

Now, once again, what is the significance of lighter fuel and heavier fuel? If u consider a lighter fuel, its mass flow rate is less. If we consider two fuels, which have similar say heating value: one is lighter, other is heavier; then the specific impulse defines us thrust per flow rate. Now, as long as you are adding the same amount of energy, the thrust is going is be the same if you have the same rocket design. So, if I look at the specific impulse, specific impulse is defined as… This is what the definition of specific impulse is. So, what we see is the specific impulse is inversely proportional the fuel flow rate. Therefore, a lighter fuel will be giving us higher specific impulse because for the same amount of same energy it is supplying, it is lighter or the weight is less or less amount is consumed; less mass is consumed. Therefore, we know that, the specific impulse for a lighter fuel is greater than the specific impulse for a heavy fuel. So, then if I look at these variations; if the specific impulse is greater, what happens to our equivalent velocity everything remaining same? Equivalent velocity is higher. Therefore, we can also conclude that, the equivalent velocity provided by the lighter fuel is going to be greater than that provided by a heavier fuel.

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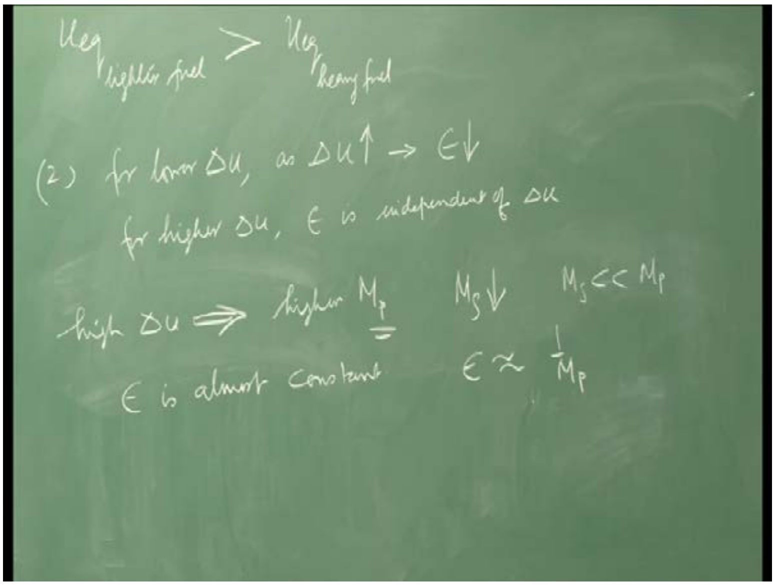

So, since specific impulse is higher, the equivalent velocity for the lighter fuel is greater than equivalent velocity for the heavy fuel. And therefore, now, what we have already discussed that, the structural coefficient is function of both the delta u as well as u equivalent. So, for the same delta u, we see that, the lighter fuel has higher equivalent velocity. Therefore, the structural coefficient is higher for lighter fuel compared to a heavier fuel. So, that is the physical explanation of this behavior. So, this is the first point. Let us look at second point here. What we see… If I take any of these graphs – any one of them, any particular fuel we are choosing; what we see is that, at lower value of delta u, when the velocity increment is less, then there is a decrease in structural coefficient as delta u is increased. But, beyond certain value of delta u, the structural coefficient becomes constant. So, let me write it down. For lower delta u, we see that, as delta u increases, the structural coefficient decreases. And, for higher delta u – higher velocity increment, structural coefficient is independent of delta u. Then it is a function solely of u equivalent as we have just discussed; this function of solely of u equivalent.

