

Jet and Rocket Propulsion
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Lecture - 22

Welcome back to this course on rocket and spacecraft propulsion. For the last few lectures, we have been discussing flow through nozzles. Actually we have been discussing flow through variable area duct and in that we have shown that a converging diverging shape of the variable area duct will be able to expand a subsonic flow to a supersonic flow, where at the minimum area or the throat of the duct. The mach number will be equal to 1. In the last class, we have discussed how the flow properties inside the nozzle vary with the variation in back pressure. We have discussed the over expanded nozzle, under expanded nozzle, and ideally expanded nozzle. So far this discussion on nozzle was quite generic, not specific to rockets. It is applicable even to gas turbines or any nozzle flow for that matter.

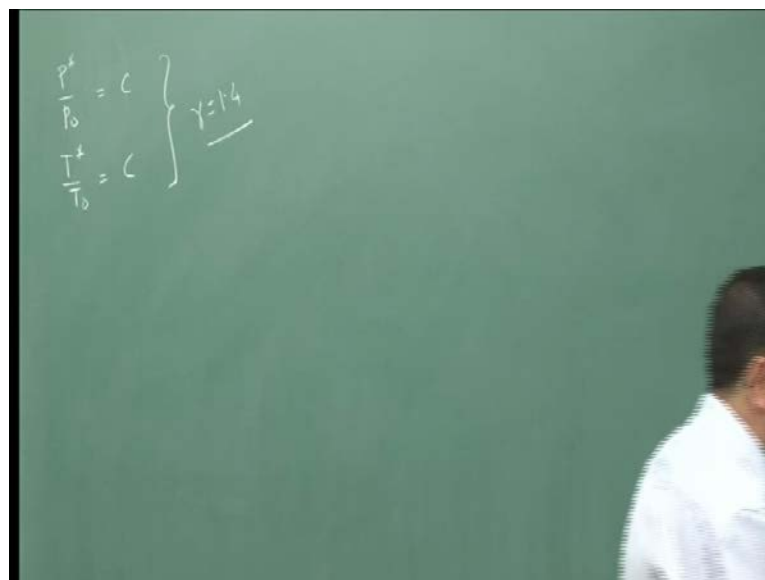
Now, let us come back and discuss the specifics for a rocket. So, now we go into the performance analysis of the rocket. In the nozzle flow discussion so far we have been silent about the velocity at the exit, but we have seen that for the rocket propulsion, the parameter that is important is the exit velocity. Of course, there were two terms in the thrust equation. One was the momentum thrust which was because of the velocity. Other was because of the pressure term. So far in the nozzle we have discussed the pressure term. We have seen how it is going to vary at different conditions. Remember at the beginning of this course, we have also shown that for the ideal expansion, the thrust is going to be maximum. So, now let us come back to our original course, which is Rocket Propulsion.

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So, now we talk about performance analysis of rocket motors. So, performance of rocket nozzles, we will consider an ideal performance. First, let me list what are the assumptions we make to consider a nozzle to be an ideal nozzle. So, the first assumption is that the working substance which is essentially the product of combustion of the propellant is homogenous and invariant in composition throughout the rocket chamber, throughout the nozzle and also the rocket chamber. Let us understand what I mean by that.

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When we discuss the flow through the nozzles, remember that we have got the conditions $P^* / P_0 = \text{constant}$ and $T^* / T_0 = \text{constant}$, but for both of them, we have considered $\gamma = 1.4$. That means, we had considered the working fluid to be air, but now when we come to rocket, this is not going to be the case because we do not have any air in rocket. That is the primary. At the beginning, we have said that rocket do not interact with atmosphere. So, therefore, these constant values are going to be different and what constant value it will take, depends on the propellant that we are using, right. So, therefore, first of all in order to define this γ and other thermodynamic properties of the propellant like c_p , c_v , R etcetera, we have to estimate that value.

