Jet and Rocket Propulsion
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Lecture – 23

So, welcome to this class 1 the rocket and space craft propulsion. In the last lecture, we started discussing the performance of ideal rocket nozzle. We have listed the assumptions that we use for ideal rocket nozzle. Look at the validity of those assumptions. Then we defined a parameter called thrust coefficient and we got an expression for the thrust coefficient. After that what we did was we derived an expression for the mass flow rate.

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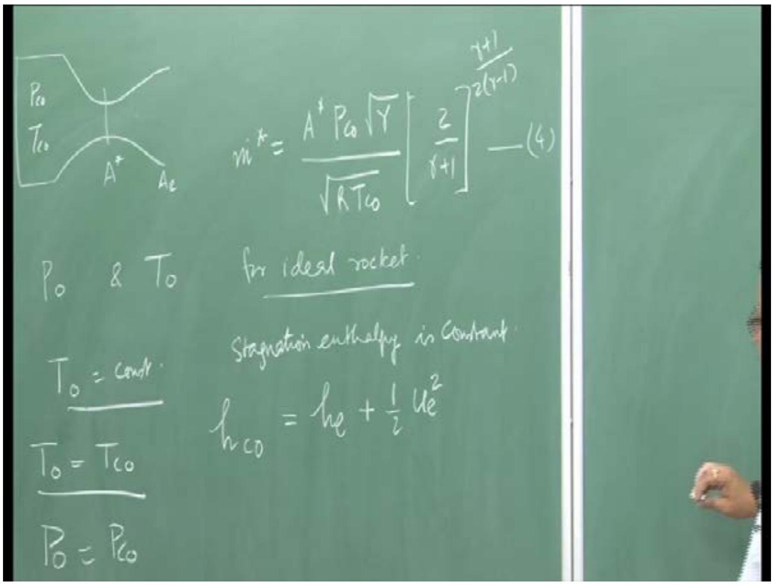

Remember that a rocket nozzle always is a converging diverging nozzle as I have mentioned. So, this is the rocket nozzle. We are considering is the throat area a star, the exit area is a e, the combustion chamber pressure and temperatures are P c naught and T c naught. The expression for the mass flow rate, the general expression for the mass flow rate for compassable flows had the term P naught and T naught. That is the stagnation pressure and stagnation temperature.

Now, the least of assumption if you take a closer look in the least of assumption for ideal rocket, we have assumed that the process is isentropic all through and if it is isentropic, then it is adiabatic and for adiabatic, process are adiabatic flow stagnation temperature is constant. This is something that is a very important thing, because even a shock wave is a adiabatic process. Therefore, across a shock wave also stagnation temperature remains constant. Therefore, for any isentropic processes are adiabatic process, the stagnation temperature is constant. Therefore, the stagnation temperature anywhere in the rocket is equal to the stagnation temperature in the combustion chamber. So, therefore, T naught is equal to T c naught. Secondly, we are assuming the float to be isentropic and there is no walk done in here anywhere, right. It is flowing on its own.

So, therefore, since the process is isentropic, the stagnation pressure will remain constant. If it was non-isentropic or if there was a work done, the stagnation pressure will change. That is why if you have a shock wave, the stagnation pressure is going to change. Stagnation will drop if you have a shock wave. So, therefore, in the least of assumption, we said that we do not want a shock flow. Rather the presence of shock wave there that was to ensure that the stagnation pressure remains constant. So, if the stagnation pressure is constant everywhere, then P naught is equal to P c naught. Therefore, best on this we had derived the critical mass flow rate which is the throat mass flow rate is equal to a star P c naught by square root of gamma divided by square root of r T c naught 2 upon gamma plus 1 to the power gamma plus 1 upon 2 upon gamma minus 1, and this was continuing from the previous lecture. Let me call this equation 4. Only difference between equation 3 and 4 is putting this as star. We are considering this as a critical mass flow rate.

Now, let us continue our discussion on the ideal rocket. So, for ideal rocket, the stagnation enthalpy how is that going to vary? If you look back at our derivation of quasi 1d flow, the energy equation we have shown that for a quasi 1d flow, the stagnation enthalpy is constant. So, for this case, since with all the assumption, this is a quasi 1d flow. The stagnation enthalpy is constant which essentially means that the stagnation enthalpy at the chamber which is h c naught is equal to the static enthalpy at the exit plus the kinetic energy half u e square. So, this is coming from state from the derivation that we have shown that stagnation enthalpy is constant for a quasi 1d flow. We have this condition now, where h c naught is stagnation enthalpy in the combustion chamber, h e is the static enthalpy at exit and u e is the exit velocity.