So, coming back to this, why is this? First, let us look at the higher delta u. How do we achieve a higher value of delta u? By burning more propellant for a longer period. So, higher delta u means higher propellant mass M P and since we are talking about the initial mass being fixed for all the cases. So, the initial mass is fixed. Also, we do not want to change the payload. Then where do we accommodate the higher propellant? Only way is by cutting down the structural mass. Therefore, at higher ((Refer Slide

Time: 28:33)) means, there is a reduction in structural mass. So, higher delta u can be achieved for keeping everything same by reducing the structural mass. And therefore, what we are seeing is that, primarily, the structural mass is much less than the propellant mass. Hence, since we are achieving this structural coefficient vary… rather the delta u variation by varying M P only, and the structural coefficient is typically inversely proportional to the propellant mass. Therefore, at very high values of delta u, epsilon is almost constant, because the marginal dependence decreases. We have already reached such a high value; we cannot change it further – structural coefficient further. So, the structural coefficient is almost constant.

And, once again coming to back to this discussion that, why for the lighter fuel, we have higher structural coefficient; what we see here is epsilon is typically inversely proportional to the propellant mass, because if you have more propellant, of course, the structural has to been ((Refer Slide Time: 29:59)). So, structural coefficient is less. So, again, coming back to this then if we take a lighter fuel, propellant mass is less. Therefore, the structural coefficient we can go to higher value. So, we can work with higher value of structural coefficient if the propellant mass is less. And, that is why we see this increase. Therefore, what we can conclude from this discussion is that, first of all, if we are operating with a lighter fuel like hydrogen, we can get satisfactory performance, that is, higher delta u at a higher structural coefficient. That is what this discussion show. However, if you are working with heavier fuel, then the structural coefficient is less in order to achieve the same delta u.

Now, what is the significance of that as far as the rocket designer is concerned? If you are allowed to have higher structural coefficient, you can make the structure stronger. So, the factor of safety improves. Therefore, the reliability of the mission or the robustness of the rocket improves if you are allowed to work with higher structural coefficient, because finally, the structure is the one that will have to withstand all the other loads – external load acting on it. Therefore, working with a lighter fuel is helpful for the designer to design, because they are allowed to design a stronger or heavier structure; and, that helps in improving the reliability of the rocket.

Second point here is that, beyond a certain value of velocity increment, we have to work with a fixed structural coefficient; structural coefficient does not change. However, we can increase the value of delta u. However, initially, as we increase the delta u, structural coefficient sharply decreases. So, the structure has to be made lighter and lighter in order to attain the higher velocity. But, once we are here, then it is independent of structural coefficient. Now, this point here that, for a wide range of delta u, typically, the structural coefficient does not change; it is something that we have to keep in mind, because later on, when we go to multistage optimization, this is something that we will be using that, typically, the structural coefficient becomes independent of delta u or any other parameter when we go to higher delta u values. So, this discussion shows us the dependence of structural coefficient on delta u.

Now, let us go back to the discussion that we had regarding the flight dynamics for various cases. What we are trying to find out is how much velocity increment that we can get. So, we are providing the vehicle with certain amount of thrust and the vehicle is flying, and it is getting certain velocity increment after certain burn out time. It is carrying certain amount of propellant, which is getting burnt and it is getting the velocity increment. What we have done during the discussion so far is we have considered different cases and made certain assumptions in those cases. Now, let us look at how these assumptions affect the final performance. So, what will be the effect of those assumptions in the final design? So, for that, let me draw the variation of delta v verses the thrust.

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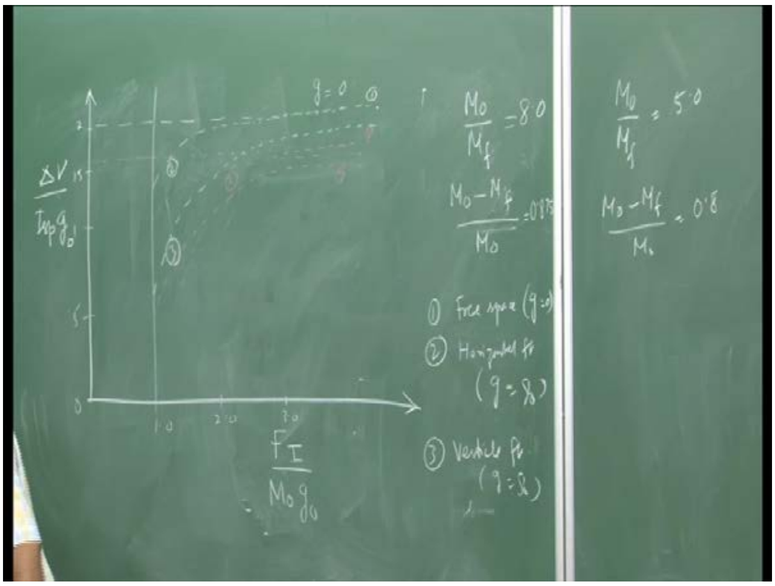

