Jet and Rocket Propulsion
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Lecture – 21

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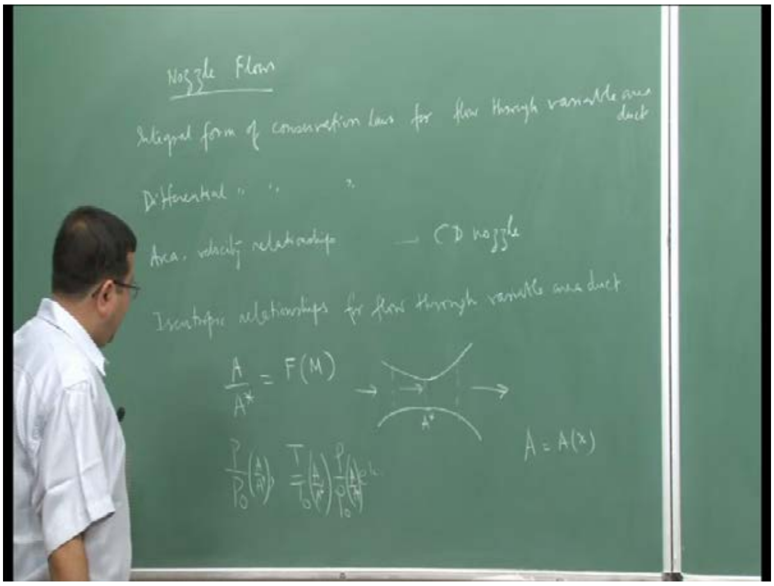

Welcome back to this lecture on Rocket and Spacecraft Propulsion. So, for the last few lectures we have been discussing nozzle flows, under this topic what we have done so far is derive the expression in the integral form of the conservation laws for flow through variable area duct, this is what we first did. Then starting from this integral forms, we derived the differential form of the conservation laws for variable area duct, after that we derived the area velocity relationship.

And through this area velocity relationship, we have shown that in order to expand a subsonic flow to a supersonic flow, we need to go through a minimum area and that is why we need to have a converging diverging nozzle; we have discussed and proved this. After that in the last lecture we derived the isentropic relationships for flow through variable area duct.

We had derived the relationship for the area ratio as a function of Mach number, where A is the, so we are discussing a flow through a duct like this converging diverging duct, this area is a star, the minimum area is a star which is the throat area. And at any location where area is A, we have a Mach number M coming from isentropic relationship. We have also discussed that looking at the nature of this relationship is a quadratic in nature, therefore for every area ratio there going to be two possible solutions for Mach number.

One of them is going to be subsonic so is going to come on this side, other is going to be supersonic, so it will come on this side of the throat ((Refer Time: 03:27)). We had derived those relationships after that using the isentropic relationship considering flow to then ((Refer Time: 03:34)). We have derived expression for P by P naught then T by T naught and rho by rho naught etcetera, where P, T, rho are the values at a certain point.

Of course, since it is an isentropic relationship, these are function of Mach numbers therefore we have got this pressure ratio in terms of area ratio, temperature ratio in terms of area ratio, density ratio in terms of area ratio etcetera. At the beginning, we have said that the area a is a function of x, therefore when we combine these two, what we get is the variation in pressure, temperature, density, etcetera as a function of x.

