Jet and Rocket Propulsion

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Lecture - 5

So, welcome to this lecture on rocket propulsion. Today we will be discussing rocket vehicle dynamics. Now, before we do that, let us kind of recapitulate what we have discussed so far.

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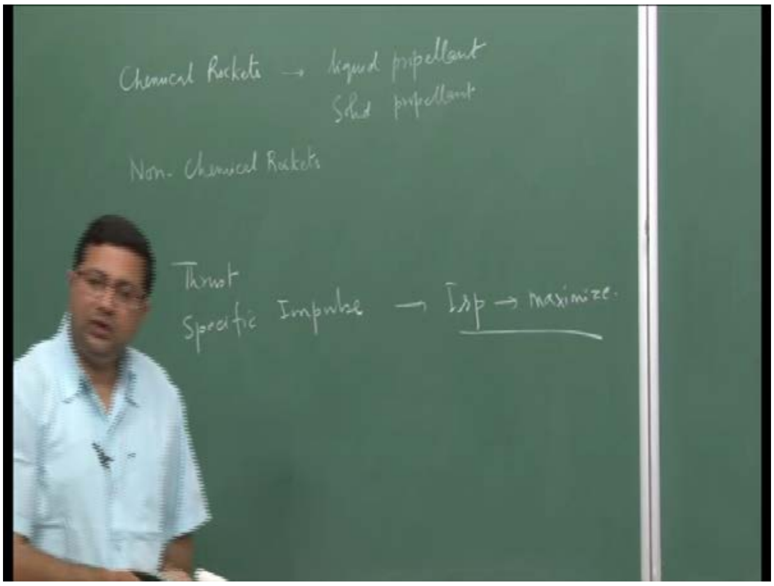

We have talked about history of rocket propulsion and then in the last lecture we talked about chemical rockets, which included little bit of discussion on liquid propellant rockets and solid propellant rockets. After that we talked little bit about non-chemical rockets as well, which were either thermal base or electric propulsion systems. We had little bit of discussion on all of them. After that we started deriving expression for the rocket performance parameters; we derived an expression for the thrust and specific impulse.

Specific impulse, as I have said in the last lecture, is a very important parameter as far as rocket performance is concerned, because of the fact that it is the measure of how much thrust is produced per unit consumption of fuel. So, ideally for any practical application, what the designer would like to do, is to produce maximum thrust out of burning minimum amount of fuel. Therefore, any machine requirement, we would like to maximize the specific impulse. So, specific impulse, Isp, we would like to maximize.

Now, I have also discussed yesterday, that the choice of rocket depends on the mission requirement. Therefore, every type of rocket has his own specific impulse and they are used for different missions. So, now, before progressing further let us summarize this different type of rockets with respect to their specific impulse and their limitations.

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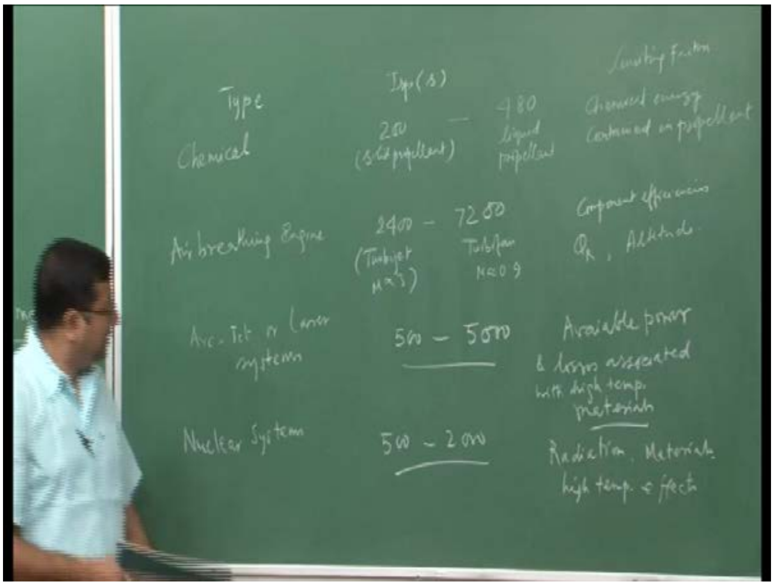

So, first let us look at type. We will discuss the chemical rockets. Then, we will discuss also air breathing engines in this. Now, here I would like to point out one thing, once I come to air breathing engines, then let us say arc jet or laser base systems and nuclear systems. Let us discuss this four type with respect to their specific impulse in seconds.