I will draw two curves. One is here. In the x-axis, we will have thrust divided by or non-dimensionalized by initial weight. So, thrust non-dimensionalized by the initial weight. Then we take any rocket with any initial configuration; it will be applicable. On the y-axis, we will take velocity increment divided by I s p g naught. So, this is the variation we are looking at. Let us for the sack of convenience, consider a particular mass ratio M naught by M f. Let us say this mass ratio is 8. And, let us also consider a payload fraction. This is payload fraction, because if I say M naught minus M f… No, this is a propellant fraction. So, M naught minus M f gives us the propellant fraction, which is equal to 8, 0.875. So, these are the conditions for which I am taking a single-stage rocket and I am plotting the variation of delta v essentially with respect to the thrust that I am providing.

So, first of all, let us consider the free space. For the free space, this is g equal to 0 case. Say this 1, 2, 3, etcetera. So, 0.5, 1, 1.5, 2, etcetera. So, what we are seeing is that, when we consider this particular rocket, once we have these values fixed, everything fixed, then the variation of delta u with thrust actually will be a constant line. Why? Because delta u is just u equivalent ln R. R is fixed here; u equivalent is this ((Refer Slide Time: 36:44)) this. Therefore, it will not depend on thrust. So, in the free space, no matter what amount of thrust we are providing, it is a function of only u equivalent and the mass ratios. Therefore, it will be a constant delta v; it does not depend on thrust. So, this is a very important observation in the free space outer space. And, that is why, in outer space, we do not need huge chemical rockets. What we need is something with higher I s p. So, that is, that will give us the required variation, because of the fact here, because in the outer space, we do not have acceleration due to gravity, we do not have drag. So, this is the case we are considering.

Then, the second case – let us say is for… This is case 2. So, case 1 we have said here is free space, where g is equal to 0. Case 2 – we consider horizontal flight when g is equal to g naught. We are considering horizontal flight. For horizontal flight, initially, as we increase the thrust, there is an increase in delta v; but, gradually, it reaches the free space condition asymptotically as we increase the thrust a lot, because what happens is if the vehicle is flying horizontally, the weight is acting vertically downward. If we keep on increasing the thrust, this essentially becomes insignificant compared to the thrust. But, initially, when the thrust is less, then it will have a tendency to bring it down or the delta u, which is we are in interested in this direction will come up come down. But, as we keep on increasing the thrust, thrust becomes much more than the weight; then it will not have that impact. Therefore, it asymptotically reaches this value.

Now, the third case we consider is vertical flight including… Now, this is the vertical flight, which is g equal to g naught again. If I consider vertical flight, it will go like this. When we are talking about vertical flight, g is always acting downwards, always trying to slow down. We are increasing the thrust that will try to compensate for the effect of gravity, but gravity is always present. Therefore, it will leverage this condition and it will always be less than this. But, as we again keep on increasing the thrust, the effect of gravity becomes less and less dominating. That is why we get this variation. And, the same thing if I look at for a different let us say ratios; like instead of this, if I look at another case; where, M dot by M f is 5 and M dot minus M f by M dot is 0.8. So, the same plots if I plot, then… Let me plot it on the same graph, but with a different color. We will see that, the free flight condition will be less, because now, R is less. Therefore, the free flight velocity is going to be less. At the same time, every other parameter now will be less. So, this is 1; this will be 2; and, 3 is even less – somewhere here. So, what we see is that, as R is decreased and the propellant fraction is decreased, there is an overall decrease in the velocity increment, which is essentially according to the equation that we have derived that, as the thrust is… The velocity increment will increase as R increases. Therefore, this is in agreement with what we have derived theoretically. Therefore, then again…

