Jet and Rocket Propulsion
Prof. Dr. A Kushari
Department of Aerospace Engineering
Indian Institute of Technology Kanpur

Lecture – 26

Good morning. So, in the last class we have been discussing the nozzle flow, we looked at various aspects of nozzle flow, and we compared the performance of a converting diverging nozzle with the performance of a just a converging nozzle, and we have shown that the advantage of the diverging portion is essentially restricted up to the optimum design.

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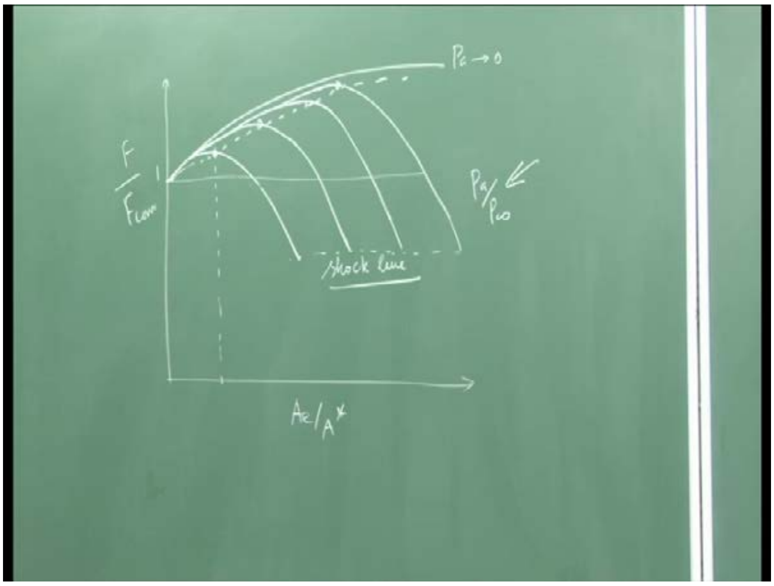

So, we have drawn this diagram yesterday which was the area ratio, which is the exit area by the float area, and the thrust ratio corresponding to the actual thrust for a converging diverging nozzle, non dimensional by the thrust produce by a converging nozzle alone. And we have seen that for a converging nozzle of course the thrust, if the nozzle is converging that is the exit area is equal to the throat area, then this ratio is going to be one, so that is the base line. After that we have discuss that for a limiting case when the atmospheric pressure tends to zero we get the ultimate thrust, and that is going to behave like this. Then, for other values of atmospheric pressure the thrust increases which is maximum, and then starts to decrease; this is for increasing P a by P c naught. And there is a point corresponding to each one of these P a, which gives us the optimum thrust, this will be the thrust produced when the area ratio is such that P e is equal to P a; that is the exit pressure at the nozzle exit equal to that atmospheric pressure we get the maximum thrust because in that case the expansion is ideal, and we have shown for the ideal expansion the thrust is going to be maximum.

We have also discussed that all this curves, by the by we see that beyond a certain area ratio the thrust decreases below the thrust produced by a converging nozzle therefore, the diverging portion is not giving any advantage beyond this point. Second point, we have seen from this is that, there is a shock line at which for all this pressure ratios it stop the nozzle operation because there is a shock that either sits at the exit or it goes into the nozzle, therefore, the flow is no longer isentropic, the flow becomes subsonic as is goes across the shock wave, normal shock wave. So, therefore, anyways it is going to match with the exit pressure, but the performance is going to be very poor because of lot of losses, so we ended our discussion yesterday on this point, shock line. So, we had said that this condition which will give us the shock wave can be estimated, so let us now look at this condition.