Typically, for chemical rockets, for solid propellant rocket, the specific impulse is about 200, pretty low, 200 to 250. Whereas, in the other side of the spectrum, for liquid propellant, particularly for cryogenic, it can be as high as 480. So, this is for the liquid propellant rocket. So, this is the range of specific impulse we can obtain for chemical rockets.

Now, let us look at the limiting factors as well, as far as the applications are concerned. Chemical rockets, as we have discussed yesterday, both liquid propellant as well as solid propellant, essentially is limited by the chemical energy contained in the propellant. So, every propellant has a maximum chemical energy associated with it. So, we cannot possibly extract more energy than that. Therefore, the specific impulse is limited by that factor.

When we come to the air breathing engines, they have very high specific impulse as compared to rocket engines. Their range between 2400 to 7200, where 2400 is typically for turbojets, at say, Mach number equal to 3, and turbofan at Mach number equal to about 0.9, it will be about 7200.

Now, if we have such a large amount of specific impulse, why we do not use them for rockets? Primarily, the specific impulse is high because the main propellant is atmospheric air, which this engine need not carry with it. It ((Refer time: 06:07)) the air and pushes it out. So, what amount of fuel, that has been, the amount of weight variation it has been considered in this specific impulse variation is only the weight of fuel, which is a very small fraction. Typically, for jet engines the fuel to air ratio is about 0.2, right. So, small amount of fuel is required because the main propellant is air, that is why, the specific impulse is very high. However, for the same reason it cannot be used for rocket propulsion because the main propellant is air. So, when air is not present this cannot be used.

As the name suggests, this is an air breathing engine. Therefore, when the specific impulse is high it cannot be used as a rocket. So, the performance is limited by, of course, component efficiencies as well as the heat contained in fuel and altitude because as we go up, high up in the altitude, the atmosphere becomes thinner and thinner and then the amount of air available reduces and then it cannot be used towards the ((Refer time: 07:30)) of the atmosphere; it is very, very thin. So, therefore, air breathing engines, conventional air breathing engines, therefore, are not much suitable for outer space application.

But to add something more to this air breathing engines, air breathing engines are being used as per missile applications, particularly for cruise missiles, those are ramjets. Ramjets are used as cruise missiles, that is, the air breathing engines. So, efficiency is very, very high; the specific impulse is high. But again, the ramjet performance is also limited by the flight Mach number and the component efficiency.

What happens in the ramjet engine is, that the vehicle has to fly at a very high speed - Mach 4, Mach 5. So, this high speed air, then is slowed down in the intake to about Mach 0.3. This slowing down process actually gives rise to the pressure because of the ram effect and that is how the high pressure required for the propulsion is obtained, as we have discussed in the last class, however. So, therefore, this ramjet engine cannot start from rest because it has to have the high Mach number.

On the other hand, the limiting factor is also this Mach number because as we go high up in the Mach number, there are shock waves for machines; there are losses across the shock wave. Secondly, the air is decelerating to a large extend, from Mach 5 to Mach 0.3. There is a massive deceleration and deceleration process is not very efficient, so there are huge amount of losses in the intake. So, the intake design becomes difficult and that adds to the development cost. So, but even then they are good as missiles, but again they cannot be used as rockets and because of the slowing down process we are losing some energy.

On the other hand, there is, another variation of ram jet is scramjet, supersonic combustion ram jet, where air did not be slowed down to such low Mach number 0.3 or so but it can be maintained at relatively high Mach number in a supersonic speed within Mach 1 and 2. The catch word here is supersonic combustion because that the combustion has to occur at supersonic speed, only then it can be called supersonic combustion. So, therefore, the combustion has to occur at a very high speed flow.

So, then you need not slow down the incoming air, that much you maintain lot of energy, kinetic energy of the incoming air energy as is little more by adding chemical energy and then let it exhaust. So, therefore, supersonic combustion ram jet will be efficient up to Mach 5. If we have to go beyond that you have to go to scram jet up to Mach 7, Mach 8 scram jet can go. But when we talk about rockets, they are extremely high Mach numbers, can be as high as 30, typically reentry vehicle comes in about 30 mark. So, therefore, up to mark 10 using, 7 not 10, is a 7 or 8 using scram jet we can go, but beyond that we cannot go with the, without having a rocket propulsion. And this scram jet and ram jet we can go to this high Mach number because of the fact, that they do not have any turbo machinery, they do not have any compressor.