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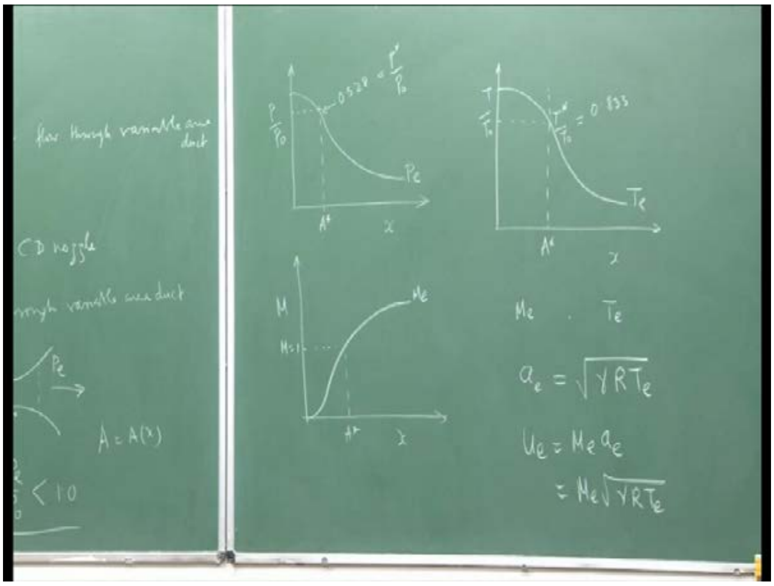

After, this what we have done in the last class is, we have plotted the variation in the properties, that is the pressure we have seen that at the throat, where area is A star for air this pressure ratio is going to be equal to 0.528; then it expands up to the exit, where the pressure is going to be P e. We have also discussed that there is only one possible isentropic solution for supersonic flow at the exit.

We had also plotted the variation in Mach number with x, we have seen that it goes like this somewhere here, we have A star Mach number is equal to 1, and here it is M e exit Mach number, which corresponds to this pressure P e. We had also plotted the variation in temperature T by T naught versus x and temperature also decreases like this. And once again we have at A star the temperature is T star, so T star by T naught was equal to 0.833 for air. This value was equal to P star by P naught, but P star is the static pressure at the throat, and T star is the static temperature at the throat; so these are the things we had discussed in the last class.

Now, to take this forward what we have to see is that there is a given exit pressure P e, so in order for this flow to be established, we need to have this exit pressure corresponding to this inlet pressure. So, this P naught and P e is going to dictate what kind of flow, we are going to have. So therefore, in order to establish a flow through this first of all, we need to have a difference in pressure between this and this, and that pressure can be given. So, here the pressure is P naught at the exit pressure is P e, the pressure ratio then P e by P naught will dictate, what flow we get at the exit.

Now, in order for the flow to be established there must be a force in the direction of flow, therefore P naught must be greater than P e. So, therefore this ratio must be less than 1, only then we will have a flow, so these are the things that, we had discussed till the last lecture. Now, let us look back at the same problem again, what we have to realize is that as I have said at the beginning that, when the rocket is flying on the, when it is on the ground it experiences the sea level pressure.

As it flies the exit pressure or the ambient pressure keeps on dropping, therefore the rocket experiences different pressure at different at different altitude. Now, we are saying here, that the exit pressure plays a very important role in what kind of Mach number will be established. And then I would like to point out one more thing, once we have this Mach number and this temperature T e, this speed of sound at the exit will be given as gamma R T e. And therefore the exit velocity will be M e times a e, so this will be equal to M e square root of gamma R T e.

So, therefore, what you see is that the exit velocity is essentially a function of exit condition, because P e is appearing here, and M e is the Mach number, which I have just said that will depend on the pressure that is there. And pressure in turn depends on this area exit area from here, so therefore, exit area will dictate what pressure and temperature we will had have at the exit. And once we have the pressure and temperature the exit velocity is also determined from there.

Therefore in the nutshell, we can say is that the area of the exit is going to tell us what kind of velocity we will have, and this is what is going to produce our thrust. So, therefore, it is important to know how this area rather, when we are designing a rocket we will design it for a fix area, but now what is happening is that when the rocket is going up, the ambient pressure is changing. We have seen from this discussion that once the area is fixed these parameters fixed for a given P naught these parameters are fixed.

So it is going to give us certain velocity, but now, if the rocket starts to experience different pressure at the exit, then what happens, how will the flow react to this variation in pressure, which is a very important parameter to be noticed during design; so that we design the rocket in such a way that it should be able to withstand this variation in pressure dynamically with stand, without losing its efficiency too much. So, therefore, the next thing what we will we are going discuss. Now is, how the variation in back pressure affects the performance of the rocket, how the flow will adjust itself to the varying back pressure.

