Jet and Rocket Propulsion
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Lecture - 25

Welcome back to this course on rocket propulsion. So, if you recall, in the last class, we have been discussing the design of rocket combustion chamber and we have solved a problem also on rocket combustion chamber design. Now, to continue our discussion on the chemical rockets, we had discussed various flow regimes for a nozzle flow. We will continue with the nozzle flow. So, first thing what we are going to do today is define some efficiency parameters for the nozzles. So, let us define some efficiencies for the nozzle. There are several efficiency parameters, which are often used to describe a rocket performance.

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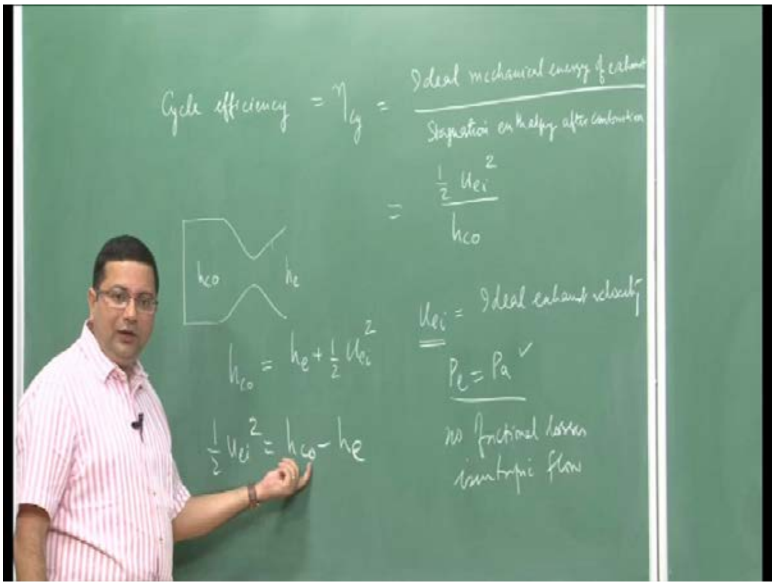

One such parameter is called cycle efficiency, which is designated by eta c y. Cycle efficiency is defined as the ratio of ideal mechanical energy of the exhaust, that is, the hot gases that is leaving the rocket and the stagnation enthalpy after combustion. So, let us see what does it physically mean. The stagnation enthalpy after the combustion is the total enthalpy – total energy available with the hot gases after the combustion has occurred.

The cycle efficiency essentially represent out of this total enthalpy is available, how much is being converted to effective thrust. So… And, that is, we are talking about the ideal mechanical energy of the exhaust, which essentially is the kinetic energy of the exhaust. So, if I look at the rocket that we had been discussing so far; let us say this is the combustion chamber and a converging-diverging nozzle representing the nozzle. Then, after the combustion, thus total stagnation enthalpy in the combustion chamber is h c naught; where, this subscript naught represents the stagnation state. And then at the exit, the static enthalpy is h e.

Therefore, from energy conservation or first law of thermodynamics, we have shown it before that, the total enthalpy here is nothing but the static enthalpy plus the kinetic energy term. So, that is, h e plus – now, what is the kinetic energy? The exhaust velocity is u e; and, i represents ideal state without any losses. Therefore, the total stagnation enthalpy after the combustion is given as h e, which is the static enthalpy at the exhaust plus the kinetic energy of the exhaust. Therefore, the cycle efficiency now can be defined as this divided by this. So, that is, half u e i square by h c naught; where, as I have been mentioning again and again, u e i is the ideal exhaust velocity.

Now, first of all, let us recapitulate what we have said as the ideal rocket. We have said that, an ideal rocket is a rocket for which there is no pressure forces. So, the expansion is ideal. Therefore, for ideal exhaust, P e is equal to P a. The exit pressure at the exit of the nozzle is equal to the ambient pressure. That is the ideal rocket for us. At the same time, we have also said that, there are no frictional losses and isentropic flow – isentropic flow in the nozzle. Let us first understand the physical significance of these assumptions. Even if the flow… Let us say we have seen the different cases at different back pressure. There are different type of flows that exist in the nozzle.

Let us for the time being assume that, the back pressure is such that there is a shock wave inside the nozzle. If that happens, across the shock wave, the flow becomes subsonic. And then the subsonic flow will reach ambient pressure. Therefore, this condition will be satisfied, but is it ideal? No, because the flow is not isentropic. If you have the shock wave sitting in between, the flow is no longer isentropic, because shock wave is a irreversible process. Therefore, in order to assume the ideal expansion, we have to assume that, the flow is isentropic, so that there are no shock waves actually inside. And then a part from that, there will be a shock-induced change irreversibility. Also, in the presence of friction, there are some frictional losses. So, that also we are neglecting. So, with this assumption, then the velocity that we will get is a theoretical velocity, which we have already discussed in detail. Therefore, that is our ideal exhaust velocity. So, now, the cycle efficiency is defined with respect to that ideal kinetic energy and the stagnation enthalpy at the exit of the combustion chamber or at the inlet of the nozzle.

