Jet and Rocket Propulsion
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Lecture - 2

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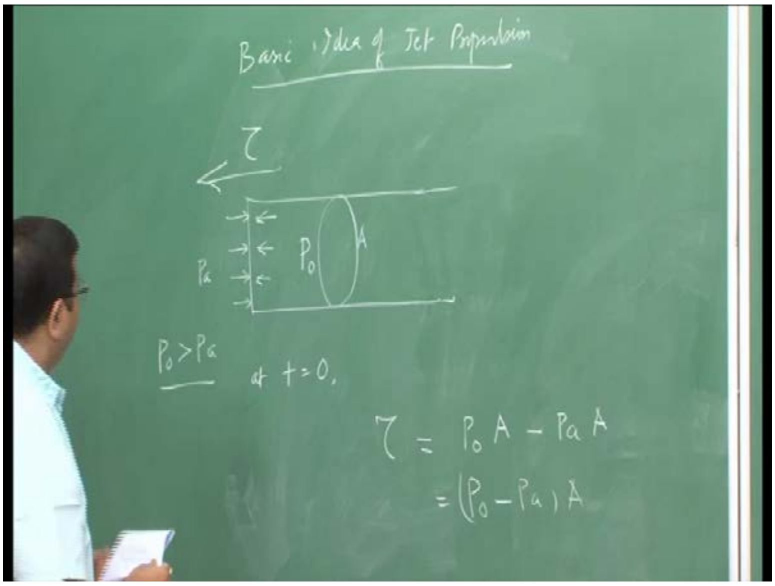

So, after the introduction of the rocket propulsion, let us now look at the basic principle of operation of a reaction-based engine or a jet engine – jet propulsion. This is applicable to both rockets as well as gas turbines, because the primary energy for propulsion is coming through the jet. Let us consider that we have a tube, which is closed at both ends like this. Let us say that the cross-sectional area of this tube is A and it is kept in the atmosphere. Let us say the atmospheric pressure is acting on all sides of this tube. P a is the atmospheric pressure. Now, let us say that we maintained a very high pressure P naught inside this. So, now if we look at this system we have walls closed at both sides. Therefore, all the pressure forces are balanced. There are no unbalanced forces in this. So, initially, even though we have high pressure here, all the forces are balanced in this system.

Let us now consider a scenario, where we removed one side. Let us say this side is removed. So, at time t equal to 0 – at t equal to 0, we remove one side. As soon as we remove this side, what we see is that, now, on this side of the system, P naught is greater than P a, because as I have said, this is a high pressure. So, P naught is greater than P a.

Therefore, the force acting in this direction is greater than the force acting in this direction. So, there is a net force acting in this direction. So, if we calculate or estimate that force; let us say that force is given by tau; then, that force is equal to the pressure forces acting on this face, because of the pressure P naught, which is equal to P naught times A; where, A is the cross-sectional area; and the pressure force is acting from outside, which is equal to P naught P a times A. Therefore, the net force is P naught minus P a times A. So, what we see is that, we have a net force acting on this system towards the left. So, this force if we do not do anything and if this vehicle is kept… – this system is kept on frictionless surface; because of this force, it starts to move. So, let us say in order to prevent this from moving, we exert a force in the opposite direction tau, which prevents it from moving. So, that is the reaction force. So, the instantaneous force thrust then is this reaction force given like this.

And, let us say that once we have removed this, there was a pressure differential on this side. So, the gases will start to escape, because there is a high pressure here; there is low pressure here. So, gases start to escape from this system. Let us say the velocity of the existing gases is given by u e – exit velocity. Now, as the gases start to move out, then some of the mass is being removed. If we consider this as a fixed volume; as the mass reduces, the density is reducing. Therefore, the pressure will start to drop. So, the P naught will now starts to drop. So, after some time, the P naught starts decreasing.

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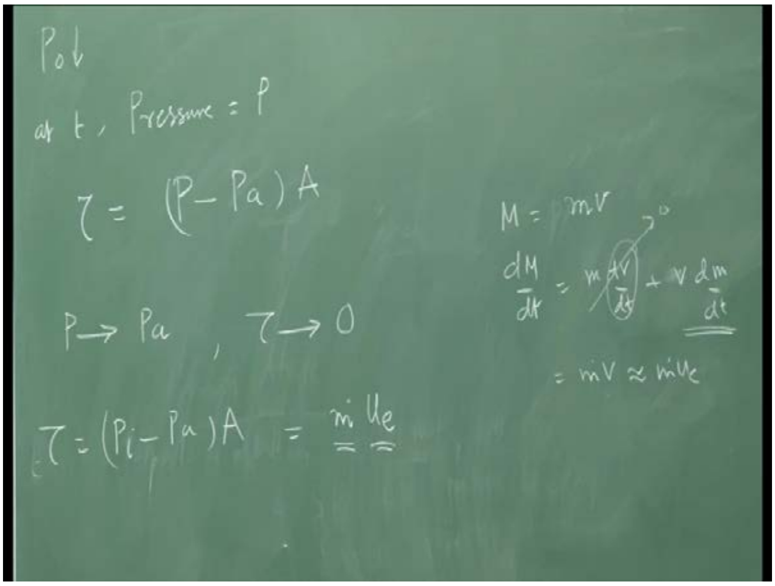

And then, at sudden instant of time t, let us say the pressure is equal to P. Then, at that instant of time, the thrust force is equal to P minus P a times A. So, the pressure has reduced to value P; atmospheric pressure or ambient pressure P a remains same. Therefore, the thrust has now reduced to this value. Now, it keep on opening this; more air or gas will pass out of this system, go out of this system; pressure will continue to decrease. Till at certain instant of time, the pressure becomes equals to P a – approaches P a.

