Jet and Rocket Propulsion
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Lecture - 27

Good morning. So in the last class we have started discussing the shape nozzles, we have discuss that why the shape of the nozzle is important, we have…

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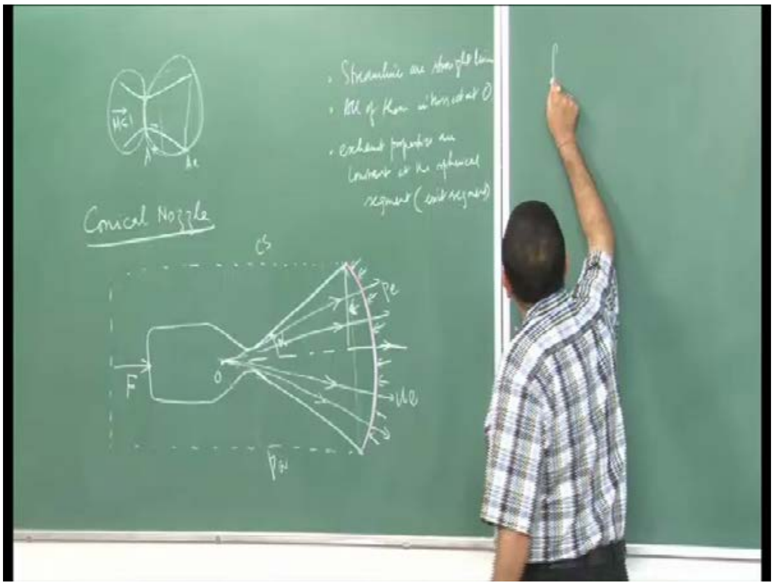

And so far we have been discussing just the throat area and the exit area, which we have derived from the area relationship. In the last class, we have discussed that why the entire shape of the nozzle is important. So, now today what we are going to do is we will discuss how do we estimate the shape. If we look at this converging diverging nozzle there are two parts to it, one is a converging part up to the throat and then a diverging part up to the exit.

So, when we talk about the full shape of the nozzle, we have to discuss both the converging part as well as the diverging part. However, if we look at the converging part itself, the flow velocity in this side is subsonic mark number is less than 1, so this is a subsonic flow in a converging passage. So, therefore this side, the converging side is not of much importance, it is just a subsonic flow in the converging passage and since the passage is converging there is no possibility of flow separation or anything.

So, we can choose design with a simple geometric convergence that will be good enough for us, because of the fact that here we have favorable pressure radiant; the pressure radiant is decreasing in the flow direction. So, therefore reasonably smooth come to provides pity good subsonic flow, there is no question of separation nozzles are also not very high.

So, therefore this side is not of much importance, shape of the diverging portion is however, important this side the diverging portion although here also, the flow is accelerating. So, is a favorable pressure radiant, but how much acceleration will be there or do we will there will be a shock wave or not or if there is huge special loss or not all of them depend on the shape, because the area ratio at every location is important.

So, at every location we have to have a given area ration, which will give us the flow, on the other hand if we look at the contour here, it is diverging and the flow is supersonic here. So, when it goes around a curve the flow will accelerate further, but there will be a expansion fan coming here, so we would like to have this expansion fan because expansion fan is isentropic.

So, then we have to design essentially considering a series of expansion fans which are giving us the proper concur, but the strength of this expansion fan or whether we are going to have an expansion fan or not and whether it is going to have a isentropic or not depends on the design, how much curvature we are providing. So, we cannot choose ivory curvature it has to be specific curvature, therefore the design of this portion is more important than the design of this portion.

So, in the next couple of lectures actually we will be focusing on, the diverging portion of the nozzle. Let us now, start with various shape nozzle the simplest one is a conical nozzle, so first let us look at a conical nozzle. A conical nozzle is a converging diverging nozzle, but with a very simple geometry, so let me consider a rocket this is the combustion chamber, small converging portion, then a smooth transition through the throat and then a diverging portion like this is the typical conical nozzle.

So, in a conical nozzle let me first draw the diagram then I will explain this, let me first draw the complete diagram this is our control surface, let us say that, let us change it back this point o this is the axis, this half angle is alpha. So, now let us look at this nozzle this is by rocket combustion chamber, this is the converging section of the nozzle, the diverging section we are considering is a conical section with at half angle alpha here.