So, now, if I combine these two, what we can see here is half u e i square is nothing but the difference in enthalpy between the inlet and exit of the nozzle, because at the inlet of the nozzle, the enthalpy is the stagnation enthalpy, because the flow is stagnant here. Therefore, the inlet of the nozzled enthalpy is stagnation enthalpy and the exit of the nozzle we have the static enthalpy h e. Therefore, the kinetic energy for the ideal expansion case is nothing but the difference between these two enthalpies.

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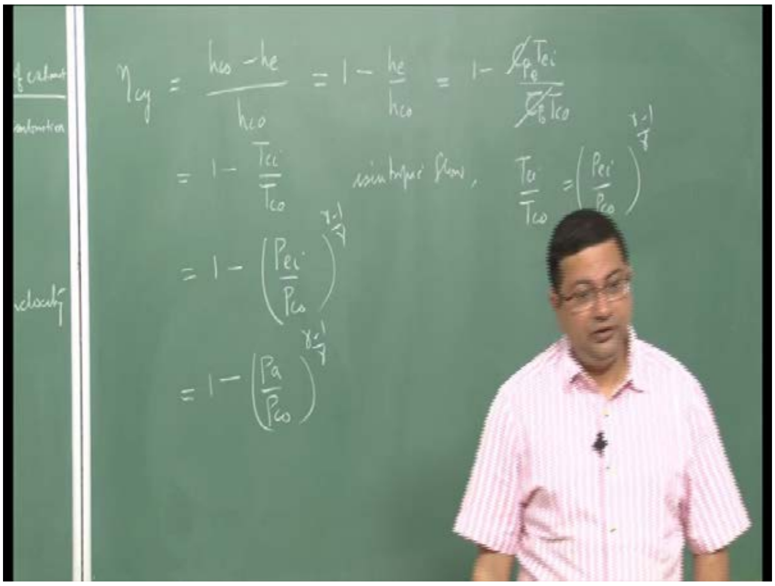

So, we can write this expression then; the cycle efficiency as h c naught minus h e by h c naught. Now, we can write this as 1 minus h e by h c naught. Now, the working fluid at… Remember at the beginning of our discussion on chemical rockets; we made a list of assumptions. One of the assumptions is regarding the state of the working fluid. We assume the working fluid to be perfect gas; that means it is calorically perfect as well as thermally perfect. Therefore, the enthalpy can be expressed as a function of temperature only. So, if we do that, this is equal to 1 minus C p T e i – this is the ideal temperature divided by C p T c naught; where, C p is the specific heat at constant pressure.

Now, we have also assumed that, the composition does not change in the nozzle. We have assumed that, it is a frozen flow. And, we have assumed also that, the C p is constant, because temperature variation is not much; there is no recombination, no dissociation of the gases. Therefore, C p is constant through the nozzle; that means the value of C p at the inlet and exit are same. Therefore, this C p – let me put it as i and o just to differentiate between the exit and inlet… e and o – let me put it like this. Now, these two are same. So, I can cancel this off. So, the cycle efficiency now is nothing but 1 minus T e i by T c naught.

Now, this is the temperature ratio now. And, we are assuming the process to be isentropic. That assumption is already there. Therefore, the temperature ratio can be expressed in terms of pressure ratio; which for the isentropic flow, since the flow is isentropic, we had earlier discussed the isentropic relationship that, T e i by T c naught equal to P e i by P c naught to the power gamma minus 1 by gamma; where, gamma is the ratio of specific heats, which we have discussed. Here P e i is the exit pressure for the ideal case and P c naught is the stagnation pressure in the combustion chamber. Therefore, this is how for an isentropic flow we can describe the temperature ratio.

So, now, we can put it back into the efficiency definition. This becomes 1 minus P e i by P c naught to the power gamma minus 1 by gamma. And, now, P e i is a function of the shape, but we are considering ideal expansion. So, what we have already assumed that, P e is equal to P a. And, ambient pressure is something that we know. Therefore, here in this expression, we can replace P e i by the ambient pressure P a. Therefore, this can be written as 1 minus P a upon P c naught to the power gamma minus 1 by gamma. So, now, this is the definition of cycle efficiency. As we can see that, this is a function of the pressure ratio; nothing else. This cycle efficiency physically is the measure of the fraction of energy produced by combustion in thrust generation. The total energy it is produced essentially is this – the stagnation enthalpy. Out of this fraction, how much is use for thrust generation. So, that is what the cycle efficiency represents. And, we have got an expression for the cycle efficiency. So, as a designer, what we want to do is always maximize cycle efficiency.

