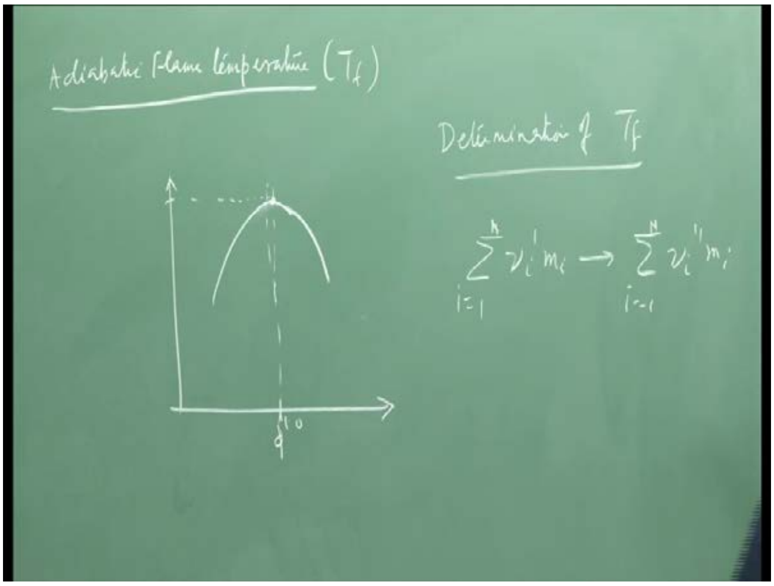
Jet and Rocket Propulsion
Prof. Dr. A. Kushari
Department of Aerospace Engineering
Indian Institute of Technology, Kanpur

Lecture - 34

Good morning. So, in the last class, we discuss the heat of reaction then towards end of the class we started discussing adiabatic flame temperature.

(Refer Slide Time: 00:30)

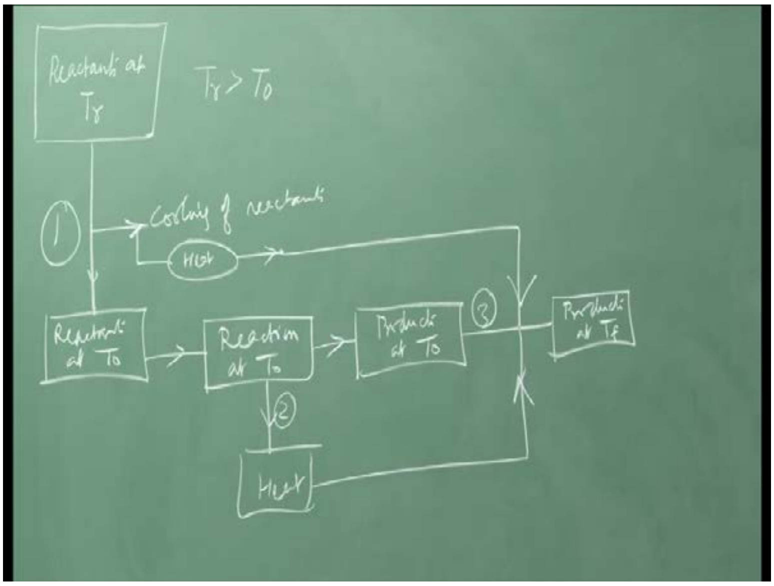


We said that in a chemical reaction some heat is liberated, typically all our combustion reactions are exothermic reaction. So, that evolve liberation of some heat. Now, if you do not allow this heat to live the system then it is going to heat of the products. So, therefore, the final product temperature depends on how much amount of heat is liberated, and what is the composition of the product. And the maximum temperature attend by the products will be corresponding to a condition when the system is adiabatic. Therefore we said that the condition is adiabatic flame temperature; the temperature is called adiabatic flame temperature. We also said that this temperature can be very high and if the temperature is very high, there is always a possibility of dissociation of the products. Dissociation reactions are endothermic reactions.

Therefore, they observe some heat and because of this absorption of heat the final temperature can be less than what the adiabatic flame temperature should have been. We also discussed that the flame temperature is going to be maximum for a psychometric mixer fraction or the stichomythic ratio. Because of the fact that at psychometric mixture fraction, all the reactants are consumed, there is no unbound oxidizer are un bound product sorry, unbound oxidizer and unbound fuel. Therefore, the only thing is available present after the reaction is product. So, therefore, the temperature is going to be maximum here, at equivalents ratio of 1. On both sides of this, when the lean side we have more oxidizer therefore that observes some heat. So, the temperature is going to be low.

On the other hand to the right side of it, that is toward the least side we have more fuel than that is required. So, that absorbs some heat. So, the temperature is going to be low. Then we said that typically for rocket application you would like to operate at psychometric condition, because that gives us the responsible temperature. Now, we have discuss then what is the adiabatic flame temperature, but in the last class we did not discuss how it is estimated. So, the next thing what we are going to discuss about is determination of the adiabatic flame temperature. So, we talk about determination of T f, for that lets consider a chemical reaction. Once again in the last class also we consider a chemical reaction. So, let us consider a general chemical reaction given like this. Let us say that this is a combustion reaction. So, we have some fuel, some oxygen or oxidizer they are burning giving out some heat, which is the heat of reaction.