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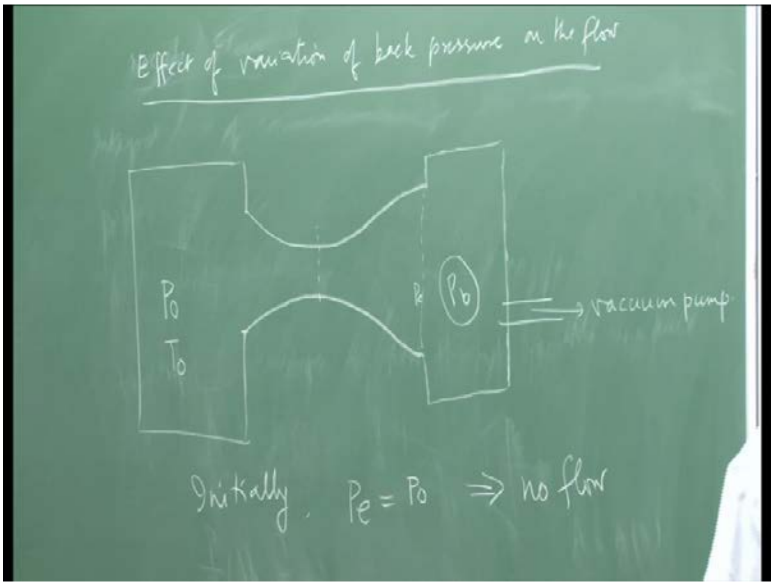

So, this is our topic of discussion today, we are going to discuss the effect of relation of back pressure on the flow able area duct of course. So, for this let us conceptualize an experiment, let us say that we have a reservoir which is maintained at a high pressure P naught, this is the stagnation pressure, because there is no velocity in the reservoir. And the temperature of the gas is here is T naught; let us, assume that a converging diverging nozzle is attached to the exit of this duct.

And let us say that at the other side of it, we have another duct, which is connected to a vacuum pump, now let us say we fill this up, and then leave it without operating the vacuum pump. Now, what happens is that, since there is an let allow it enough time to settle down, then the flow will come here, it will reach equilibrium, then everywhere the pressure will become equal to P naught, because vacuum pump is not operated.

So, everywhere means, now this is our nozzle exit, so this is my P e exit of the nozzle, so at the beginning when the vacuum pump is not operated. So, initially we have P e is equal to P naught, because everywhere the pressure is same, which essentially means, that there is no pressure differential across the nozzle. And if there is no pressure differential there is no driving force to establish any flow, so that therefore, in this condition it will mean that there is no flow.

Now, after this let us now start operating this vacuum pump, so what we do is we start to reduce this pressure here in this chamber, let me call this pressure T b it is called back pressure, which the nozzle is experiencing. So, let us start to reduce this pressure P b as P b is reduced slightly below the P naught value, now this pressure is P b this is P naught there is going to be a flow because P naught is greater than P b.

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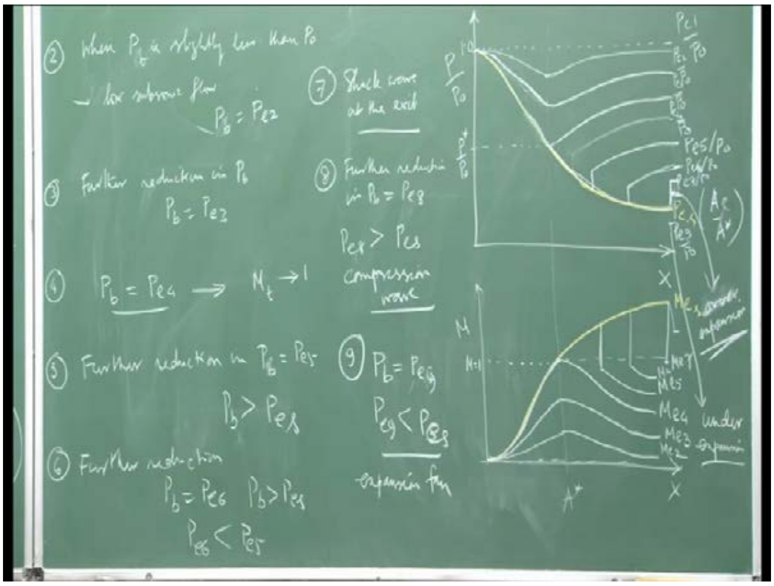