While we come to the turbojet or turbofan because of the presence of compressor at very high Mach number, there will be shock wave permission on the compressor blade and that will lead to huge losses, massive losses, plus the shock exerted forces on the blade was also very high. It may lead to failure of the blades, therefore this engines are very, very inefficient at high Mach number, but at the lower mark number they are very good.

As we can see, the specific impulse is very high. If you come to the other alternative, non-chemical sources like R jet or lasers, there have a range of specific impulse ranging between 500 and 5000. Again, that is also very good compared to chemical rocket because they start where chemical rockets end. As you can see, about 480 chemical rockets, there they started about 500, then go up to 5000. So, this is the best combination possible, kind of almost overlapping. Therefore, this is the combination is actually used where we have the electric propulsion systems and the chemical rockets. So, this is something which is used because electric propulsion systems are also included in this category.

The limit, limiting factor for this, the R jet or laser, are essentially the available power and losses associated with high temperatures. As I have said, for example, the R jet, R jet requires very high voltage. You know, that to produce that high voltage you require very high power. So, that is the limiting factor. Secondly, the, secondly the arc can go to very high, can be at very high temperature, up to about 12000, 13000 kelvin. So, there are huge temperatures and because of this high temperature there are losses associated with the high temperature because there is always be a thermal flow from high temperature to low temperature region. So, therefore, there is losses associated with high temperature, is a limiting factor for this type of devices.

Other problem is the materials. The materials have to withstand this high voltage, high density of electrons, then high temperature, etcetera and therefore material becomes an issue. And also insulation, spot of the material is insulation is also because this need to be insulated. If you are having such a high voltage source within your satellite where there are lot of electronic instrument, if this is not very well insulated, we are going to fry up the entire satellite immediately. Therefore, the insulation also becomes a very important issue, issue in the use of R jet or laser thrust systems.

Nuclear systems, on the other hand, again has relatively high specific impulse, but not as high as the R jet or laser electric propulsion systems. So, their range between 500 to 2000, the limiting factors for nuclear, as I have discussed yesterday, is primarily the radiation. Then, of course, the materials because of the high temperatures and the high temperature effects, because in the nuclear systems also the temperatures are expected to be very high. So, that also limits the application of this systems. So, with this is the end of summary of all type, different type of rocket systems or space propulsion systems that we can talk about. Now, this brings us to the end of the first chapter.

Now, let us look at the vehicle dynamics. So, the next topic we are going to talk about is vehicle dynamics. Here we will consider a rocket vehicle and first we will identify different type of forces acting on them. Then, we will follow two approaches to get to the same equations. It show the commonality between this. First will be a ((Refer time: 15:24)) approach where we will just do force balance, identify different type of forces from first principles and use that to get the equation. And secondly, we will consider the mass of the propellant going out of the system separately, take care of that in the estimations and from there we will derive the same equations.

The equation we are trying to derive now here is the velocity increment. When the rocket is operating how much increase in velocity we can get because that is going to decide how much height the rocket is going to get or how far it is going to, to go or how much time will it be required to burn the rocket completely. So, therefore, velocity increment is the very important parameter. So, the first approach, that we are going to follow is an instantaneous approach where we consider the rocket at a given instant of time and from there we start.

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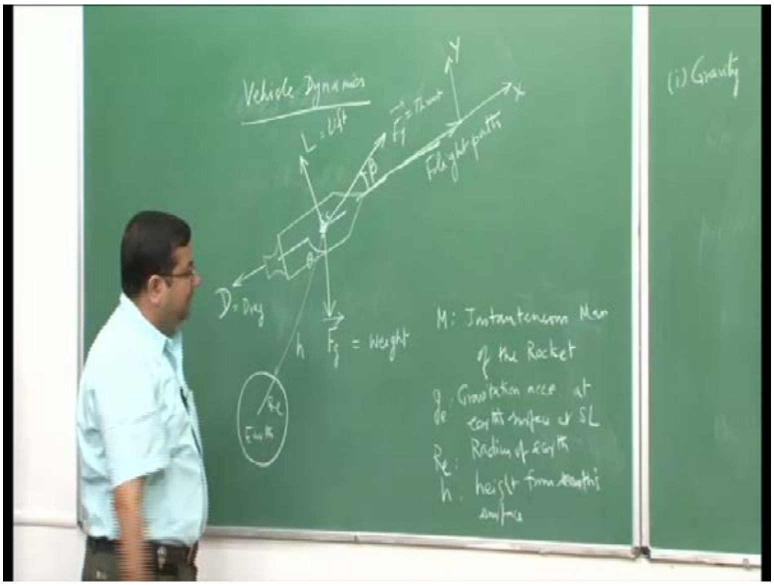

Now, what we are going to do is we are going to talk about vehicle dynamics. Let us consider a rocket vehicle in a flight. This is the axis; let us say that this is our flight path; this is the direction of flight. And let us consider a reference frame, like this is x direction, this is y direction. Now, let us identify different forces acting on this rocket.

