Jet and Rocket Propulsion
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Lecture - 8

So, in the last lecture we were discussing the vehicle dynamics for a rocket vehicle, and we had considered the equivalent exist velocity as the prime factor in moving the vehicle.

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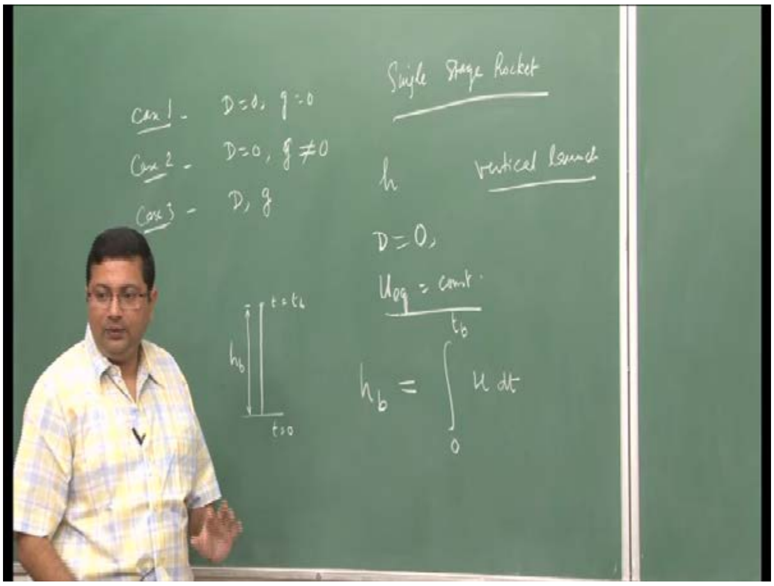

And, we derived expression for the change in the vehicle velocity for 3 specific cases; case 1 was where we have no drag and no gravity. Case 2 - we neglected drag, but the gravitational force were naught 0. And case 3 - we considered both drag as well as gravity. We have discussed these 3 cases in the last lecturer. Now, let us continue our discussion on the vehicle dynamics for a single stage rocket. So, let us now a talk about a single stage rocket specifically. What we want to do is; we want to calculate the height that is h to which a single stage rocket will rise, if we neglect drag and assume that effective velocity is constant. And now we are talking about a vertical launch. So, we will talk about vertical launch; we neglect drag and we considered that the equivalent exist velocity is constant.

So, this is typically what a sounding rocket will do by the way. If you recall that I have talked about little bit of history the first launches by ISRO, they are sounding rockets. Sounding rockets typically what they do is; if you fire the rocket there will be a certain stage till the propellant burns. Once the entire propellant is burnt then by that time it will reach certain height and then it will continue rising further, because it still has lots of kinetic energy. It will continue to rise further, and when this happen since, there is no more trust produced for the next of the duration. The kinetic energy will start decreasing, and potential energy will start increasing. A point will come when the velocity of the rocket will be zero, after that it will start to come down. So, the maximum height reached by this vehicle h max is essentially the combination of powered flight and the ballistic flight, where we have no trust produced.

So, this is what we are discussing now. So, let us consider first we are considering a vertical launch. So, we are considering a vertical flight; first let us look at the burn out condition h b. So, for the burn out we have let say we are launching from at time t equal to 0, and it reaches its maximum height like this. So, this is our launch at time t equal to 0 at time t equal to t b it reaches this height. So, this is equal to h b. So, if we considered an instantaneous velocity is u then the height is equal to u dt. Remember u is not constant, the vehicle velocity is not constant vehicle is accelerating. What is constant is exhaust velocity and we are considering there is no drag. So, now this u integral over u dt over time t equal to 0 to t b gives us the height attained by this vehicle during this flight signals.

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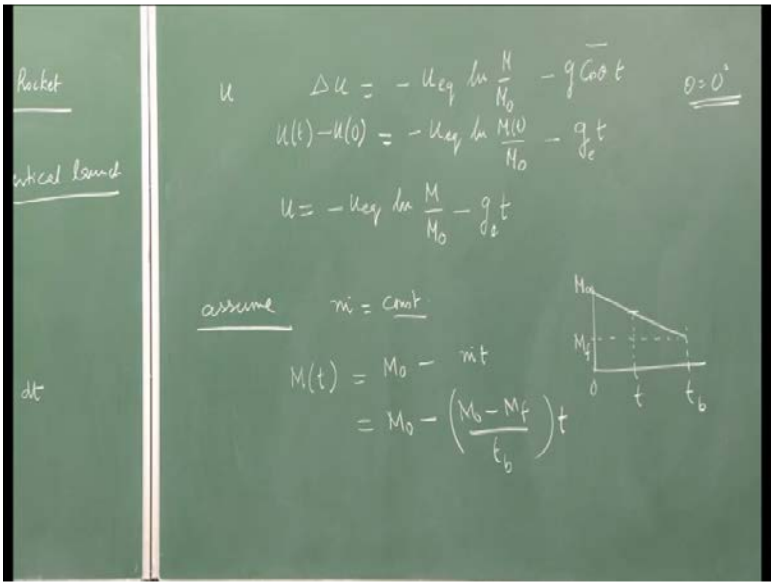