Now, we also assume that the flow essentially stream consist of streamlines all of them emitting from this point o, this are the flow direction. So, there are multiple streamlines all emitting from this point o, so here all our streamlines, so streamlines as straight lines, so streamlines or straight lines and all of them intersect at the point o which is the origin here. Let us, consider that this is our control surface as I have shown here and the control surface passes through this spherical segment here.

So, the flow is coming out like this and the pressure here is my exit pressure p e everywhere else the ambient pressure p a is acting, let us say the velocity of the flow coming out is u v. Let us, also consider that the radius of this section maximum radius is r, rather let me take it as capital R, so the maximum radius at the exit is capital R. So, this is the geometry of the conical nozzle that we are going to talk about and we consider that the flow is coming out like a spherical segment as shown here.

Let me just highlight this spherical segment so which the flow is coming out, this is my spherical segment through which the flow is coming out and we assume that the properties are constant exhaust properties are constant at the spherical segment which is the exit segment, so whatever our exit properties the exit velocity and pressure. So, all along this exit segment this properties are constant, then now we want to analyze this flow and estimate the thrust produced by this nozzle with the conical section.

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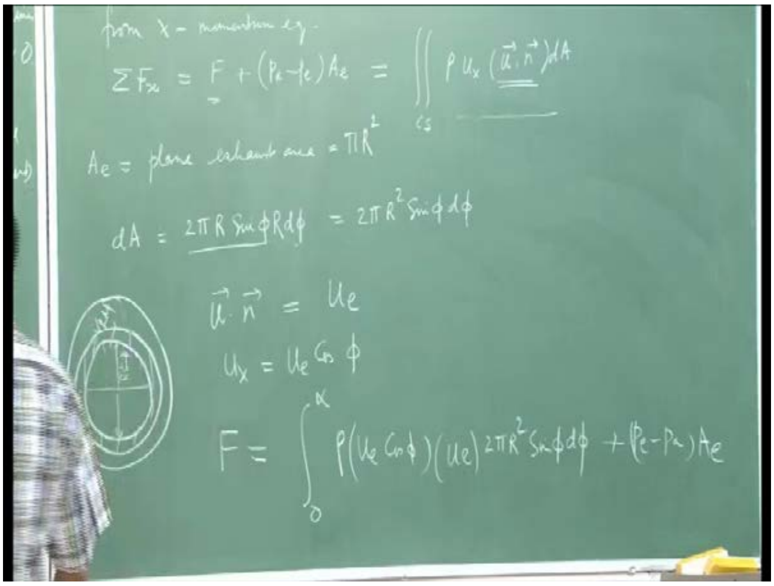

So, let us first look at this is our thrust let us say is F reaction force, so the thrust essentially will be in the axial direction, so we will look at the x momentum equation. So, from x momentum equation from second the momentum equation the sum of all the forces acting in the x direction is equal to what are the forces that are acting the reaction force F and the pressure force pressure force will be equal to p a minus p e, times A e at the exit area because everywhere else it is closed.

So, the only pressure is acting on this surface this is equal to the rate of change of momentum, so how do we get the rate of change of momentum here, the integral over the control surface rho u x, u dot n, d a, that is the x momentum, u x is the x component of velocity, u dot n is a normal component normal to the area, d a is the in front means small area, rho is the density. So, this is the momentum flux or of the control volume, so this is our x momentum equation then.

Now, from this equation we will try to get the expression for this reaction force F that is our thrust, this is what we want to estimate. So, first of all let us look at the area A e, this area is the plane exhaust area what is the plane exhaust area this area is the plane exhaust area. So, this area is our A e, so the plane exhaust area A e, then will be given as equal to pi R square that is the area, because R is our radius at the exit section.

Now, let us consider a small elemental area here, so this is the small elemental area the angle here let us say is b alpha, then rather let we take it as d phi because alpha I am using here and this angle is phi. So, from the axis to the initial segment here is phi, then the small included axis angle is d phi, we want to estimate this area, because this is the area normal to which the flow is going out. So, that is our d A, so we want to estimate d A for that this is the geometry that we are considering then d A is going to be equal to 2 pi R sin phi d phi, this is R sin pi.

