**Jet and Rocket Propulsion**

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**Lecture - 18**

Welcome back. So, let me recapitulate first what we have discussed so far in this course. We have talked about the performance of a rocket, we have talked about the vehicle dynamics both for single stage rocket and multi stage rocket, we have talked about orbital mechanics to find out why do we need a given velocity increment from a rocket.

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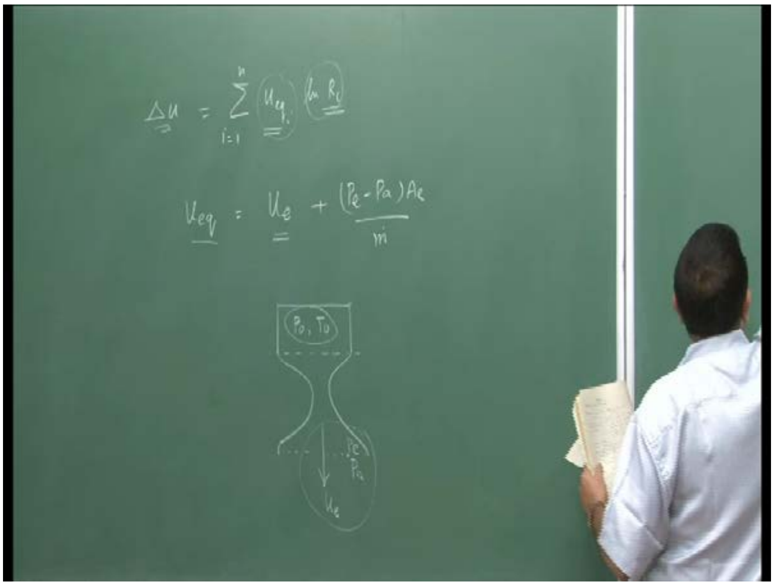


Now, when I say velocity increment, we have prove this expression delta u is equal to sigma i equal to 1 to n u equivalent i l n R i, where delta u is the velocity increment for a multi stage rocket with n stages, u equivalent i is the equivalent velocity for i th stage and R i is the inverse of mass fraction, payload mass fraction. Now, coming back to this term, so far we have focused on this when we talked about the velocity and the multistage dynamics and all.

We have focused on the mass distribution to optimize, we had said that equivalent velocity is given, the next thing what we are going to discuss now that how do we give this value of equivalent velocity, what is equivalent velocity. Now, if you recall the vehicle dynamics, the performance that we have discussed, we have shown that the equivalent velocity is given by exit velocity plus the pressure term. Now, both this terms exit velocity as well as the pressure term, we have discussed that the pressure term is going to be 0, if we have ideal expansion when we have under expansion or over expansion, then this term is going to be non zero.

And we have also shown that the thrust is going to be maximum when we have ideal expansion, but that discussion at that point of time was precursory, we just talked about it and went along. Now, let us take a step backward and focus on that, because both this exit velocity as well as this pressure term depends on our nozzle design. So, what we are going to start now is discussing chemical rockets, when we talk about chemical rockets essentially there are two components.

One is a combustion chamber followed by a nozzle that gives us the required amount of thrust, now the combustion chamber will give us the high pressure and high temperature and then the gases are expanded through this nozzle, we get the high velocity u e. And depending on the expansion here, we may have some pressure here P e some pressure P a based on that we get the pressure term.

So, what we are now seeing is that the equivalent velocity essentially depends on this, what we are getting at the exit of the nozzle. So, the first thing now we want to discuss, before we go into the more details of chemical rockets is the performance or the functioning of a nozzle.

So, the next topic that we will take up it is part of our chemical rocket is a nozzle, I would like to point out here that the way am going to cover this topic is first we talk about nozzles, because now as I said am discussing chemical rockets. First we talk about the basic principles of nozzles, then we go into the nozzles as they are used for rocket propulsion, there we get the characteristic velocity, the thrust coefficient etcetera, the nozzle performance parameters.