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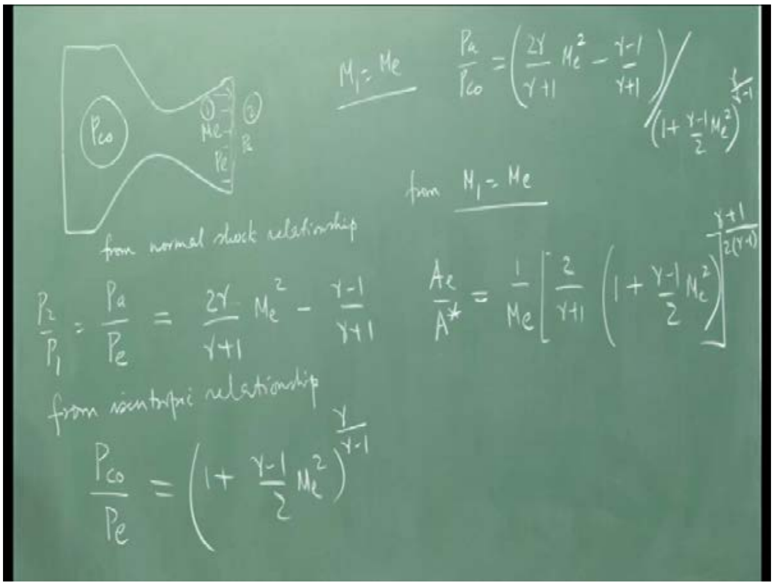

Let us consider that we have a rocket with a converging diverging nozzle like this, and let us assume that a normal shock wave given here sits at the exit of this nozzle. So, the condition just ahead of this shock are mock number is M e, the pressure is P e, and after this shock wave; so this is the flow direction, after this shock wave the pressure is equal to P a. If that is the case, only then the shock will sit at the exit of this nozzle, so across the shock wave that is going to be a degeneration of the flow and the pressure will reach the ambient pressure, and this pressure is going to increase between P e and P a because the static pressure increases across a shock wave. So, this is the condition we want to derive now.

Let us consider first the normal shock relationship, we know that from normal shock relationships which you can find in any text book and gas dynamic or fluid or aerodynamics, the normal shock tables are also given because these are very important properties of the flow. So, for normal shock relationship we can get the pressure change across a normal shock, for this case this is two, this is one. So, this is equal to P 2 by P 1 which is equal to P a by P e, for our case is a function of the mock number at 1. So, at 1 our mock number M 1 is M e because that is the mock number coming up to the exit of the nozzle. So, then we can write the normal shock relationship as this, 2 gamma up on gamma plus 1 M e square minus gamma minus 1 upon gamma plus 1, we can write the normal shock relationship like this. At the same time the flow before the shock wave from the nozzle inlet to the nozzle exit is isentropic, so therefore, for this part of the follow we can write isentropic relationship.

So, from isentropic relationship we can write P_c naught because the stagnation pressure here is P c naught, so P c naught by P e; this is the static pressure at the exit, is equal to 1 plus gamma minus 1 by 2 M e square to the power gamma upon gamma minus 1. Now, if it combines these two relationships, we can get this pressure ratio P a by P c naught which corresponds to these points of these curves. So, combining these two, we can get P a by P c naught equal to 2 gamma upon gamma plus 1 M e square minus gamma minus 1 upon gamma plus 1 divided by 1 plus gamma minus 1 by 2 M e square to the power gamma upon gamma minus 1. So, from this we get the pressure ratio here which will give us a shock wave standing at the exit, so that is one point.

Second point, we want to find out is let us say for this case what is the corresponding area ratio because when we are designing the rocket nozzle it is the area ratio which is more important because we are designing for the area. So, therefore, we want to estimate what will be the corresponding area ratio which will give us this normal shocks standing at the exit, for that what we do is we use the area relationship which we had already derived before. So, in this case the area ratio will be deriving based on this mock number because this area ratio will give us this exit mock number M e. So, for M 1 equal to M e, the area ratio will come from the area relationship A e by A star equal to 1 upon M e 2 upon gamma plus 1 into 1 plus gamma minus 1 by 2 M e square whole to the power gamma plus 1 upon 2 gamma minus 1. So, this expression gives us the area ratio corresponding to which the exit mock number is going to be M e.