Let us say, this is the center of gravity of this rocket. So, the different forces acting on this rocket when it is flying is, first of all its weight F g acting downward. And this is a vector, so let me describe it as a vector. Then, we will have lift force acting normal to the flight path at the flight direction given by l. And let us consider, that the angle that the flight path makes with the gravitational for the vertical direction is theta. This angle that is made by the flight path l is the lift acting normal to the flight path.

In the direction opposite to the flight will be the drag acting because drag will try to slow down the rocket and it will act in direction opposite to the flight path. So, drag is given by D. So, here this is the weight of the vehicle, this is the lift force, this is the drag force acting on it. Now, the rocket, let us consider is in the cruise state. So, therefore, there is some thrust, which is being supplied. So, this thrust will be acting in the direction, forward direction, of course.

Now, if you recall, last, in the last step, couple of lectures back we have discussed, that in order to provide stability to rockets the thrust is slightly effected. So, let us consider like that, the slightly effected thrust.

So, the thrust is not considered to be acting like this in the, flight, direction of flight path, but in a slightly effected way F t. So, this our thrust and let us consider that the angle that the thrust vector makes with the flight path is beta. So, this is the free body diagram of the rocket. Now, let us consider the different forces separately and then we will put them together to get the velocity increment.

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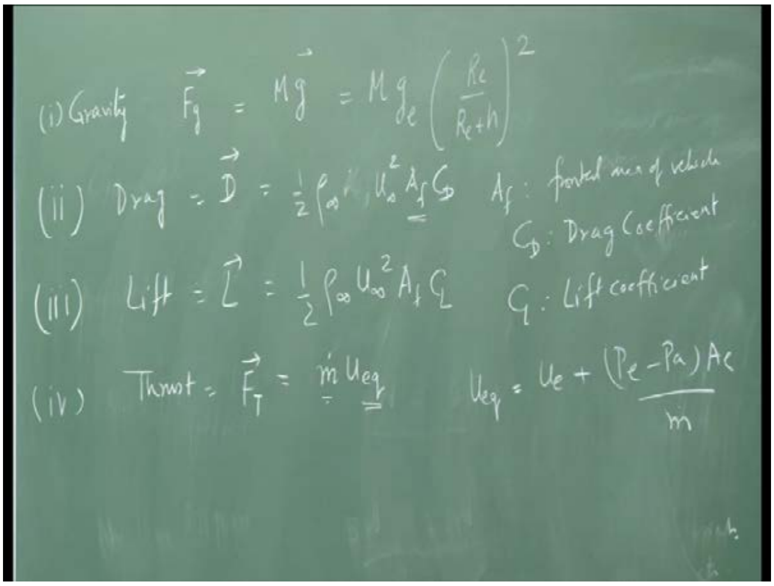

So, first let us talk about the gravitational force. First, let us talk about gravity, which in this diagram is represented by this F g or weight. So, when we talk about the gravitational force with respect to R theta, call it the weight, but if you are talking about the celestial body, still the gravitational force may not means the weight, but is the total force acting on it because of the gravitational forces. So, we will consider that as F g. So, then from Newton's law of gravity this is equal to mass where mass M is the instantaneous mass. So, let me write it here, M is the instantaneous mass of the rocket, M is the instantaneous mass of the rocket, and g is acceleration due to gravity, gravity, g is a vector.

Let us go a step further and break up this g into the gravitational acceleration at the earth's surface and then thus effect of height, ok. Here, g e is gravitational acceleration at earth surface at sea level, R e is the radius of earth and h is the height from earth surface. So, essentially, what we are saying is, this is the earth, this is R e and this is h. So, this is the schematic of the things we are discussing. So, M is once again instantaneous mass of the rocket and the on-boat fuel, it contains the on-boat fuel as well instantaneous mass.