Now, unless we consider that propellant after combustion products are homogenous, we cannot easily estimate. Then, locally there will be variation. Then, it will become a function of the location which we do not want to have in an ideal rocket. So, that is why we are considering a homogenous composition. Secondly, invariant composition. What do we mean by that? When the gasses expand through the nozzle, remember that this is product of combustion, right. So, the gasses are coming from the combustion. Combustion reactions still continue as the gasses expand the temperature drops, right. As the temperature drops, the chemical kinetics is going to change. Rate of reactions are going to change because from combustion, we have seen in the combustion courses we know that the rate of reaction is a function of temperature.

So, as the rate of reaction changes, the reaction constants also change, right. So, the reaction, if the reaction constant change, then the overall composition is also going to change because that is the function of the reaction constants. So, the overall composition can change here. We are assuming that the composition does not change. If the composition changes, once again γ , R , c_p etcetera is not going to be constant throughout. Then, at every location it is going to vary. We do not want that to happen. Now, let us see that how valid is these assumptions. When we look at a rocket chamber, we would like to have very uniform combustion, otherwise there will be hotspots, there will be high temperature somewhere, low temperature somewhere. So, if we want uniform combustion, then after combustion, of course it is going to be homogenous. Therefore, this assumption is quite coming to this invariant composition. Remember that the flow through the nozzle is supersonic, primarily only little bit of subsonic portion,

then primarily supersonic. If the flow is supersonic, characteristic flow time is quite small, right.

So, the characteristic flow time is given by T_{flow} is quite small. If this characteristic flow time is less than the chemical time, this characteristic chemical time is because of the chemical kinetics, the time taken by the reaction to be completed, right. If the flow time is less than the chemical characteristic time, then the flow is moving faster than the chemistry can bring about the composition change. Such a flow is called a frozen flow, right, frozen flow. Now, typically these are the conditions which prevail in rockets because the velocity is supersonic, the flow velocity is supersonic. So, therefore, we usually have frozen flow in the rocket nozzle. So, therefore, we can assume that the composition is invariant. Therefore, these two assumptions are quite valid as far as rockets are concerned. So, that is one first assumption. What does this assumption do? It gives us constant values of γ , c_p , R etcetera all through the rocket. So, now, this is one condition which has been identified.

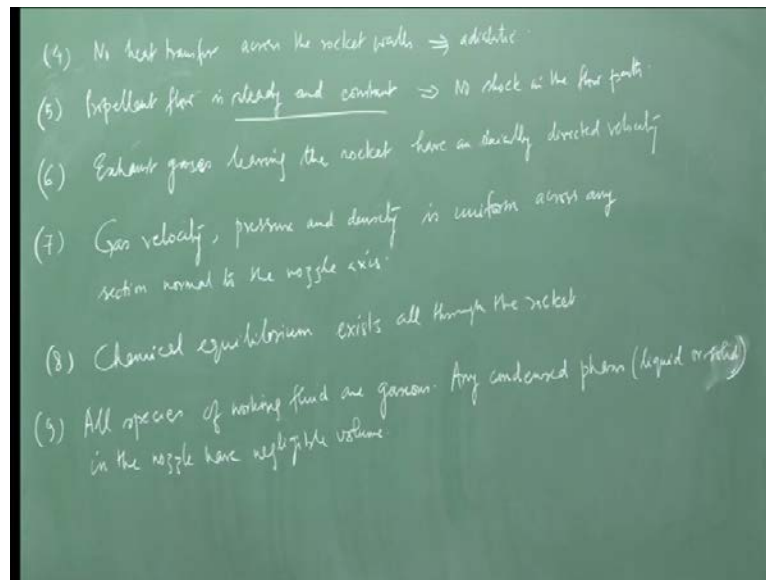
Next, let us consider the working substance to be a perfect gas. Once again what is our working substance? It is the product of combustion which is a mixture of various gasses. We can assume the mixture to be a perfect gas. The advantage is, then it will follow the perfect gas equation of state which is $P = \rho R T$. This is the assumption which is very universally used everywhere in any gas flow. We assume the working substance to be perfect gas. So, this is something reasonably good approximation. Third, there is no friction.