How do we maximize the cycle efficiency? By increasing P c naught. That is why for rocket application, we always want to get as high pressure as possible in the combustion chamber, so that the cycle efficiency will be more. But, cycle efficiency like any thermodynamic cycle cannot be more than 1; that is for sure. Therefore, this cycle efficiency cannot be more than 1. In the limiting case, when we have absolute vacuum, P a is tending to 0; then this ratio will tend to 0; cycle efficiency will tend to be 1. So, that is the ultimate thrust that we have already discussed. So, in the case of ultimate thrust, the cycle efficiency will be leading towards 1; it will not be 1; there will be some differences. But, it will be very close to 1. So, either as… Therefore, it says that, as we go up – as the rocket goes up, the cycle efficiency increases, because the ambient pressure continuously decreases at the rocket is launched higher up. Therefore, this is how we define the cycle efficiency. The cycle efficiency as we can see is the efficiency of the entire rocket cycle, which includes the combustion chamber as well.

Now, let us define another efficiency parameter, which is called nozzle efficiency. If you look at the cycle efficiency, we have considered the nozzle to be ideally expanded. But, in practical cases, we will not have ideal expansion; that is for sure because we have seen that, there is only one case. Out of all the ambient pressures, there is only one solution, which will give us the isentropic flow. So, that is very difficult to achieve in practical big cases, because we would not be continuously varying P c naught. So, then P a is varying continuously. Therefore, it is almost impossible to have a continuous application with ideal nozzle. Therefore, the next step that comes in is to define the efficiency of the nozzle. So, when we say the efficiency of the nozzle, we do not make these assumptions. We say the nozzle is not ideal. So, if the nozzle is not ideal, how much fraction is getting converted; that is the nozzle efficiency. So, next, let us define the nozzle efficiency.

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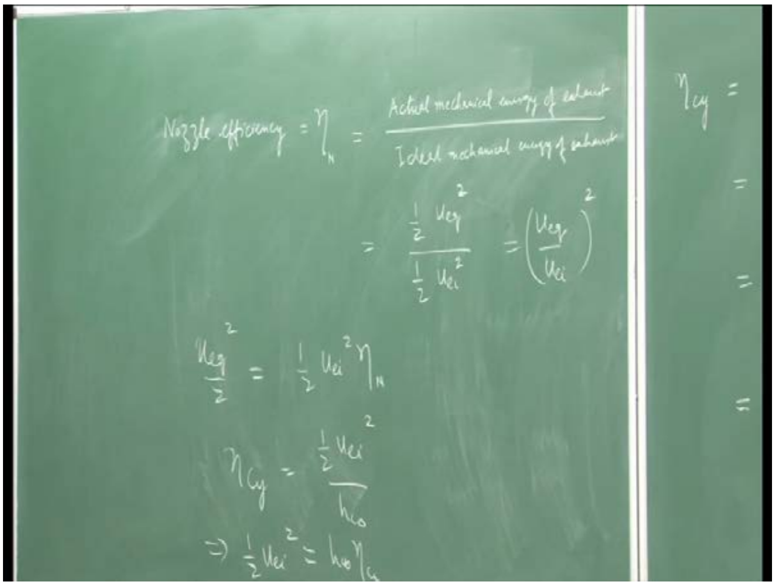

So, the nozzle efficiency is designated by eta n, is defined as actual mechanical energy of exhaust by ideal mechanical energy of exhaust. So, now, in order to define this actual mechanical energy; what is the actual mechanical energy will be represented by? Here P e is not equal to P a. So, if you recall at the beginning; when we defined the thrust or specific impulse, we have defined an equivalent velocity. So, that contains the information regarding the non-idealness of the expansion. Therefore, the kinetic energy that will be there if the expansion is not ideal will be represented by that equivalent velocity. So, the actual mechanical energy then will be half u equivalent square. So, this is the kinetic energy of the exhaust for the actual nozzle, which is not ideal. And, for the ideal, we have already seen that, this is equal to half u e i square. Therefore, straight away we can see that, the nozzle efficiency can be given as u equivalent by u e i square.

So, what does the nozzle efficiency now physically representing? If you look at the cycle efficiency; cycle efficiency tells us that, out of the total energy it is produced because of combustion, how much is getting converted for an ideal expansion for the exhaust. Now, how tough that is ideally supposed to have been converted; how much is actually getting converted; that gives the nozzle efficiency. Therefore, nozzle efficiency is the measure of the losses occurring in the nozzle during the expansion of propellant through it. However, how much is the losses because of the non-idealness? Which will be frictional losses; which will be shock-induced losses; which will be because of over expansion, under expansion, everything; all those things included in the definition of nozzle efficiency.

So, now, with this, what we can see is that, u equivalent then can be written as… u equivalent can be written as u equivalent square by 2 is half u e i square by eta n; where, eta n is the nozzle efficiency. And, we have defined the cycle efficiency as half u e i square by h c naught. Therefore, half u e i square is h c naught times cycle efficiency. Now, if I combine this and this, I get an expression for the equivalent velocity in terms of the cycle efficiencies and the total energy that is available for the exhaust, that is, the stagnation enthalpy after the combustion.