After that we go back again to the nozzles, we talk about the various types of nozzles and little bit of nozzle design. Once we had done with that, we will see that the nozzle performance actually what we are doing is we are taking reverse approach, first we have found out from the machine requirement, orbital mechanics what is delta u that is required, in order to get this delta u what should be the distribution of the mass fraction and the equivalent velocity that we get from the vehicle dynamics and optimization.

Then we say how do we get this equivalent velocity, for that we look at the nozzle and as we go along we will see that this parameters u e, P e they depend on this P naught and T naught, that is the rocket combustor, rocket combustion chamber properties. So, that we will take at the end, because that is how it has to be designed, first we specify the machine and then go backward to get the operating conditions. So, now for the next few lectures we will focus on the nozzle, as we said that let me now go into the discussion of nozzles.

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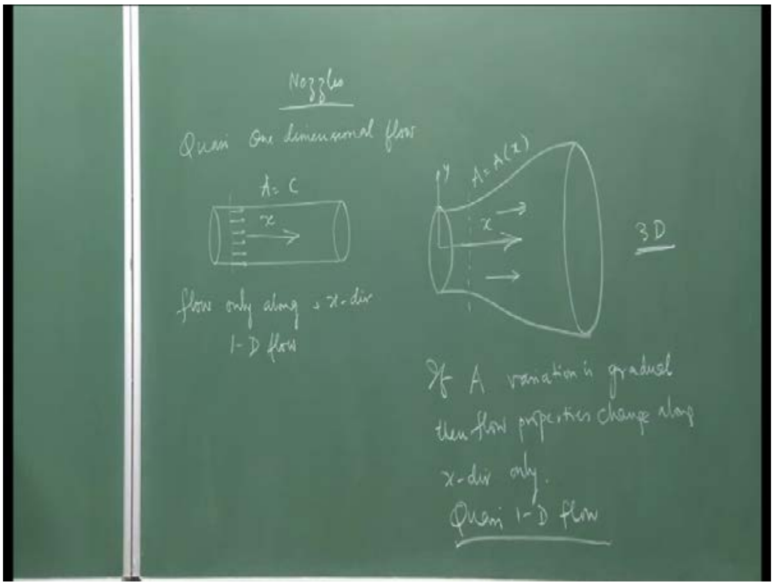


If you recall at the beginning of the course, I gave a couple of lectures on the history of rocket propulsion and I talked about the work of Robert Gadard. As I mentioned that one of the path baking patents that he had on rocket propulsion was the use of the laval nozzle, which kind of changed the way the rocket work. So, in the next few lectures we are actually going to look at how de Laval nozzle works and why it works in such an efficient way, that is going to be the...

First of all will say what is de Laval nozzle and then look at it is performance, so before we do that we need to know, what is the nozzle flow. So, we will look at this nozzles in a simplified manner, we look at a quasi one dimensional flow, let me explain what is the quasi one dimensional flow. Let us consider that we have two stream tubes through which some steady flowing, one of the stream tube is a straight cylindrical stream tube, that area is constant everywhere. And the flow is along this direction let me consider this direction as x, so the flow is in the positive x direction.

So, this case flow only along positive x direction, so this type of flow where it is only along the one direction is called a 1 D or one dimensional flow. So, in a one dimensional flow, the flow properties will change only in one direction thus that the flow is only in one direction. On the other hand, if I look at another case where I have a variable area like this ((Refer Time: 07:49)) and the area is changing gradually, and the flow is still along one direction.

In this case the area is constant here ((Refer Time: 08:02)) my flow area is function of x, the x let us start from this and moves in this direction. So, if I look at let us this is my origin, if I look at this flow, in this flow strictly speaking since the area is varying in the x direction as we progressed along the flow, strictly speaking it is not a one dimensional flow, it is a three dimensional flow. However, if this variation in area a x is very gradual, is the variation is small and gradual, then the flow properties will change along the x direction only; so if area variation is gradual, then the flow properties change along x direction only.