Now, when the flow goes through the nozzle, there is going to be a boundary layer. In the boundary layer, there is going to be some friction, but the flow is going at a high speed and we have a diverging passage. If I look at the total extent of boundary layer, it is quite thin compared to the full flow passage to the boundary and the velocities are. So, high. So, because of that the boundary layer is limited to a very thin zone at the wall, and that is where the friction is occurring. So, therefore, the most part of the flow, there is no friction because it is essentially gas flow friction is there. The inter-fluidic friction is there, but this is very small. Most of the friction is confined to the wall.

So, therefore, for the most part of the flow is friction less. So, we can assume there is no friction. The advantage of assuming this is that there is no friction, then the flow is or the

process of the flow is reversible, right. So, if the process is reversible, one condition for isotropy is made. Isotropy has two conditions. One is adiabatic, other reversible. If there is no friction, one condition is met. So, therefore, this is again a reversible approximation for tactical cases then.

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As I have just said, one condition will be of isotropy will be met by considering no friction. What is the other condition? No heat transfer. So, let us assume that also, that we have no heat transfer across the rocket walls which essentially mean that the process is adiabatic. We assume there is no heat transfer across the rocket wall. So, the process is adiabatic.

So, now we are considering adiabatic and reversible. So, therefore, my process is isentropic, but there should be a word of caution here this something that very rarely is fulfilled in practical cases because the temperatures are so high that there is going to be huge heating, lot of heating if no heat transfer is allowed. So, actually heat transfer prevents damage to the rocket, particularly at the throat area where the temperatures are going to the, heat transfer rates are going to be very high because the area is small and unless we allow that heat to go out, it will melt the throat and change the shape of the throat. Therefore, heat transfer is something that is actually needed for the operation for the nozzle to survive the high temperature gasses flowing through them. Therefore, this assumption is questionable that there is no heat transfer. Actually for all the kind of

rockets, there is some kind of cooling for the nozzle which essentially takes the heat away from the nozzle wall to maintain a reasonably lower temperature and that is essentially through heat transfer. So, it will actually enhance the rate of heat transfer to the cooling part, so that the rocket wall survives the high temperature particularly at the throat. So, therefore, this is a questionable assumption, but once again for back of the envelope calculations are the first order calculations. This is a reasonable assumption, ok.

Then, the next assumption for an ideal rocket is that the propellant flow is steady and constant. That means, that the flow of a propellant whether it is a liquid propellant or if it is solid propellant, then the pyrolysis process and the evaporation process, this is constant. It is coming at a constant rate and steady, particularly for a liquid propellant. It is very important to have steady and constant flow rate essentially means that we do not have any shockwaves or other disturbances in the flow path of the fluid or the propellant coming to the combustion chamber.

So, this implies we have no shockwaves. No shock in the flow path and I would like to mention one thing here that if we are talking about let us say cryogenic or semi-cryogenic rockets, where one of the propellant is cryogenic like for example, if it is a cryogenic, it is liquid hydrogen and a liquid oxygen and liquid kerosene. The cryogenic fuel when it is sent through the rocket chamber, it absorbs heat from everywhere, even pump and everything and by that time it reaches the combustion chamber, it may have turned gaseous and now, if it is coming at a fairly high speed, it may be coming at a supersonic speed and then we have and all to control the flow rate, right. Now, this can create shockwaves which will essentially make it unsteady or not constant flow rate. So, the purpose of this has to be designed in such a way that we should not have shockwaves in the flow path. At the same time for liquid propellant rockets, we need to have the steady and constant flow rates, for that is why we use it, so that the disturbances in the chamber do not propagate backward and change start to change the propellant flow rate.

So, we get a steady propellant flow rate into the chamber. So, therefore, this is a very important assumption that the flow rate is independent of rocket chamber conditions. Then, the sixth point is the exhaust gasses leaving the rocket have an axially directed velocity. Now, what do we mean by that, that the flow of the gasses coming out of the rocket is predominantly in the axial direction essentially allows us to consider the flow to

be quasi 1d. In reality, it is a 3d flow as like we have discussed, but if we are considering the primary dimension direction of the flow is axial direction, we have discussed a quasi 1d flow. So, this allows us to assume quasi 1d flow. Therefore, whatever discussion we had in the last few lectures can be used to this flow. That is why this is important assumption that we make for the ideal rocket. Then, the seventh assumption is the gas velocity. Then, the pressure and density is uniform across any section normal to the nozzle axis. Once again remember that this is the same assumption that we had made when we talked about the quasi 1d flow that we have uniform properties normal to the axis at a particular frame. The properties are uniformed.