Now, when this pressure inside this tube becomes equal to the atmospheric pressure, then the thrust at that instant will tend to be 0. So, the production of thrust stops as soon as the pressure attains the equilibrium. So, as soon as the system reaches mechanical equilibrium, the pressure will be equal to the atmospheric pressure; the thrust production stops. So, then the vehicle will stop moving. So, how do we attain as a state, but the vehicle will be continuously moving? Or, in order to do that, what we have to do is we have to somehow maintain a higher pressure inside. See if we can somehow maintain a high pressure inside and a lower pressure outside, then there will always be a pressure differential that will allow us to produce the thrust to have a sustained motion, because as soon as thrust goes to 0, we do not have sustained motion anymore.

Now, how do we achieve that? By let us say modifying this. What we do is we add a secondary supply of say compressed air, which can maintain the pressure at P i. So, we have a secondary source of air, which supplies air into this system even without the right-hand side wall. And, it can maintain a high pressure here. Now, if that is possible, how does we maintain this pressure here as P 1? We can produce a thrust tau equal to P i minus P a times A. So, if the pressure inside the tube can be maintained at a constant of value P i, then we will produce a thrust given by P i minus P a times A. This shows that if we know the pressure distribution inside the cluster, we can calculate the thrust. So, if we can get this pressure distribution, we can calculate the thrust produced. So, the easiest way is that, if we know the pressure distribution, we can get the thrust. But, that may not be possible always to know a priori what is going to be the pressure distribution. So, this is essentially the force balance. Force can be projected as rate of change of momentum. So, this is the force that is acting. If we look at the rate of change of momentum, then the thrust can be equated to rate of change of momentum.

Now, in this case, rate of change of momentum is m dot u e. The next thing we are going to derive that. For the time being, let us just look at phenomenologically how this is correct. The momentum M is given as mass times velocity. So, the rate of change of momentum equal to m dv dt plus v dm dt. So, as long as we want to produce a constant thrust, no acceleration. If there are no acceleration, this term is 0. Therefore, this is equal to m dot v roughly equal to m dot u e. Here this velocity I am talking about is the exit velocity; the velocity of the exhausting gases. So, this is a very ((Refer Time: 09:12)) way of talking about it; but, just using the Newton's second law, we can show that, rate of change of momentum for this system is equal to rate of mass flow rate multiplied by the exit velocity. So, here m dot is the mass flow rate and u e is the exit velocity. So, roughly speaking, we can say that, the thrust produced essentially depends on the pressure differential, which can be represented as the product of mass flow rate and exit velocity.

So, the primarily, what we need to do is then maintain this high pressure as the primary function. So, with this background, as I said, this is applicable to any type of jet engine; whether it is a rocket or gas turbine, this is what is applicable. And, that is why nozzle becomes a very important component, because this exit velocity u e is produced by the nozzle. So, during this course, we will discuss that as well. So, once phenomenologically we have shown this relationship; now, let us go into more rigorous mathematical treatment of the same. So, next thing we discuss is the performance of rocket vehicles.

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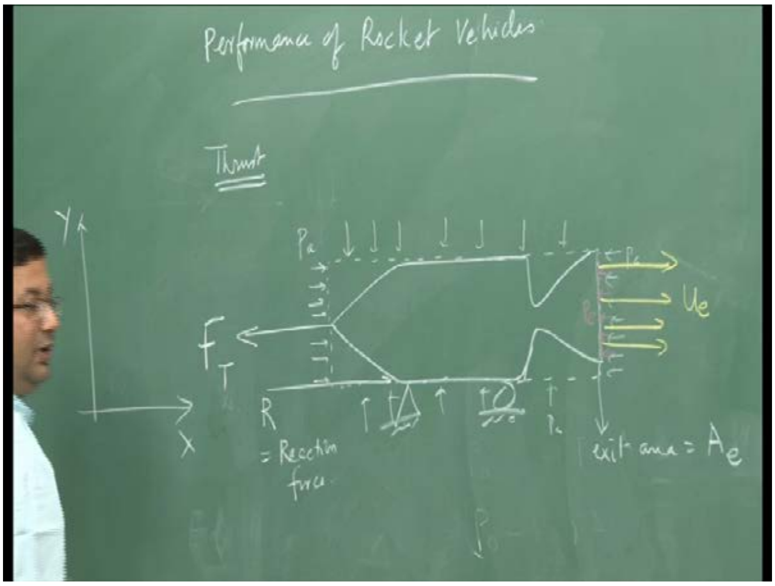

So, next, let us talk about performance of rocket vehicles. So, we will define the performance parameters for the rockets. The first and foremost, what we want to know is the force, which is thrust. So, the first parameter we define and try to estimate is the thrust. So, we begin our study of performance by studying the rocket under static state condition; that is, the rocket is put on a test stand like typically is done in LPSC in a test stand, it is not flying; it is on a test stand and we estimate the thrust from there. So, let us consider that we have a rocket; let us say given like this; this is the rocket, which is on a thrust stand.