So, when P e is reduced slightly below P naught when P e is slightly less than P naught or rather P b let me put it like this P b is slightly less than P naught, we will create a low subsonic flow very low subsonic flow through this nozzle. So, if it is a low subsonic flow let me start, plotting now what happens to the flow properties x P by P naught and I will also plot Mach number here, our x is starting from this going up to this point so this is my origin this is my x this is the exit of my nozzle.

Now, initially we had everywhere pressure equal to P naught, so this will be equal to P naught, so therefore this everywhere the pressure is same, so this ratio is going to be one. Now, what we have done is we have slightly reduced the back pressure or the exit pressure, as I have just discussed because of that, so initially the Mach number everywhere will be 0 it was here. Now, once we have slightly reduced the back pressure or exit pressure a very low subsonic flow will be established.

So, when the subsonic flow is established, here ((Refer Time: 16:09)) we start to create a flow here a subsonic flow, so till the throat here is our throat a star the flow will accelerate, because it is subsonic and is a converging passage, so flow will accelerate it reaches certain value here, but the differential is not strong enough to create a supersonic flow or take it to the sonic flow.

So, it will reach a certain value here, the pressure will decrease up to this point, and then this side the flow is still subsonic as the flow is still subsonic this is a diverging area, so pressure will increase velocity will decrease. So, at the end we will have this pressure equal to P e which is of course, less than the back pressure, but it is not P naught. And a flow and it is neither equal to the pressure here, and the velocity will be slightly higher than 0, so Mach number is going to be higher than 0.

So, if I now plot it here this is let me call this as case 1, this is case 2, so for case 1 my exit pressure is equal to P naught, so that represents this plot here. Now, for case 2 my let us say my exit pressure P e or P b equal to P e 2, so now, what is happening is that let us say this is our throat from here the flow accelerates. So, the pressure drops reaches a certain value which is not the critical value, and then when it goes to the sub the diverging section it will come up again, and reach a value like this, so this ratio is P e 2 by P naught.

If I now, look at my Mach number increases reaches a maximum value at the throat, and then in the diffusing the diverging portion it will reduce, so the Mach number will start to reduce and we get M e 2. So, this is the condition that will prevail when it is slightly reduced, we next what we do is we continue to reduce this pressure we continue to reduce this pressure or back pressure. So, case 3 let us say it is further reduced, so there is further reduction in P b, so in this case P b equal to P e 3.

But, still the pressure is not enough, to give us the critical flow at the throat or the sonic flow at the throat, so then it will continue to decrease like this. And once again it will increase up here giving an exit pressure P e 3 by P naught, so Mach number once again will increase reach a maximum value here and then drop again M e 3. Only thing here is that we have reduced this P e 3 pressure, so the exit pressure is now less than the previous case. So, the throat pressure will also be less the velocities are going to be higher everywhere corresponding compare to the previous case, but still it has not reached the sonic case.

Now, we continue to reduce this pressure, so let us in the forth instance we reduce the exit pressure in such a way that, now the throat pressure there will be so back pressure is reduced to a value P e 4, such that the Mach number at the throat is just 1 it has just reached the critical state. In that case what happens from here to here there will be a drop in pressure like this it reaches this point here. Now, it has reached Mach 1, so here it has reached Mach 1 this pressure value, now is equal to P star by P naught, but it has reached Mach 1 at the throat it does not have additional energy to push it forward.