So, once again this assumption will allow us to use the quasi 1d derivation that we have done so far. So, these two together will allow us to use the equations we had derived so far, and the analysis of the rocket flow and everything now will be applicable. So, like what were the assumptions, there we have made, we had considered steady. Here, we are considering steady, right. We had considered isentropic here. We have this and this together gives us isentropic. We have considered no friction which is here. We had considered quasi 1d flow which is here and of course, the body forces will come. So, essentially most of the assumptions that we had made for the quasi 1d flow or the isentropic flow through variable area duct are there in the ideal performance of rocket nozzles assumptions. So, therefore, whatever we have discussed so far will be applicable.

Let us continue from here that eight chemical equilibriums exists all through the rocket. This is something that if we have chemical equilibrium, then at every point of time we have the chemistry has reached equilibrium. So, the composition has taken its value which is required value. So, this is something, actually this assumption and these assumptions are in contradiction to each other. If you consider chemical equilibrium at every point, then if the temperature changes, chemistry is going to change, but we can consider one thing that the process has reached equilibrium in the combustion chamber and then the composition is invariant in the nozzle. So, the chemical equilibrium has occurred in the combustion chamber that can be assumed.

Now, another issue is that one thing that very frequently happens in rockets particularly that when the temperature starts to increase to a very high level in the combusted chamber, there is a possibility of dissociation. We do not want to consider the dissociation. Therefore, we say that chemical equilibrium has occurred. Now,

dissociation will be actually prevented when you go into the nozzle because the temperature is dropping, but in the chamber dissociation can happen.

So, chemical equilibrium assumption essentially is not allowed dissociation. So, once again as I said that this and these assumptions are contradicting each other, but if we consider that the chemical equilibrium has occurred in the rocket chamber and the nozzle flow is frozen, then these are not contradicting. It is perfectly valid. So, this is something that we will look back again when we talked about the combustion chamber characteristics, and then number nine is all species of working fluid are gaseous in any condensed phases which can be either liquid or gas. Sorry, or solid or solid in the nozzle have negligible volume. Let me understand what we mean by this. Our working fluid is the product of combustion typically for rockets we rarely use gaseous propellants. The propellants are either solid or liquid and cryogenic is part of liquid.

Now, when the combustion takes place, however, again this I have said again and again in the combustion courses which we have attended that the combustion will only be in the gaseous phase. So, when we have the propellant, let us say liquid propellant or a solid propellant, they first have to gasify. If it is a solid propellant, pyrolysis. There will be vapor formation for liquid, there will be atomization and evaporation to form the vapor and then the reaction will occur in this gaseous phase. What we are saying here is that all the species are in gaseous phase always if any condensed phase like liquid or solid remains. Now, what happens is that the process of pyrolysis or the process of atomization and evaporation is not perfect. So, some of the solid may have come out from the propellant bar and a small thing may remain in the flow which goes out with the flow. Similarly, the atomization is occurred, but evaporation is not completed when it goes to the nozzle. So, therefore, that small point of either solid propellant or liquid propellant has an order of magnitude higher density, right.

So, therefore, locally there is a massive density gradient. There is a massive variation in density because both liquid and solid are much denser than a gas. So, if that happens, then these properties, the perfect gas equation will not be valid. These properties will have different values because the specific heats will change. So, what we are saying here that most of the working fluid is gaseous. All the working fluid is gaseous. Now, just to compensate for any eventuality, we say that even if some liquid or some solid remain, the volume of that is much smaller, negligible volume, right. Essentially what we mean

here is negligible volume means that the volume is negligible, density is high. So, mass is small, right.