And, let us say that, the thrust is equal to F T. Let us draw first of all the control volume here. So, let us say the control volume is the entire rocket from root to tip and consider a rectangular control volume. Then, the forces acting on this rocket when it is on the thrust stand; the atmospheric pressure let us say is acting everywhere, which is given by P a, is the atmospheric pressure acting everywhere including these surfaces. And then, out of this rocket, the hot jet let us say is coming out with a speed u e. And, let us consider the exit area of this rocket nozzle is exit area is equal to A e. And, let us further consider the pressure forces or the pressure of the exit jet at this plan is equal to P e; this is the exit pressure. Now, this is our configuration; we have a rocket on a thrust stand. And, let us say that, the reaction force is R, which is the reaction force. Let us say our axis system is an initial frame given by XY. Now, in order to estimate the thrust, first, let us make some assumptions for this problem.

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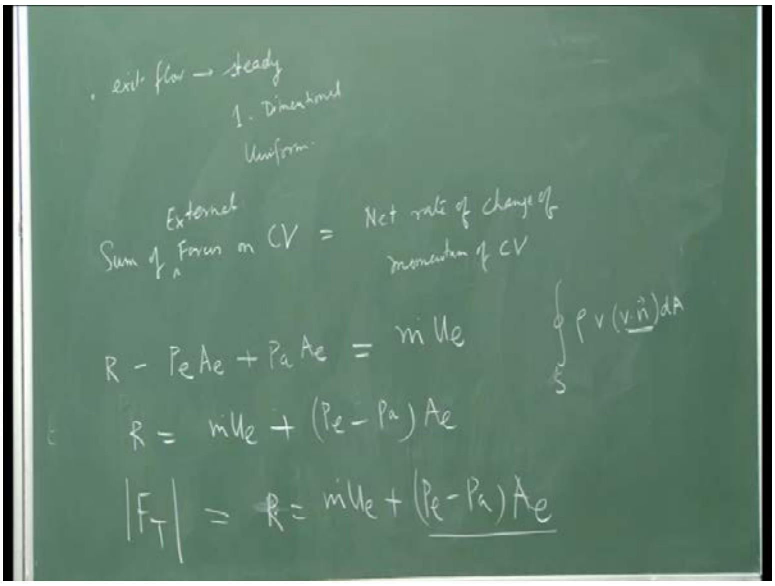

Let us consider that, the process – the exit flow is steady. It is 1-dimensional only along the X direction as shown in this figure and it is uniform. These are the assumptions that we make regarding the exit flow. Now, from the Newton's third law of motion, which we have already discussed, the action is equal to the reaction – equal and opposite to the reaction. Therefore, the reaction force is equal and opposite to the thrust generated by this rocket and this is what we want to estimate from given parameters. So, let us look at the momentum form or the control volume form of the momentum equation. Momentum equation as we know is essentially Newton's second law of motion applicable to a fluid flow. So, using Newton's second law of motion then; on this control volume, they can get the forces.

So, first, what does the Newton's laws of motion says? That the sum of forces on the control volume; where, this is our control volume as I have just discussed is equal to net rate of change of momentum of the control volume. So, the sum of the forces acting on the control volume are the external forces. External forces acting on the control volume is equal to net rate of change of momentum of the control volume. Now, the net rate of change of momentum have two components: one is instantaneous component, which is a time-dependent term; other is the convective term. Since we are considering steady flow; so, the time-dependant term will be zero. So, only the convective term will remain. And, the external forces acting here are the pressure forces and the reaction force. So, now, if we apply the forces and the moment balance; then, what we see is that, R is the reaction forces acting in this direction.

Now, we consider everything positive in this direction, because momentum equation is a vector equation. So, we consider everything positive in the positive x-direction. So, R is positive in this direction; then, minus – at this plane, the pressure acting is P e, which includes P a – atmospheric pressure is included here. So, the pressure acting is P e times A e is the exit area. We have considered this is a rectangular control volume. Therefore, this area is also equal to this area. So, this area is also equal to P a. And, the atmospheric forces are pushing them on this side. So, the atmospheric pressure forces will be acting as P a times A e. So, this is the net sum of all the external forces acting on this control volume. And, this is equal to net rate of change of momentum. And, the momentum is changing because of the exit velocity u e and mass flow rate. And, the integral form if you look at that; it will come out to be equal to m dot u. So, in this case, it will be m dot u e. Actually, this term in the integral form will be integral over the control surface rho v v dot n d A; where, v dot n is the normal component of velocity normal to the control surface c s. And, rho v dot n and d A is the mass flow rate. So, that becomes mass flow rate times velocity. Therefore, the right-hand side here is m dot u e, which is mass flow rate times the exit velocity. So, this is the basic thrust equation.