So, now let us first look at u. What is u? For the case that we are talking about no drag considering gravity is case 2 here, right. So, in case 2; we had got an expression for delta u is equal to minus equivalent l n M by M naught minus g cos theta t right, this is the expression that we have got for delta u is already integrated in time. So, now we consider at any time instant t the velocity is u. So, delta u is what is the velocity at time t minus velocity at 0 right, that is what our delta u is equal to minus u equivalent l n M at time t by M naught minus g here. Let say we are talking about a vertical launch from earth surface; typically these rockets will not go much far therefore, the acceleration due to gravity can be considered that equal to the gravity at the sea level.

So, therefore this can be written as g equal to g e and theta is 90 degree which is one. So, therefore this is equal to g e t. Sorry, we are talking about vertical flight so theta is 0 right. So, therefore theta is 0 degree in this case. So, therefore cos theta is 1, we get g e t. So, this is the expression for u, and we are starting from 0 speed. So, therefore u (0) is 0. So, we get u equal to minus u equivalent l n M by M naught minus g e t. This is the expression for instantaneous velocity. Now, we take this and put it back in to this equation. So, we can before we do that let us consider one more thing; we have to get an expression for M also, in order to do that; let us assume that the mass flow rate is constant, which means that m dot is constant.

So, the flow supply is been burned at a constant rate, or the propellant is been burned at a constant rate. Then, if that is the case; then the instantaneous mass M (t) is nothing but the initial mass minus the m dot times t right because that is the total time that it has reached. Now, m dot is constant which means that the slope is constant for the, slope of mass is constant between time t and t b. So, if we consider this is the variation of mass. Since, they are fuel is been consumed at a constant rise, at time t equal to 0 the mass was M naught, at time t equal to t b which is here the mass is M f, right. Then, from this triangle considering M naught to be constant, we can find out the mass at any instant of time t. So, let me say this is the mass at time t. So, which is equal to M naught minus the slope of this which is M naught minus M f by t b, this is the slope of this curve multiplied by the time t. So, this is the instantaneous mass. So, now what we can do is; we can put it back in to this equation.

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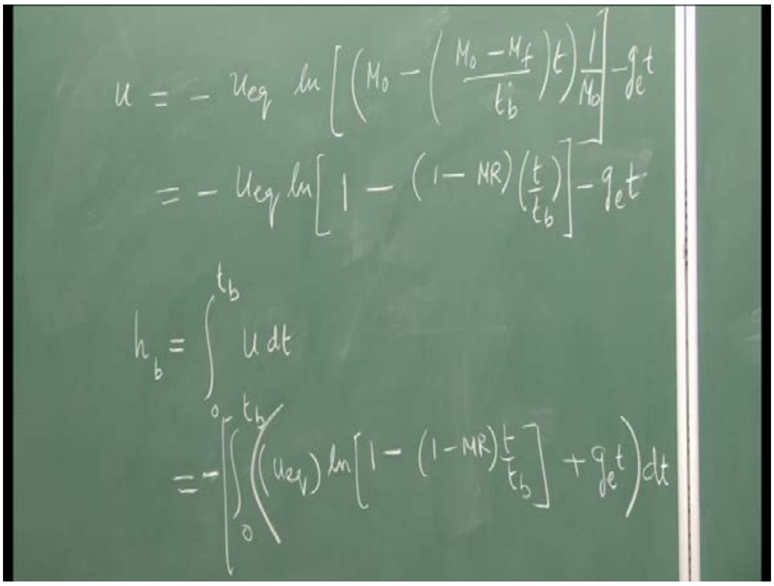

So, when we do that our expression will be. So, the expression for the change in velocity then; will be equal to or the expression for the velocity will be equal to u equal to minus u equivalent l n. Here, in place of M we write this. So, this is equal to M naught minus M d naught minus M f by t b times t, this is the instantaneous mass, whole of this divided by 1 upon M naught. So, this is the first term in this equation minus g e times t. So, this is the expression for instantaneous velocity. Now, what we can do is; this we can take M naught inside this bracket and we can write this as 1 minus. So, M naught divided by M naught is 1 here also M naught divided by M naught is 1 minus M f by M naught.