Since as I said, delta v is something that is the mission requirement, which will become apparent when we talk about space dynamics. Therefore, in order to achieve that, we need to choose a particular value of R. And, once that is fixed, we need to choose a particular value of structural coefficient that I have just discussed. So, if the mission is given; if the propellant is chosen, every other parameters will come up from this analysis. So, then that is what the rocket design is. That first should be the mass distribution of various components. And then we actually going to build it. But, the first step is to find out the mass distribution to achieve that mission. So, this brings us to the end of our discussion on single-stage rocket performance. But, before we stop, what I would like to do is I would like to solve a problem. I would like to solve a problem that will essentially focus on the single-stage performance that we have discussed so far. So, let me define the problem.

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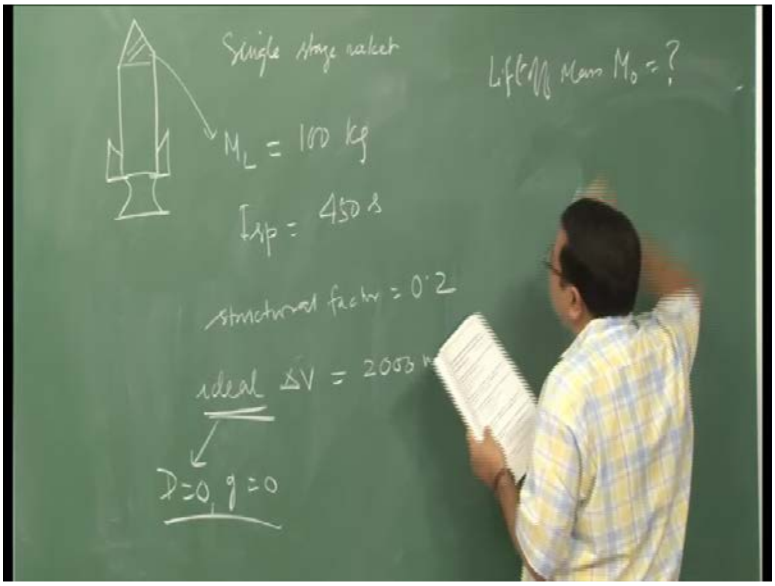

Let us consider a single-stage rocket. This is a single-stage rocket. Let us say that, this is the our payload. Let us consider that, the payload mass is 100 kg. Let us consider that, the I s p for this rocket is 450 seconds. And, let us consider that, the structural coefficient or the structural factor… I will define what is structural factor; it is 0.2. And, ideal delta v is equal to 2000 meter per second. Now, first of all, what is ideal delta v? Becomes evident from this plot. Ideal delta v is the maximum velocity that we can achieve. And, it should not depend on thrust. So, it corresponds to which case? g is equal to 0. So, ideal case corresponds to no drag, no gravity – the outer space. So, for this case, you are asked to find out, calculate the lift of mass, which is M dot. This is equal to what? That is the question. So, this is essentially what we have been discussing so far.

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So, let us now look at… What is given is M L equal to 100 kg. I s p equal to 450 seconds. Structural factor – let us say we call it s, which is equal to 0.2 is defined as M S upon M S plus M P. So, this is equal to M S upon M naught minus M L. And, we have to get delta… We have given that, delta v equal to 2000 meter per second. So, first of all, we know that, for this case, delta v is equal to u equivalent ln R. This we have proved. And, u equivalent is equal to I s p times g e. So, I s p times g e ln R is equal to M dot minus M f. So, first of all, let us look at this expression little more closely.