Now, if you look at this two equations P a by P c naught and A e by A star what we see is that both of them are functions of the exit mock number and gamma. Understand one thing, this exit mock number by the way is not the exit velocity because there is a shock wave here. So, the mock number is going to be M e just before it, when the flow goes across the shock wave the velocity is going to decrease, right. So, the thrust producing velocity is going to be less that is why we get such a huge drop in thrust because there is a shock wave sitting here which will reduce the velocity.

So, this M e would have been exit velocity if P a is equal to P e otherwise, it is not going to be. So, coming back to this description now, so according to these two equations that we have just derived, we see that for a given value of gamma; now where and on what parameter gamma will depend, it will depend on the composition of the propellant after combustion or after the energy production. So, for a given value of gamma there is only one pressure ratio P a by P c naught for which there is a normal shocks standing at the exit because once gamma is fix, M e of course decreases by this, right, so therefore, there is only one value of pressure ratio for which the normal shock will standard at the exit. For other values of pressure ratio, other values of ambient pressure, the normal shock either enter or it will not be a normal shock, but an abolish shock at the exit.

So, therefore, for a given exit area ratio A e by A star, there is only a unique value of exit pressure or ambient pressure for which there will be normal shock at the exit, so that corresponds to these points in this curves. So, for this value of P a there is a particular area ratio for which we get the normal shock at the exit, so that is how we obtain this shock line. So, one of the parameters that is repeatedly appearing in all our description of chemical rockets is the pressure at the chamber P c naught, is the chamber pressure, this is a very important parameter.

So, let us now take a look at the P m I pressure, now by the way chamber pressure is not something that evolved because of combustion, whereas we see that in order to increase the thrust we have to; discussed it again and again, we have to increase the chamber pressure, but chamber pressure is something that you supply to the chamber. And then the design is such that because of the presence of this throat, this flow cannot go out so that the pressure is maintained, right. Because of the throat it will just become the ambient pressure everywhere, the throat actually chokes the nozzle, so therefore, the pressure is maintained at a high value. And now, let us see that how we choose this value of P c naught. So, this is the next topic we are going to discuss that how this P c naught is chosen because this is a very important performance parameter as per as chemical rockets circumstances.

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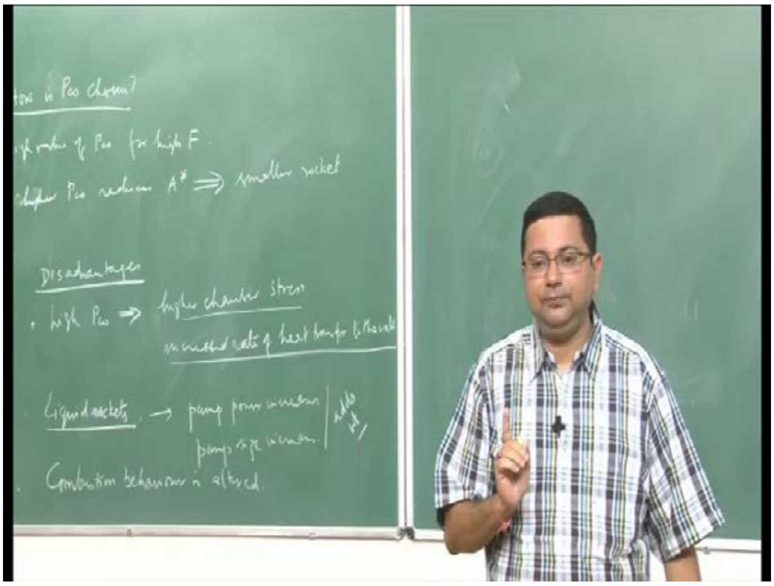

First of all what is desired, the desired P c naught is a high value of P c naught because if P c naught is high P a by P c naught is going to be low, and we have shown that it will produce more thrust. So, and we want to produce more thrust, so therefore, for high thrust we like to have as high value of P c naught as possible. Now, if we go to higher thrust or other higher P c naught, what will be the advantage is that not only the thrust is increasing if we go to P c naught, we remember that the m dot mass flow rate is also a function of P c naught and is inversely proportional, so as P c naught increases m dot decreases, therefore, essentially the throat size decreases.