So, what we are looking at remember this is the conical area, so if I look from this side it is a circular side I am trying to find out a small area concentric area like this is the area I am trying to find out. So, this area will be given as this is the area I am talking about, so this radius is R sin phi, So, the perimeter is 2 pi R, sin phi times, this depth what is this depth is R d phi. So, this will be equal R d phi, so this the one more R here.

So, this perimeter is 2 pi R sin phi and this depth is R d phi, therefore the total area is this, so this is equal to 2 pi r square sin phi d phi, that is the small elemental area that we are considering through which the flow is going out. Next, let us look at this term u dot n u dot n in this case, according to our description here this is the segment through which it is coming out and the streamlines are all normal to this surface coming with exit velocity u e, so therefore u dot n here is my u e exit velocity.

If this is the exit velocity, if I look at the x component of exit velocity, this is the component we are looking at, so this is u x, therefore u x is the x component of exit velocity which is u e cos phi at every phi location I have this. So at the axis phi is 0, so u v is equal to u e at every other location phi is changing, so u x is also varying, so therefore, these are the properties the physical properties at the exit section.

So, now let us put all of this back into this expression for thrust, we are interested in getting an expression for this thrust F. So, my thrust will be equal to now since we have already transferred the d A to in terms of d phi, the integration over this area essentially is integration over this angle. Now, how is phi changing phi is changing from 0 here to alpha here, that is the half angle for this conical section.

So, therefore, this term will be equal to integral 0 to alpha rho u x is equal to u e cos phi, u dot n is equal to u e, d a is equal to 2 pi R square sin phi d phi. So, that is this term and this will now be taken to the right hand side, this becomes equal to p e minus p a times a e where A e once again is this area the projected area here. So, this is our thrust expression, so now if we can integrate this we have an expression for the thrust.

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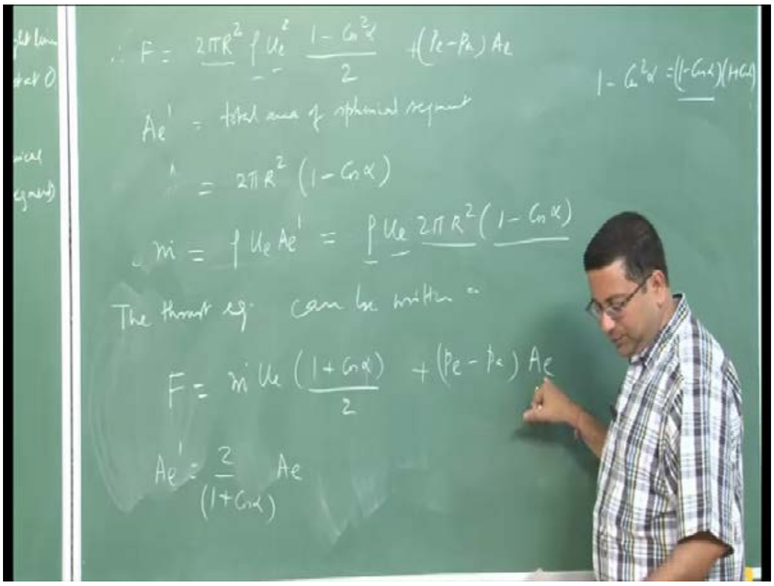

So, let us see that how we integrate this. So, it is pity straight forward integration after integration we get this is equal to 2 pi R square rho u e square 1 minus cos square alpha by 2 plus p e minus p a times A e. So, this term comes after integrating this is a pity straight forward integration, now this is the expression for the thrust. Let us, have a closer look if I look at this area here we have defined everything in terms of this area.

This is the segmental area, let us say A e dash is the total area of spherical segment, then this is equal to you can do the math yourself it is pity easy 2 pi R square 1 minus cos alpha this area. So, now if I express this in terms of if I look at the mass flow rate m dot m dot is going to be normal to this segmental area because the flow is normal to this area right say m dot will be defined normal for the flow normal to this area. So, m dot is going to be equal to rho u e, A e star A e dash.

So, therefore, this is equal to rho u e 2 pi R square 1 minus cos alpha, now I see that sum of this terms here are included in this expression. So, let us take a closer look at this expression then, so the thrust equation can be written as F let us see here 2 pi R square, I have 2 pi R square rho I have rho u e. I have u e 1 minus cos alpha and this is 1 minus cos square alpha, 1 minus cos square alpha can be written as 1 minus cos alpha, 1 plus cos alpha, so this term is also included here.