So, that 1 minus M f by M naught is mass ratio M r according to our definition, M R t by t b minus g e t. So, now what we have is let us have a re look at this equivalent velocity is constant. Mass ratio is something it is fixed; we know the initial mass, we know the final mass, if we define M naught as constant, and we know the burning time then t b is also fixed, g e is also constant. So, now what we have is u as a function of time only the only variable here is time, everything else has been constant everything else is constant. So, now let us take this equation and put in the expression for h b ok. So, this is equal to nothing but integral 0 to t b minus let me write it as minus equivalent l n 1 minus 1 minus M R t by t b plus g e t dt. So, this is the expression which we need to integrate now.

So, as we are considering equivalent is constant everything else is given now, this is very simple to integrate. So, if we integrate this; the final expression that you will get it before we get the final expression let me define 1 more parameter. Remember in the previous expression when we obtain delta u, we got delta u equal to minus u equivalent l n 1 by R 1 by M R right.

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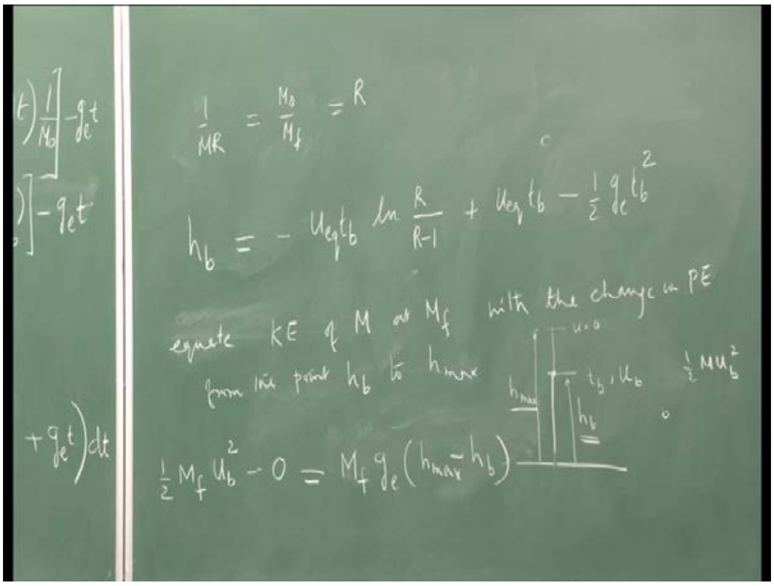

So, this one by M R term keeps on popping up one by mass ratio is nothing but M naught by M f, right. So, this is something that keeps on popping up again and again. So, instead of having work with this 1 by term, we define this as a new parameter R. Now, then this expression will be represented in terms of R. Now, after that let us integrate this and get the final expression, this final expression is equal to minus u equivalent t b l n R upon R minus 1 plus u equivalent t b minus half g e t b square. Where, R is given by 1 by R. So, this is the expression for the burnout height that we will be getting by integrating between 0 and t b. Still our work is not completed; burnout height is only up to the powered phase. Now, have to look at something beyond that after the power is completely fuel is completely used up, propellant is completely used up it was still continue to move. So, now let us see what will be the additional height it will gain in the ballistic phase.

So, what we will do is; let us equate kinetic energy of the mass M at M f that is the final mass of the rocket after the burnout with the change in potential energy from the point h b to h max. Let me explain what I mean by that; this was the power flight we reach this height h b at time t b, and at this point all the fuel is burnt. And, at this point the vehicle as this height h b and it has certain velocity let say u b ok. Now, because of this u b it has certain kinetic energy, and that kinetic energy is equal to half M u b square, right. Now, behind this the vehicle does not have the power to maintain this kinetic energy.

So, now what will happen is that; since, the thrust is not present this will start to decrease, and as we go up further, u b decreases till it reaches a velocity 0, u equal to 0. This is the maximum height h max, why is this decreasing because now, only this term is acting acceleration due to gravity and this is slowing it down because it acts downward. So, now in this during this slide acceleration due to gravity will slow it down till it reaches zero velocity, after that gravity still acting on it so it was start to come down. Therefore, this height reach is the maximum height, and that can be obtained by equating the total energy here which is the sum of kinetic energy and potential energy to the total potential energy, here by equivalent velocity we already talked about.