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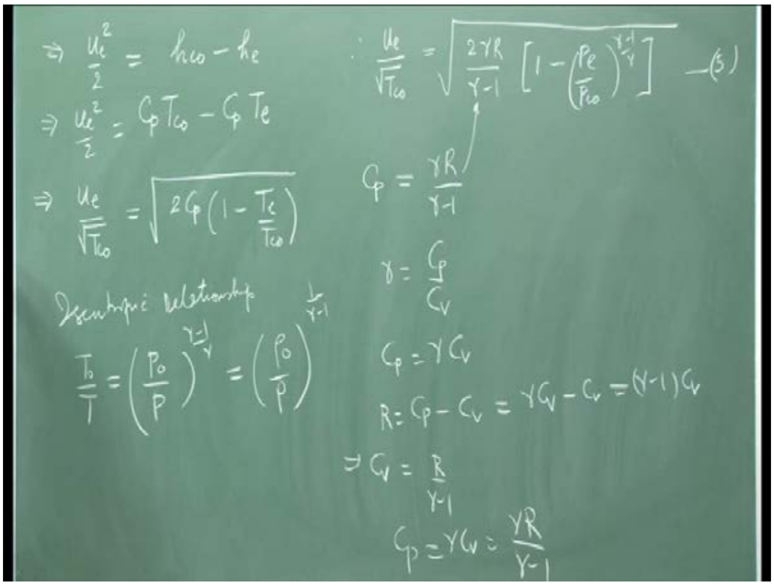

Now, from this equation we can write u e square by 2 is equal to h c naught minus h e and since, we are considering the working fluid to be a perfect gas which is homogenizes and everything. So, therefore, C P is constant, right and c c thermally perfect gas h is equal to C P t. So, therefore, we can write it as C P T c naught minus C P T e plus C P is the specific gas constant at constant pressure. Now, this equation will give us an expression for u e by square root of T c naught is equal to 2 C P 1 minus T e upon T c naught. This equation now can be rephrased in terms of the pressure ratio. Here, it is in terms of the temperature ratio T by T c naught. If you look back at our discussions on the effect of back pressure and the isentropic flows, T e is something that we do not know. We cannot possibly measure also with very much accuracy, but exit pressure we know that is why we talked about the effect of exit pressure, not effect of exit temperature.

So, what we would like to do is this we will represent in terms of the pressure when the pressure is something that we can very easily measure. So, next what we do is, since the process is isentropic, we can use the isentropic relationships. So, from isentropic relationship what we can get is that T naught by T is equal to P naught by P to the power gamma minus 1 by gamma. This is also equal to row naught by row to the power 1 upon gamma minus 1. This is the isentropic relationship which you must have seen in aerodynamic courses are any fluid mechanic course we should have seen the isentropic relationship. These relationships are very important. Therefore, for most practical gasses, these are either given in isentropic tables. So, therefore, this is something that is applicable. Now, notice is one thing that since we made this assumption that the flow is isentropic, we have every simple relationship getting pressure, temperature, density etcetera. Without this we cannot do this. Without assuming the isentropic, we cannot have this simplifier relationship. So, therefore, this is again very important assumption because this isentropic process from thermodynamics, we know that the process has to be defined in order to get the state properties and these are the state properties pressure temperature etcetera.

So, the change in state properties depend on the process and here, with the assumptions that we have made, you have considered the process to be isentropic. Therefore, we have a simple relationship relating the state properties have this given here. So, now, we take this equation and put it back in to this. We get our expression for u e by T c naught square root of T c naught is equal to square root of 2 gamma r upon gamma minus 1 minus P e by P c naught to the power gamma minus 1 by gamma. Let me call this equation 5.

Now, notice what we have done. We had a term here T e by T c naught. This we replaced by P e by P c naught to the power gamma minus 1 by gamma which is appearing here. Then, we had a term C P here and by definition C P is equal to r gamma upon gamma minus 1, we can prove this also gamma is equal to C P by c v, right because gamma is ratio specific rates. Therefore, C P is equal to gamma c v and r is equal to C P minus c v. So, what I can do here is I can write this as gamma c v minus c v. So, this is equal to gamma minus 1 c v. Therefore, c v is equal to r upon gamma minus 1 and C P equal to gamma c v is equal to gamma r upon gamma minus 1, right. So, this is can be very easily proved.