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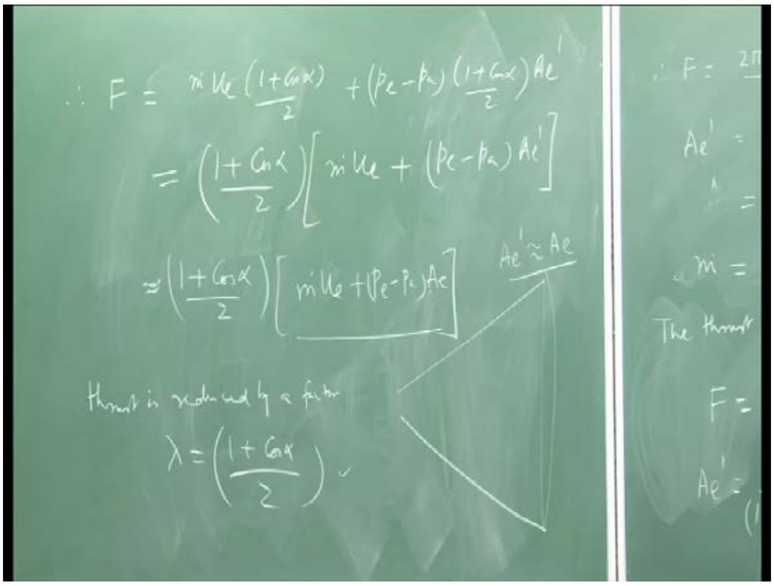

So, therefore m dot is present here it becomes equal to m dot u e 1 plus cos alpha by 2 plus p e minus p a times A e if I look at this A e dash this is also nothing but equal to 2 1 plus cos alpha times A e, now therefore, this term here can be written in terms of A e dash. Now, this thrust equation I will rewrite as therefore, F is equal to m dot u e 1 plus cos alpha by 2 plus p e minus p a times, 1 plus cos alpha by 2 times A e dash.

So, essentially this exit area projected area we have written in terms of the segment area, now here I can take this common this becomes equal to m dot u e plus p e minus p a times A e dash. So, look at the form so far all over the equations were actually this, now only thing is added is 1 plus cos alpha by 2, and where is this coming from where is this 1 plus cos alpha by 2 coming from. I will come to that, let me just little bit focus little more or less.

Typically, we have seen that the area ratios are very large, A e by A star can be 30, 40, 60, 100, so this area ratios are very large. So, if I look at this large area ratios even bigger than this, then this area and this area are almost equal. So therefore, typically a dash is almost equal to A e, so we can replace this a dash now by A e here, so in that case, this is equal to 1 plus, so almost equal to let me write it as cos alpha by 2 m dot u e plus p e minus p a times A e.

Because, typically for the practical rocket this area ratio is large, so A e dash is almost equal to A e, so we can write it like this. Now, let us look this equation, this the our original thrust equation which we had derived this portion here, this is our original thrust equation, which we had derived assuming the flow to be one dimensional.

Now, it is no longer a one dimensional flow, so what we are seeing is that, the one the thrust equation is modified from A one dimensional flow common as so essentially for an ideal rocket, with half angle alpha, alpha is our half angle for a ideal conical rocket with half angle alpha, the thrust is reduced by a factor lambda. The thrust is reduced by a factor, this factor is lambda equal to 1 plus cos alpha by 2, why are we saying that the thrust is reduced? Because, cos alpha is bounded between 0 and 1, and I am minus 1 and 1.

So, if cos alpha is 1, in that case lambda is equal to 1, otherwise cos alpha is less than 1, so lambda is less than 1, so the thrust is reduced. So, straight away what we are seeing is that, considering the 3D flow, not that 2D that we have been discuss 1D flow that we have been discussing, considering a 3D flow there is a reduction in thrust. In the limiting case, when alpha is equal to 90 degree no alpha is 0, when alpha is 0, then this will be equal to 1, but alpha is 0 means it is straight flow.

So, that is something that we will come to that ideally we would like to have a straight flow going out that is why we have to give a shape we will come to that. So, for here we are considering a conical nozzle, so therefore, the flow is at an angle and because of that angle variation, this is the reduction in thrust the factor that we reduce the thrust. So, primarily this change or reduction is because the flow is no longer one dimensional, but as we can see that this analysis is quite easy, very straight forward analysis.