So, if we do that let us see now, what is happening the kinetic energy at the burnout point is half M f u b square right. And, the kinetic energy at the maximum height is 0. So, this is the total change in kinetic energy in the ballistic phase when there is no power, this is equal to the change in potential energy between this point and this point. And, that will be now; the mass of this vehicle now is only M f. So, M f times g e h max minus h b right. That is our change in kinetic energy the potential energy between these two points. So, if I simplify this expression then I get the expression for maximum height.

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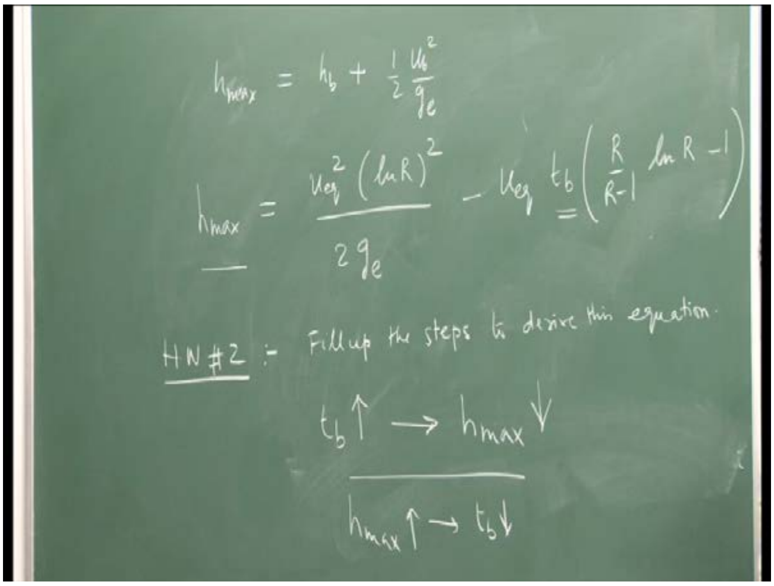

So, the h max term is equal to h b plus half u b square by g e right, it is coming from this equation, simplifying this equation. Now, we just combined this with this we get the total height h max is equal to u equivalent square l n R square by 2 g e where this is coming from the burnout velocity, plus the minus u equivalent t b R upon R minus 1 l n R minus 1. This is coming from this term right, as you can see here equivalent t b l n r by R minus 1 is here is coming from this term. So, this is the maximum height that will be attained by the vehicle during its flight. Now, once again some of the skips steps I have skipped, for example; the estimation of u b excreta. So, I will give them as homework home work number 2; fill up the steps to derive this equation. So, you fill up the steps to derive this equation. So, that finally, we get this expression.

Now, this is now our expression for h max. So, we have done the mathematics we have obtained how much it will go. Now, let us looks at some parametric dependence how the h max depends on the flight parameters. One thing that is important to notice here is that the height that will be reaching depends on t b, the burnout time. So, burnout time is an important parameter, and second point that to show here is that this term is positive u equivalent is positive. So, as t b increases h max will decrease because the effect of this term is to reduce the height. So, from this expression what we can see is that as the burnout time increases, the height that will be attained by the vehicle decreases.

This is something that is straightforward coming from this expression. Now, this is a important observation; ideally for any operation would like to maximize the height that can be attained. So, what this expression shows that; in order to get higher height we have to have a smaller t b. In other word; if you want to increase M h h max we have to decrease t b, this is quite evident from this equation. So, hence it is required and desired to reduce the burning time as much as possible, in order to meet emission requirement.

However, the reason is by the way; why do we say that we will be getting advantage if you burn faster? It because what is happening is that; the higher height will be attained if we have to lift less weight, because we are producing same amount of energy. If you are burning faster the rate of reduction of mass is also more, therefore as the burning rate increases there is a faster decrease in mass. So, the vehicle is becoming lighter in a faster way lighter faster, and because of that it will attain a higher height. So, therefore a short burning reduces the energy consume in simply lifting the propellant right. This is the reason why short burning time is preferred. However, there are some practical problems associated with very short burning time.