Essentially what we are saying is that, if we look at any location, any x location, at that x location the flow properties are uniform, I like to point out here that in both this cases, we are not considering the ((Refer Time: 09:36)), so this is in visits flow. Therefore, what we are talking about at uniform flows that is at any cross section, the velocity is same everywhere, therefore the pressure will be same, temperature will be same everything at a particular x location. Whereas when we come here, as we can see that the if the area of variation is small at a particular location we have uniform flow and then the only property variation will be along the x direction.

So, such a flow even though it is 3 D, it can be consist of the variation in components only along x direction, it can be considered to be a one dimensional flow, but it is said a quasi one dimensional flow, so this is called a quasi 1 D flow. So, therefore, if I now summarize what is a quasi one dimensional flow, in a quasi one dimensional flow all the flow properties are essentially function of only one direction.

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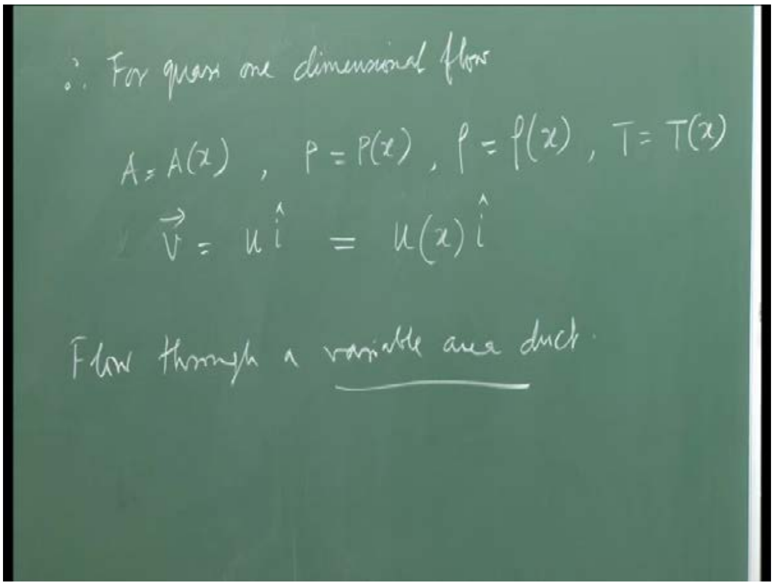


That is therefore, for quasi one dimensional flow, we have area as a function of only x, therefore pressure is function of only x, then my density is function of only x. Now, if I consider the working fluid to be a perfect gas, that if pressure and density are function of x, then temperature is also a function of x only and the velocity is only along x direction, so I write it as u. So, velocity vector V is only one dimensional, so this is u i only and this is a function of x only, so the u velocity is function of x only.

So, therefore, what we sees that the all flow properties for a quasi one dimensional flow is function of only x direction or one dimension. So, the quasi one dimensional flow as we are discussing is actually an approximation, it is not a reality; in reality this is going to be a three dimensional flow. And such a flow if we consider a three dimensional flow, the governing equation becomes more complex, we have to solve the full Navier's stokes, so we have to solve it numerically.

So, if we consider the flow to be one dimensional, the governing equation simplify quite a bit and then we do not we do not need to solve them numerically, analytical solutions can be obtained, that is the advantage of working with the quasi one dimensional flow, it is reasonably good approximation and gives us the initial design point. So, from there we can build up a more vigorous design, so therefore the quasi one dimensional flow can be considered to be a very good engineering approximation; and typically this is what is used for analysis of diffuses on nozzles.

So, essentially what we are looking at is flow through a variable area duct, where the area variation is small, so it is gradually changing, so with this background let us now come to the actual topic. So, as I just said that quasi one dimensional flow analysis is flow through a variable area duct, so now having established or discussed what is the quasi one dimensional flow. Let us now start the actual topic, which is flow through a variable area duct, I would like to point out at this point that this is applicable both through the nozzle as well as through the diffuser.