So, then what it will do is it will fall back, so it will fall back to the subsonic domain because the additional energy is not available it has just reached sonic, and then since it does not have the additional energy to push the flow forward with the same energy it will fall back little bit. So, now, here it has reached the sonic speed, but here it becomes subsonic again, because it is losing some energy, so when it is subsonic here again this will work as a diffuser because it is a diverging passage, so therefore, the velocity will decrease pressure will increase.

So, here then again the pressure will increase and to the exit it reaches P e 4 equal to P naught. So, in all these cases what we have seen that the exit pressure of the nozzle is equal to this back pressure P b. And now, it will take this value P e 4, so the Mach number will fall and reach some high subsonic Mach number M e 4, but it is still not supersonic. So, up to this now up to this case then what we have seen is interesting up to this case for every value of P e there is an isentropic solution.

For every value of P e there is a certain well defined flow pattern which is isentropic because so far is all isentropic, so up to this pressure everything is isentropic. Now, if you further reduce this pressure, then the things start to change, so at the point so here, now we have infinite number of isentropic solutions, for the subsonic flow through a nozzle which, we have just discussed both the critical area ratio that is A by A t, so this what will dictate the flow is this A by A t and P e by P naught.

Both of them will dictate that particular location what pressure and what Mach number we are going to have. So, for these are the two critical parameters that will dictate what is going to happen inside the nozzle, but now let us come to the supersonic case. So, case 5 is further reduction in P e, so when the exit pressure or back pressure is P b, which is equal to say P e 5 exit pressure or back pressure is further reduced, now according to the estimates we have already reached the sonic speed here.

So, it is further reduced up to this the flow will not change, because it in this portion we have the same area ratio and we know that the Mach number is a function of area ratio only, so up to the throat the flow is not going to change. So, we will reach still Mach 1 here, and of that is the case at every location here the pressure is also going to be same as what was there here, so therefore, now beyond this point from here to the throat we get the same conditions this remain same that does not change, whatever we do here this is not changing, the changes will occur after this in the diverging portion.

So, that is first point such a condition, then is called that the throat is choked which means that the back pressure is such that the this area is choked which means that the flow has reached Mach 1 at this point or sonic speed at this point, so therefore, from here to here no matter what we are doing here this flow does not change. So, in other word we can say is that the converging portion or the subsonic portion is decoupled from the diverging of supersonic portion of the nozzle.

Now, what will be the consequence of that if the flow parameters are same up to this point, and this is the minimum area up to this point flow parameters are same, which means my density does not depend on this my area anyway does not depend on this. And my velocity, now here, is a star is also does not depend on this which essentially mean, that all these three parameters are independent of the back pressure. And we have shown that the product of these three from continued equation is the mass flow rate.

So, therefore, it shows that the mass flow rate through the nozzle at the throat is independent of the back pressure, once we cross this state, and reduce the pressure down further down. Now, this is one point, second point I would like to make here is that continuity equation says that the mass flow rate is constant everywhere. Therefore, if this is constant here it is constant here everywhere and we will have the same value.

So, essentially what it means is that no matter what how much, we reduce the back pressure the mass flow is not going to change and that is what is called choking. So, we say that the nozzle is choked because mass flow rate is not changing, if it was just a converging nozzle, then we cut it here and we apply these conditions here, directly we can get P star and compare with P star here, it is slightly different because here the pressure is not P star. So, what pressure it will have here P e depends on this area which is already defined by the area.

Now, the problem here is that the pressure here defined by this area variation, will have only one isentropic solution in the supersonic zone that, we have already discussed therefore, for only a specific value of exit pressure we get an isentropic supersonic flow, but what if that is not there. So, let us continue our discussion and come back here, now we have reduced the back pressure slightly below P e 4, we have discussed that at that condition up to the throat there is no change, but beyond the throat what happens, now the flow beyond it is supersonic.