Now, the next force, that we will be considering is the drag force. So, let us say drag is given by D here and drag is given by half rho infinity A infinity U infinity square A f. So, this expression tells us, that the drag depends on rho infinity, which is the density of air or the medium around it and the density at the point, sorry, U infinity is the flight speed or vehicle speed. So, let us say the vehicle is at that instant moving with speedy U infinity in a steady atmosphere and A f is the frontal area of vehicle, orbited area of the vehicle, which applies the drag force.

Then, the third type of force acting is the lift, which is normal to this, normal to the flight path. So, lift is equal to L, this is half rho infinity U infinity, oh forgot to write C D, drag coefficient. Yes, C D will also come in here, which is the drag coefficient and half rho U square A f C L, where C L is the lift coefficient. So, this are the three external forces that are acting on the vehicle.

And then apart from that the fourth force is our thrust. So, fourth is thrust, which is given by F T. Now, yesterday we have derived that expression for this thrust, which is nothing but m dot U equivalent when M dot is the mass flow rate going out of the engine and U equivalent is the equivalent velocity. So, we have derived this expression; we have derived this expression yesterday. So, then this gives us the expression for the thrust.

So, we are considering, that the flight path is at an angle theta relative to the gravitational vector here. So, theta is the angle between the flight path and the gravitational vector and we are considering, that the thrust is vector by this angle beta relative to the flight path now with this description of all the forces.

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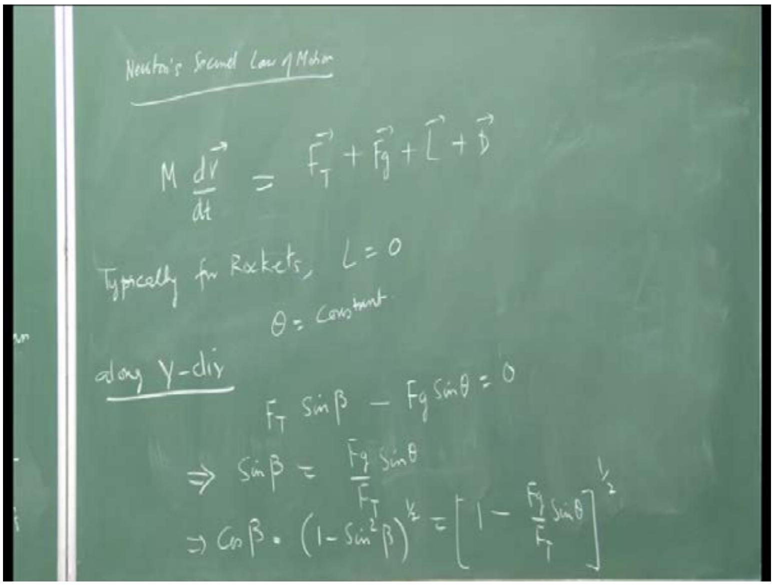

$$M \frac{d\vec{v}}{dt} = \vec{F_T} + \vec{F_g} + \vec{L} + \vec{D}$$

$$F_T \sin\beta - F_g \sin\theta = 0$$

$$\Rightarrow \sin\beta = \frac{F_g \sin\theta}{F_T}$$

$$\Rightarrow \cos\beta = (1-\sin^2\beta)^{1/2} = \left[1 - \frac{F_g}{F_T}\sin\theta\right]^{1/2}$$

So, now let us, once we have defined all the forces, now let us balance the forces and get our expression for their velocity increment. So, for that we use Newton's second law of motion. Now, what is thrust on Newton's second law of motion says? It says, that the rate of change of momentum, so in this case what is the rate of change of momentum? Momentum is mass times velocity, so rate of change of momentum is mass times acceleration, right. So, therefore, we are considering instantaneous mass M, let us say the instantaneous velocity is v, so rate of change of momentum is M dv dt. This is equal to the sum of all the forces, the external forces acting on it. So, we have four forces, as we have just indentified. So, let us just add them, F t plus F g plus L plus D, these are the four forces acting.

As we can see, that these are vector equation. We are considering a two-d reference frame, so essentially we will get two equations out of it. So, this are the equations. This is the equation. These are the two equations that we are going to consider to express the vehicle dynamics.

Now, let us consider, typically a rocket vehicles does not have lift because we do not need to have lift for rocket vehicles, we are not, we do not have wings. So, typically, for rockets L is equal to 0. Actually, L we give a side force, which we do not want. So, therefore, typically we like to have lift to be 0. Then, if we consider theta to be constant, which has the attitude to be constant, then let us first look at normal to the flight path in the y direction.