(Refer Slide Time: 03:56)



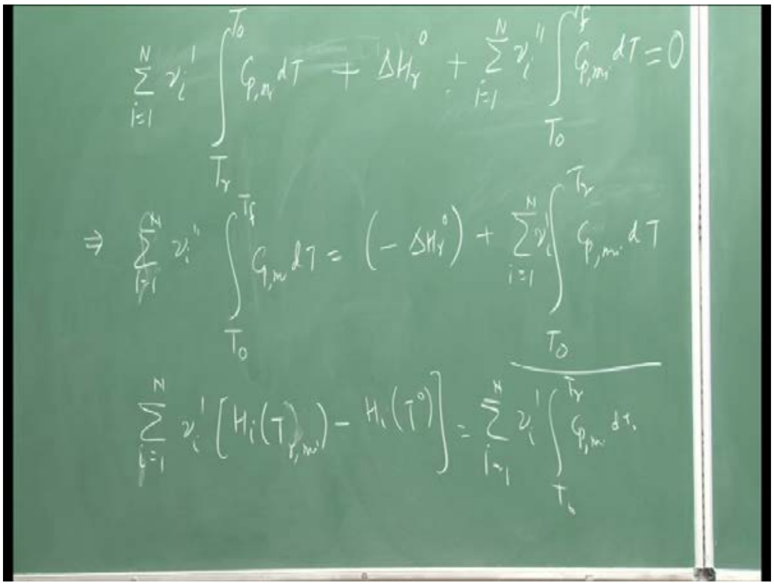
Let us now, look at the reaction path. For that, let us assume that initially the reactants are at some temperature T r. So, initially the reactants are at some temperature T r. Let us say that, this T r is greater than T naught, T naught is the standard temperature. Now, as we have said that the standard temperature heat of reaction as standard conditions are easy to estimate. So, we assume that our reactions are occurring always at standard state and the reactions are isothermal. Whether it is isothermal or not does not matter, because we can take any path and finally, go from the initial path to the initial state to the final state. So, we can allow the reaction to occur at any condition, right.

So, let say we are assuming it that the reactions are occurring at standard condition at standard temperature and at one atmosphere pressure. In that case what happens that the heat of reaction for that can be estimated from heat of formation data which is typically available in the thermo chemical tables. So, therefore, what we say is that our reaction is excepted to occur at standard temperature T naught. So, if that is to happen first we have to cool this reactants, which were at a higher temperature to T naught. So, first thing is cooling of reactants. So, some amount of heat is taken out, let me say that heat that is taken out is given here heat it is taken out of the system. So, that now, the reactants are cool to temperature T naught. So, now, we have reactants at temperature T naught. Then we say that the reaction is occurring now at this temperature. So, next thing is we have isothermal reaction at temperature T naught.

Because of this isothermal reaction some additional amount of heat is liberated. So, for the time being let us keep that heat also and because of the reaction after the reaction we get the products at the standard temperature T naught. Now, this heat is add to the products and this is heat is also add to the products and now these two together go to heat up the products. So, that finally, we get products at temperature T f and all the heat that is taken out are generated is added to heat up. So, finally, we do not have anything no heat is going out of the system. So, we get the final adiabatic flame temperature of the products. So, let us now, mathematically represent every step. What is the step, first step is this, were we are cooling the reactants to the temperature T naught. Second step is this reaction at temperature T naught, which gives me some heat of reaction and some product.

And the third step is we have we are heating this products by supplying this to heats to the final temperature right. So, these are the three steps that are taking place. So, let me now, get the total change in enthalpy for all this three states. First let us look at the first step which is cooling of reactants from temperature T r to temperature T naught.

(Refer Slide Time: 07:52)



$$\sum_{i=1}^{N} \nu_i' \int_{T_r}^{T_0} C_{p,m_i}\, dT + \Delta H_r^0 + \sum_{i=1}^{N} \nu_i'' \int_{T_0}^{T_f} C_{p,m_i}\, dT = 0$$

$$\Rightarrow \sum_{i=1}^{N} \nu_i'' \int_{T_0}^{T_f} C_{p,m_i}\, dT = \left(-\Delta H_r^0\right) + \sum_{i=1}^{N} \nu_i' \int_{T_0}^{T_r} C_{p,m_i}\, dT$$

$$\sum_{i=1}^{N} \nu_i' \left[H_i(T)_{r,m_i} - H_i(T^0)\right] = \sum_{i=1}^{N} \nu_i' \int_{T_0}^{T_r} C_{p,m_i}\, dT$$

That will be equal to we have N species. So, this integral from initial temperature T r to the final temperature T naught, C p m i d T. So, this is the total change in enthalpy in cooling the reactants at T r to temperature T naught. Then the second step, where we have reaction at temperature T naught and because of that heat of reaction is liberated. So, that will be delta H r naught right or we can write a delta H r T naught also. So, this represents the heat of reaction at standard temperature.