So, the mass of this is very small compared to the total mass. So, it does not have very glaring impact in either the perfect gas flow or the flow properties. So, again we can still assume it to be homogenous. That is the point because this comes directly from this that we have homogenous. Unless we have gaseous mixture, it is very difficult to assume the flow to be homogenous. So, therefore, this assumption essentially is a subset of the assumptions that we have made here that in order for these three assumptions. First, these two assumptions, first and second to be correct. This assumption must be there and this is again a very practical assumption. Typically, it will be like that and actually not it will be like that this is what we want to attain and last, but not the least, we should have a steady flow. This steady flow assumption is not the assumption here. Here, the propellant flow was steady. Here we are talking about the overall flow is steady.

Now, the unsteadiness can come from combustion instability from shockwaves presence which will start to move right from acoustic. So, we are saying that those disturbances are not present; we have steady flow to the entire system. So, these are the ten assumptions that we make which will allow us to consider the rocket nozzle to be an ideal rocket and now, for this we will start our discussion, but before we do that let us see that we have made these ten assumptions. How do we actually achieve these conditions? That is also important to know that what should be the design. These are our design goals. We would like to attain this, so that we want to be as closer to ideal as possible. How do we attain this?

First of all let us look at this assumption. First assumption and the ninth assumption as I said that these two assumptions are kind of related. We want to have a homogenous propellant flow. At the same time, we do not want any condensed phase present for a liquid propellant rocket. How do we attain that is by having very fine atomization, right. If the atomization is very fine, evaporation rate will be faster and then the mixing will be faster. We get a better mix thing, same thing and that atomization rate is faster. Atomization is fine. In general, the droplet volume is much smaller. So, this condition is also met. So, even if it is not evaporated, the volume is very small. So, therefore, this assumption one and nine can be attained by having a very fine atomization for a liquid rocket which can provide a homogenous gaseous, mixture perfect gas. Here is a

reasonable approximation. So, first of all if we are having a homogenous gaseous propellant flow, then we can assume it to be a perfect gas.

So, the direct consequence of meeting these two requirements, one and nine is that we have a perfect gas. So, this is also a reasonably valid assumption. Now, like I have said if we are considering the flow to be frictionless and adiabatic three and four, then what we have is an isentropic process. So, this is something would like to attain and for the assumption here it is an assumption. With that assumption, we can use the isentropic relationships which we had derived so far to analyze the flow.

So, therefore, these two assumptions are very critical to this entire analysis. It is a very critical part of the entire analysis. Without these assumptions, whatever we have discussed in the last four lectures is not going to be applicable to this problem. So, it is very critical part of it. Then, the fluctuations inside the propellant flow rate as I have mentioned in practical system, you have the experience that you use this essentially are use of two purpose. First of the metering of the flows and secondly, to decouple the fuel feed system from the rocket chamber, so that the fluctuations are minimized, right.

So, therefore, this is something that is inherently designed in the system. So, therefore, this is a very good assumption. This is something that we want to have. We do not want to have fluctuations because if we have fluctuations in the propellant flow rate, then what happens is that the heat release rate is going to change. Because of combustion, the combustion is going to change because this propellant flow rate essentially dictates the fuel air ratio or the fuel oxidizer ratio or the composition of the propellant coming in, reactant coming in and the product will depend on what reactions we are putting. So, if the reacting composition changes, the product composition changes, the temperature changes and since, we are talking about almost a constant volume here because we have a chamber and a throat. So, that makes it almost a constant volume. So, if the temperature starts to change, the pressure will also change, right.

So, the pressure and temperature starts to change, everything starts to change. We do not get a steady flow. So, therefore, this is something that has to be guaranteed in order to make this assumption also. So, the propellant flow must be maintained steady, and constant. Second point is that as we will go along, we will see that the mass flow rate is a very important parameter like for our thrust equation. The first term was mass flow rate,