So, therefore, we have C P is equal to gamma r upon gamma minus 1 which we have put here. So, this is now our expression for the exit velocity. This is what we have been trying to get. We have been talking about the velocity increment from flight mechanics. What we needed was this exit velocity. Now, we have got an expression for exit velocity. As we can see that exit velocity is function. Of course, the fluid property is gamma and r, it is function of exit pressure, function of stagnation, temperature in the combustion chamber and the stagnation temperature and pressure at the combustion chamber. So, now this is the equation that will give us the exit velocity which we needed to estimate the thrust.

So, that is why we followed this process because finally, this is what is going into the thrust. In the last class, we have defined thrust in terms of thrust coefficient. So, now, let us go back to the definition of thrust coefficient and then, in the thrust coefficient, remember that the first term had the exit velocity term appearing. So, there I will replace that with this equation if I go back to the definition of the thrust coefficient.

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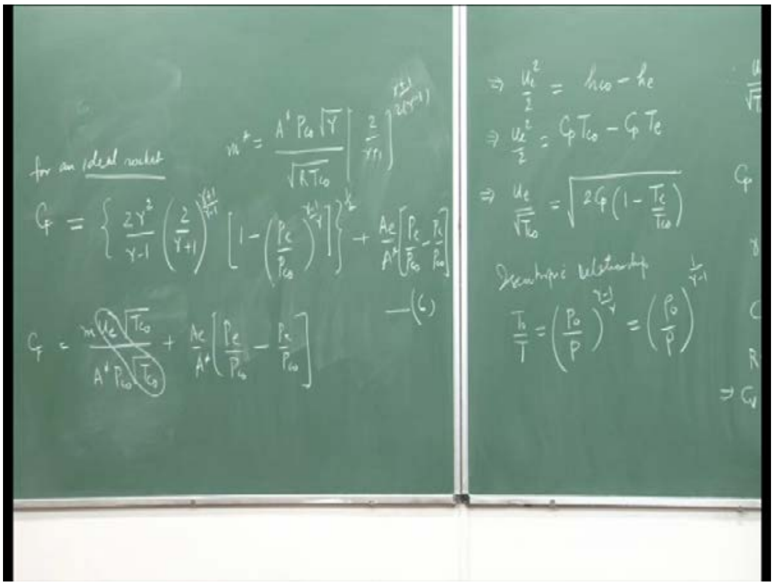

So, for the ideal rocket, the thrust coefficient then will be given as 2 gamma square upon gamma minus 1, 2 upon gamma plus 1 to the power gamma plus 1 upon gamma minus 1, 1 minus P e by P c naught to the power gamma minus 1 by gamma to the power half plus a e by a star P e by P c naught minus P a by P c naught. Let me call this equation 6.

Now, let us see how did you get this? Let us see how did we get this equation? This equation comes from the equation which we have derived. In the last class, we said that c F is equal to M dot u e square root of T c naught divided by a star P c naught by square root of T c naught plus the pressure term, which is this, this is the equation we had derived for last class for the thrust coefficient.

Now, let us look at this equation. We have a term here u e by T c naught, right. We have derived an expression for u e by T c naught here. So, this is what we have put in. So, this term appearing here is coming from their and we have one term M dot square root of T c naught by a star P c naught. Look at this here. From this equation I have deleted some part of it. I have deleted some part of it from this equation. We get M dot square root of

T c naught by a star P c naught appearing here will be a function of gamma and r, right. So, I put this back into this equation.

I put this into this equation. then little bit of algebra will give me this equation. So, this is the expression for the thrust coefficient for an ideal rocket. Now, let us see what does it convey? According to this equation here, we have eliminated T c naught completely. So, temperature is not present there. So, always whenever we talk about combustion, always people think about temperature has the most important parameter, right, but here you see temperature has no significance. It has been eliminated completely. What is important here is the pressure, right.

So, therefore, for the rockets, temperature is not important. What is important is the pressure, right. So, therefore, now looking at this equation, the dependence on temperature is there. So, this parameter gamma temperature is going to dictate gamma, otherwise temperature has no significance. Here, it is essentially a function of pressure to this thing by the way for most of the aero engines. That is why when we talk about combustion for aero application, pressure is the more important parameter. Then, temperature, you do not even report temperature in most of the practical cases essentially.