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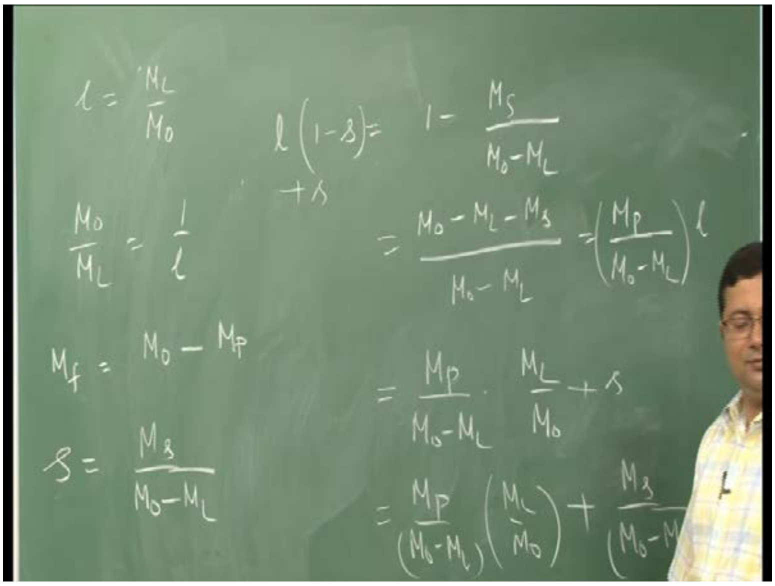

Let us say we define a parameter l, which is the propellant mass fraction L equal to M dot by M L. So, from here, we can write that… Just a second. M dot by M L equal to 1 by L. And, M f equal to M dot minus M P at… And s is… Let me do it from here; s equal to M S upon M dot minus M L. Therefore, let me first look at this one. Let me do 1 minus s. So, 1 minus s is equal to 1 upon M s – M dot minus M L. So, this is equal to M dot minus M L minus M S divided by M dot minus M L. What is this? M dot minus M L minus M S – M P. So, this is equal to M P upon M dot minus M L. Now, let us multiply this by l. So, let me multiply this by l. Then what I get is; this is equal to M P upon M dot minus M L into L is defined as M L upon M naught. Now, to this, let me add s. So, let me add s to this; plus s. So, this will be equal to M P upon M naught minus M L into M L upon M naught plus M S upon M naught minus M L. What we see is that, this is common in these two. So, after I simplify this, this will be equal to…

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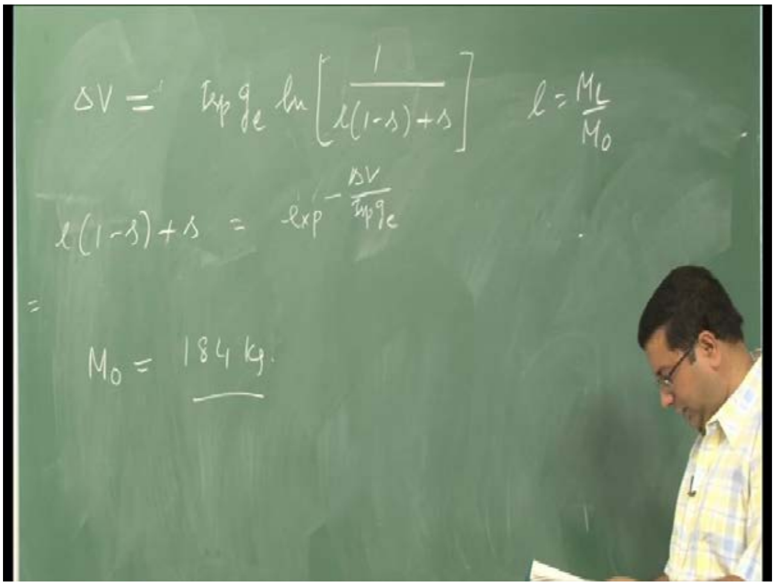

Delta v we have got like this. So, delta v will be equal to I s p g e ln 1 upon l 1 minus s plus s; where, l is the propellant mass fraction and s is the structural factor. Now, from here then we get l 1 minus s plus s is equal to exponential minus del v upon I s p g e. Once we simplify all these, we will get the overall mass is 184 kg. But, let me try it out little differently also. Let me try it out little differently. Since I am starting to do with this, this is the final solution, but I will do it little differently. Once again let us look at this ratio – M dot upon M f.