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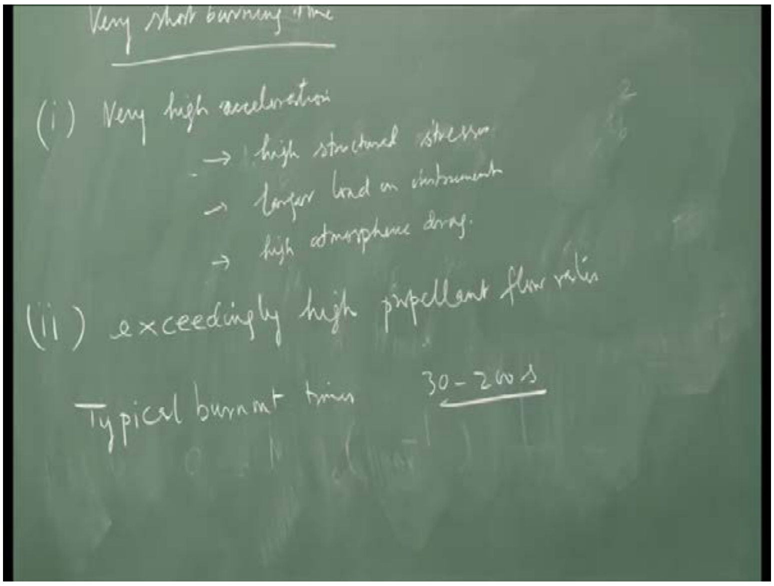

If we consider very short burning time then what are the problems first; burning time is how much time you are having for the power flight? You reduce the burning time we are moving faster attending the burnout height faster means we are accelerating more, right. So, shorter burning very short burning time is very high accelerations. So, this is very high acceleration and acceleration is not good for the structures, because acceleration impact load. So, there can be severe stresses on the structures, and also the instruments essentially we are increasing the g level right, the perceived acceleration is the g level. So, if we are accelerating at a very high rate you are putting lot of load on the structure as well as on the instruments first point. Second point is we have discuss the drag right, the variation of drag in practical situation; when we are lifting from ground we have seen that the as the velocity increases the drag will increase.

Now, if we are moving in a very faster there is a massive acceleration velocity increases faster therefore, the increasing drag is also faster. So, there is higher atmospheric drag acting on the vehicle. So, very high acceleration will lead to first of all high structural stresses, larger load on instruments. This is the structural load I am talking about or acceleration load and higher atmospheric drag. All this essentially puts a limit to the burning rate that we can achieve, because we do not want a very high structural this is the structure and buckle may fail. We do not want to have lots of loads on the instrument because instrument will not perform the way they are suppose to if they are subjected to very high loads or acceleration.

We do not want very high atmospheric dry because it wills slowdown the vehicle, right. So, therefore very high acceleration because of the short burning time is not something that you want to have in practical systems, this first point. Second point; how do we achieve the short burning time? We are carrying certain amount of propellant with us, if we have to burn it burn that at a shorter time we have to make it flow at a faster rate. So, shorter burning time means faster propellant flow rate right. So, therefore a very short burning time can be exceedingly high propellant flow rate. So, high propellant flow rates now, that can be limited by your hardware, because first of all; how much you propellant is you can pump in depends on how much capacity of pumping is available with you, right.

So, therefore the size and capacity of the machinery needed to provide such high flow rates may be a limiting factor. Similarly, for solid propellant rockets the burning rate depends on the chemistry or the chemical composition. So, that is for a given propellant fixed you cannot change it. And, for liquid you can change it, but the point is as the liquid propellant the problem is that as we can as we go to liquid propellant higher flow rate then the amount of propellant required will increase. So, the pump and other system have to work more you require a bigger machinery to pump at a higher flow rate, which may limit the maximum flow rate that can be attained. So, because of this even though this equation shows that if you have a shorter burning time you can attain a higher velocity or a higher altitude by attaining high velocity at the burnout at the end of burnout time.

There is a limit of how fast can be burned. So, the typical limit; typical burnout times that is attained in practical systems are between 30 to 200 seconds. Now, the maximum stage will be burning not more than that because beyond the less than this 30 second. If you have to burn this will be something that cannot be handled. More than 200 second will not reach enough height, because the burning is so slow. Essentially what happens is that you do not have enough acceleration. Faster burning means higher acceleration we are reaching the exit velocity at a shorter time. So, faster burning provides us the higher acceleration which will take it further, slower burning reduces the acceleration therefore, it will not go very high in the ballistic phase.