So, as we increase P c naught the throat size decreases, as the throat size decreases in order to maintain the same area ratio, let us say the overall size of the nozzle is going to decrease, so the rocket becomes smaller, so this higher P c naught reduces A star, implies a smaller rocket. So, both of them are advantageous, we are producing higher thrust at the same time we are reducing the size of the nozzle or the rocket, so the weight is going to decrease, and we have seen that that weight is the part of the structural mass, right. So, if reduce the structural weight, either you can carry more payload or can carry more propellant and can go further, right, essentially both of these are advantages as per as the performance is considered. However, the question is, if that is case we can continuously keep on increasing P c naught, and we can get better performance, but is it possible, the question is are there any disadvantages if we keep on increasing the P c naught.

So, we see here that P c naught is advantages, but is there a limit to which we can increase it, so let us look at the disadvantages when we increase P c naught. First of all when we have high P c naught, we get higher chamber stress, right, because this high pressure gas has to be contained within the combustion chamber. So, increase in the P c naught means that more pressure is acting on the wall, so therefore, the chamber stresses are going to increase. So, the material or the structure has to withstand the increase pressure, so there is a limit to it like how much it can withstand.

Second point is, so first of all it gives higher structural stresses, second point is as pieces are increasing, the rate of heat transfer to the wall also increases, so increased rate of heat transfer to the wall. So, now, out of the heat produced by the combustion more is going to the wall, so therefore, available energy for thrust production is reducing at the same time the structure experiencing higher thermo stresses also, so we have to have more efficiency or more cooling. So, the energy spend in cooling is increasing, so overall energy content of you complete system is constant. Now, you are spending more in cooling and at the same time you are losing some of the energy because the energy is going their waste, so because of this, the higher P c naught puts a limit to how much thrust can be produced.

Now, these are two disadvantages which are essentially universal for any type of rocket motor, either solid propellant or liquid propellant, any chemical rocket that if we go to higher pressure that is going to be higher stresses at the same time increase rate of heat transfer. Now, let us come to specific type of rockets for example, for liquid rockets first of all how do we create this high P c naught that is a question, it comes from the supply, right, we send the propellant at high pressure and then that pressure is maintained because of the choke, so if we are talking about liquid rockets the propellants have to be send at high pressure.

So, if you have to pressurize it more, the liquid propellant have to pressurize more, the pumping requirement is going to increase, right, we have to push it at a higher pressure, so the pump has to push at higher pressure. So, therefore, the pump power increases, so essentially we have to pump more at higher pressure so that power requirement for the pump is increasing. Now, once again this power is not coming from anywhere, it is contained in the rocket, so you are losing some energy there, right, so that pump power is increasing. Secondly, pump size will also increase, because now since we said to withstand higher pressure I have to push it at higher pressure, the piping also has to be stronger, the pump has to be bigger, the power requirement is more.

So, all of this essentially adds weight, and these are all structural weights. So, as we keep on increasing P c naught, at one point of time the power requirement for pump or the size of the pump and the other fuel feed system become so bulky that we do not get much of advantage, because structural weight has to increase too much. So, therefore, all of this puts a limit, because we do not want the structural weight to go beyond a certain value because the structural weight will eat up either our payload carrying capability or our fuel carrying capability. So, therefore, this is something that puts a limit to how much P c naught we can get.