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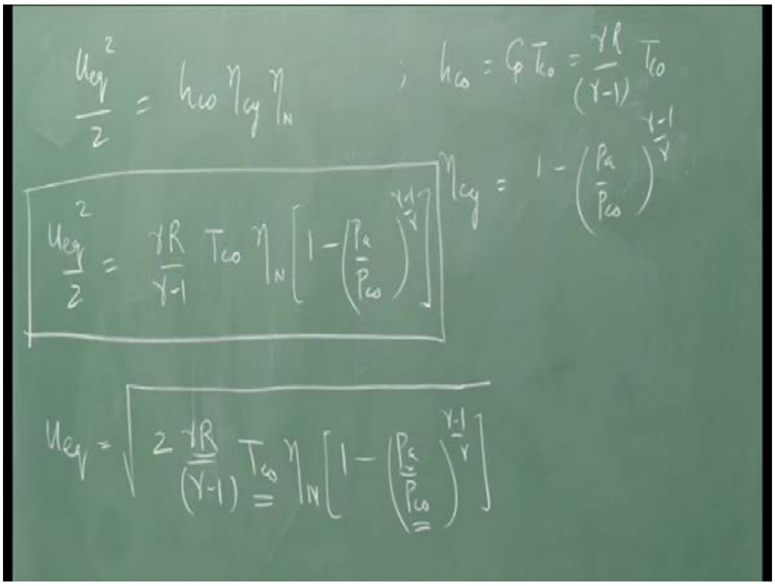

So, if I combine these two, what I will get is u equivalent square by 2 equal to h c naught eta cycle upon eta nozzle. Now, h c naught is the total stagnation enthalpy at the inlet of the nozzle. Once again, the stagnation enthalpy is equal to C p T c naught. And, C p – we have shown this before is equal to gamma R upon gamma minus 1. Therefore, h c naught is gamma R upon gamma minus 1 times T c naught; where, T c naught is the stagnation temperature after the combustion. So, it is the temperature that exists after the combustion has occurred. So, then… And, we have shown that, cycle efficiency is equal to 1 minus P a upon P c naught to the power gamma minus 1 by gamma. Now, let us combine all these and get an expression for the equivalent velocity. So, the equivalent velocity will be equal to h c naught, which is gamma R upon gamma minus 1 times T c naught times eta n times the cycle efficiency. So, this is the expression for equivalent velocity. Remember that, the specific impulse is a function of equivalent velocity.

So, now, just to recap what we have discussed; we have shown that, the performance of the rocket; that the flight mechanics part is a function of the specific impulse. We started all our discussion on chemical rockets essentially to estimate the specific impulse for a given propellant system and the nozzle design. Now, what we have seen is that, there is an expression for equivalent velocity in terms of the nozzle efficiency and the combustion chamber conditions. And of course, gamma and R also depends on the composition. So, after the combustion, the composition of the propellants will dictate what will be the value of gamma and R. So, now, we are inching closer to our goal.

Now, we have an expression for equivalent velocity. This can be further written as 2 gamma R upon gamma minus 1 T c naught eta n 1 minus P a upon P c naught to the power gamma minus 1 upon gamma. This is the expression for equivalent velocity. And, as we see that, this is a function of T c naught, P c naught, the ambient pressure and gamma and R, which are the function of the composition. And, eta n – eta n is essentially the rocket nozzle design, because that will dictate what kind of losses will be there in the nozzle. So, this expression then gives us all the information required to estimate the specific impulse.

Now, I would like to point out few more things here. Let us assume that, the nozzle is not fully expanded, is under expanded nozzle. And, at the same time, there are no losses, because typically, the losses are because of the boundary layer; frictional losses are because of the boundary layer. The boundary layer is so thin in these cases that, it is confined only very close to the wall. Therefore, the momentum loss because of the boundary layer is insignificant compared to the total momentum. So, we can assume that, the frictional losses are very small. So, under that scenario, if the frictional losses are very small, we can still have under expansion; and, at the same time, there are no losses. What happens to the nozzle efficiency under that scenario?