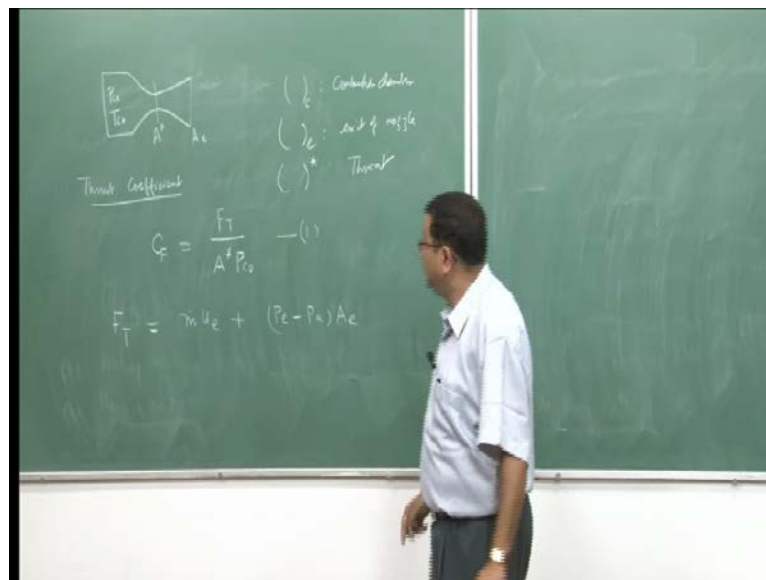
right. $\dot{M} u v$. So, therefore, unless that mass flow rate is constant, the thrust will also start to vary, right. If the mass flow rate is not constant here, the entire flow rate is the propellant flow rate. So, that starts to vary. The thrust will start to vary and that is something you do not want to have. You want to have a constant thrust because all our flight mechanics derivation, our orbital mechanics derivations, everything was with the assumption that the thrust was constant during the entire operation, right and not only that if you look at our flight mechanics derivations, we had considered the mass flow rate to be constant. \dot{M} to be constant, right and based on that we derived our equations. Now, if that starts to vary, we have to go back and redo the entire exercise to take care of that variation which we do not want to do. Therefore, we want to ensure that this is constant.

Then, coming to the flow is steady and also, this portion axially directed velocity uniform across all this. All these will be possible if you have a quasi 1d flow and then one more thing. Here is the steady and shock free flow would like to have steady shock free flow, right. So, there are no irreversibility's in the flow. So, all these, it will be made possible only if we have a converging diverging nozzle that we have discussed. We want to have a supersonic flow at the exit, but we need to have a well designed converging diverging nozzle, so that we can get an isentropic flow through that, so that there is no shockwave.

So, therefore, these assumptions, this, this, this and the other, there is no shock in the path etcetera. All these will be valid only if we have a good design of a converging diverging nozzle and level nozzle which will allow for a isentropic flow to be established. So, therefore, this is another condition that we should have a good converging diverging nozzle. Now, one thing that is also here mentioned that we are assuming the gasses to be perfect gas. What does a perfect mean? A perfect gas mean essentially the gasses are thermally perfect as well as calorically perfect. So, therefore, the properties are function of only temperature. Now, if you consider the temperatures are not varying very large extent, then the properties are constant, right. So, therefore, we can consider that for the temperature variation which will be seen in the rocket nozzle, the properties can be considered to be the average value. So, typically the property of interest here is for gamma.

So, we can consider average value of gamma and the value of gamma typically will lie between 1.1 and 1.3. This is the value of gamma we get, we can expect in a rocket propellant and this low value of gamma is because of the fact that the temperatures are very high. Temperature can be as high as 3000-4000 kelvin sometimes. So, because of the high temperature, the value of gamma is low, but typically the gamma will be in between this zone. So, these are the basic assumptions that we will need to make in order to analyze an ideal performance of a rocket. So, I have listed the assumptions. I have discussed the validity of these assumptions. Now, let us go to the actual performance estimation. So, before we do that, we will define now some parameters. So, next we will now go to the performance by defining some parameters which will be useful in estimation of the performance.

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Let us consider a rocket first. Given like this, this is our rocket. Let us say that the throat area is a star, the exit area is a e. Let us say that the chamber pressure is P c naught, chamber temperature is P c naught. So, the rocket chamber pressure is P c naught is a stagnation pressure and stagnation temperature of the rocket chamber is P c naught. First parameter we will define is called thrust co-efficient. It is designated as c f and defined as total thrust divided by a star P c naught. Let me call this equation 1. So, in this equation or in this definition, c f is our thrust coefficient. P c naught is the stagnation pressure, sorry stagnation pressure in the combustion chamber. Everything with the subscript c represents actually the, yeah combustion chamber conditions, everything with

subscript e represents the exit condition, everything with superscript star represents the throat condition. So, first let me mention that subscript c is combustion chamber, subscript e exit of nozzle and superscript star is throat condition.