Now, last but not the least; again this is applicable to all type of chemical rockets, combustion behavior is altered. When we go to higher pressure what happens is that your chemical reaction changes, because as we go to higher pressure the rate of reaction increases. Typically, the reaction rate is the function or pressure, as we go to, if we look at a chemical reaction how does it happen is intermolecular collision, right, not only inter molecular collision this collision has to be energetic enough, only then the chemical reaction time can take place. So, there is a finite probability of having some energetic collisions within a certain volume.

Now, if you increase the pressure, essentially what we are doing we are packing more and more molecules within the same volume, so therefore, the chances of energetic collision occurring is increasing that is energetic collision which will lead to chemical reaction is increasing. So, as we go on increasing the pressure the chemical reaction becomes faster, so there is a direct influences of increasing pressure on the chemical characteristic time. Now, that chemical characteristic time or the time of the reaction plays an important role in combustion dynamics; how the combustion is taking place, and it may lead to combustion instability, because if the heat release which is because of the chemical reaction and the pressure, because they are going to be certain perturbations in the pressure.

Also, there are acoustic phenomena also occurring because this chamber essentially is like a acoustic resonator, right, so there is special oscillation because of acoustic and there is a heat released which is also, now the time is changing. If there are small perturbations then the heat release starts to change with time, pressure is also changing with time, if this two are in phase then there can be a feed back to pressure by the heat released leading to an increase in pressure oscillations. This phenomena is called combustion instability which can be very dangerous because now what we have is that we are seeing at high pressure we have high chamber stresses, now this stresses are periodic, now if high stress is applied in a periodic manner that are more dangerous from the structure than a steady high stress.

So, therefore, the periodic phenomena which are combustion in stability can set in if we go to high pressure combustion. So, at the same time even sustaining the flame may become difficult because now the flame is burning at a very fast rate, we may not be able to supply the fuel at the same rate, in that case the flame will go off; in diffusion flame it is called flash back, but it does not happen in diffusion flame, but the flame may not be sustained. At the same time, since the burning is so fast the stresses acting on that, the strain acting on the flame is more, so flame may not sustain itself, flame may blow off. Thirdly, when this oscillation occurs; this oscillation essentially leads to periodic change in the chamber pressure.

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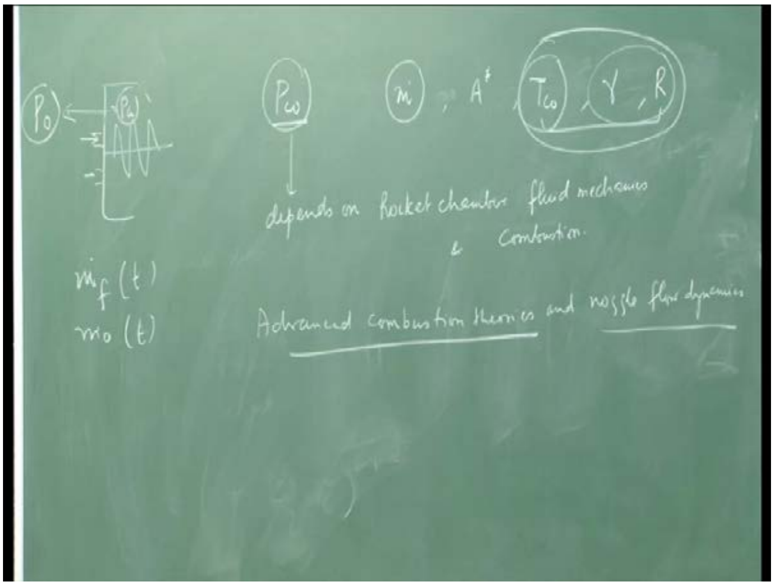

So, if I look at this chamber pressure, chamber pressure is oscillating like this. Now, the fuel and oxidizer are coming at a fix rate because we maintain a higher pressure here, and this pressure is lower than this pressure, so because of this pressure differential the flow of oxidizer as well as fuel comes into the combustor. And now, if this starts to oscillate, we keep this fixed then this differential starts to oscillate. As a differential starts to oscillate, the flow rate also starts to oscillate, so m dot of fuel and m dot of oxidizer both of them become function of time, so now there is oscillation of fuel flow rate and oxidizer flow rate.