Now, what we can do is we can get an expression for R from this which is equal to m dot u e plus P e minus P a times A e. So, that gives me the expression for the reaction force. Then, Newton's laws of motion says that, the thrust is equal and opposite to the reaction force. So, the thrust is given as R, which is equal to m dot u e plus P e minus P a times A e. So, the overall thrust or the net thrust produced is m dot u e, which is the momentum thrust; and P e minus P a times A e; this is called pressure thrust. So, what we see is the thrust has two components: 1 is because of the flux of momentum out of the rocket, because this is the momentum going out of the rocket. So, this is the momentum flux of momentum going out of the rocket and work done by the pressure forces. So, this is the rate of change of momentum because of the pressure forces, which is P e minus P a times A e. Therefore, this pressure force term – particularly for rocket application, rocket propulsion is very important.

Typically, for gas turbine applications, we neglect this term. But, we cannot neglect that much so easily for rocket propulsion – the pressure term. So, now, total… This gives us the expression for the total magnitude of thrust along the negative x direction, that is, in this direction. We can modify this expression little bit and define a new velocity called equivalent velocity m dot u equivalent and the significance of this will become clearer when we talk about specific impulse.

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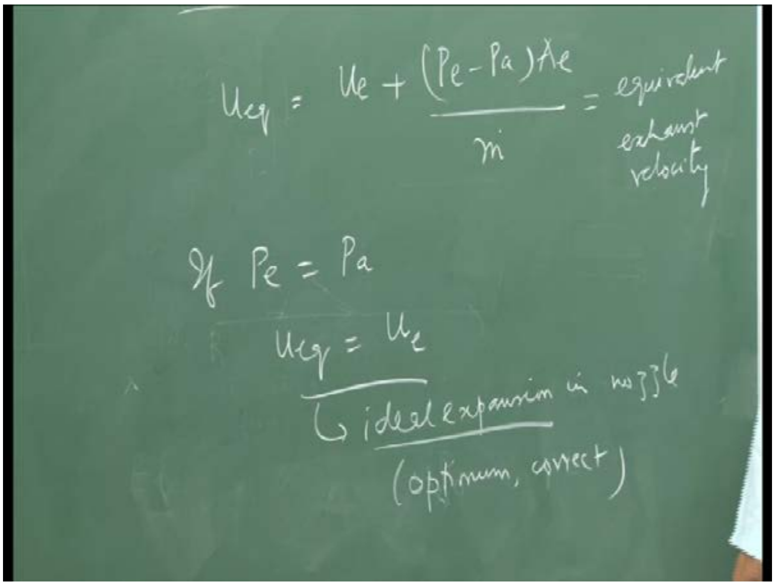

So, if we define the equivalent velocity like this; then, the expression for equivalent velocity is equal to u e plus P e minus P a times A e divided by m dot. This is called equivalent exhaust velocity. So, this essentially is a term, which includes the pressure forces also. So, we can express the thrust by just one expression. That simplifies the calculations later on. As we go along, we will see that, that simplifies the calculation substantially.

Now, from this expression, we can see that, if the exit pressure is equal to the ambient pressure, then equivalent velocity is equal to the actual exit velocity. So, certain cases when the exit pressure is equal to the atmospheric pressure, the equivalent velocity is equal to the exit velocity. This is true if we have ideal expansion in the nozzle. So, this condition is applicable for ideal expansion in nozzle. Again, we will discuss this in detail later when we talk about the nozzle flow. So, ideal expansion actually is optimum expansion also called correct expansion. When the expansion is correct, then the pressure at the exit will be equal to the atmospheric pressure and we get ideal expansion; velocity is equal to the equivalent velocity. But, if the expansion is non-ideal; in that case, there will be a contribution from this term. So, let us look at the significance of this ideal expansion.

First of all, typically, what happens for nozzles? For a practical rocket applications, whether the exit pressure will be equal to the atmospheric pressure or less than the atmospheric pressure or greater than the atmospheric pressure. What is typically the scenario?

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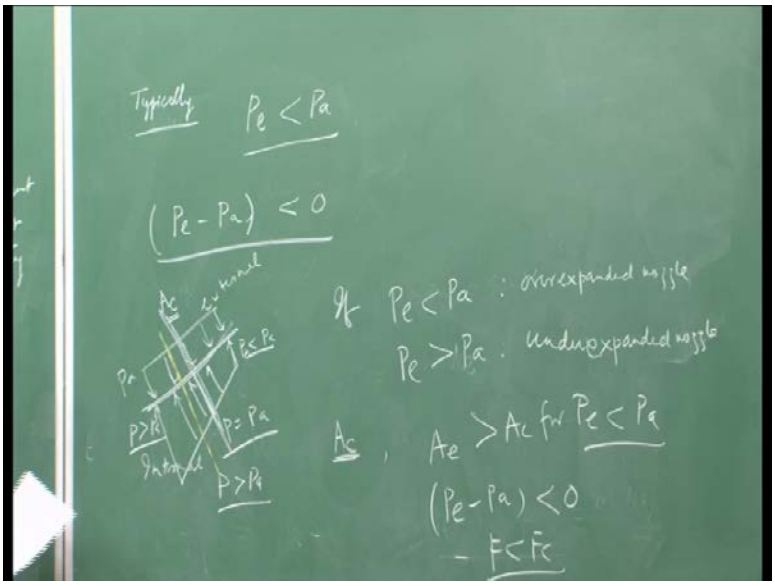

Typically, we will see exit pressure is less than atmospheric pressure. Now, there is a reason to it. So, this type of nozzle is called over expanded nozzle. There is a reason to it that, when we talk about nozzles, we will see that, if this condition is not maintained; if P e is greater than P a, then there is a possibility of shock reformation or expansion wave formation at the exit. But, with this case, we have shock wave formation at the exit. Now, across the shock wave, which is very easy to match the exit condition. But, against the expansion wave, it will take few expansion waves crossing; only then we will match the exit condition. Therefore, this is typically the condition that is maintained.