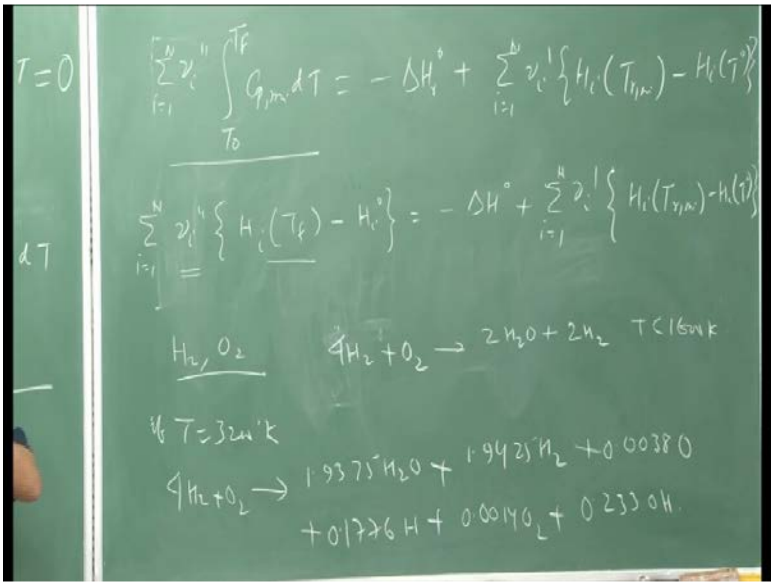
And then the third step is when this to heats are added to the products. So, the product is heated from temperature T naught to final temperature T f. So, that will be plus sigma i equal to 1 to N all the products T naught to T f C p m i d T. So, this is the total change exchange of heat in the entire system during the reaction. Now, what we are saying is that none of this is living the system therefore; the total change in heat is equal to 0, delta H is equal to 0, that we have already shown before so, this is equal to 0. Therefore, now we can get sigma i equal to 1 to N mu i double prime T naught to T f C p m i d T is equal to minus del H r naught plus sigma i is equal to 1 to N integral mu i dash integral T naught to T r C p m i d T.

Notice one thing I have done here, that I have taken this two terms to the right hand side that, because of that we get the negative sign here, this should also come with a negative sign, but then I switched over the limits of integration, because of that this becomes positive. And now T naught is less than T r right, that is what we have assume. So, therefore, this is essentially give a positive quantity. So now, this is an equation typically on T f. So, we can solve this equation either algebraic equation, because all this algebraic equation to estimate the value of T f. There is another possibility that some time like in cryogenic rockets here fuel and oxidizer locks an L H 2 they are put in to the combustor at different temperatures, they do not come at the same temperature. So, that can also be consider here, we can consider them separately.

Only thing is that all of them has come to the standard temperature. So, at what temperature we are putting it in to the combustor is a material as long as, we consider that all of them are reacting at standard condition. So, that can also be considered in this case. So now, this is my expression for the adiabatic flame temperature, this term here can also be slightly modified, we can write instead of C p we can write in terms of the enthalpy. So, I can write it as the heating term. So, I am just modifying it nothing else. We can write this term, as we heating the reactants from the temperature T r to T naught sorry, cooling the reactant as the change in enthalpy also. So, once again for every products species of the reactants, we know the enthalpy at the standard temperature from the tables we can estimate the, we can get the enthalpy at any given other temperature.

So, we can write it like this also. So, since we know both T r and T naught we can estimate the total enthalpy change. Either we do it like this or we are do it like this these are almost similar. There is just a different technique to get it, but if I do it like this then my expression for the adiabatic flame temperature will be slightly modified.

(Refer Slide Time: 13:17)



$$\sum_{i=1}^{N} \nu_i'' \int_{T_0}^{T_f} C_{p,mi}\, dT = -\Delta H_r^0 + \sum_{i=1}^{N} \nu_i' \{ H_i(T_{r,mi}) - H_i(T^0) \}$$

$$\sum_{i=1}^{N} \nu_i'' \{ H_i(T_f) - H_i^0 \} = -\Delta H^0 + \sum_{i=1}^{N} \nu_i' \{ H_i(T_{r,mi}) - H_i(T^0) \}$$

H_2, O_2 $\quad 4H_2 + O_2 \rightarrow 2H_2O + 2H_2 \quad T < 1600\,K$

If $T = 3200\,K$

$4H_2 + O_2 \rightarrow 1.9375\, H_2O + 1.9425\, H_2 + 0.0038\, O + 0.1776\, H + 0.0014\, O_2 + 0.233\, OH$

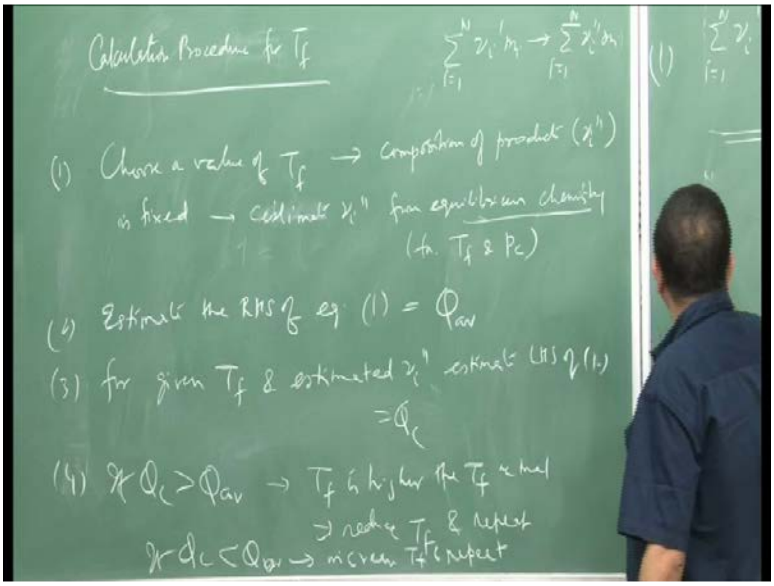
And I will get sigma i equal to 1 to N mu i double prime integral T naught to T f C p m i d T is equal to minus delta H r naught plus sigma i equal to 1 to N mu i dash H i T r m i minus H i T naught. And this can be further simplified by changing this also; instead of writing in terms of C p you can write in term enthalpy. So, this can also be written as, it can also be written like this. Once again for every products species also, we know from thermo chemical tables typically the value of enthalpy at standard state the value of enthalpy at any other temperature typically known.