The next result is the oscillation of composition fuel; here fuel oxidizer ratio is changing, so the equivalence ratio is changing. Now, as the equivalence ratio keeps on changing the composition keeps on changing, so no longer we constant gamma or constant c p. So, everything locally keeps on changing and that is something that leads to catastrophic failure either we may have a flame out flame may go off or the structural stresses can be so high that it will rupture the combustor. So, these are few things that needs to avoided, and that is why we do not want go to exceedingly high pressures, we want to operate at high pressure, but not exceedingly high pressure and as we go to higher pressure we have to ensure that we take care of this combustion instability problem because if the instability creeps in we have major problem in our hand.

So, what do we see here is that these are some constraints that of choosing at high pressure, and these needs to be addressed in the design state itself that how much pressure we can allow the system to go to. Now, however, if I look at these effects, the structure say their increase in hit transfer rate, the pumping power, the combustion instability, none of them are linear, none of them are straight forward, and all these essentially are also coupled to certain extent.

So, therefore, the analysis is not very simple, it is a highly coupled because here we are talking about the structures, here we are talking about heat transfer, here we are talking about power requirement for the liquid fuel, let us say the pumping requirement of a liquid phase, here we are talking about combustion instability. All of them put a limit to P c naught, but all of them also occurring together cannot be analyzed easily. So, the analysis is very complex and difficult, there is not simple way to analyze how to handle all these problems together.

So, therefore, because of the complex nature of these disadvantages as well as their inter-link nature, because, as I said that if combustion instability occurs, the structure stresses will be periodic, so they inter-link. So, there is no simple treatment possible to pin point that if we do this, if we changes the pressure so much this is going to be, the effect is not a simple statement. Essentially, it will be obtained by trial and error, we choose a particular value of pressure and see whether it is safe or not. And there is some limited analysis also available for, say combustion instability for pumping requirement, for structural stresses, but again coupled analysis is at the present state of the art is very difficult to do.

Now, if I look at P c naught, first of all what are the parameters that dictate the value of P c naught; P c naught is governed by, here first of all mass flow requirement m dot, what is the requirement of mass flow rate and the throat area A star. So, if we have a specific requirement of m dot and we choose a particular A star, and then after combustion there is a particular value of T c naught, then P c naught is fixed. Now, m dot is related to the thrust, so the total thrust will depend on how much m dot will have A star, and we have seen that exit pressure is dictated by that T c naught, the chamber combustion temperature, so therefore, that dictates on the reaction that is occurring.

All of these together will fix the value of P c naught, so P c naught is not a parameter that appear independent of other parameters. So, this parameter then we choose that if we have to give this much of mass flow rate at this temperature to this area, this is required for P c naught, and then we maintained that P c naught in the chamber. P c naught, then this depends on the rocket chamber fluid mechanics, and combustion. These are the two things on which the chamber pressure will depend.

So, then how do we estimate this, we require advanced combustion theories and nozzle flow dynamics. Remember that as we are seeing here, these are the nozzles parameters; m dot and A star are nozzle parameters, so we cannot decide on P c naught independent of the nozzle, we have to specify the nozzle performance also, only then we can decide on P c naught. So, it is not that we can design the chamber separately and the nozzle separately, we have to first do a coupled analysis, so we have to have the nozzle flow, we have to have the combustion, all of them together will tell us how much P c naught is going to be there. Now, here is the catch, after doing this we decide the P c naught, but in order to do that we need to know T c naught as well as another parameter that will be coming again and again is gamma, right, gamma and r that is the propellant properties.