Now, another thing that we will discuss again and why we need to have this condition. So, typically, what we see here then that, P e minus P a is 0, is less than 0. Therefore, this term is less than 0. Therefore, the equivalent velocity is less than the exit velocity. First of all, as I said that, first of all there is a question of expansion wave. Secondly, if we let us say somehow we maintain an isentropic condition and all; even then, what should be the pressure to give us the optimum thrust? That is what we should have a look out. So, let us look at various possibilities and see which one will give us the optimum thrust. So, let us look at first of all a nozzle wall like this and let us consider this as a flat plate. The nozzle wall is typically curved, but let us consider it as a flat plate. Now, external pressure is acting on this nozzle wall is the external pressure. And, internal pressure is also acting; but inside the nozzle, the flow is expanding. So, there is a decrease in pressure in the flow direction. So, pressure may be higher here; it is decreasing reaching this point somewhere here. So, the internal pressure is having like this. Now, this pressure of course, is greater than atmospheric pressure.

Then, let us look at this pressure variation. This is greater than atmospheric pressure. As we go along towards the exit, the pressure is decreasing. Further movement – let us assume that, the pressure here is less than the atmospheric pressure. In that case, somewhere in between, it has crossed the atmospheric pressure. So, somewhere in between we had P equal to P a; everywhere else P was not equal to P a. Now, with this scenario, let us see this is the exit pressure P e. If P e is less than P a, which is this condition here, which is represented here as this condition; then of course, this is called an over expanded nozzle. And, another scenario can be that, the nozzle is actually truncated before it reaches this condition. So, let us say the nozzle is truncated here. Then, in this case as we can see at this plane, P is greater than P a. Therefore, the expansion is still not complete up to the exit pressure. So, that case is called an under expanded nozzle.

Now, let us look back at this scenario. If at this point, the pressure is getting equal to the atmospheric pressure; therefore, the area ratio available here is the critical area ratio for which the expansion is ideal. So, let us say that, the area for this exit area is equal to A c – ideal area. Once again I am saying that, we will discuss nozzle flow in detail later; that we will discuss these things again. So, A c is the area, which provides us the ideal expansion. Then, if I look at this case – the first scenario, where P e is less than P a, we will see that, the exit area is greater than this critical area. So, for over expanded nozzle, the exit area is greater than the critical area. And, because of that… And, P e minus P a is less than 0. So, looking at the thrust equation then, we can clearly see that, the thrust produced by an under expanded nozzle is less than the critical or the optimum thrust. So, that is a straightforward thing. On the other hand… So, this is straightforward; we should agree to this. But, now, let us look back here when P e is greater than P a. Then, according to this expression, the thrust should be more. Now, let us look back at this diagram and consider this case, where the nozzle was truncated here.

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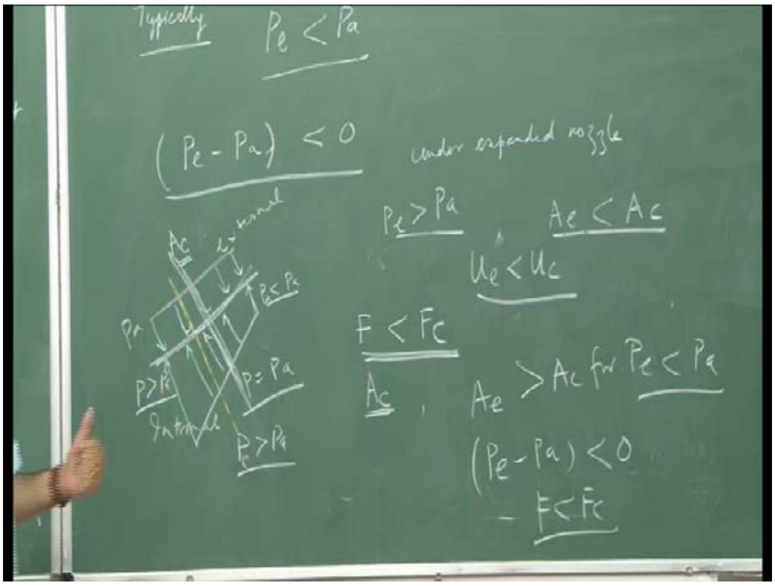

In that case, what is happening now that, for an under expanded nozzle, what we have is P e is greater than P a as shown in this yellow line. So, P e is greater than P a; nozzle has been truncated before it has reached the ideal expansion. Therefore, from this figure only, we see that, A e is less than A c. So, the exit area is less than the critical area. Now, the velocity depends on the exit area. For the constant mass flow rate, if we have larger area, the velocity will be less. But, we are talking about supersonic flow; we are talking about supersonic flow. So, that will be other way round. So, in the smaller area, we will have… Smaller area will have a lower velocity; larger area will have a higher velocity. So, because the…