What is temperature? It is the important parameter. So, now, let us look at this. So, our thrust coefficient is the function of this pressure ratio and this pressure ratio is function of our nozzle geometric, right that we have shown. So, therefore, this dependence on nozzle geometry and here, also dependence on nozzle geometric and of course, the initial pressure and gamma is a very simple expression. Again, let us point out that it has two terms. First term is this and the second term is pressure term. So, thrust coefficient has two terms. First is the momentum term; other is the pressure term. Now, let me just look at this equation and write it in some functional form. So, what I will do is like to have a little closer look at this equation.

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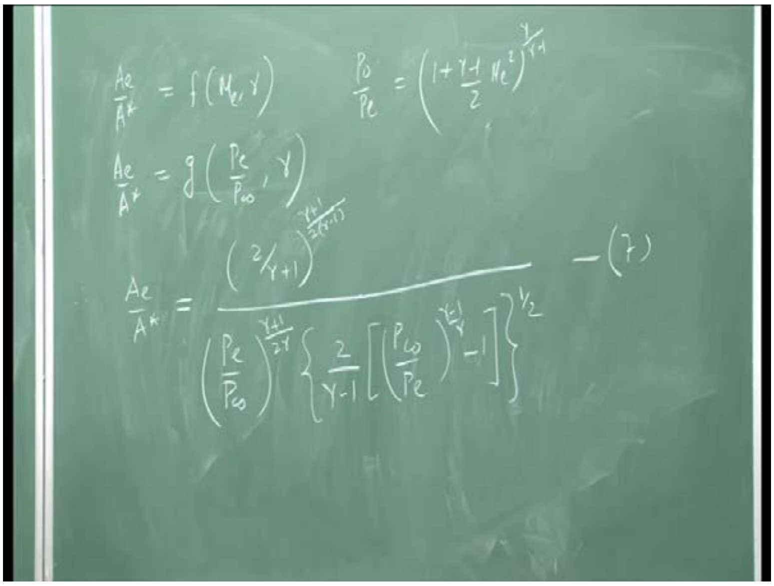

So, let us look at first the area ratio. Area ratio is given here. We have shown that the area ratio is a function of the mach number and gamma, right, exit mach number and gamma. We also know that if you are considering the flow to be isentropic, then P naught by P e is equal to 1 plus gamma minus 1 by M e square to the power gamma upon gamma minus 1. This is for isentropic flow.

So, therefore, looking at this, what we can see here is that the mach number is a function of P naught and P e P naught by P e, this pressure ratio and gamma. Therefore, putting this back into this equation, we can say that the area ratio is also function of P by P naught and gamma. So, a e by a star is a function of P e by P c naught and gamma. Here in this case why P e is exit pressure? P c naught is the stagnation pressure as we have discussed earlier. So, therefore, this gives me the area ratio. Now, area ratio in terms of this can be written as again this. You can do yourself homework. I am not going to the details of this. I have already derived the expression for the area ratio in terms of mach number, right. In that equation you will replace mach number by this and then, simplify to get this P c naught by P e gamma minus 1 minus 1 to the power half. Let me call this equation 7.

So, let me see what we have done here. The area ratio is a function of mach number and gamma and mach number is a function of pressure ratio. So, in this area, mach number relationship which we had derived earlier, if I replace the mach number by this pressure ratio, then after little bit of simplification we get this relationship. Here, area ratio is represented in terms of pressure ratio and gamma. So, now, what is the advantage of this equation? If I take this equation and put it back into my expression for thrust coefficient equation 6, this area ratio is now replaced by this. So, what we have now is the thrust coefficient as a function of only the pressure ratio and gamma, nothing else. Everything else has been eliminated.

So, using this now we have, if I put it back into this equation, we get the thrust coefficient as a function of pressure ratio and gamma. Let us now take a look at this term here is my area ratio and the combination of this and this will give me my pressure ratio. What I will do now is look at little more on this. Remember at the beginning of this course when I derived the thrust equation, I had shown from analysis looking at various things that we have discussed that the thrust is going to be maximum if you have ideal expansion. Now, let us mathematically prove it for that. What I am going to do is I have this area ratio expression here and I had the thrust equation which I have just deleted.