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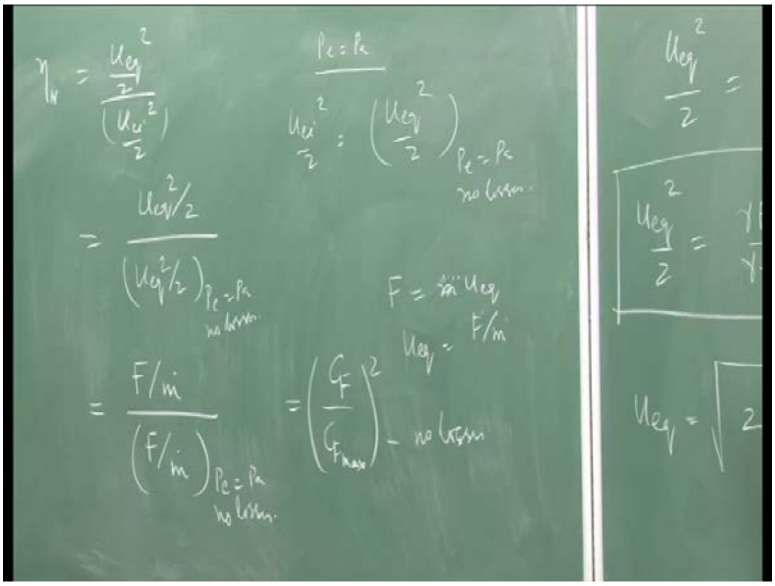

So, let me look at that case. First of all, eta n is equal to u equivalent square by 2 upon u e i square by 2. This is the ideal expansion case. Now, we are assuming that, there are no losses. On top of that, if the expansion was ideal; if the expansion was ideal, then this value… Then, eta n will be equal to 1. Therefore, this value is equal to this value. So, what is the ideal expansion? That P e is equal to P a. Therefore, this number – u e i square by 2 essentially is the exit velocity without any losses and ideal expansion. So, that is essentially u equivalent square by 2 when the expansion is ideal – P e equal to P a and no losses. Therefore, the nozzle efficiency definition then is equal to the actual case u equivalent square by 2 divided by u equivalent square by 2 for ideal expansion, which is P e equal to P a and no losses. So, I can write the nozzle efficiency like this.

Now, what is equivalent velocity? This is the expression for the thrust – F equal to m dot by u equivalent. That we have derived at the beginning our discussion. So, from here u equivalent is nothing but F upon m dot. But, F is the total thrust that is produced and m dot is the mass flow rate of the propellant. With this definition, let us go back to this and put this here. So, then nozzle efficiency becomes F upon m dot for the actual case divided by F upon m dot for ideal expansion with no losses. Now, remember that, we have discussed the thrust coefficient. We have discussed the thrust. When we discussed the thrust, we have shown both mathematically as well as with some arguments that, the thrust is maximum when the expansion is ideal. And, for all those cases, we assume there were no losses. Therefore, the denominator here corresponds to the maximum thrust. So, maximum thrust also means maximum thrust coefficient. Therefore, and also, this term will be proportional to the thrust coefficient. Therefore, this is the actual thrust coefficient divided by maximum thrust coefficient. So, we can write this as C F upon C F max. And, it will be squared, because thrust coefficient is proportional to u equivalent. Therefore, the value of nozzle efficiency… Remember all these we are doing with no losses – no frictional losses. So, for no losses, the nozzle efficiency is given by C F upon C F max squared. This is for no losses.

So, once again, the value of C F max is the thrust coefficient when P e is equal to P a; otherwise, it is C F. Therefore, we have defined now the nozzle efficiency when there are no losses. I would like to point out here one more very critical thing. This is the maximum thrust that is produced for a given ambient pressure. It is not the ultimate thrust produced by the rocket, because ultimate thrust will be produced when the cycle efficiency is 1. And, cycle efficiency is 1 when this term goes to 0; which means P a tends to 0. So, that is the vacuum. Therefore, this C F max is not the ultimate thrust. So, let me just rephrase it here.

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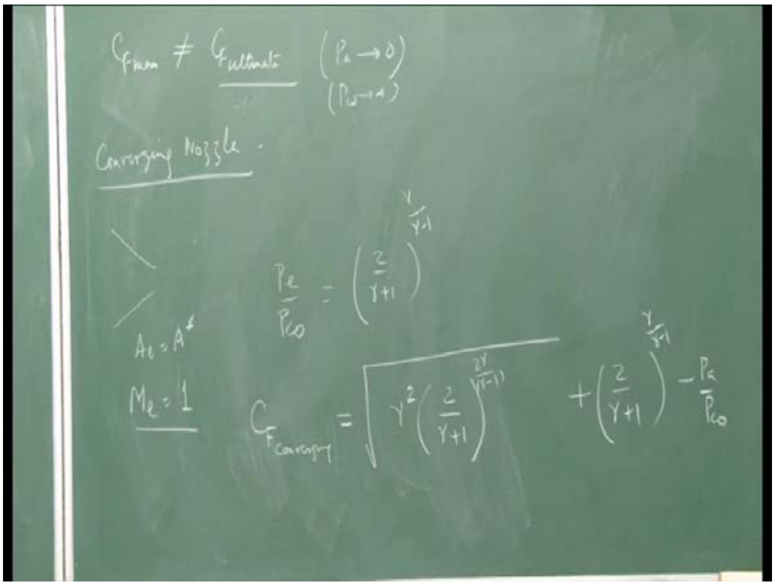

In this case, C F max is not is equal to C F ultimate, because ultimate thrust is the condition when P a tends to 0 or P c naught tends to infinity, but that is highly unlikely. So, we will take P a tends to 0. So, we are going to almost vacuum. Then, the thrust produced is the ultimate thrust. And, that is not equal to the maximum thrust we are discussing here. Maximum thrust is we have shown that, when P e is equal to P a, when the expansion is ideal – optimum expansion; it is neither over expanded nor under expanded. So, this is the definition of nozzle efficiency. All these definitions we have been deriving so far for the nozzle efficiencies were considering converging-diverging nozzles. So, this is all for converging-diverging.