Based on this, we choose value of P c naught, but the question is as how are these known because these are also dependent on P c naught because our reaction rate is going to be depend on P c naught. Therefore, the gamma and r that is the final composition of product is going to depend on pressure, and if that is depend on pressure the temperature is also dependent on pressure, so these parameters, all of them are dependent on pressure. So, this again, initially we can choose a value of this and analyze this, but are they correct choice that is the question, because here of course if we are implying advance combustions theories, the chemistry has been considered Otherwise, we have to an integrity processes.

We choose a value of P c naught, derive this estimate, these quantities, check whether this meets the requirement or not, and iteratively we get all this parameter. So, once again the pressure as we can see is coupled; not only pressure all the properties are coupled with each other, so therefore, that makes it a very complex problem. So, just summarize what we discuss now is that we know that the chamber pressure is an important parameter, because if we got higher pressure we can get higher thrust, at the same time if we go to higher pressure we operate with a smaller throat area, therefore, the nozzle size decreases, so we get an advantage of reduced weight also.

However, we cannot keep on increasing P c naught as we want to, because there are some disadvantages associated with it. First of all as we got to higher pressure the chamber stresses increase, there is increase rate of heat transfer. In order to accommodate this chamber stresses the structure has to be stronger, so structural weight will increase. Secondly, if we are talking about liquid rocket, the pumping requirement is going to increase, we need more power for the pump as well as the pump size, and the piping, everything has to be increased, and that is all add to the weight. And thirdly we have the combustion dynamics as function of pressure, so that may be altered, so that may lead to onside of combustion instability, it can be devastating for the rocket performance, so because of this we cannot increase P c naught as much as we want to.

Then, coming to, on what parameters P c naught depends; they depend on rocket chamber fluid mechanics, and the combustion. Therefore, the analysis also should take into account both of these that is we should have the rocket chamber combustion has to be modeled properly as well as the nozzle flow dynamics has to be modeled properly . So, now, we are discussing primarily the nozzle flow, in this lecture and as well as next couple of lectures we will primarily focused on nozzle flow dynamics, but keep this in mind as I have just discuss P c naught and these parameters will also dictate how the nozzle is going to behave, right. So, therefore, this needs to be estimated as well which will be done through this, combustion analysis. So, after we are done with the nozzle flow dynamics, the discussion on nozzle flow dynamics we will go to the combustion analysis.

In the combustion analysis what we will be focusing on is estimation of this that is the composition of the propellant after combustion which will give us what is the gamma and r, and what is the temperature T c naught. And we will see the dependency of these parameters on P c naught as well because remember as I have just discussed that the combustion depends on pressure. So, we will see the dependency of this parameter on P c naught when we go to combustion analysis, at present we continue our discussion with the nozzles. So, we have establish the importance of P c naught, let us now continue our discussion with the nozzles. So far, we have not talked about the shape of the nozzle, we said just is a converging diverging nozzle or something like that, now lets us come to the shape of the nozzle.

So far, our analysis is essentially is just estimation of the area ratio and all. Now, this area ratio we can take any shape and get that area ratio, right, but what is the optimum shape, so now, come to the shape of the nozzles. So, the next topics we will be discussing is the nozzle, we have shown that how to estimate the area, now we will be see what is optimum shape having the desired area that will give us the best performance.