Once again let me refresh. When the mass flow rate is constant; then, for a given flow, typically, when the area is larger, velocity is lower; when the area is lower, velocity is larger. But, in this case, if we look at this condition, here the flow was expanding. So, for subsonic flow, what I said was correct. But, here the flow is expanding, because it is a supersonic flow and the expansion grows as we move away from the initial point. So, there is a diverging area and the expansion grows in the diverging side. Therefore, the velocity increases or there is acceleration in the increasing area side. Therefore, looking back at this condition, when the exit area is less than the critical area, the exit velocity is less than the critical exit velocity. And, because of that, the overall thrust, which is the combination of this and this – even if this is positive, this thing is decreasing; the exit velocity is going down. And, that makes the thrust in this case less than the critical thrust. So, what it shows is that, for the ideal expansion, the thrust is maximum. On both sides of it, the thrust will reduce. However, the reduction in thrust for this case is because of lower exit velocity, because the expansion is not complete; whereas, for this case, it is the other way round. This is because of the decrease in pressure. So, inertial, what we have shown here is that, for the ideal expansion, the thrust produces maximum and we have got the thrust equation.

Once again we will have to point out here that, few lectures down the line, we will take up the nozzle flow in detail and we will discuss all these things in detail there again. Now, one point that, from the practical application point of view is very important that, why we want to have this; because a rocket is launched from say from ground and has to go to very large altitude. The pressure is very high on the ground; and at a very high altitude, the pressure is very low. So, the pressure has to vary from the longing point up to the intended final location and the pressure is decreasing.

Now, if the pressure is decreasing, then if the exit pressure is less than atmospheric pressure at certain point in between; initially, we will start of probably with a higher pressure. But, sudden point in between, actually this pressure will fall down drastically. Then, we can get into a condition, where P e becomes greater than P a. And, after that, it is always greater, because the pressure is decreasing here. So, if we start the design with an under expansion; then, for a long duration of operation, it is operating under expansion condition. And, that will reduce the thrust substantially. But, if we design with an over expansion condition, we design with a higher P e.

Now, as we go up, P e is decreasing; at one point of time, we actually hit the ideal condition. And then, little bit of let say under expansion. But, quite a substantial period of time is a possibility of having ideal expansion as well. Therefore, overall thrust will be more, because the thrust essentially is from low to high and then coming down. Whereas, here it is monotonically coming down. Therefore, the overall thrust is more in this case. So, that is why typically as I said that, over expansion is something that is preferred for practical applications. Now… So, one of the performance parameters we have discussed is the thrust. Let us look at some other performance parameters, which are important for rocket propulsion. And, the…

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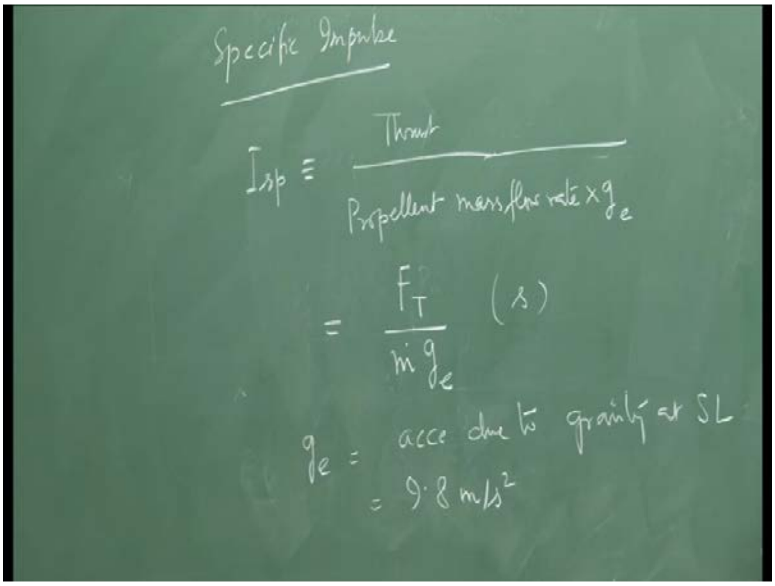

Perhaps the most widely used parameter is the specific impulse. When we compare various rockets, we actually talk in terms of specific impulse. So, this is the very important performance parameter. This parameter is defined as I s p, is the thrust produced per unit rate of propellant consumed. So, the thrust per unit mass flow rate of propellant multiplied by the acceleration due to gravity. That is the definition of specific impulse. So, if we consider thrust as F T, then this is equal to F T upon m dot, which is the mass flow rate times g e, which is the acceleration due to gravity. What is the unit of the specific impulse? Seconds. So, here g e is the acceleration due to gravity at sea level, because we need some reference point. So, it is acceleration due to gravity at sea level. Typical value of g is 9.8 meters per second square. So, now, if we combine this with the expression for the equivalent velocity and the thrust; then, in terms of the equivalent velocity, the thrust is written as m dot u equivalent. So, the expression for the…