However, this method is little more difficult, because here this is my unknown right. So, we have to look up at the table and iteratively get it, instead if I use this method where C p can be express like a polynomial in temperature then we get an polynomial equation in T f which we can solve numerically and get the final value. So, depends on what is available to us we can solve for this. So, I like to point out here one thing, that this term the final composition is something that it dependent on the flame temperature also. So, this two are kind of inter linked therefore, that makes a difficult to analyze solve this problem directly. If this is known, if the posen composition is known, we can easily solve for it, but if it is not known then both of them have to evolve as the solution. Just let us look at an example that how this depends on the final temperature.

Let us consider the reaction of hydrogen and oxygen typically cryogenic fuels in your cryogenic rockets. If we consider a fuel reach a reaction the 4 H 2 plus O 2 then that will give us this and the temperature is going to be less than 1600 Kelvin. So, if the temperature is less than 1600 Kelvin, we get this as the final composition, but the same thing it is carried out at higher temperature. So, if temperature is a 3200 degree Kelvin. So, just double then that. Then the same initial composition will give us a completely different final composition. It will give us not 2 H 2 O, but 1.9375 H 2 O it will give us so much hydrogen. And now, some oxygen will also will start to come up. So, all different type of products also come up.

So, what we are seeing here is the depending on the final temperature the composition is changing, but we need to go the composition to estimate the temperature. So, that is something that we have to iteratively then evolve. So, any way let us now, look at let us now summarize that how do we calculate the final temperature, I have described how it is done now let be just summarize. So, I will just list the different steps involved in estimation of final temperature. So, this equation here is the basic equation that will give us the value of adiabatic flame temperature. So, let me just written this equation.

(Refer Slide Time: 18:04)



Calculation Procedure for T_f

$\sum_{i=1}^{N} \nu_i' m_i \rightarrow \sum_{i=1}^{N} \nu_i'' m_i$

(1) $\sum_{i=1}^{N} \nu_i$

(1) Choose a value of T_f → composition of product (ν_i'') is fixed → estimate ν_i'' from equilibrium chemistry (fn. T_f & P_c)

(2) Estimate the RHS of eq (1) = Q_{av}

(3) for given T_f & estimated ν_i'' estimate LHS of (1) $= Q_c$

(4) If $Q_c > Q_{av}$ → T_f is higher than T_f actual → reduce T_f & repeat

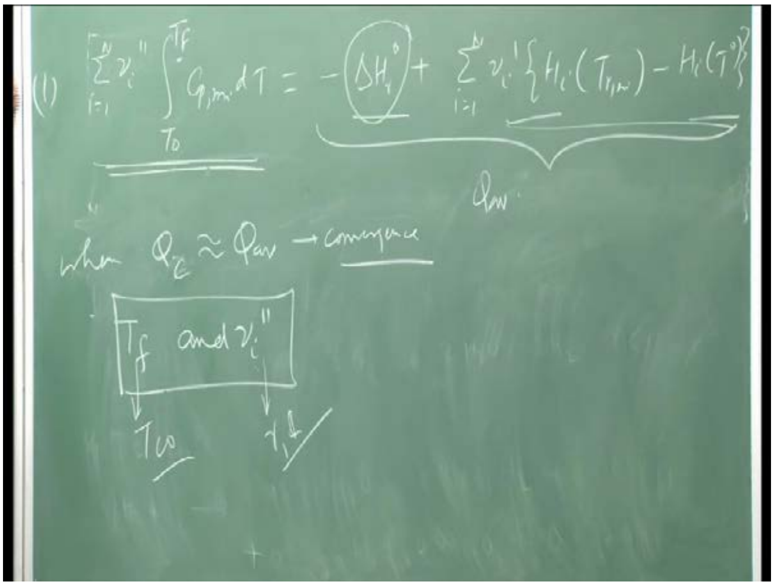
If $Q_c < Q_{av}$ → increase T_f & repeat

So, now let us look at the calculation procedure for T f. So now, we have to use this equation, this is my chemical reaction or the combustion reactions. So, the first step here is let us choose a value of T f. So, we choose a value of T f here. Now, as we choose this value of T f this is known so, the right hand side is known. Once we choose the value of T f then the composition which we will discuss later, that becomes fixed, because all the rate of reactions etcetera become fix. So, when the value of T f is chosen the composition of products that is mu i double prime is fixed. So, then for the given mu i prime and T f the initial conditions or the given initial conditions we can now, estimate for this given chosen value of T f what is mu i double prime. So, we can calculate or estimate rather not calculate estimate mu i double prime from equilibrium chemistry. You can consider the reactions are at equilibrium we can estimate; I will discuss that after this after we discuss the adiabatic flame temperature and discuss how to estimate this equilibrium composition.

So, we get the equilibrium composition now from equilibrium chemistry. Now, of course, it will be this equilibrium chemistry corresponds to the chamber pressure as well as the final temperature. So, this is the function of T f and p c, where pc is the chamber pressure. Once that is done then first of all this side here in this equation let be call this q average, this is second estimate the right hand side of equation, let me this equation 1, equation 1 which is equal to q average. Now, here since we are assuming that the reaction is occurring at standard state this term is known, this is known, this is known. So, everything here is known right hand side. So, we can get the right hand side directly. Next the third step is for given T f and estimated mu i double prime estimate L H S of 1 and called this say Q c. That is, this is the left hand side of this equation now, if the value of T f we have chosen for that we have estimated this mu i double prime. So, now, everything here is known.