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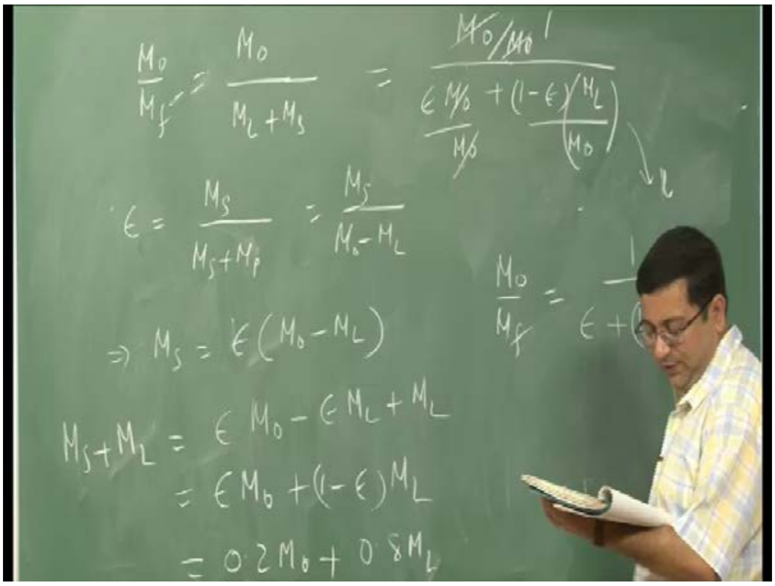

So, M dot upon M f is equal to M dot upon… Finally, what remains is the payload mass and the structural mass. And, we have defined epsilon as this M S upon M S plus M P. So, this is the epsilon; s is the epsilon. So, this is equal to M S upon M dot minus M L. Therefore, we get M S equal to epsilon times M dot minus M L. We can get this expression. So, M S plus M L is equal to epsilon times M dot minus epsilon times M L times M L. So, we are adding M L to this equation. Then this can be written as epsilon M naught plus 1 minus epsilon times M L. So, let us take this now and put it back into this equation. So, this will be then M dot upon epsilon M dot plus 1 minus epsilon M L.

Now, let us divide both of them by M dot. So, we get this is equal to 1 and M L by M naught is equal to L. Therefore, M dot by M f is equal to 1 upon epsilon plus 1 minus epsilon times l. So, now, let us take this; come back to this equation and put it back here. So, what we get is this is equal to… Instead of doing it here, we can also start from here right away. M L is given; epsilon is given. So, I can write this as equal to 0.2 M naught plus 0.8 M L. So, now, if I take this and put it back into this equation, what I get is… If I put this back into this expression for delta v, I get 2000, which is the ideal delta v equal to 450, which is the I s p times 9.8 acceleration due to gravity multiplied by ln M naught 0.2 M naught plus 80.

Now, in this equation, the only unknown is M naught; I can easily solve for this. So, after solving for this, I get M naught is equal to 184 kg. So, as we can see that, we get the expression that we have been looking for; and then we get the initial mass that will give us this velocity increment, which will put a 100 kgs payload to certain velocity increment. So, with this we complete our discussion on single-stage rockets.

Next class what we will do is; we will start with multistage rockets. We will define various parameters for multistage rocket. We will see what is the advantage of multistage rocket to begin with; why we need multistage rocket; and then what are the various parameters; and then how to optimize the performance of a multistage rocket. After that, then we will go into the space dynamics; that is, why do we need certain velocity increment delta v. So, that time… Then we will talk about the mission requirements. So, thank you very much. We will stop here today.