This relationship that we have derived here, compares very well with experimental data, experimentally it can be proved or observed that in this the reduction in thrust follows this relationship. So, therefore, we can assume that this relationship is quite valid over the wide range of operating conditions. So, establishing that this is a valid relationship, let us now look at the physical consequences of this relationship. So, let us consider the conical rocket that we are discussing with certain half angle alpha.

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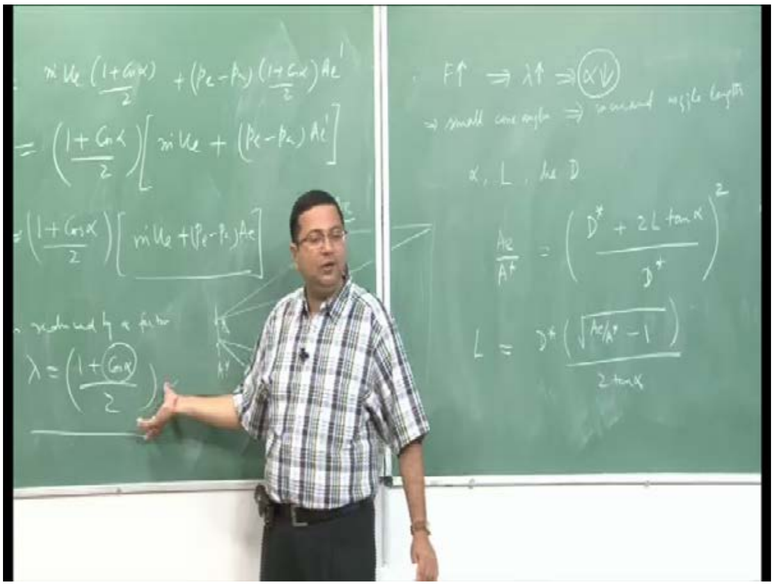

First of all, if you want to increase the thrust for everything else just want to change the shape and increase the thrust, how we can do? it is by increasing lambda, so if we increase lambda the thrust is going to increase. Now the question is, how do we increase lambda? Lambda will be essentially depending on alpha, in order to increase lambda, what is the minimum value of lambda here? it is going to be half, when cos alpha is 0 it is going to be 1 when cos alpha is 1, when cos alpha is 1 for 0 degree.

So, when we are reducing alpha lambda is going to increase, so therefore, by reducing alpha we can increase the thrust. So, for a conical nozzle you can increase the thrust by reducing the half angle alpha. So therefore, we would like to have small cone angles, but now comes the tricky part, we would like to have a small cone angle at the same time, in order to get proper expansion. Our area ratio is fixed, so this exit area to this throat area is a star A e, this is something that is fixed.

Now for this fixed, if I reduce this area let me look at this if I reduce this half angle, how do we get this exit area ration? Because that we need to get only way to get it is by extending the length. So, earlier we were getting this area ratio within this length, if we reduce the half angle we have to increase the length. So, smaller cone angles means increased nozzle length, so the nozzle length now needs to be increased, to a great degree.

In order to increase the thrust, and that is something that we do not want to do because this is going to add weight, just to give an idea for a conical nozzle, let us say for a conical nozzle with angle alpha and length L diameter D, D is the diameter at the exit. If, I estimate this area ratio A e by A star, it will be a function of the throat diameter D star plus 2L tan alpha by D star square, this is the area ratio expression.

So, area ratio is a function of D star, which is the throat diameter here D star length of the nozzle L and tan alpha which tan of the half angle. So, the area ratio is a function of this, now if I look at this equation in this equation my D star is fixed if you decrease alpha and we want maintain same A e by A star L next to be increased and it needs to be increase substantially, in order to maintain the same area ratio. So, therefore I can get an expression for L also L by manipulating this equation, we can get this is equal to this is the expression for the L.

So, once again we are seeing that as alpha decreases, L has to be increased, in order to maintain this two constant D star and A e by A star, let us look at an example, if we want to maintain A e by A star equal to 100, typical values for rockets, we want to maintain it at 100. Let us say that L by D star is 7.8 for alpha equal to 30 degree, so if you take the half angle to be 30 degree. The length throat diameter ratio is 7.8, maintaining the same area ratio, if we reduce the value of alpha to 15 degree, when alpha is equal to 15 degree, then the corresponding L by D star is going to be 16.8.