So, if it becomes supersonic it will follow certain thing let me put this as a virtual path, so this is my isentropic case this is P e s M e s. Now, let us understand this as I have said that there is a specific value of back pressure at, which we get an isentropic supersonic flow. Let us consider that this is the value and this value of course, is a function of A e by A star that we have discussed.

So, for the given area there is a specific value of back pressure, at which we get an isentropic solution coming like this and the corresponding Mach number variation is this. But, now what we are saying is that the pressure we have further reduced, but we have not taken it to this value so; that means, P b is still greater than P e s, then what happens it is not going to give us an isentropic solution. Now, my exit pressure is somewhere here P e 5 by P naught, the mechanical equilibrium must be established at the exit.

So, therefore, somehow the flow has to readjust itself, so that it can get to this point, so how will it do it is following this path it falls below this pressure, now it has to somehow increase this pressure, how will do it by going across a shockwave. So, there will be a normal shockwave somewhere here beyond that there is an increasing pressure not only that across a normal shockwave, let us say we have a normal shockwave sitting here the supersonic flow becomes subsonic, so across this the flow becomes subsonic.

So, let us say here we get a normal shockwave the flow becomes subsonic as the flow becomes subsonic, now the rest of the diverging portion we have a subsonic flow in a diverging passage. So, pressure will increase, so it goes like this and the Mach number is going to decrease, so it reaches M e 5.

So, now, what we see is that in this case we have a shockwave sitting inside the nozzle therefore; this flow is no longer isentropic because shockwave there is going to be

Irreversibility's in the shockwave, shock wave is not reversible, so therefore, it is no longer an isentropic flow. Now, as we further reduce this pressure actually, if the pressure was here the shockwave will occur before as we further reduce this pressure.

So, case 6 further reduction P b is equal to P e 6 still P b is greater than P e s, but now P e 6 is less than P e 5, so we are further reducing the pressure is somewhere here.

So, now what we see is that we actually need less rather because the pressure is falling here in falling the isentropic path it has to increase, but this increase is less, so therefore, the shockwave will come down further. So, the shockwave will start to move downstream it will come here somewhere and the corresponding case will be something like this. So, as this pressure is less the Mach number is going to be higher, so now as we further reduce the shockwave which was created here, after the throat will start to move downstream it will move towards the exit.

Now, once we reach a condition as we keep on reducing it a condition will come where the shockwave will stand just at the exit. Then, across the shockwave, so if we further reduce it two 7 let us say shockwave at the exit. Now, so this is say my P e 7 by P naught a shockwave is standing just at the exit, the flow is becoming subsonic just at the exit it is still a subsonic flow, here this is M e 7 shockwave is standing just at the exit. So, before the shockwave this entire portion the flow remains isentropic the flow is isentropic before the shockwave.

Therefore, what we see is that the flow is coming down it is attaining this pressure almost at the exit, but beyond that it does not have the steam which has lost all the steam, and suddenly there it sees that the pressure is higher outside, so it will form a normal shock jump over that and create a shockwave here, so that this pressure is equal to this. So, now, for this case how do we solve the problem that is very interesting problem for this limiting case, what we have to do is we considered the flow to be isentropic up to this.

So, for the given area ratio we have an isentropic flow up to the exit therefore, for this area ration, we can estimate what is the Mach number at the exit, what is the static pressure at the exit, because the total stagnation properties are given, once we have that then across the shockwave we use normal shock relationship. So, now, we have a normal shockwave sitting here, it is a normal shockwave this is M 1 it becomes M 2 here we know P 1 T 1 P naught 1 T naught 1 everything is known on this side.

And everything we estimate using the isentropic relationship because we know the exit area exit area is known; now we use the normal shock relationships to get the properties here, so you can get u 2 P 2 T 2 etcetera, all the properties across the shockwave. So, then for this value of P pressure exit pressure which is shown here P 2 equal to P e we get the normal shock at the exit, after that the flow becomes subsonic, so still it is non isentropic.