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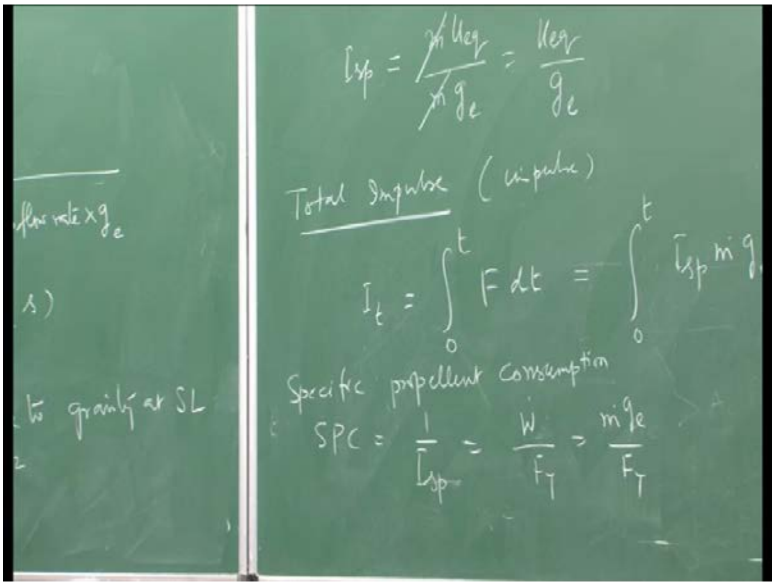

I s p can be then written as m dot u equivalent by m dot g e; m dot will cancel off; which is nothing but u equivalent by g e. So, equivalent velocity divided by acceleration due to gravity; that is the definition of specific impulse. We will define some other parameters also. One is total impulse; also, sometimes called just impulse, which is given as I t, which is the overall thrust integrated over the period of time. So, this t is the flight time; F is the thrust at an instantaneous thrust; that is, many times the thrust may not be constant; and this is the time increment. So, this is equal to then… Thrust from this equation is I s p times rate of change of weight. So, we can write it here as I s p m dot g e dt, that is, the total impulse.

Now, another performance parameter is specific propellant consumption. This actually is reciprocal of I s p. So, specific propellant consumption is given as 1 upon 1 by I s p. So, it is essentially is the measure of how much weight is required to produce unit thrust. So, that is, w dot by F T, which is m dot g e by F T. So, the unit of specific propellant consumption is per second. So, this is essentially the measure of how much rate of fuel will be required to produce per unit thrust. Some other parameters, which are very important as far as the performance of rockets is concerned; when we talk about vehicle dynamics, they will become apparent are the mass ratios. So, let us define the mass ratios as well right now.

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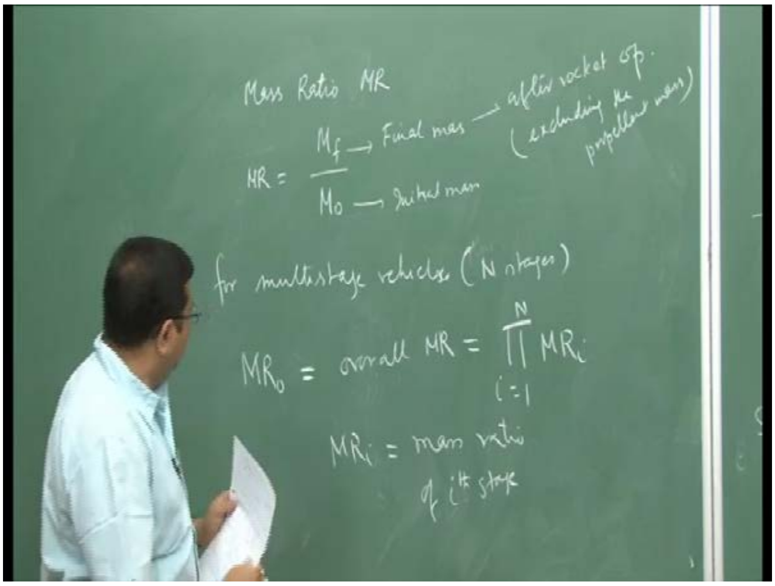

First of all, the overall mass ratio MR. MR is the ratio of final mass after the rocket operation and the initial mass of the vehicle. So, M f by M naught; where, this is the final mass of the vehicle; this is the initial mass of the vehicle. Now, when the vehicle is under operation, what changes happens? Therefore, this is after the rocket operation, which is essentially excluding the propellant mass. So, the mass ratio is given by M f by M naught, which is a very important design parameter. Typically, this mass ratio applies to single stage and multi-stage of vehicles both. For multi-stage vehicles… This is for single-stage vehicles.

For multi-stage vehicle, let us say with N stages; every stage will have its own mass ratio. So, first of all, we have an overall mass ratio given by M R naught, which is the overall mass ratio for the entire vehicle; which is nothing but product of individual mass ratios; where, M R i equal to mass ratio of i-th stage. Total number of stages is N. Every stages of mass ratio is M R i. So, overall mass ratio is… So, let us see that, why that is true. Take a step and show that this is correct.