So, once again as I am saying the D star is same we do not want to change D star that is the throat condition we do not want to change that, because this D star is the throat diameter remember our discussion in the previous class. The chamber pressure is maintained by this; so that we cannot change otherwise the chamber pressure is going to change. So, we have this chamber pressure p c naught maintained by this D star, which will dictate then chamber pressure the combustion conditions as well as the specific impulse, we do not want to change that.

So this remains same, now in order to maintain the same area ratio if we reduce the half angle by half, the length has increased by more than double. Because, earlier the length was 7.8 times D star, now it is 16.8 times D star, so it is more than double. So, what we are seeing is that the nozzle length and now another point I would like to mention here is that, if we do not make this change what happens, if you maintain the same area ratio and reduce this, what will happen.

The expansion is not going to be complete, we get an 100 expanded nozzle, because this condition A e by A star is for our ideal expansion this an ideal expansion, and we are not truncating it here, so we get an 100 expanded nozzle. So, the mark number is going to be less, exit mark number is going to be less than the design exit mark number. So, our exit velocity is going to be less will be producing less thrust, so therefore, if we do not change this length and reduce the half angle.

The thrust produce is going to be less, because our exit mark number has reduced. So, what we are seeing here, now is that the exit mark number which is dictated by the length of the nozzle and as well as the included half angle. So, half angle plays a very important role in the exit mark number, reducing the exit a half angle by 15 degree more than doubles the nozzle length, and typically the weight of the nozzle is proportional to its length.

So, if we are increasing the length by more than double, its rate is going to increase the weight will be more than double, so therefore, when in all counts increasing this length like this is going to increase because if the length is doubled, the weight is doubled, the structural coefficient is going to change.

And when, we are losing once again either on the performance or on the payload carrying capability, so both of them is something we do not want to do. So, therefore what we have shown here is that a conical nozzle although is very simple to get the expressions, but because of the 3 D effect there is a loss in our performance given by this factor.

This loss can be minimized by reducing the included half angle, but as we reduce the half angle our length is increasing. So, therefore, this is not very advantageous, so we have to look at some other alternatives that although a chemical a conical nozzle is easy to design easy to fabricate, but the losses because of the float turning we may you may say is probably not acceptable in most of the practical cases. So, we have to then look at some other alternative to address this issue.

So, one of the ways then will be instead of giving a fixed angle, what if we vary this nozzle angle over the length, we give it a shape that the acceleration is here if I look is a fixed divergence. So, acceleration is almost constant everywhere, instead of that what if we give a variation smooth variation in such a way that we can get our flow to turn the way we want to first advantage, second we have shown here that if we make alpha equal to 0 degree then lambda is 1.

So, alpha equal to 0 degree essentially means that if we make the flow parallel to its axis, then lambda is 1, may not be able to do it for the entire nozzle, but even if we do it for a section towards the exit then we have improve the performance a lot. And in that case since, we are not changing lambda we are not changing we do not need to change alpha it is straight section now. So, therefore, this advantage we can get without having to reduce this alpha.

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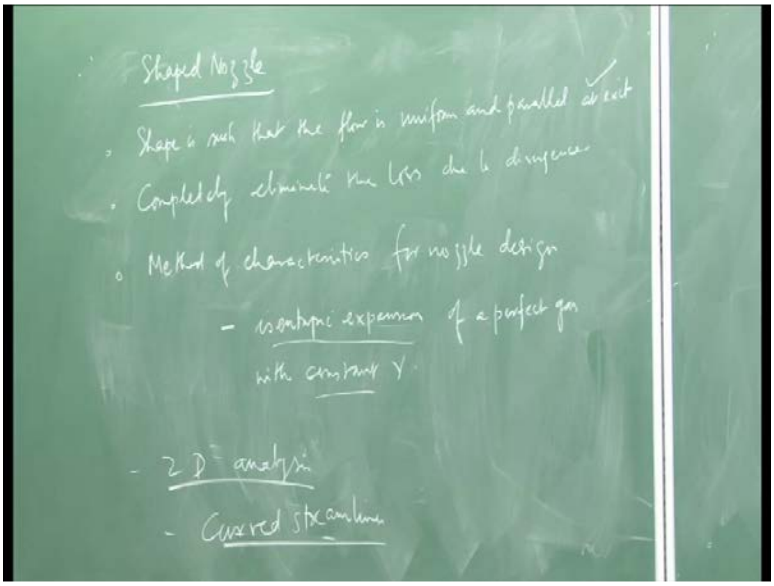