Now, after this still it has a long way to go between this point and this point let us say that if the my Mach number is 3 at the exit. Then, there is a substantial reduction in static pressure substantial increase in static pressure, when it goes across the shockwave therefore, this P e s still much less than P e 7, what happens in between these two. So, up to this point from here to here. Every time the exit pressure at the throat by some means is balancing itself to the back pressure P b.

So, up to this point here the flow was becoming supersonic the subsonic, then with the subsonic diffusion, it is reaching the exit pressure, which is equal to back pressure. So, everywhere here the exit pressure was equal to back pressure, so we have the ideal expansion. This point here, still it is equal to the back pressure, because it is going across the normal shockwave and reaching the back pressure, what happens in between these two.

Once, we reduce this pressure further 0.8 further reduction in P b to say P e 8 still P e 8 is greater than P e s. We are now somewhere here, so we do not have conditions for establishing a normal shockwave also at the exit, because pressure is less than that, but still it is higher than the exit, the isentropic pressure. So, now somehow it has to increase to this point, but it does not have the shockwave normal shockwave there still it needs to expand.

So, our expansion is still not ideal it has not reached the isentropic, so expansion is under. So, this case here is my under expansion is, still not complete means need to more have more expansion to reach the isentropic case. So, now, what happens here, the exit pressure then is more than the pressure that it would take, so from here to here there needs to be a pressure jump. So, if that pressure jump has to be there that pressure jump will come only if there is a compression wave.

So, under that scenario there is going to be a compression wave here, oblique shocks will be created across this oblique shock, there will be rise in pressure the flow can be supersonic here, flow will remain supersonic here. So, there will be oblique shock here the flow is, now supersonic somewhere here, and we reached this condition here. So, expansion is still not completed.

So, this there is going to be a compression wave this is over expansion till the exit, let me put it little more differently till the exit of the nozzle is over expansion till the exit, of the nozzle it has gone down. Now, from here to here it has gone down and reached a condition where it is the exit pressure is, so here it has expanded up to this point till the exit of the nozzle and beyond this it has to come up now.

So, there must be a compression wave and that compression wave essentially is, if I compare these two points the point here just outside and the point at the throat at the exit, it has expanded more than that is required right it has expanded more than that is required, so it has to now come up. So, that will happen, so this condition is called an over expansion that will happen only if there is a compression wave. So, there is a compression wave or an oblique shock which will allow for the pressure to rise from here to here till it reaches this condition.

Now, we further reduce this pressure, we continue to reduce this pressure and bring it below this is my P e 9 by P naught, so we have P b equal to P e 9 where P e 9 is less than P s now P e s. So, now what is happening if I look at the exit point here in my nozzle, this pressure back pressure is now less than this, so the exit reaches certain pressure here, back pressure is further less than that, so therefore it needs to expand further to reach this back pressure. So, such an expansion is called under expansion because now my expansion is less than the required expansion to reach the mechanical equilibrium, so such an expansion is called under expansion.

So, now in this case since the exit pressure less than the isentropic condition, which is my, now up to this point as we can see there is no change all through the nozzle there is no change there is an isentropic solution.

At the exit of the nozzle it has reached this condition P e s, then we are realizing that we need expand it further and that can be done through an expansion fan. So, now at the exit of this nozzle there is going to be expansion fan through, which further expansion of the flow takes place, such that it reaches this pressure P e 9.

So, therefore, what we are seeing here now that we have from this condition P e 7 where a normal shockwave exists at the exit of the nozzle till the isentropic condition, we need to have some pressure rise outside, so we need a compression wave that is the over expansion. So, over expansion will be, let me now summarize it here over expansion case will be when P b is less than P e 7 greater than P e s where P e 7 is the condition, where we have a normal shock exiting standing at the exit of the nozzle, this is the over expansion case.