What if we have just a converging nozzle? So, in the next case, we will be discussing is just for a converging nozzle. And, that will bring out the efficacy of providing the diverging section once we discuss the converging nozzle alone. So, for a converging nozzle… Essentially, what is a converging nozzle? That a nozzle which terminates at the throat; it does not have the diverging portion; otherwise, it would have been like this. But, the diverging portion is not there; it is terminated here. So, then the exit area here is the throat area; exit area is equal to the throat area. Now, under that scenario, what is the Mach number at the exit? It is 1. So, the exit Mach number for a converging nozzle is equal to 1.

Now, we have shown for the isentropic process or an isentropic flow, the pressure relationship as a function of Mach number. So, when we put the exit pressure, a Mach number equal to 1; that essentially corresponds to a unique exit pressure. And, that will be of course, depending on the P c naught. So, for this case, P e upon P c naught is equal to a function of only gamma; that we have shown before. So, P e by P c naught is equal to 2 upon gamma plus 1 raise to the power gamma upon gamma minus 1. With this then the thrust coefficient converging for just the converging nozzle will reduce to… We can look at the expression for the thrust coefficient will be gamma square 2 upon gamma plus 1 plus 2 upon gamma plus 1 to the power gamma upon gamma minus 1 minus P a upon P c naught.

Let me explain this expression. We had derived the expression for the thrust coefficient before. Now, we are considering only a converging nozzle. So, for the converging nozzle, the pressure ratio… This is the exit pressure, which… Now, we are saying that, it is not equal to the atmospheric pressure or ambient pressure, because we do not have an ideal expansion in this case let us say. The exit pressure is a function of only gamma; and, there will be now because the expansion is not ideal, there is a pressure term appearing here. If you look at these two, this is nothing but this. So, this is nothing but P e minus P a upon P c naught. So, it is coming from there. And, this term raise to the mass flow rate m dot and u equivalent. So, that will come like this. So, bottom line is if I look at this expression for the only converging nozzle; then the thrust coefficient is function of gamma and this pressure ratio – P a by P c naught, because P e by P c naught is function of only gamma in this case. So, the thrust coefficient for the converging nozzle is given like this.

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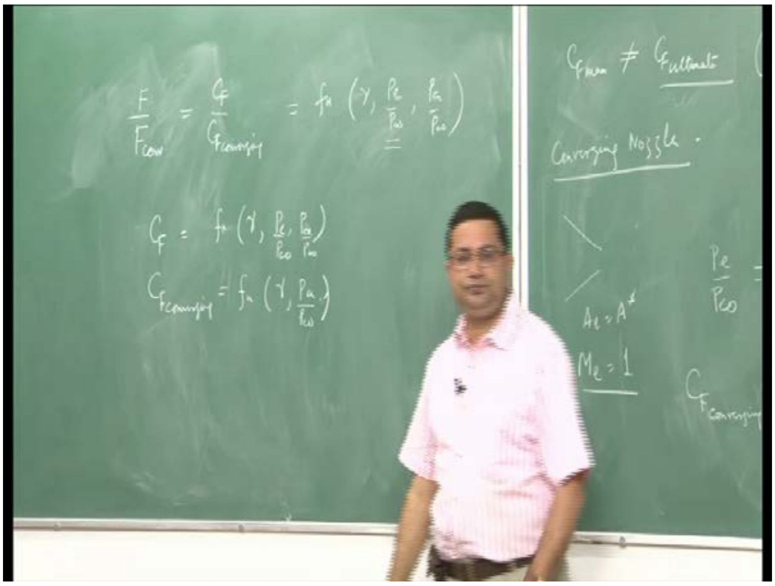

Now, let us define some more parameters. All these parameters are actually used for design. Let us look at another parameter, which is the ratio of thrust for actual nozzle, which is converging-diverging. So, the converging-diverging nozzle divided by a corresponding converging nozzle; that is, we have cut that nozzle at the throat. So, the ratio of thrust produced by these two will be nothing but the thrust coefficient for the actual nozzle divided by the thrust coefficient for the corresponding converging nozzle. Now, the thrust coefficient for the actual nozzle will be – we have shown this before – function of gamma, the composition of the propellants, the exit pressure and the ambient pressure. So, the nozzle design as well as the composition. So, we have shown this before that, this is a function of gamma P e and P naught.