So, we can maintain a shorter length and get the advantage of the increase, the thrust by eliminating this loss or this drop, because of the three d effect by giving a proper shape to the nozzle. So, that is what now we are going to discuss a shape nozzle. So, the next topic is the shape nozzle, so as we can see here that the conical nozzle gave us the direction the analysis of the conical nozzle gave us the direction towards which we need to proceed to optimize the performance of the nozzle.

So, next thing we are going to discuss is a shape nozzle, in the shape nozzle the ultimate goal is to make alpha as low as possible towards the exit. Ideally you would like to have a uniform parallel flow at the exit, so the primary m is that the shape is such that the flow is uniform and parallel at the exit. This is the ultimate goal, that we turn the flow gradually we do not turn in it in one go we turn the flow gradually in such a way that towards the exit of the nozzle the flow is uniform and parallel.

This, if can be done properly it will completely eliminate the loss due to divergence what is the loss due to divergence in the conical nozzle we have seen that there is a loss in thrust and that loss was because of the 3 D effect which is because of the divergence. So, that loss is called divergence loss if we can provide a uniform parallel flow at the exit it will completely eliminate that loss lambda we can get lambda equal to 1.

So, that is the m it will completely eliminate loss due to divergence, now the thing is that how do we get this that is the important factor parameter that how do we get this variation the method used is called method of characteristics, method of characteristic is typically used in designing contour for a supersonic flow. When, the supersonic flow goes through a curvature it turns and then we use method of characteristic because that assumes the flow to be isentropic and maintains isentropic nature of the flow at the same time it will accelerate the flow.

So, that is the method now we will we can use for the nozzle design for nozzle design, so what is the assumptions involved in method of characteristics, first of all it is isentropic expansion of a perfect gas with constant gamma. So, method of characteristic actually assume the flow to the isentropic and expanding flow this are the two things that has required for using method of characteristics. So, here we have an expanding flow in the nozzle.

So, we essentially maintain isentropic expansion, we assume the fluid to be perfect gas with a constant value of gamma and we can use method of characteristics to get the proper acceleration and it can also be used for variable gamma and chemical reaction can also be incorporated, even the wall friction can be incorporated that we will come to later.

At present we will consider that the value of gamma is constant, we will not consider any chemical reaction, we will not consider any wall friction, we will use the classic method of characteristics to get the contour, that is what we want to do we want to get a basic contour, that will give us this. Then, after that that basic contour can be modified by bringing in more complexities that is something not very difficult to do.

One advantage of using method of characteristic, from the previous analysis is it is no longer 1D, it is essentially a 2 D analysis, we will be doing a 2 D analysis to get the characteristics. So, that is what the advantage that we are already considering the flow to be 2 D and we are designing best on that. So, therefore this divergence losses as somehow taken care of during the design itself, because we are considering the flow which is almost real nature.

Therefore, it is already considered in the design portion and one of the thing that method of characteristics does since we are considering a 2 D flow, it can provide significant curvatures to the streamlines, since the streamlines will be curved not the straight streamlines as we have seen in the case of the conical nozzle here the streamlines will be curved.

Now, if we are considering curved streamlines then we know that the steam lines curve there is a normal component of the pressure. So, therefore, the component of pressure perpendicular to the streamlines are important, that needs to be considered when we are using this method of characteristics. So, the perpendicular component of steam line because the streamlines are curved quite significantly that needs to be considered in our analysis.

So, this is what we are going to discuss in the next class now that how do we get this shape, how do we use method of characteristics because we have shown that the conical nozzle although is easy to make, easy to analyze the losses are quite can be quite significant because of the 3 D effect. So, I would like to eliminate those losses by giving a proper shape to the nozzle.

So, that is what we are going to discuss in the next class, essentially we will discuss method of characteristics for nozzle design this is what we are going to discuss in the next class that how do we use method of characteristics for the nozzle design. So, I will stop here now, in the next class we will continue from here.

Thank you.