Now, this is the nomenclature. We are going to follow all through. Now, whenever we use the subscript c, it represents the combustion chamber. Subscript e will be the exit of the nozzle and superscript star will represent the throat. So, now, coming back to this equation, then P_c is the stagnation pressure in the combustion chamber, A^* is the throat area and f_T is the thrust produced by this rocket. So, f_T is the thrust. We had derived the thrust equation. What was the thrust equation? Thrust equation was equal to $\dot{M} u_e + P_e A_e - P_a A_e$. This was our thrust equation.

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$$C_F = \frac{\dot{m} u_e + (P_e - P_a) A_e}{A^* P_{c0}}$$

$$\Rightarrow C_F = \frac{\dot{m} u_e \sqrt{T_{c0}}}{A^* P_{c0} \sqrt{T_{c0}}} + \frac{A_e}{A^*} \left\{ \frac{P_e}{P_{c0}} - \frac{P_a}{P_{c0}} \right\} \quad (2)$$

mass flow rate

$$\dot{m} = \rho u A = \text{const} \quad (1-D \text{ flow through out the nozzle})$$

$$\dot{m} = \frac{M A P_0 \sqrt{\gamma}}{\sqrt{R T_0}} \left[\frac{1}{1 + \frac{\gamma-1}{2} M^2} \right]^{\frac{\gamma+1}{2(\gamma-1)}} \rightarrow \text{HW}$$

General expression for 1-D compressible flow.

$$\frac{f}{P_0} = f(M) \quad P_0 = \frac{P_0}{R T_0}, \quad u = M a = M \sqrt{\gamma R T}, \quad \frac{T}{T_0} = g(M)$$

So, now if I take this equation and put it back into the definition of the thrust coefficient, then the thrust coefficient can be written as $\dot{M} u_e + P_e A_e - P_a A_e$ divided by $A^* P_{c0}$. Now, as we can see the thrust coefficient has two terms. One is that the momentum thrust term; other is the pressure thrust term. So, let me split it into two components. First is this term. So, I will write it as $\dot{M} u_e$. Then, let me multiply and divide this term by T_{c0} , where T_{c0} is the stagnation temperature in the combustion chamber or this will be combustion temperature. After the combustion, the product of a , the temperature of the products which we will be estimating later through combustion analysis.

So, this is the first term, and the second term is a_e by a^* . Then, P_e by P_c naught minus P_a by P_c naught. Let me call this equation 2. Now, let us look at the second term a_e by a^* . So far in the last few lectures, we talked about the area, relationship area mach number relationship, right and we have shown there that the exit mach number is a function of the area ratio a_e by a^* . Therefore, this term here is the function of exit mach number or exit mach number will depend on this which we have already shown. So, once this is specified, the exit mach number is specified. P_e by P_c naught is P_c naught is our stagnation pressure.

Now, So, P_e by P_c naught is the exit pressure divided by the stagnation pressure. Once again that is a function of this for isentropic and P_a by P_c naught is P_b by P_c naught. That we will discuss little later. If it is ideal expansion, then this term will be 0. This entire term will be 0 because P_e will be equal to P_a , otherwise we have seen for over expansion and under expansion. There will be some components coming here. So, that we will discuss later. So, now, this is the expression of the thrust coefficient. Let us look at this term. Now, focus at this term \dot{M} . \dot{M} is our mass flow rate. So, let us look at the mass flow rate \dot{M} . Remember that we have a converging diverging nozzle d level nozzle we are using. So, we have a minimum throat area. Now, let us look at the mass flow rate. Keeping that in mind, when we talked about quasi 1d flow, we had shown that $\rho u a$ is constant for a quasi 1d flow, right and from continuity equation $\rho u a$ is \dot{M} .