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Let me just plot the variation of thrust coefficient c F verses this area ratio. Now, notice one thing that whether we are going to have an ideal expansion or under expansion or over expansion depends on what the exit pressure, and the exit pressure depends on the exit mach number which depends on exit area. Therefore, this is the parameter that is going to dictate whether we have an ideal expansion, under expansion or over expansion.

So, now, what we do is let us say that we vary this parameter area ratio and plot our thrust coefficient. If I do that, I will get something like this. This plot is for a given value of P c naught P a by P c naught. That is back pressure for a given value of back pressure P a. Here is our back pressure if I plot the variation.

Now, what is happening if you choose a particular value of a by a star gamma is fixed, then P e becomes a parameter which can be estimated from here, right. Once we estimated this P e, we can put into the expression for specific thrust coefficient and we can get the value of thrust coefficient, but for that we need to specify this. So, this value is specified left side and now, we get this plot. Let us look at plot. Now, what we notice here is that there is a maximum point here. There is a point corresponds to a given area ratio here for which the thrust is maximum and it so happens that this plot, this point corresponds to P e equal to P a. You can show that you can maximize that equation with respect to P e, right. That is another way. If my every equation maximize it with respect to P e, you will see that for P equal to P a c F is maximum. So, this point here, the thrust coefficient is maximum and this is of course our ideal expansion, right.

So, once again we are proving that for the ideal expansion, the thrust is maximum now if the area is less than this ideal expansion. Now, we have shown that. So, this is my ideal expansion area, right. If the exit area is less than this, then the expansion is not complete. It has to be more expanded to get to the ideal expansion, right. So, the expansion is under, right. If this area ratio is less than the critical area ratio, we have under expansion. So, therefore, this side is my under expansion and if my area is more than this, then the expansion is more than that is required because this is my ideal expansion. Expansion is more. So, this is my over expansion. So, in the under expansion here, since we have cutting the nozzle before it is used, the ideal condition, the pressure is higher at the exit of the nozzle. The pressure is higher than the ideal expansion pressure.

So, therefore, it has to go through an expansion and to further expand it. When we are taking it further away, then the pressure is lower than the ideal expansion. So, it has to be compressed which has to be done through as an expression way where abolish shock wave to bring it to the ideal case, other bring it to the atmospheric equation. So, this is the plot of various cases we had discussed already, right. So, this is something this plot once again (( )) the fact that for the ideal expansion, the thrust produced is going to be maximum now and also, from this plot we have discussed that for the under expansion which is here we need to have an expansion (( )).

So, because of the fact, this is P e, this is P a ideal for under expansion P e greater than P a. So, we need to have expansion fans to further expand the flow, so that finally it returns this P a. So, that is for the ideal expansion also. We have discussed that for over expansion, the pressure here is less than the atmospheric pressure. So, therefore, we need to have a shock wave or comparison wave to increase this pressure. So, this is going to be a shock wave. So, increase this pressure, so that finally the pressure becomes equal to the atmospheric pressure. So, this is for over expanded nozzle. Now, one point I like to mention here is all these things, all the discussions here are for a given value of P c naught stagnation chamber pressure. Now, does the thrust coefficient depend on the stagnation chamber pressure? If I look at expression for the stagnation, the thrust coefficient you can see that the stagnation chamber, the thrust coefficient can be increased by increasing the chamber pressure.

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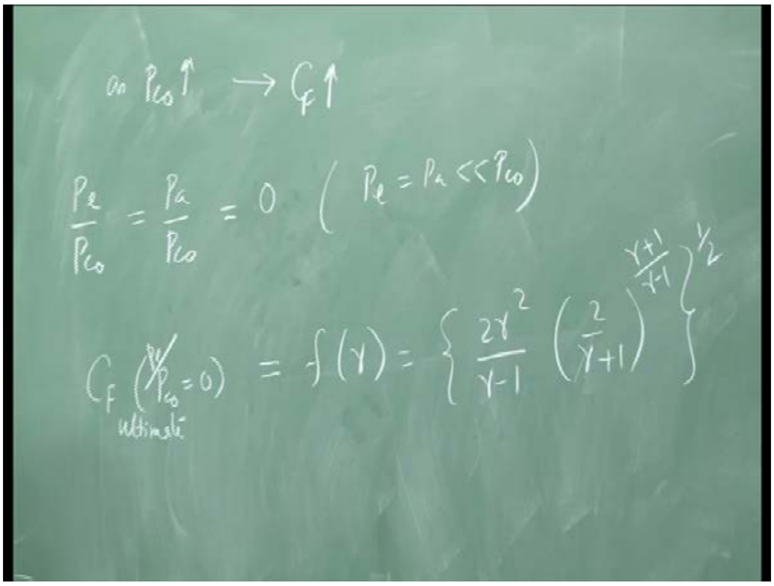