So, what we have seen here is that typical burning time is limited between 30 and 200 seconds. One more thing that we can I have to point out here is that; in the absence of drag and gravity, if you are operating in the outer space then delta u will not be dependent on t b, because delta u turn will only this. It does not have this term present because if gravity goes to 0 the time dependent goes off. So, only u equivalent l n R right, so in the outer space. So, this is the expression that we have obtained first for the burning velocity. Once again the effect of burning velocity is failed if and only if we have the drag or gravity, but if you do not have drag or gravity under circumstances.

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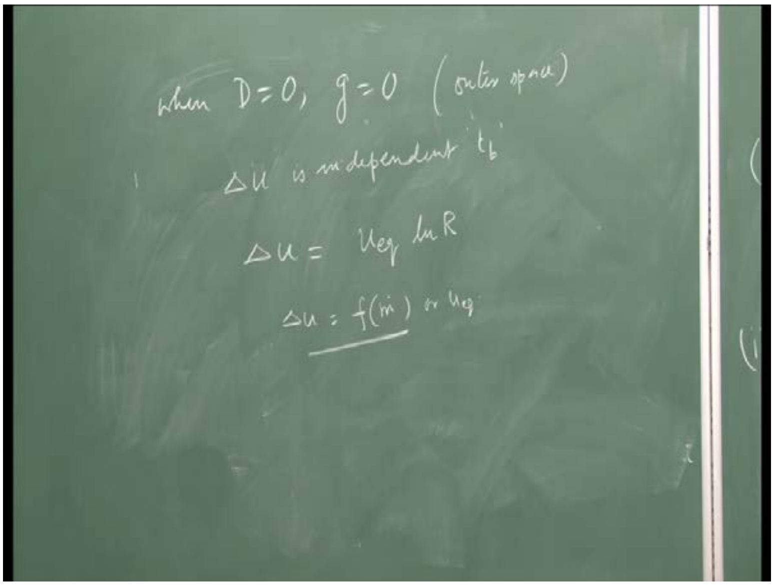

So, when D equal to 0 and g equal to 0 this is typically in the outer space, actually g is not zero, but very small. In outer space we see that delta u is independent of t b. This is very important, it is the independent of the burning time, because of the fact the delta u under circumstances will be u equivalent l n R. In that case; delta u is a function of only the mass flow rate or u equivalent. So, in the outer space delta u does not depend on the burning time. Now, if you have a vehicle moving at a very high speed in outer space, we need very small acceleration you have to maintain the speed or increase the little bit that is why electric propulsion systems are useful there. Because we can just small amount of energy can be used and that will give the increase. So, we can give that small amount over a period of time, you do not have to give it together. The small amount of increase in the period of time and slowly it can build up and go to the every high velocity like typically is a maneuver to change from one orbit to another orbit it takes about 24 hours.

We have that window to move it from one orbit to another orbit a prolong period of time, you can do it slowly by providing small amount of thrust and getting from one altitude to another altitude, because you do not have the dependence on t b, right. And, that is possible only in the outer space, because it depends only on this factor nothing else. So, this is a very important observation for rockets. Now, with this we come to an end of our discussion on the vehicle dynamics for single stage rockets. But some more parameters are like to defined at this stage which will be useful in the next topic, where we take the multi stage rockets. And, after that what we will do is we look at the practical performance of single stage rockets. How the practically the performance depends on various operating parameters. So, here we have defined the mass ratio, we have defined the inverse mass ratio. Let us define some other parameters as well.

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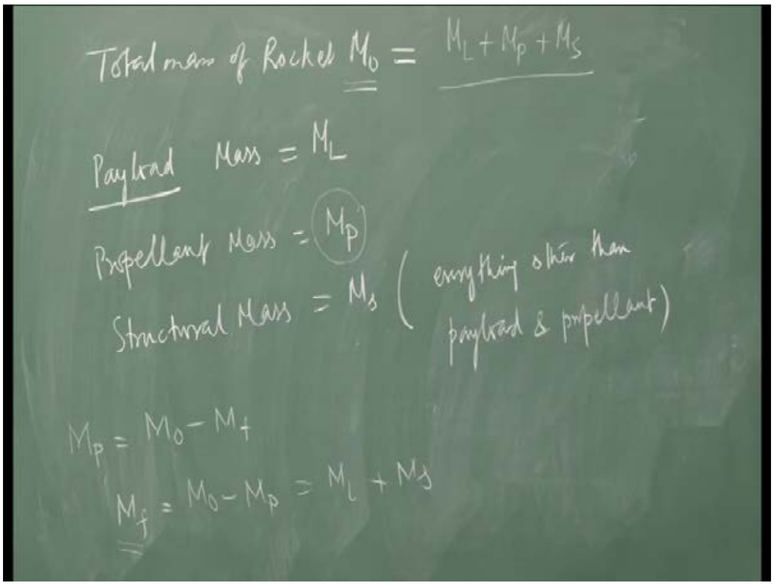