So, you can estimate this value. Now, what we do is compare fourth if Q c is greater than Q average and that is this term is greater than this term, which means that our estimate initially guesses of the final temperature is higher then what it is. So, therefore, what we have to do is we have to reduce. So, T f is higher than T f actual, which implies reduce T f and repeat the process. So, we reduce the, we come back to one reduce the value of T f and repeat the process. On the other hand if Q c is less than Q v, this term is less than this then our initial guess is less. So, we increased T f. So, therefore, Q a v increase T f and repeat. So, that way now, we go on iteration we choose different values of T f and then iterate over this entire thing and get the final value. So, once we have the convergence.

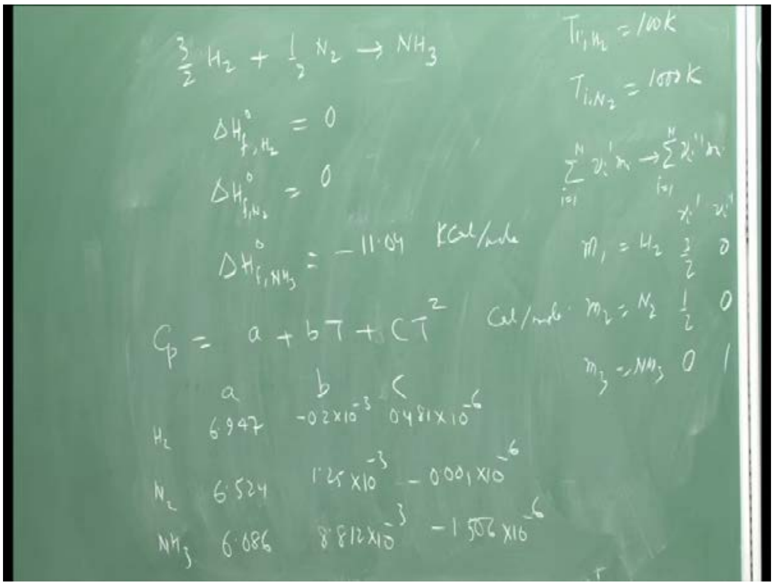
(Refer Slide Time: 23:15)



So, when Q c is almost equal to Q a v we have convergence, which means our chosen value of T f is correct, not only that our chosen, our estimated composition is also correct. So, the solution will give us T f and mu i double prime. So, as the solution of the problem we get both the composition as well as final temperature. And therefore, now we get whatever we need for the rocket analysis and because this is our T c naught and this gives us our gamma, R, etcetera, because the composition is known. So, by solving this process problem like this we get the final temperature as well as the composition,

But like I have just discussed it has to be done iteratively, because this two are interlinked. Let us now, look at an example. In this example of course, I am not going to estimate the composition, what I am do going to say is that the composition is known. Let us estimate the flame temperature adiabatic flame temperature. So, composition part as I have said there are I will take up later then it will become clearer that how do we get the composition. Now, look at an example of estimation of flame temperature.

(Refer Slide Time: 24:50)

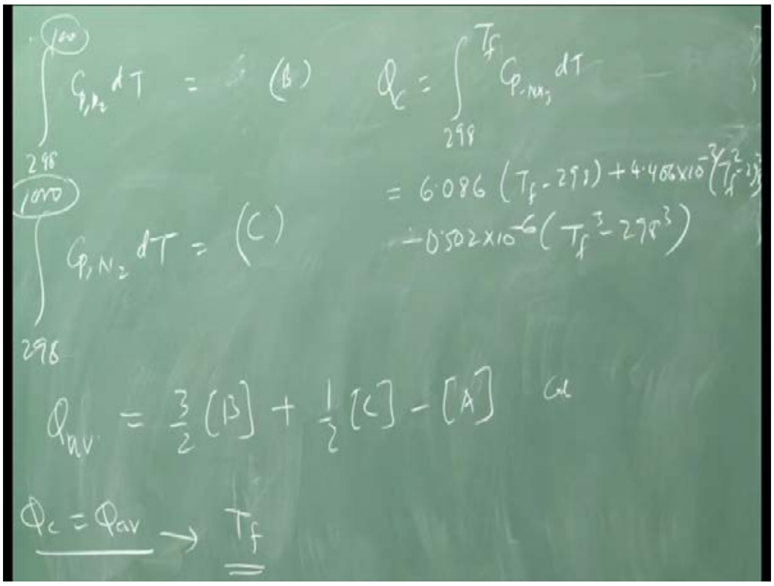


Let us consider a chemical reaction of hydrogen and nitrogen forming ammonia. Let us say that the every temperature the same reaction occurs is independent of temperature. Now, for this since hydrogen naturally occurs as hydrogen gas in nature at standard temperature. So, heat of formation of hydrogen is 0.

Similarly heat of formation for nitrogen is also 0; however, for ammonia we have certain heat of formation value, this is equal to 11.04 calorie per mole. This is the value we can get from the tables. Let say the C p for all three of them as given as a polynomial, second order polynomial and it is in calorie per mole. So now, for hydrogen, nitrogen and ammonia this only these values of a, b and c these constants are different. Let us say that I know the constants also so, a, b, c for hydrogen, nitrogen and ammonia. So, this is equal to for hydrogen I will write down first, for nitrogen and then for ammonia. Notice one thing that, if I look at this coefficients by for the most important one of the most dominant one is this constant one this and this are very small quantities right.