So, it can be shown that as P c naught increases, the thrust coefficient will increase and we would like to get higher thrust coefficient, right because that will give us higher exit velocity. So, the higher thrust which will essentially mean that we can carry more pay load or go to a longer distance. So, this is something we want to increase, but as we can see, here is the thrust coefficient increases with increasing in the stagnation chamber pressure, then the question is can we say that whatever we do, it will be more than it will be increasing. We keep on increasing the chamber pressure and the thrust is going to increase always. Then, we can get infinite thrust. Technically, is it possible? No. Because even though as we increase P c naught, the exit velocity increases, c F increases, but with the same at the same time there is a change in our pressure ratio and that attends a limit. So, let me look at this. If I look at this ratio P e by P c naught, right since we are talking about ideal expansion, let say this is equal to P a by P c naught. What will be the limiting ratio? It can be very small, but it cannot be negative, right. Pressure cannot be negative, absolute pressure because talking about an absolute pressure always. So, pressure cannot be negative. So, in the limiting case, it can be 0.

So, if my exit pressure which is equal to (( )) pressure is very small. With respect to the chamber pressure, it can attend A 0 velocity A 0 value. It cannot go less than that. Then, this ratio 0 can go back to our equation for the thrust coefficient c F for P c naught equal to 0. Now, this pressure ratio is 0 in that. Then, this will be a function of only gamma, right. If you look at that equation, you will get this to the power half. So, this is the maximum thrust coefficient. You can get when this ratio is 0. You cannot get more than this. So, this thrust coefficient is called ultimate thrust coefficient.

The limiting case where, sorry not P c naught is 0. P e by P c naught is 0. In the limiting case when the pressure ratio is 0, we get the maximum possible thrust. Coefficient is this and that depends on gamma. So, that depends on the kind of propellant we are using. So, therefore, this discussion shows that there is a maximum value of thrust. We cannot get more than that. It is limited, right. So, with this, we complete our discussion on the thrust coefficient. There is another important parameter for the rocket proportion which is called characteristic velocity.

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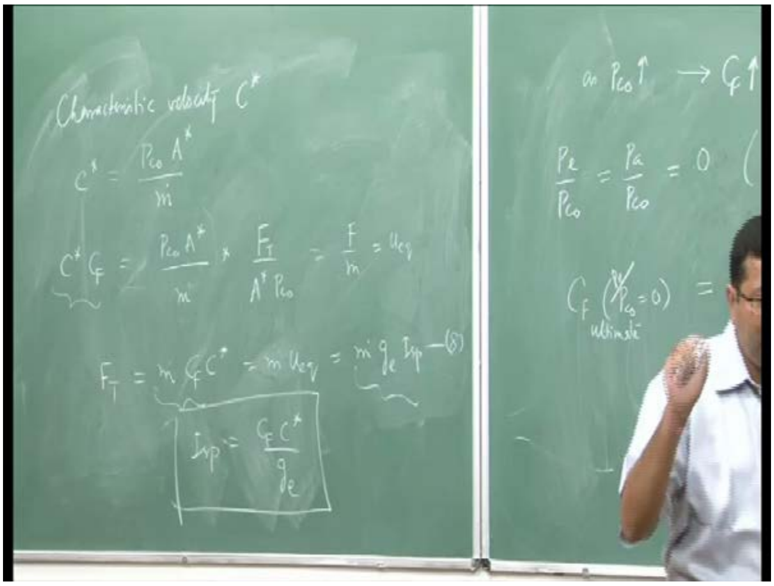

So, next let us define the characteristic velocity and then, we will see that the performance actually is a function of these two parameters thrust, the coefficient and characteristic velocity. So, the next thing we discuss is characteristic velocity c star is discussed by c star characteristic. Velocity c star is defined as P c naught upon a star by M dot. Now, the thrust coefficient if you look at essentially depends on the nozzle design as a parameter of nozzle design whereas, the characteristic velocity is a parameter representing the combustion characteristics because this P c naught and M dot depend on the combustion. So, therefore, the combination of these two is the complete rocket, the combustion chamber as well as the nozzle. So, thrust coefficient gives us the nozzle performance, characteristic velocity gives me the combustion chamber performance.