So, this is for a 1d flow throughout the nozzle. We have discussed this in detail and shown this expression. So, \dot{M} is $\rho u a$. Now, what we can do is, we can convert this to a different representation by using the definition of stagnation properties and getting a different expression for the velocity. So, I will just write this equation, and then you can do it yourself. \dot{M} can be written for a combustibile flow like this here. What we have done is that a remains here. The density can be written in terms of mach number because we know that this is a function of mach number, right. So, from here we can write density as a function of mach number and ρ naught and ρ naught equal to P naught by $R T$ naught from the perfect gas equation of state. So, we can write then density as a function of mach number P naught and T naught which are appearing here. So, that is how we replace density. Similarly, the velocity can be written as M times a and this is

equal to $M \gamma R T$, right and the temperature T by T naught is a function of mach number, right.

So, here the density we can replace by ρ naught by $R T$ naught times a function of mach number coming from isentropic relationships. The velocity we can replace by mach number, the function of mach number put all of these back here, we get this equation. So, this is the homework, you derive this equation. I will just write down that this is the general expression for 1d compressible flow. This is a very general equation. You must have seen it in aerodynamic courses also. So, just I have given that process. How do you derive this equation? You must have done this in aerodynamic courses. So, now, this gives me the mass flow rate. Here in this equation, M is the mach number. Then, A is the area, P naught is the stagnation pressure, γ is ratio of specific heats, R is the gas constant. So, this equation.

Now, what we can do is, take that equation and now, consider the throat, look at the throat as we can see here that this equation is function of the mach number, and the stagnation properties at the throat. What is the mach number? Mach number is equal to 1 and the area is a star.

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So, at the throat M dot will be equal to mach number. We put equal to 1 for area, I put a star. The pressure is here P c naught. Stagnation pressure is P c naught square root of γ divided by square root of $R T$ c naught, and here also for mach number I put 1.

This simplifies to $2 \gamma + 1$ to the power $\gamma + 1$ upon $2 \gamma - 1$. Let me call this equation 3.

So, this is the mass flow rate at the throat, but like I have said here, mass flow rate is constant everywhere. So, therefore, if this is the mass flow rate at the throat, it is the mass flow rate everywhere, first point. Second point, we have, we can see here that is function of a star P_c T_c does not depend on what is here, right. So, as long as the stagnation properties and the throat area is constant, mass flow rate is not going to change. It is choked. This is a choke mass flow rate. Third point, we have seen that when we reduce this back pressure because the flow up to this point remains same. Mass flow rate is not going to change. So, we get the choked mass flow rate from this equation which is going to remain same all the time. It is not going to be effected by the change in the back pressure, and if we have to change this, what do we do? We have to change either P_c or T_c . That is the only way we can change mass flow rate, otherwise it cannot change the mass flow rate.

So, these are the few things that are very important to understand when we talk about the flow through nozzles which is a combustible flow. So, what we have done today? We have listed the assumptions required to consider a rocket to be an ideal rocket. We have discussed the validity of those assumptions that how valid are those for practical processes, and how we can attain those conditions. After that we have defined, one parameter which is the thrust coefficient, we have seen that it can be split into two terms. One is the momentum thrust term; other is the pressure term. We have seen that the momentum thrust term is a function of mass flow rate and the stagnation properties which we can get an expression from here, and the pressure term is a function of area ratio is essentially means it is dependent on the mach number or essentially the area ratio dictates the pressure term, and previously we have discussed the significance of the pressure difference term. If we have an ideal expansion, this term is going to be 0 for non ideal expansion. We will have certain value for this.

So, that we will discuss again later. Let me come to the actual this, come back to this term here. So, I will stop here today. Before that I will have to remind you that this is the homework that you derive this expression. I have given how to derive it. We get an expression for density in terms of mach number and the stagnation properties we get a expression of velocity in terms of mach number, and the stagnation temperature put all of

them and the little bit of algebra will give us this equation. This equation is important to get the choking mass flow rate which we have got here. So, unless we get this equation, we cannot get this. So, I stop here now today and then in the next lecture, we will continue from here.

Thank you.