So therefore, they are not going to the much of influence; however, for very as the temperature goes up at low temperature they will not have much of influence, but if the temperature is high then they become quite influential. So, typically for low temperature this higher order terms will have no influence and the C p will be constant. So, you can take constant value. Now, let us consider that the initial temperature of hydrogen is 100 Kelvin and that of nitrogen is 1000 Kelvin. So, for this case, if I get, if I write my chemical reaction in the general form, which I have been writing so for. Then my m 1 is hydrogen for which mu i prime is 3 by 2, mu i double prime is 0, my m 2 is nitrogen for which mu i prime is half, mu I double prime is 0 ,my m 3 is ammonia for which this is 0, this is 1. So, this is the full expression ok. Now, we want to estimate the adiabatic flame temperature for this. So, first of all let us look at what is happening. My hydrogen initially was at 100 degree Kelvin it is getting heated up to 298 degree Kelvin ok.

(Refer Slide Time: 28:52)



$$\int_{298}^{100} C_{p,H_2}\,dT = (B) \qquad Q_c = \int_{298}^{T_f} C_{p,NH_3}\,dT$$

$$= 6.086\,(T_f - 298) + 4.466\times10^{-3}(T_f^2 -$$

$$- 0.502\times10^{-6}(T_f^3 - 298^3)$$

$$\int_{298}^{1000} C_{p,N_2}\,dT = (C)$$

$$Q_{av} = \frac{3}{2}[B] + \frac{1}{2}[C] - [A] \quad cal$$

$$\underline{Q_c = Q_{av}} \rightarrow \underline{\underline{T_f}}$$

So, there is a change in C p enthalpy for that, which will be given as 298 to 100 C p H 2 d T right. So, if I put all these values of a, b, c and integrate, I can get this value, let me call this value some A, you can put those values and get it. Sorry,, let me call, let me say A is different. First of all delta H r is let me call this A, this is the heat of reaction or standard state, this is my chemical reaction. So, here these values are 0 right. So, only this is only heat of formation of ammonia. Ammonia is one for the product 0 for the reactant. So, I get heat of reaction as standard state is equal to the heat of formation of ammonia, which is equal to this value let me call this equation A or value A. This one here is the enthalpy change in heating sorry, yeah in heating hydrogen from hundred to 298 degree Kelvin is of course, comes with a negative sign.

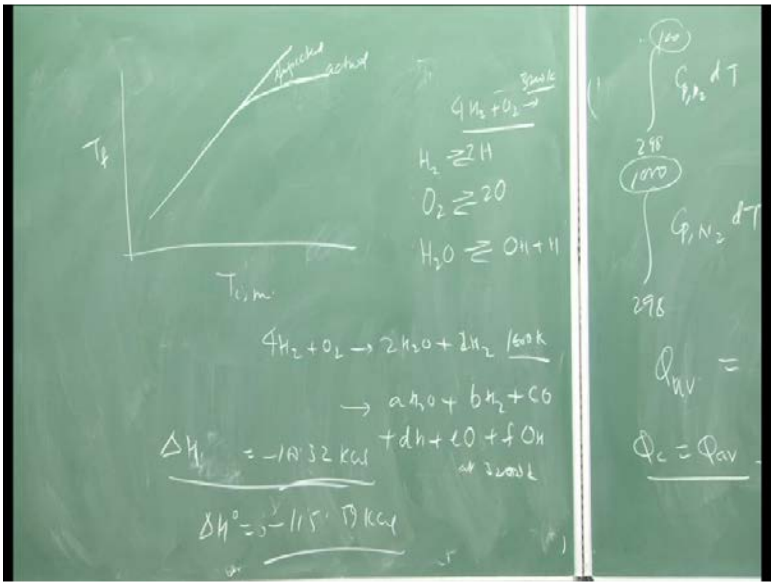
So, let me call this B, certain value that will come. Similarly my nitrogen is cooled from 1000 Kelvin to 298 degree Kelvin. So, this is the change in enthalpy for that let me call this C. So now, what is the right hand side of the Levine equation it is nothing but this plus this and this right. So, the right hand side which was equal to Q a v according to our description will be equal to 3 by 2 moles of hydrogen must present. So, will have 3 by 2 times B plus half a mole of nitrogen was present half C minus heat of reaction which was A, right.

So, this is my calorie is my right hand side. My left hand side on the other hand is Q c is heating ammonia from 298 to the final temperature. So, this is equal to 298 to final temperature C p N H 3 d T. So, if I expand this with the given vales of a, b and c this comes out to be equal to minus 0.502 into 10 to the power minus 6 T f cube minus 298 cube ok. This is my left hand side.

Now, I equate this two this value is known only unknown here is T f. So, we get a third order polynomial in T f, which can be solved. So, we do Q c equal to Q a v then solve for this third order to polynomial for the value of T f. Of course, this has to be solved numerically, but it can be solved, once we get this gives us the final temperature by solving, this using the method that have just discuss. One thing I would like to point out here that in this example, nitrogen was at certain temperature, hydrogen was at certain temperature. Now, the question can always arise that how does this initial temperature effect the final temperature? What is this initial temperature doing, if I look at schematic in this case forget about the hydrogen particularly nitrogen one, nitrogen is cooled right from its initial temperature to the final, to their standard temperature. So, in general if the reactants are at a higher temperature that is how we started our discussion then the standard temperature then initially what we are saying is that we are cooling, but in reality we do not cool, but actually we are supplying some energy and are taking out some energy.