So, along y direction, along y direction what do we have? This is about y direction, we have a component of, right, a component of F g, which is F g, this will be cos theta, this is sine theta, right, and a component of F t. And this is once again F t sine beta and we are considering lift is 0. Therefore, this component of thrust is balancing this component of gravitational force. So, what we get is, considering upward direction is positive, y direction is positive, we write F t sine beta minus F g sine theta equal to 0, right. So, that is balancing along the y direction. From this we can simplify and write sine beta is equal to F g by F t sine theta. Now, we can also get cos beta which we will be using later. So, I am just getting it here. Cos beta is equal to 1 minus sine square beta to the power half and this is equal to then 1 minus F g by F t sine theta to the power half.

Now, notice one thing in this equation. Since lift is 0, if we do not provide this beta what, which is square F g by F t sin theta whole square. So, if we look at this equation we want lift to be 0, if we do not provide this beta, then this term goes to 0, right. But F g is not 0, therefore sine theta must be 0. So, therefore, the only possibility is flying horizontally, sorry, flying vertically.

But now, if you have to move in forward x direction you cannot do it by sine vertically, right. In that case you have to give an angle. But if you give it this angle there, unless you have this it is not going to go or other thing is, that it will generate its own force in this direction, which will take it away from the intended path, and that is what used to happen in Mysore rockets. Without this vectoring it will deviate from its path, from the intend path. So, providing this small vectoring keeps it in the path. Now, this is the, with this expression we get an expression for the vectoring angle, that we should provide.

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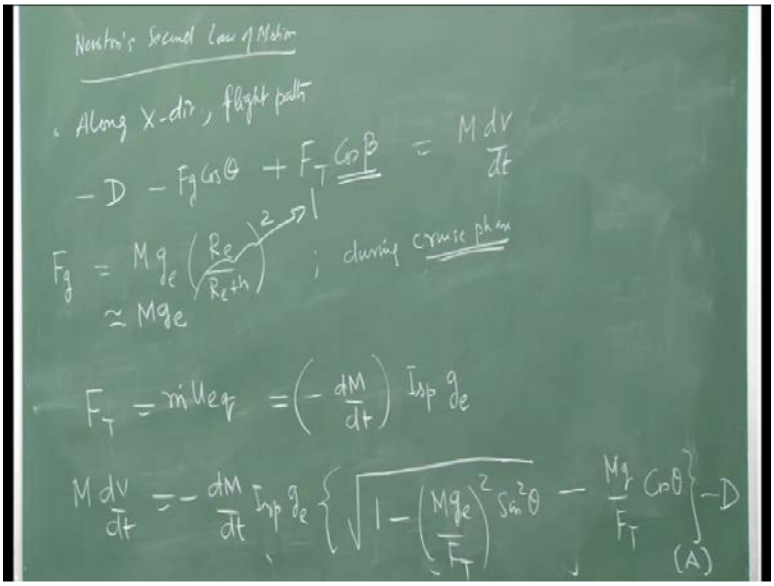

Let us come back to the second law of motion. And now, let us write it for the x direction. Along the x direction, let us write it along x direction, that is, the flight path, along the flight path. So, let us see what are the forces?

Drag is acting in negative x direction, so that will be minus D. We have a component of gravity along this direction, which will be F g cos theta, so that will be acting here, again in the negative x direction, F g cos theta. We have a component of thrust in the positive x direction given by F T cos beta. So, this is in positive x direction, so it will come with a plus sign F T cos beta. And now, along this direction we have the rate of change of momentum, right, in the y direction. We do not want any change in momentum when you force, but in this direction we have the rate of change of momentum. So, this is equal to M dv d t, where once again v is the instantaneous velocity.

Now, I would like to point out here two things. First of all at the beginning we have written, that the F g gravitational force is equal to M g e Re R e plus h whole square. This is the force acting, because of the gravitational pull. During the thrust period or cruising period, when the thrust is on, most of the rockets will be pretty close to earth's atmosphere or pretty close to the earth surface. Therefore, typically, during cruise phase or not cruise phase, I can also say powered flight phase, when the thrust is on, typically h is much, much less than R e because the radius of earth is about 6400 kilometers. So, typically, we are quite close to the surface during this phase of operation. Therefore, this expression can simplify as this goes to 1, this is just M g e. So, therefore, the acceleration due to gravity at the sea level can be considered as the gravitational acceleration and we can just write, that F g is equal to M g e; that is one thing.