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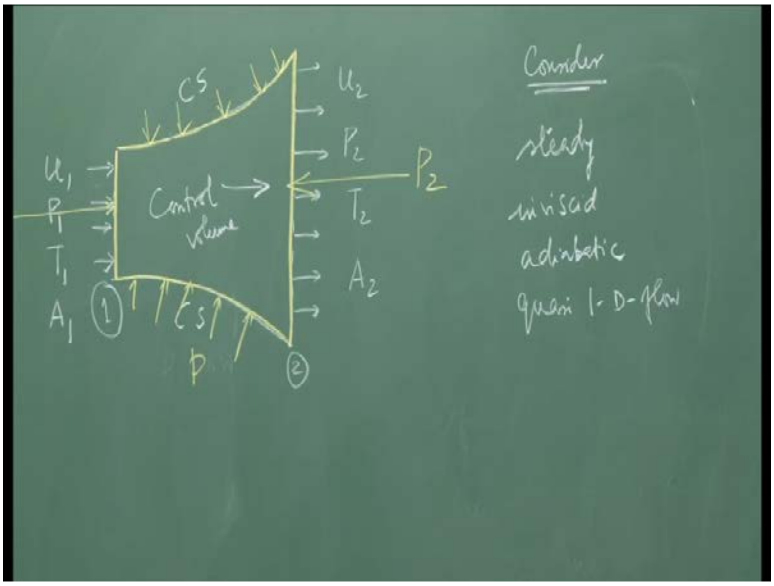


Let us look at consider a variable area stream tube, so let me consider a flow like this, let us say this is section 1 the entry to the stream tube, this is section 2 is the exit to the stream tube. By the way what is the stream tube, a stream tube is the virtual area in space or the virtual volume in space across which there is no flow. So, the flow will actually follow in this direction, but nothing will go across this stream, that is what stream tube is, so this is a variable area stream tube that I have considered this is location 1, location 2.

Let me consider that a flow, a uniform flow is entering this stream tube at 1, so the velocity I consider at 1 is say u 1, let us say the pressure at 1 is P 1, the temperature is T 1 and let us the area of the stream tube at location 1 is A 1. And let me consider that the flow after crossing this stream tube is exiting at 2, again this is a uniform flow with a velocity u 2, pressure P 2, temperature T 2 and the area of the stream tube at location 2 is

A 2. Now, I would like to analyze this flow, for that let us first start with a control volume approach.

So, let us define our control volume as this surface, this is my control volume which is the stream tube along with the inlet and exit sections, that is my control volume, therefore the surface of this stream tube is control surface. So, this surface is at the control surfaces, now at this section I know that pressure is acting P 1, at this section pressure is P 2 it will be acting normal to it in the opposite direction. So, if I now draw the pressure forces this will be P 1, this will be P 2, because pressure always acts normal to the surface.

Apart from that I like to point out here, that this is a stream tube, this is not a solid boundary, therefore at the surface of the stream tube there will be pressure forces acting and this will be normal, this will be normal to the surface of the stream tube, so these are special forces let say given by P acting on this control surface. So, the external forces now acting on this control surface is the pressure force P 1 at the inlet of the stream tube, pressure force P 2 at the exit of the stream tube; and this surface pressure P acting all along the stream tube; so this is our problem geometry and problem definition, the flow is going in this direction.

So, since for this case, let us now consider the flow to be steady, the flow to be in viscid that is viscous forces are absent, let the process be adiabatic, so there is no heat transfer out of this. And let us consider this to be a quasi 1 D flow that is the at every cross section the flow is uniform and the only variation in properties is along this direction, which is the x direction in this case. Now, for this control volume, let us apply the conservation laws, so we will first start with the conservation of mass.