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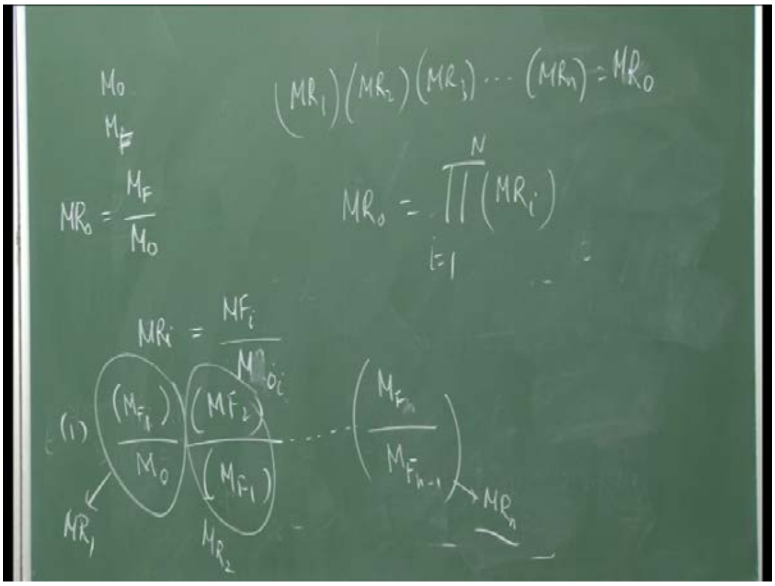

Let us say that, we have N stages; initial mass is M naught. Let us say the final mass is the payload mass; let us say M L. Then, after burning everything, let us say… Let me put it as M F. This is the final mass. So, overall mass ratio is the final mass divided by initial mass. And, for a particular stage M R i, this is the initial mass for that stage and final mass for that stage only; initial mass M naught – M naught i.

Now, for the first stage, the initial mass is actually M naught – the overall mass; and the final mass is what remains after the burn out, which is M F 1. Now, this M F 1 now becomes the initial mass for the next stage. So, M F 1. And, the final mass is M F 2. So, this term here is the mass ratio of the first step – first stage; this term here is the mass ratio for the second stage and so on till we get to the last stage. So, the initial mass for the last stage is the final mass for the last, but one stage and then final mass, which is just M F. So, this term here is M R n. So, what we see here then is M R 1 times M R 2 times M R 3, etcetera. So, M R 1 times M R 2 times M R 3 etcetera up to M R n is equal to M R naught – overall ratio, because that is what we started from. So, overall mass ratio is given like this. So, we see that, the overall mass ratio is the product of independent stage mass ratios. That is why M R naught is product of i equal to 1 to N M R i. Therefore, the overall mass ratio is proved to be the product of individual mass ratios.

Now, one more thing I would like to point out here is let us say the final mass here is M F; initial mass was M naught. The difference of these two then is what? The propellant mass, because that is what has been used.

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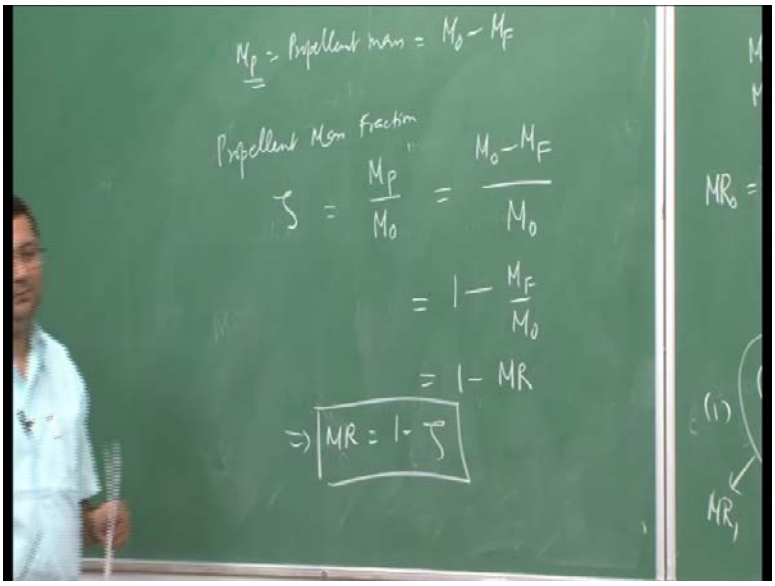

So, we can define the propellant mass here now. M p, which is the propellant mass equal to M naught minus M F. Now, this M F is the final mass that includes the guidance devices, the navigation gear, payload, tanks, structure, engine, hardware, everything. This is the final mass, except the propellant; everything else remains.

Now, once we have defined the propellant mass, then we can define another parameter, which is called propellant mass fraction given by zeta. This actually is the percentage of propellant mass in a given vehicle. So, it is defined as propellant mass divided by the overall mass. So, as we can see, this is the percentage of propellant mass in the given vehicle defined like this. So, we can take a step further; write this as M naught minus M F by M naught. And, this is equal to 1 minus M F by M naught. And, what is M F by M naught? Is the mass ratio – overall mass ratio. So, this is equal to 1 minus MR. So, zeta equal to 1 minus MR. Therefore, we can also write… If we can bring MR to this side, zeta to that side; that MR equal to 1 minus zeta. So, the propellant mass fraction is 1 minus overall mass ratio; overall mass ratio is 1 minus propellant mass ratio.