Now, the next point I would like to make here is our thrust, F t. F t is equal to M dot by U equivalent, which we had derived in last class. Now, what is M dot? Is the rate of change of mass, so that is equal to dM dt because M is the instantaneous mass. But the mass is decreasing because fuel is being consumed, so therefore, this is equal to minus dM dt, right; that is our M dot. And U equivalent is equal to Isp times g, that we had derived earlier, right. So, that, this is the expression for the thrust.

Now, here we are considering, that the change in mass is only because of change in fuel quantity or the fuel is being thrown out, after burning the fuel is going out of the system. So, now let us take a 0 LIFT vehicle like here, still considering lift to be 0. For a 0 lift vehicle, then we put this and this back into this equation, then what we get is differential equation M dv dt equal to minus dM dt Isp g e. And then here we have the F t cos beta, right and we had derived an expression for cos beta in terms of F g and F t.

Now, in place of F g we put this, in place of F T we put this, then our equation will simplify as 1 minus, let us still retain the F t term, so it will be M g e by F t square sine square theta, that is, the cos beta term, right, minus the F g cos theta term, right. So, F g cos theta term will be written as, now the M here this should be equal to actually M g only, M g cos theta. This F t is common, common form this, we have taken it common and this, then minus the drag, which is appearing here, minus drag, then this is the differential equation, which need to solve to get our change in velocity. Let me call this equation, equation A.

Now, let us formulate the problem. What exactly we are trying to do? What we are trying to do is for a given initial mass M naught and initial velocity v naught, for a given thrust and flight angle, now we can integrate this expression to find expression for later stages. As we can see here, we have dv dt and dM dt. So, if we integrate this expression we can get, from say, from t equal to 0, from particular t we can get the expression for velocity and as a function of mass and other parameters. So, now, this is the basic governing equation. Let us look at specific cases and try to get expressions representing this. So, first of all, now and first, let us look at a single stage rocket.

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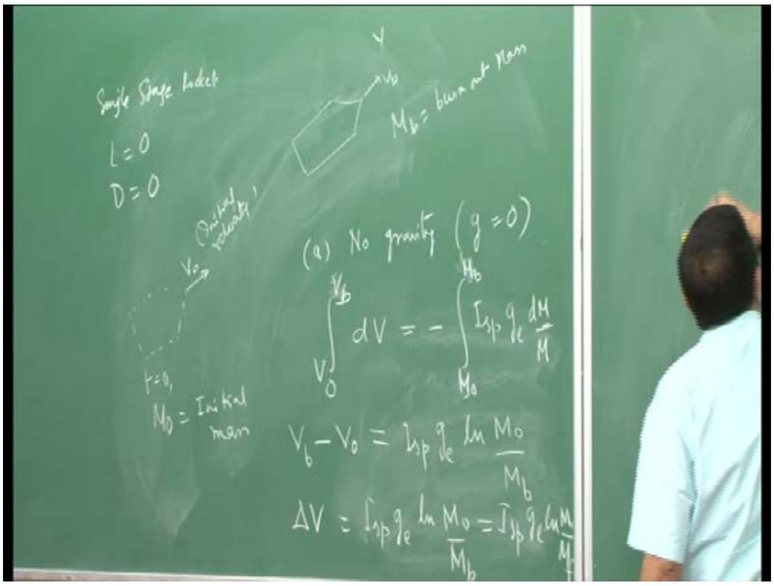

So, let us consider a single stage rocket and I like to analyze the results of this equation A. So, various solutions of this equation A for a single stage rocket. So far we had assumed, that the lift is 0. Now, just for simplicity, in order to simplify this let us consider, that the drag is also 0. So, the only forces acting here are the gravitational forces and the thrust. Then, let us look at the two stages that we are interested in, two states rather. This is at time t is equal to 0 the rocket was here, it had an initial mass M naught and it was moving with an initial velocity v naught; this is the initial velocity.

Now, we want to find out, first of all, if the rocket is carrying a certain amount of fuel and then during this operation the entire fuel burns out, then the remaining mass at the end of the burn out is the burn out mass. Let us say, that the burn out mass is M b, that is, the mass after the fuel is burnt out, the remaining mass. So, we want to find out, once it has burnt out how much velocity it has attained at burn out. Starting from this initial mass it has burnt out all the fuel, it has moved somewhat, how much velocity it has attained, that is what we want to find out. Now, we will consider several limiting cases and then we will analytically integrate them.