So, now, looking at this if I look at c F c star upon c F, the product of these two we will see that this is equal to F by M dot which is my equivalent velocity by definition, right. So, therefore, the product of characteristic velocity and the thrust coefficient gives us the equivalent velocity. So, this is, remember that this is the parameter which we have been looking for. Now, we have shown that from the flight mechanic exits the most important parameter. Now, we are looking for this parameter c F. We have already discussed c star. Characteristic velocity is coming from here. The product of these two gives us the equivalent velocity.

So, this parameter and if you are looking for the thrust, focal thrust, then it is M dot c F c star from here. So, thrust produces M dot c F c star. We can write this then as is equal to M dot equivalent which is equal to M dot g e I s P. Let me call this equation 8. So, then notice one thing what we are doing now. So, for in the flight mechanics, we talked about I s P, right. We said is the given parameter. Now, for the first time we have got an equation relating this and this which will give me I s P I s P is equal to the specific impulse is given. Now, from this equation c F c star by g e which was a given parameter for in the flight mechanics analysis, now we can estimate this. We have seen that c F is a function of P by P naught and gamma in this equation a star again is M dot by a, star is a function of P naught, right. P e by P naught again this will be a function of P e by P naught and gamma temperature will also coming here.

So, c star will depend on temperature also. So, once we have the pressure and temperature of the combustion chamber, we can put it here. We can get the value of I s P and one more thing that we have been saying again and again at the beginning. We said that I s P value for a given fuel is constant that should have been talking at the time why should it be constant. It is equivalent velocity, right. It is a velocity you should be able to increase the velocity as much as we want to by putting in more pressure, but that time I have said that I s P constant for a given fuel ratio are a given propellant combination. You did not ask any question. We have proved it here that has a thrust coefficient gets an ultimate value. We cannot possibly increase beyond that, right and that is the I s P. We are talking about remember the rockets will be operating in vacuum ideally, right. So, this condition is valid. So, therefore, the thrust coefficient has a maximum value and the maximum value will be coming through this. So, that is why I s P for a specific chemical rocket are any rocket is constant. It does not depend on anything else, right though if I have prove this here.

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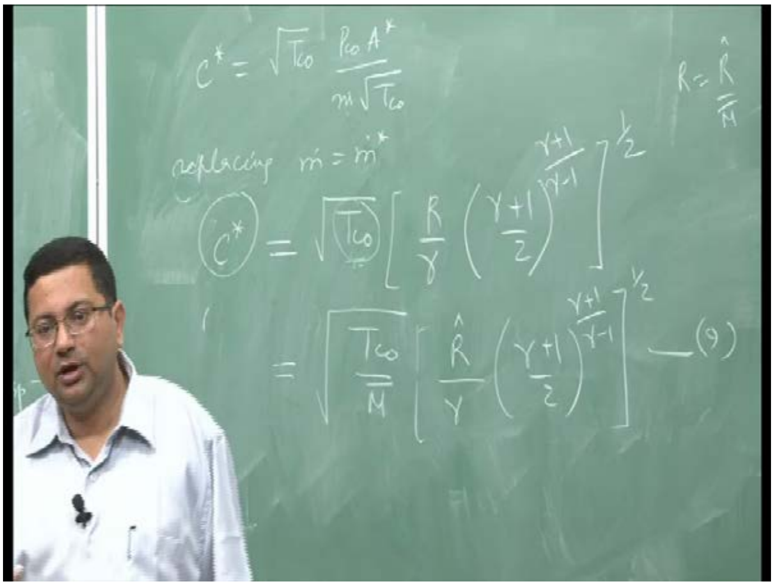

So, now let us go back to the c star characteristic velocity. First, look at c star. We can write c star as square root of T c naught P c naught a star upon M dot square root of T c naught. Now, M dot is the mass fluorite. We have shown that mass fluorite is going to be constant everywhere because the throat is choked. So, we can replace this M dot by M dot star which is the choking mass fluorite. After we do that, then this is replacing M dot by M dot star we get c star is equal to square root of T c naught r upon gamma plus 1 by 2 to the power gamma plus 1 upon gamma minus 1 to the power half. So, this is the value of c star as we can see that c star is dependent on T c naught. So, thrust coefficient was dependent on P c naught, where as the characteristic velocity is dependent on T c naught.