So, that it goes down to the standard temperature. So, therefore, essentially there is some delta H available. Now, higher this initial temperature is more delta H is available right. So, essentially if we increase the initial temperature the final temperature is going to be higher, because we have more delta H available. So, what we can conclude is that as the initial temperature increases the final temperature also increases.

(Refer Slide Time: 34:24)



So, if I plot it, it will be, if I plot the variation of final temperature with initial temperature for some species let us say, T i for m I, final temperature will expect that it continuously increases right. This is our expected trend, but in reality what happens is that after a while it starts to droop of this is what the actual trend is. So, what we are saying is that initially our temperature was higher whether the expected temperature is higher, but actually the temperature drops down. This drop is because of molecular dissociation. So, as the molecule starts to dissociate we lose some energy. So, therefore, the final temperature drops.

Now, what does dissociation do for example, if I look at the condition that I have just described initially that when we have this reaction at 3200 Kelvin pretty high temperature right. We got all type of products, it was not only water vapor and hydrogen, but all type of products, where those products come from for example hydrogen now will dissociate into two hydrogen atoms. Then the, if we have nitrogen present that will dissociate, oxygen will dissociate into two oxygen atoms. Water vapor will also dissociate into O H and H. So, bottom line is that when we go to the higher temperature, very high temperature this stable molecule starts to break or dissociate into atoms and those reactions are all endothermic.

So, they absorb some energy therefore, there is a drop in the final temperature. So, the dissociation reaction being generally endothermic reduces the final temperature. They absorb the extra heat available and hence the flame temperature goes down. And then its starts to become constant also as a dissociation increases as you can see that the flame temperature kind up that was out becomes constant because whatever extra heat is generated goes into dissociate.

So, we do not get the additional heat because of the chemical reaction. So, because of that what we see is that the temperature becomes almost constant. And that is the reason we get the different product. At high temperature the example that we have seen we have seen this example right. So, let me go back to this example, this was giving us only 2 H 2 O plus 2 H 2 at 1600 Kelvin and the same reaction will give us some different composition at 3200 Kelvin right. If I estimate the total enthalpy change in this two or total, from this to this, when I go to from this to this, I see that some hydrogen has broken given H, some oxygen has broken given O, some water vapor broken given O H and H.

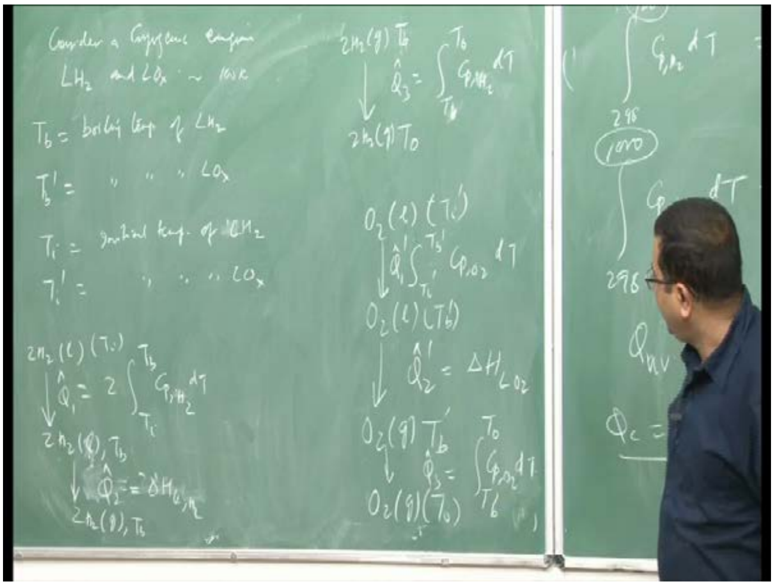
If I estimate this energy contained for this, all of this the heat of formation for O H is equal to rather the total delta H, if I estimate for this reaction this comes how to be equal to 100.32 kilo calorie for this reaction occurring at 3200 Kelvin where as the delta a the sorry, the delta H the heat of reaction at 32 degree Kelvin 3200 Kelvin is minus 100.32 kilo calorie. Whereas when this reaction occurs at 1600 Kelvin my delta H was minus 115.59 kilo calorie for this reaction and I can estimate delta H because I know what is forming right. We know the final composition also I can estimate what was delta H. So, if I look at this two numbers the heat of reaction for at low temperature was higher than heat of reaction at higher temperature which is kind of (( )).

But we see the about thirteen percent difference in this two, which is because of the fact that the dissociation has occurred, which is going to reduce the total available heat for heating of the products to the final temperature. Dissociation reactions are typically favored at high temperature and as the temperature drops there is a tendency to recombine. This is what is problem with rockets; in the combustion chamber temperatures are very high. So, we get dissociation, but when this flows through the nozzle the temperature drops. So, there is a possibility of recombination. So, when the recombination occurs then we from here let us say we go to this. So now, the composition has changed as the composition changes my gamma, R, etcetera will change. So, the temperature pressure everything will change right. The entire gas dynamics portion will change. So, if we consider equilibrium flow that is at every instant we have equilibrium then the composition will keep on changing. The advantage is typically after the throat the flow moves at such a high speed that we can assume that the flow time is much less compare to the reaction time. In that case the reaction does not enough time to be completed and bring in the changes equilibrium changes. So, in that case we can assume the flow to be frozen, but if the reaction time is comfortable with the flow time then the reaction will proceed and we will change the composition.