First, let us consider a case with no gravity so it is in the outer atmosphere without any gravity far from the earth surface, let us say. So, if we consider no gravity, then g is 0. In that case, if I come back to this equation, this term is 0, right. But this remains because this is ((Refer Time: 42:14)) to gravity of earth field, this term goes to 0, then the expression is only this. Now, we have to integrate that. So, coming to this now, oh this term will also be 0, this term is 0 and this term is 0. So, this two term will be 0. When we consider gravity to be 0, in that case we will get integration 0 to t equal to t b. t b represents the burning time. dV is the total change in velocity, is equal to minus integral 0 to again t b Isp g e dM by M.

So, once again I would like to point out here, that this term here was a F g term, this is here the F g term. So, F g goes to 0 and we regulate gravity. Therefore, only this portion of the equation remains, which you are now integrating. So, this is the expression. So, after integrating them, the, the final velocity is V b. So, we have V b minus V naught is equal to Isp g e l n M naught by M b. So, this gives us the change in velocity is equal to then Isp g e l n M naught by M b, sorry, M naught to M b and this is V naught to V b. So, integrating over V naught to V b from M naught to M b, this is what we get.

Now, recall back our discussion on the last day. M b, we have said, is equal to M F phenomenon that we use, the final mass. So, we can write it as Isp g e l n M naught by M f. So, this is the expression we will have after integrating this. So, this is the total change in velocity after integrating it from time t equal to 0 to t b. So, this was for a flight with no gravity. Now, actually if the gravity is not present, then the altitude has no effect. Theta, theta has no effect. Effect of theta comes in when the gravity is present. Let us now look at some other cases, yeah. So, what we will do is the next case that we will consider. This was flying at a certain angle theta. Now, we will consider different thetas.

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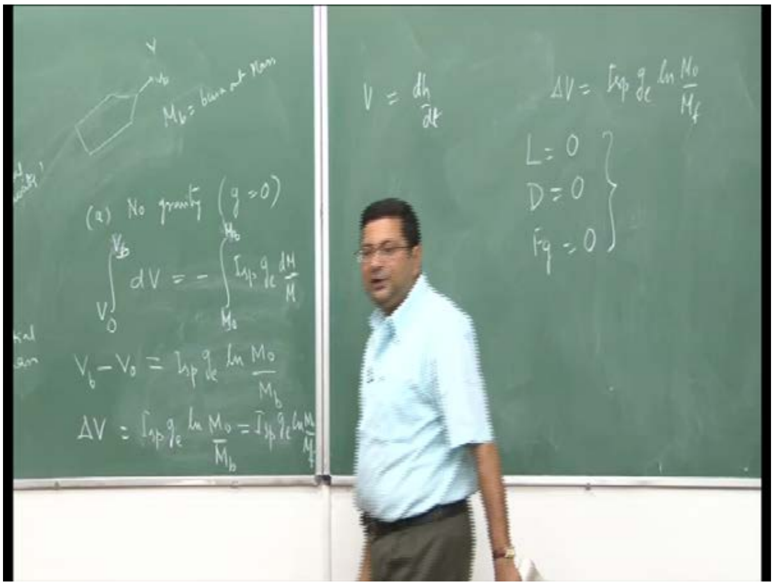

First we will consider theta equal to 90 degree, that is, it is a, vertical, horizontal flight and then we will consider theta equal to 0 degree, which is a vertical flight and we will flow the same procedure. After that what we will do is, we will revisit this and take a different approach and we will see, that we derive the same equations. So, this way we can get various solutions by integrating this and then later on we will integrate. The velocity is rate of change of displacement, right. So, V can be done as dh dt.

So, we can write velocity as dh dt. Now, we get an expression for V, we can integrate this to get an expression for h. So, you will be able to find out how much it will go either in the horizontal direction or in the vertical direction or in the flight path, how far it will go. So, this will be, essentially the vehicle dynamics.

Particularly, we are talking about single stage rockets. When we go to multi-stage we will do the same thing, but we will consider how much one stage we will take, then switch over to another stage and repeat the same. So, the basic philosophy or the basic formulation will remain same, either we are doing single stage or multi stage and that is why this is important. And this expression is very, very important, delta v is equal to Isp g e l n M naught by M F. However, this importance has to be highlighted with the assumptions that we have made. So, in this assumptions what we have is L is equal to 0, D is equal to 0, F g is equal to 0; no lift, no drag, no association due to gravity. with this assumptions we get this velocity increment.

So, let us stop here today and in the next class we will continue from here. We will first start with the horizontal flight, then a vertical flight, then we will revisit the single stage with a different approach.

Thank you.