On the other hand, for the converging nozzle alone has we are showing here that, the dependence on P e by P c naught is replaced by a dependence only on gamma. Therefore, for the converging nozzle alone, thrust coefficient is function of only gamma and… This is P a – gamma and P c naught. Now, let me write is as ratios, so that P c naught also comes into the picture. So, now, for the converging nozzle, it is no longer the function of the exit pressure, because exit pressure is already predetermined, because we are saying that, at the exit, the Mach number is 1. So, the exit pressure is already predetermined from this relationship. Therefore, this is the functional relationship.

If I now combine these two and look back at this ratio – F upon F converging; then that becomes a function of gamma P e by P c naught and P a by P c naught; the pressure ratios as well as the composition. Let us now look at the physical significance of this. For that, what I will do is I will plot… Now, one point I would like to point out here that, this pressure ratio P a by P c naught – on which parameter does it depend? It depends on the area ratio – exit to the throat area ratio. So, what will be the exit pressure? Will be dictated by the nozzle design, which is the area ratio is one of the prime factors. Of course, if you have a shock wave, it is going to change; but here we are assuming there is no shock wave; is an isentropic flow; there is a function of only the area ratio.

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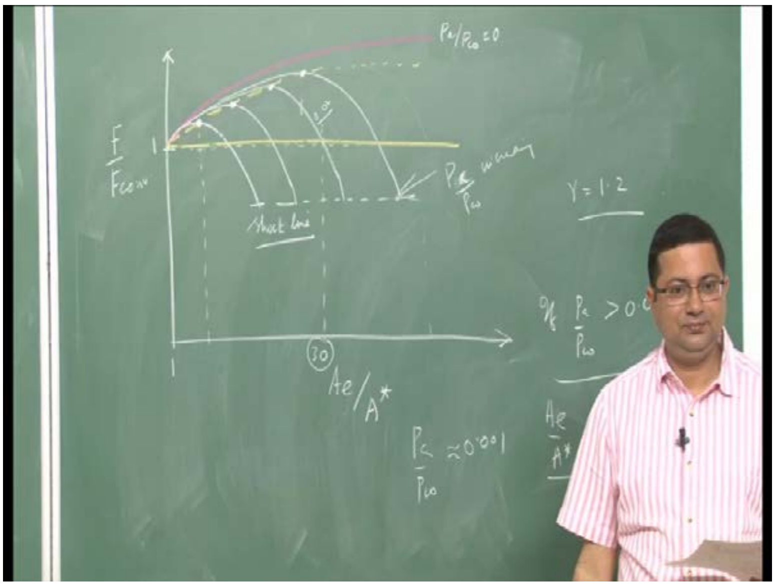

Therefore, now, if I make a plot of area ratio versus… This is A e by A star; A e is the exit area of the nozzle, A star is the throat area versus this thrust ratio – F upon F converging. First of all, for the converging nozzle, A e is equal to A star. So, when this ratio is equal to 1, then this is equal to 1 also. So, let me say that, I start from here 1, 1. Second point – we have proved that, now, in order to get the expansion, the area must increase. That we have shown before. Therefore, now, this ratio cannot be less than 1; it will always increase. So, the ideal converging case is this. I will just drop a line here representing the ratio to be equal to 1.

Now, first of all, let us look at this dependence – P a upon P c naught. So, for the given value of A e by A star, as we decrease P a by P c naught, what will happen? This thrust is going to increase and it will monotonically increase till it reaches the ultimate thrust. Therefore, if I plot this for different values, it goes like this. And, this is the ultimate thrust let us say; where, P a by P c naught is equal to 0. Remember that, we have shown that, ultimate thrust is also finite, because that becomes a function of gamma only; we do not have infinite thrust. That is why it is monotonically will approach this value.

Now, on the other hand, when we look at the variation with respect to P e by P c naught; that is a function of A e by A star. So, as A e by A star increases, initially, the thrust is going to increase; reaches the maximum for the optimum; and then drops. That we have shown that, the thrust is going to be maximum for ideal expansion. So, till the ideal expansion, it will increase; then start to drop. So, let me now just maintain this one and remove this. So, now, if I draw the variation with respect to P e by P c naught; this is for a given value of P e by P c naught; then we increase it; it goes like this. So, this is the increasing P e by P c naught. What we see here that, every one of them has a maximum point. So, I can draw a line joining these maximum points. So, this is the maximum thrust line. This is the maximum thrust line; that is the optimum thrust line. So, now, I have the maximum thrust line represented here.