So, now, in over expansion this has to increase to this point, which will happen through a compression through series of compression waves or oblique shock waves, when P e on the other hand is P b is on the other hand is less than P e s like in this case P b is less than P e s, then the required, we need more expansion, so it is now under expanded case. So, this is under expansion, we require more expansion to attain the mechanical equilibrium.

So, then at the exit of after the nozzle at the exit we are going to have expansion fans, which will further expand which will expand the flows outside creating the required conditions. So, what we see here then that starting from P e 7 onward except at, when pressure is equal to P e s somewhere in between we will reach that condition also the exit pressure of the nozzle is going to be different from the back pressure.

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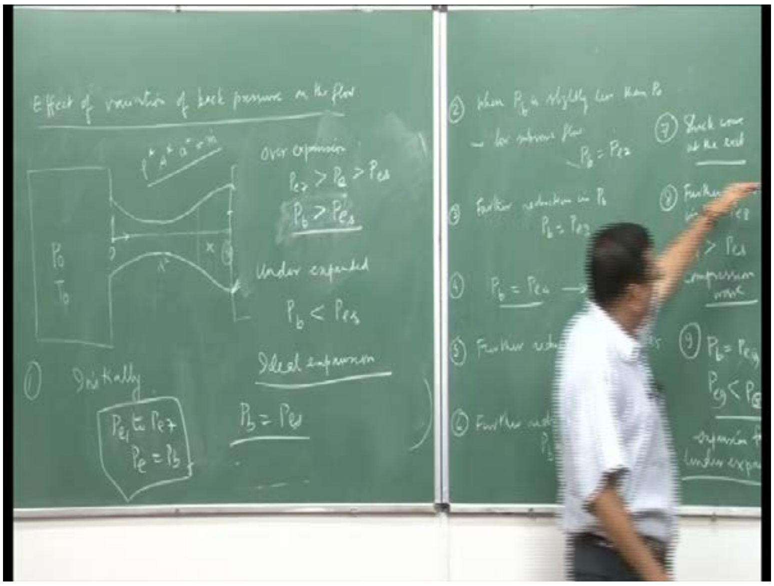

So, the three cases that I have, now I will just talk about the three cases, let me just summarize over expansion when P e 7 is greater than P b is greater than P e s. So, therefore, we have a greater than P e s right. So, in that this would not be P b this is P e exit pressure, so this exit pressure is the pressure at the exit of the nozzle, then P e 7 is my back pressure, so in this case then it will jump up and I will get P e 7 P e is equal to P b.

When, it goes across the normal shockwave I will get the exit, pressure not in this case, so here what is happening is that P e which is my exit pressure P e 7 is the back pressure P b, so the back pressure is, now greater than the exit pressure. So, in the thrust equation we will have this term coming here in the over expanded, under expanded in under expanded here, P e 9 is less than P e s or P b is less than P e s here, the exit pressure is my P e s exit pressure becomes P e s in this case also exit pressure was P e s.

And then after that we have the back pressure P b and this is less, so therefore, we will have a negative term appearing for the pressure term here. We have a positive term appearing in the pressure term ideal expansion, now ideal expansion actually is two things first of all when P b is equal to P e s we have an ideal expansion. So, P b equal to P e s we have ideal expansion, otherwise also from all these pressures P e 1 to P e 7 the exit pressure is equal to back pressure.

So, for P e 1 to P e 7 P e is equal to P b, but the point is beyond this point, where we start to get the shockwaves the flow is not isentropic inside the nozzle. So, therefore, the estimation of exit velocity has to consider the shockwave presence of the shockwave. Here and everywhere in the nozzle the flow is isentropic all our changes are happening are happening outside the nozzle everywhere inside it is isentropic.

In this case when, the flow is subsonic up to this point no problem we have isentropic, but when the exit flow is supersonic, then the flow is not isentropic even though P b is equal to the exit pressure. So, these are the few things that we have to keep in mind when we talk about the flow process through nozzles. So, I will stop here now in the next lecture I will continue from here, and then now we go to the performance try to estimate the velocity. So, I will stop here.

Thank you.