So, the total mass of rocket let say M naught; essentially is sum of various components a rocket will consist of the payload. So, first of all the payload is the most crucial and critical thing all our effort is actually to deliver the payload to certain location. So, the most important mass is the payload mass. So, first of all let us talk about the payload mass, let us defined it has M L most important mass. Other now, in order to put this payload; we have to provide certain acceleration certain velocity and that can be achieved with burning the propellant only. Therefore, the next important mass is the propellant mass.

So, next is propellant mass. So, let us write it has M P; and now the payload and the propellant all of them have to be put together they have to move together. So, you need an external structure to carry all of them. Then, the third important thing is the structural mass and further more this structure has to withstand all the external forces that is going through all the loads that are acting the structure have to withstand. So, that is also a very important factor. Now, we can have further sub classes of structural mass, but let us just focus on this three. Because payload is something that we want to deliver propellant something we use up. So, what we say is everything else, let us say is the structural mass.

So, then every other mass we put it under this category of structural mass, which we include everything other than payload and propellant. That is it will include the casing it will include the motor it will include the pump it will include the instruments everything is part of the structural mass. So, therefore the total initial mass now is the sum of all these. So, total initial mass is M L plus M P plus M S. So, total initial mass of the rocket is the some of the payload mass propellant mass and structural mass. And, now after the flight is over sorry, not after the flight is over after the propellant is burned out, the final mass is what remains. What remains after the propellant is burnt out is, this is gone only this and this.

So, we have seen that M P is M naught minus M F or the final mass is M F equal to M naught minus M P. So, this is equal to then the payload mass and the structural mass. So, our final mass is the some of the payload mass and the structural mass. And all the propellant has been used up.

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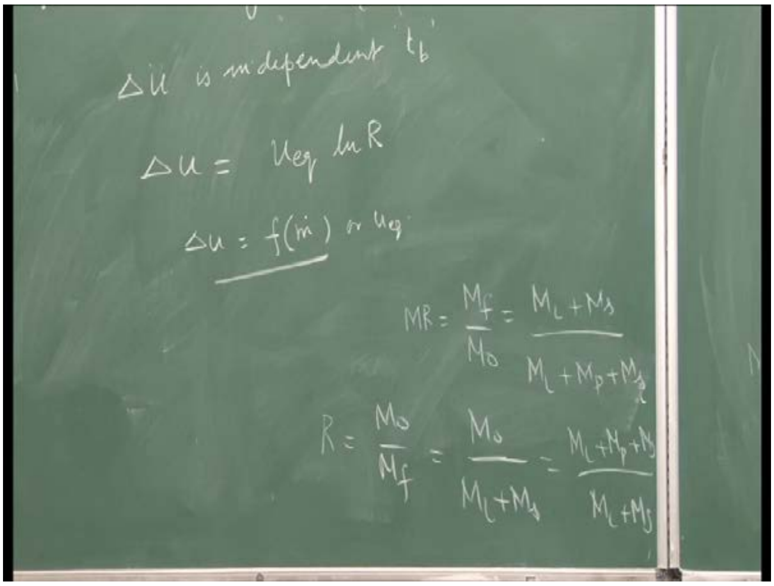

So, therefore, this and this if you combined them, we write M R which is the mass ratio as we have defined this as M F by M naught is equal to M L plus M S by M L plus M P plus M S. We can define this mass ratio like this, or we can also change it little bit we have defined R; we have said that R is inverse of M R. So, R is equal to M naught by M F. So, we can write this as M naught by M L plus M S. So, R is equal to M naught by M L plus M S can also be written as M L plus M P plus M S by M L plus M S. So, R can also be written as then 1 plus R can be written as then 1 plus M P upon M L plus M S right, we write R like this.