And, let us say this is we are doing for gamma – particular gamma value of 1.2. So, what we are seeing here is that, this is the converging portion that is limiting here. Now, the diverging portion starts. Diverging portion depending on this area ratio and the atmospheric pressure of course, gives different contribution to the… Actually, this is P a by P c naught; this is P a by… Let me put it P e only because we are considering expansion to be proper. So, the diverging portion gives a contribution to the thrust in a nonlinear manner. Initially, it increases; which is the optimum value when the expansion is ideal and then it starts to drop. So, for each value of P a by P c naught… As we can see here, this is different values of P a by P c naught. For each values of P a by P c naught, we have an optimum point. And, this optimum point corresponds to a specific area ratio. Now, this area ratio then is our ideal expansion area ratio. So, when we are getting this maximum, at this point, the exit pressure P e is equal to this pressure. Therefore, this ratio is equal to the ratio that we are getting here. So, for a given value of P a by P c naught, there is an optimum value of area ratio, which will give us the maximum thrust, because that gives us the ideal expansion. Thrust…

Now, what we are seeing is that, if we add an area larger than this, what we have is over expansion. And, over expansion is going to reduce the thrust. So, there is an optimum area. If you add area more than that, the thrust is going to reduce. So, that is proved here. Now, for a reasonable pressure ratio, let us say if P a by P c naught is greater than 0.02; this is the reasonable pressure ratio we are considering. It can be shown that, for this type of pressure ratio, this reduction in thrust will not happen unless A e by A star is greater than 30. So, for this pressure ratio – ambient to total, when area ratio becomes greater than 30 only, then the thrust is going to reduce. So, I can show it here. Let us say this is 30; this is 0.02. So, this line here corresponds to this pressure ratio P a by P c naught corresponding to 0.02. This is the thrust that a converging nozzle had produced, would have produced.

Now, what we are seeing is that, if we are adding a diverging portion, beyond a certain area ratio, the thrust produces actually less than the converging nozzle – a thrust. Therefore, that ratio is about 30. So, beyond that value, adding the diverging portion does not benefit at all. Before this value, even though there is a reduction in thrust, it is not the optimum value; but still it is greater than the converging thrust. So, we are having some advantage. But, beyond this, there is no advantage; there is a disadvantage; the thrust is reducing. At the same time, by increasing the area, you are increasing the weight. So, there is an optimum value, beyond which there is no advantage; optimum value of area ratio, beyond with there is no advantage in adding the diverging portion. Less than that, even though the thrust is not maximum, it is still advantageous over the converging portion.

Now, for very small pressure ratios; let us say if the pressure ratio is close to 0.001; then as we can see that, as we are… because this is pressure increasing. So, decreasing will be in this direction. If you have very small pressure ratio, this area ratio is very large. So, we can have huge areas or very large areas associated with the maximum thrust. So, for the very small pressure ratios, we need to have large nozzle exits in order to get the maximum thrust; which may not we all this possible. So, we may not have the maximum thrust, but we may have something still more than the converging portion. So, that is the advantage that we will be obtaining.

Now, let us look at another scenario. As we keep on decreasing the pressure ratio, what happens? What happens we have discussed as we reduce the back pressure. What happens? Shock wave forms. So, as we keep on increasing this pressure, the shock wave initially will be outside, slowly it will move in. So, when the shock will stand at the exit, after that the flow becomes subsonic. So, that is the limiting point of ambient pressure or limiting point of area ratio. We do not want to operate beyond that because the flow becomes now non-isentropic; there is a shock wave in the nozzle. So, there is a limiting value here. This is called shock line. So, for each value of this pressure ratio, there is a point, beyond which we can have a shock wave going into the nozzle.

So, this line represents the pressure ratio and area ratio for which a shock enters the nozzle. And then we can… We absolutely do not want to operate beyond that; we do not want to have a shock going into the nozzle, because the flow becomes subsonic, you will use huge amount of thrust. So, we do not want to operate at all in that condition. And, that condition can also be derived mathematically. We can derive an expression for this condition, that is, the pressure ratio and area ratio for which a shock will be sitting at the exit. If you increase the… If the ambient pressure is little more than that, the shock will go in. Or, if the area ratio is little more than that, the shock will go in. So, this is another limiting line, which we can derive.

So, what we will do now is a… Rather I think we have exhausted the time today. So, we will stop here today. In the next class, we will start from deriving this expression, that is, the expression for the pressure ratio and area ratio for which a shock will be sitting at the exit of the nozzle. So, that will give us the shock line. So, just two recapitulate what we have discussed today; we have defined the nozzle efficiencies; we have defined the cycle efficiency; we have defined nozzle efficiency; we have defined the ratio of thrust for the converging area only. We have shown that, the advantage of adding the diverging area is limited to a certain range. If the area ratio is more than that, we do not get any advantage; actually it is disadvantageous to have the diverging portion beyond a particular area ratio. But, that is a function of how much pressure ratio we have. And, we have discussed this plot, which essentially is the performance plot I would say. So, in the next class, we will first discuss the relationship or we derive the expressions when the shock will sit at the exit of the nozzle; and then we will take on from there going to the shaped nozzles. So, we will stop here.

Thank you.