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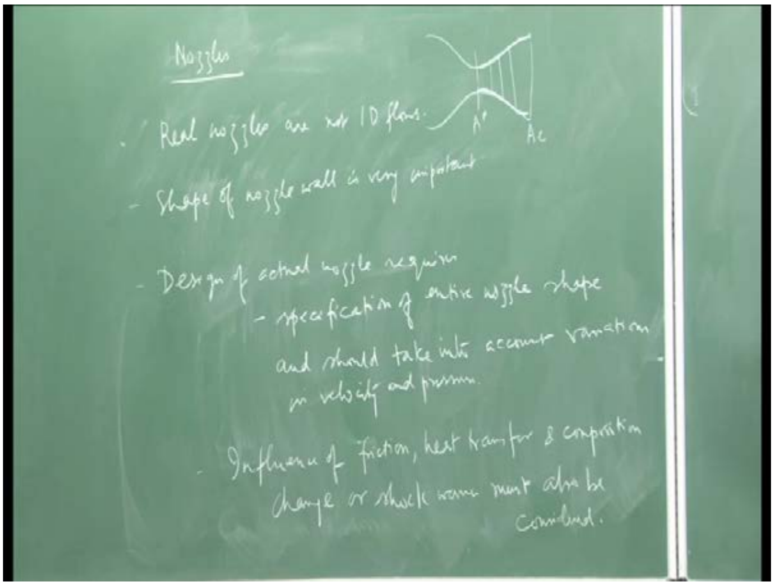

So, next we look at nozzles, but now we look at the detail geometry of the nozzle. So far, if I recall what we have been interested in the nozzle is only this area A star and this area A e, but as we can see here at every location there is a different area, right. So, how do we estimate this area that is what we will come to know, so for that let us look at the nozzle. First of all, when we estimated this area relationship what was the first assumption that we made that it is cosine only, but nozzle flows as we have said that at that time also they are not one dimensional flow, they are usually 3d flows, so in reality real nozzles are not one d flows. Now, if it is not a one dimensional flow then the shape of this wall plays a very important role in the amount of loss that will incur as well as the flow acceleration, and since we are talking about supersonic flow, the shockwave;

whether we get a shockwave or not will be dictated by the shape. So, therefore, the shape of the nozzle wall is very important.

Now, when we talk about the nozzle as part of the design of the rocket, when we talk about design of the nozzle, then nozzle design should actually give us the entire shape at every location, the geometric variation of all the walls that is what the nozzle design is. So, the design of the actual nozzle requires first of all specification of entire nozzle shape, and should take into account variations in velocity and pressure that is how the velocity is changing across the nozzle length, how the pressure is changing across the nozzle length. All of these should be taken into account when we talk about the nozzle design, at the same time the influence of friction heat transfer and composition change or shockwaves must also be considered, right.

We assume that flow is frictionless, but in reality it cannot have frictionless flow, so the effect of friction should also be incorporated in the nozzle design heat transfer. As we have just said that as the pressure increases, heat transfer rates are going to increase, so heat transfer is something that also needs to be considered when we are designing the nozzle. At the same time, the composition may not be frozen because as the flow is expanding, so the nozzle, the temperature drops, and if the reactions are still continuing then the combustion keeps on changing. At the same time, we have talked about the formation of shock waves, when the shock waves are present then how they are going to alter the flow field, and do we need to have a change variation in a design to first of all ensure that there are no shockwaves. So, all these things must be incorporated in the nozzle design.

So, nozzle design is not just estimating the exit area and the throat area, it is the variation everywhere because of the fact that if this curvature is not proper, if the acceleration is sudden, then we can suddenly get into shock waves or the frictional loss can be higher or if the throat is not design properly, if we have a sharp throat then we can have very high heat transfer; it will melt the throat. So, all these things must be incorporated when we are talking about the nozzle design. So, these are the issues that we are going to address now that what is the proper shape of the nozzle, and for a given shape how do we estimate the performance of the nozzle, this is what we will be focusing on for the next couple of lectures.

We will first start with the determination of the suitable shape, shape of the nozzle is not what we are going to talk about, we will initially will not have the extra complications like the effect of friction, heat transfer etc. Actually, we will not go into details of this in this course, just give some passing remarks on this, we will primarily focus on the require shape of the nozzle, so that is what we are going to discuss in the next couple of lectures. So, let us stop here now, and in the next lecture we will first start with a basic shape, look at the performance of the nozzle with that shape, and start with a conical shape, then we will go to how to determine the exact curves shape that is employed in practical rockets, we will just discuss that later. I will stop here now.