So, gamma and R and depending on that pressure, temperature, everything will change, it will not conform to the calculations that we have made. Now, this is a very common occurrence in rockets the dissociation do occur because temperatures are high. And again up to the converging portion the flow, within the converging portion up to the throat the velocities are not that high. So, recombinations also do occur only beyond that after the throat to the nozzle. The flow accelerates beyond the point acceleration becomes, so much and the mach number is very high then in that portion we can consider the flow to be frozen, but before that we cannot. So for better estimates of all the properties we need to have the chemistry also being solved as we move along the nozzle so, because gamma and R etcetera will be changing. This problem is more severe in ram jets and scram jets, because their as the temperature drops the incoming flow is supersonic, there is a change there is lot of loses out going flow is supersonic particularly in ram jet there are more loses.

But for rockets, since incoming flow does not have that impact so, we can be essential actually that is no incoming flow does not exists rocket everything its carrying itself. So, therefore, dissociation does not have so much impact and another point is that typically rockets are operated for very short duration right. So, therefore, also dissociation will not have very drastic impact, but it helps in estimating this, because if you are thrust is not matching are i s p is not matching with what we are expected one culprit can be the dissociation. So, therefore, that needs to be accounted for when we do the calculation for the performance. Now, this so far we have been discussing typically single phase reactions right, where all the philosopher gaseous, but if we go to practical systems like liquid rockets or cryogenic rockets or gas turbines for example, but here let us focus on the rockets. Then the fuel and oxidizer may be in liquid form, in solid form, in solid propellant rockets, but very rarely in gaseous form right. So, we have to then consider what happens, when we have a different phase either a liquid or a solid propellant rocket or a cryogenic rocket.

(Refer Slide Time: 43:29)



So, let we look at an example of a cryogenic rocket then how do we estimate this temperature. So, let we look at cryogenic. So, I will just keep the algorithm for estimate. So, consider a cryogenic engine. So, the propellants are liquid hydrogen and liquid oxygen. Now, what happens is that initially both of this will be at pity low temperature right, close to about hundred Kelvin also. So, the first thing that has to happen is that these has to be taken to the standard temperature. So, that will be, but at standard state both of them will be gas right. So, it will not go directly to the standard state first it will go its burning temperature. So, first let us look at T b is the boiling temperature of L H 2 and let us say T b dash is the boiling temperature of L o x, T i is the initial temperature of L H 2 and T i dash is the initial temperature of L o x. So, what happens first is that the liquid hydrogen absorbs some amount of energy.

Let us say which was initial at temperature T I, absorbs some amount of energy and it goes to the boiling temperature. So, liquid hydrogen is a absorbing some amount of energy going to the boiling temperature. After then, but still boiling temperature is still less than the standard temperature that it requires some more energy. So now, what we have is H 2 liquid or gas at boiling temperature then some more energy is added, which takes it to the standard temperature. So, sorry just a second, once it goes to the boiling temperature is still as a liquid, it has to evaporate boil right. So, that energy has to be supplied latent heat the vaporization has to be supplied. So, that it gasifies converts into gas. So, H 2 liquid then goes into H 2 gas at the boiling temperature and that requires some amount of energy, let us say it is equal to the latent heat of vaporization of hydrogen delta H l. Now, this hydrogen gas let me write it here, which is at the temperature T b for then needs to be heated to hydrogen gas at standard temperature for that I need another d T ok, this is H 2 and then the reaction will occur.

Similarly, if I look at the oxygen part, liquid oxygen part, initially the oxygen was at temperature T i first it needs to be taken to the boiling temperature for that I have integral T b dash C p O 2 d T then oxygen liquid at boiling temperature evaporates gives gaseous oxygen at boiling temperature. So, Q 1 dash, Q 2 dash, which is equal to the latent heat of vaporization for oxygen. Then this oxygen gas now, has to be further heated to oxygen gas at standard temperature. So, Q 3 this will be equal to integral T b dash to T naught C p O 2 d T. Now, this process and this process brings liquid hydrogen and liquid oxygen to the standard temperature after that the reaction takes place and then finally, we get the heat of reaction, which will then heat the product. What we are seeing here is that all this brings in a change.

So, the final product temperature will depend on all these steps. This stay heating, this heating, this heating, this heating, this heating, this heating everything so, everything must be accounted for before we get the final temperature. So, we have to add all these steps right, all of this is part of initial heating that takes place, all of these must be incorporated in the heating process. And then after we have now accounted for all the heated exchanges we put delta Q total change in heat is equal to 0 so, delta H is equal to 0 then we get the final temperature, which can be solved.

Of course, numerically at higher temperature the dissociation reaction can must also be accounted for this will give it take to high temperature. So, dissociation is always a possibility. So, this is how we account for the phase change as we can see here this two, Q 2 and Q 2 dash this accounts for the phase change, where liquid hydrogen converts to hydrogen gas, liquid oxygen converts to gaseous oxygen. So, this takes care of the phase change as well. So, this is how we estimate the adiabatic flame temperature for a complex system like a cryogenic rocket. So, this completes our discussion on adiabatic flame temperature, we now know how to estimate the adiabatic flame temperature. The next topic what we have to start now is the composition. So for here we said that we get composition for equilibrium chemistry. Now, we will see how to get that composition. So, I will stop here today. In the next class we will start from estimation, go to chemistry, equilibrium chemistry to get to understand how to estimate the equilibrium composition and that will complete then our discussion on the chemical rocket, because we have all already talked about the performance part, the p c naught part.

Now we talk about s c naught and gamma. Once we get this parameters then we are in a position to choose a particular propellant get the temperature, pressure and the molecular mass of the products to estimate gamma and R that will able to estimate the total performance. I will stop here today.

Thank you.