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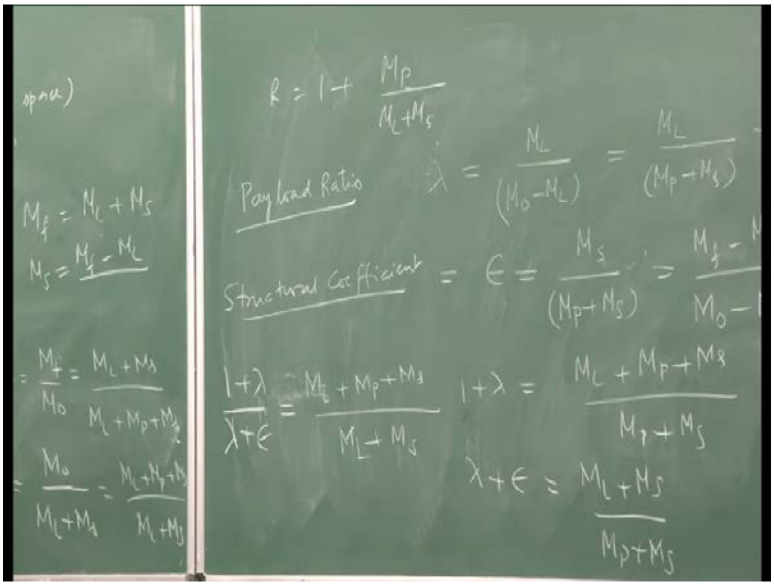

Now, we defined a new parameter which is called payload ratio. So, very important parameter let say we express it as lambda. Payload ratio is defined as the payload mass divided by the initial mass minus the payload, which is all the mass except the payload mass, and so this is the fraction. So, if I write it like this is M L and this will become because this is equal to M L plus M P plus M S minus M L. So, this is equal to M L plus M P upon M S. So, the payload ratio is the ratio of payload mass to the ratio of the sum of structural and propellant mass. This is something is a very important parameter, and typically would like to have large payload ratios. Because would like to put up as much payloads as possible with as it will expand each other propellant are structural weight as possible. Therefore, this is something we want to maximize; we want to get as much payload ratio as possible for economical operation. So, we have defined the payload ratio.

Next, let us define one more parameter which is called structural co-efficient. Payload ratio is a measure of how efficient with the payload is delivered structural coefficient on the other hand is the estimation of the structural efficiency. So, structural coefficient is represented by the term epsilon, which is defined as again the structural mass divided by everything else, but the payload. So, it is given as M S upon M P minus M S. So, here both in these definitions the denominator is M P plus M S that is the propellant mass and the structural mass. So, this now can be written as; if I look at the definition of the final mass M F, we have seen that M F is equal to M L plus M S. Therefore, M S can be written as M F minus M L right.

So, this can be written as M F minus M L divided by and mp plus M S is equal to M naught minus M L overall mass minus the payload mass. So, this can be written as overall mass minus the payload mass. So, the structural co-efficient is defined like this. Structural coefficient is the measure of vehicle designers skill in designing a very light tank and support structure. How efficiently you can make it and also it depends on the choice of material for making the system. Typically, if you are using metals it will be heavier now a day's people use composites which are much lightweight, and can withstand higher load with that the structural coefficient has been reduce substantially.

Structural coefficient something we do not want to have very high value, we want a small value of structural coefficient. Payload ratios you want have very high value, ok. So, these are the two parameters which are very important as we will see as we go along. Now, if we combine this 3 expressions R lambda and epsilon; we will see that R is equal to what is the R here M L plus M P plus M S by M L plus M S. One if I do 1 plus lambda, if I had 1 to this then the numerator becomes equal to M L plus M P plus M S divided by M P plus M S. Here, if I had one it becomes M S plus M P plus M S. Sorry, no if I add this and this, this because M L plus M S divided by M P plus M S. So, let me first do 1 plus lambda, 1 plus lambda is equal to M L plus M P plus M S divided by M P plus M S. and, lambda plus epsilon is equal to M L plus M S divided by M P plus M S.

Now, if I do 1 plus lambda upon lambda plus epsilon then this is equal to this divided by this. So, M P plus M S that is denoted both the expression will cancel of we have M L plus M P plus M S divided by M L plus M S. So, if I write it here, M L plus M P plus M S divided by M L plus M S. and, that is exactly what is R? by our definition; therefore, this is equal to R. So, what we have proved is that R is equal to 1 plus lambda upon lambda plus epsilon keeps this definition in mind, when we go to multistage rockets. We will be using this very extensively, and these expressions are also use for optimizing the rockets ok. So, with this we come to an end of this discussion. Next time we will talk about the actual performance of single stage rockets, we will discuss some performance parameters, how they vary with certain operating conditions and after that we go to multistage rockets.

Thank you.