So, therefore, the temperature effect comes into picture through this for the characteristic velocity thrust. Coefficient is not affected much by the temperature, but the characteristic velocity is. Of course, we will not be going to details of that. This is also responsible for the instabilities, combustion instabilities particulate. The bulk model instability is dictated by this characteristic velocity. Characteristic velocity here we are saying is dependent on T c naught which will depend on the fuel layer ratio that we are, sorry fuel oxidizer ratio. We are considering, we can rewrite little differently by changing r to universal gas constant.

So, we can write this as is equal to T c naught by M bar. Let me call this equation 9. So, here this r had is a universal gas constant and this M is the molecular weight. So, this r here we can change r equal to r by M bar ms. What we can see now is that if you reduce the molecular weight, what happens to characteristic velocity. It goes up as my characteristic velocity goes up, I s P goes up, my thrust goes up. So, if you can use a lighter fuel, we get more characteristic velocity. So, we get more thrust. So, if you go to hydrogen as a fuel, we produce more thrust.

So, therefore, the cryogenics fuel like hydrogen will give us more thrust. The c star here, I would like to point out it essentially the measure of how much energy essentially dictated by this T c naught, right. T c naught is the combustion chamber temperature. Now, how is this temperature created? When there is a chemical reaction, heat of reaction is released and then, this heat is absorbed by the products to give the final temperature. We will come back to this when we talk about temperature. At present we see that T c naught is essentially the temperature of the products. So, it will depend on two parameters. One is how much energy is contained in that fuel, right and secondly, how efficiently is converted.

Now, that conversion is essentially means what kind of composition we are getting. High efficient is the combustion process. If it is completely converted, we get the maximum temperature where if it is not completely converted, the temperature will be less. So, therefore, this c star depends on the composition of fuel. It’s heating value and the combustion efficiency, so that you would like to get this as high as possible, so that our c star higher and that will be attended by choosing a proper propellant. That is why for a given, first of all there is a specific flow that we use for rockets, that is we do not use alcohol. Why not? Because alcohols have pretty low heating value, although they are fuels, they have pretty low heating values.

So, we will not get enough T naught to produce enough thrust to carry that weight. So, you do not use it. So, there are specific fuels which are used and there is a reason for it that will give us the higher c star. So, c star is a fuel property and the combustion characteristics. So, therefore, the design of combustion chamber, this becomes important. Another point like to again reiterate is the fact that c star is inversely proportional to M bar is a molecular weight, so light that the propellant higher characteristic velocity we will get.

So, we will have higher specific impulse. That is why hydrogen is a good propellant for rocket application c star. Finally, the practical use of this is to get the size of the combustion chamber. How large will be the combustion chamber? The combustion chamber design is dictated by this and also, it characterizes mixing effectiveness. How effective is the mixing of propellants? So, essentially the combustion chamber design parameter will be coming from this. So, that is why we discussed it.

Now, from here we will design the combustion chamber and then, once the chamber is made, then we look at the temperature pressure etcetera. So, essentially bottom line for the machine requirement is, we want to maximize c star. For that we have to maximize this. So, either we maximize temperature or minimize molecular weight. For a given fuel, molecular weight is fixed. So, we get much temperature as possible. That is the idea. So, therefore, I will stop here.

Now what we have discussed is that c star, the characteristic velocity tells us how to design the combustion chamber. So, in the next class, what we will do is we will take up a small combustion chamber design problem that shows how we size the combustion chamber, simple thrust chamber sizing that will give us the parameters that we will require and how do we get the length and diameter etcetera. After we have done with that, we will continue our discussion on nozzles because as we have seen that the nozzle thrust coefficient is one of the very important parameter.

Once we have with the nozzles, then we will come back to the combustion. Here at present we have the temperature is a given quantity. Later on we come back to the combustion. We will discuss the combustion process. We will say that for a given fuel and oxidizer combination, how do we get this temperature and what will be the final product given the reacted. That is what we will discuss later. So, I stop here today with the discussion with the characteristic velocity in next lecture and talk about the simple design of thrust chamber and then, continue on the discussion on the nozzles.

Thank you.