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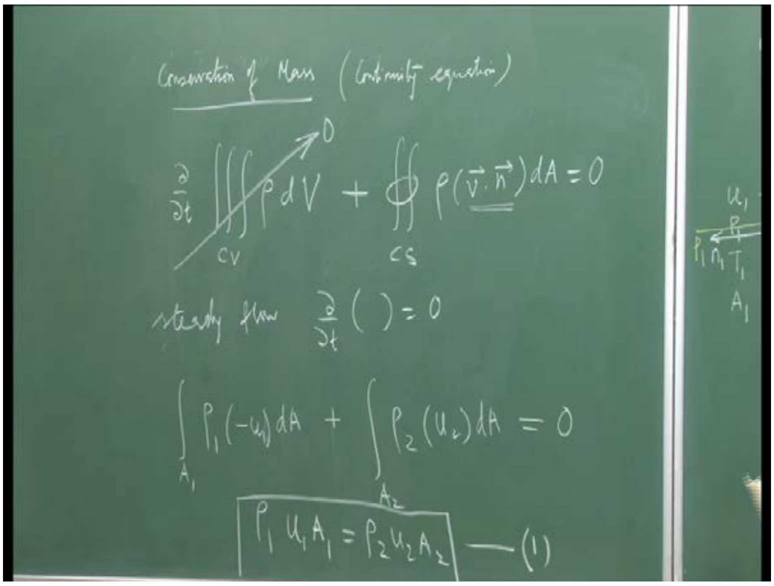


So, my first conservation equation is conservation of mass, which is also called as you know continuity equation, so we are going to apply this to our given control volume. The general form of conservation of mass as you know is has two terms, I will explain this terms, this is our conservation of mass. Here as you can see there are two terms, first let us see what is the term inside the integral rho d V, rho is the density, v is the small volume. So, if I consider a very volume here ((Refer Time: 20:21)), with in this control volume, volume times density is the mass, so therefore rho d V is the mass of a very small volume inside this control volume.

Now, if I integrate this over this entire control volume as we start here, then this is the total mass of this control volume at a given instance. Now, if I differentiate this with respect to time, what I get is rate of change of mass at a particular instance of this entire control volume. So, therefore, the first term here is the rate of change of mass of the control volume, within the control volume at a given instant. Now, when I come to the second term here what is say rho v dot n d A, if I consider a small portion here anywhere, then with it is area d A, a small area d A.

And let us at the flow is moving through it with a velocity v, so at 1 second the distance it will cover then will be equal to v, so the flow will cover distance v. So, if I extend this ((Refer Time: 21:36)) area through a volume, per second the volume of this will be A times v, am talking about the normal. So, therefore, this is something with the flow how much mass is going in or out, so because this volume is going to be A times v times density gives me the mass, so this gives us how much mass is going in or out to this control volume through the control surfaces per unit time.

So, that is essentially mass flux, flux of mass through the control surfaces into the control volume and notice one thing that I have taken v dot n, were v is the velocity, n is the normal vector to the control surface. So, for this surface the volume will be pointing outward n 1, for this surface normal again will be pointing outward n 2, because normal will always point outward to the control surface. Second point here is that, now if I take the dot product between the velocity vector and the normal, is essentially the normal component of velocity, so the normal component of velocity at the control surface; so this expressions you have seen in fluid mechanics.

Now, coming back to this equation, this is the general form of continuity equation, now we will use our assumptions that we have listed here ((Refer Time: 23:09)). So, first of all since we are assuming that the flow to be steady, which essentially means that there are no time derivative, the time derivatives are 0, the flow does not depend on time, therefore the first term here goes to 0. So, what we have left with is only this that is integral over the control surface rho v dot n d A equal to 0. Now, let us see how many surfaces do we have, we have the inlet plain 1, we have the exit plain 2 and we have this control surface for the stream tube.

Now, for the inlet plain v dot n is minus u 1, because v and u are direct and opposite manner, so v dot n term is minus u 1, the density at this plain is rho 1 and the area is A 1. So, for the first plain what I do is I integrate over area A 1 rho 1 minus u 1 d A, so that is the first term here, then I have this plain number 2. On plain 2 v dot n is u 2 dot n 2, u 2 i dot n 2, this two are direct in the same direction, so therefore v dot n in this plain is plus u 2, therefore and the density here is rho 2, area is A 2, so this will be equal to plus integral over A 2 rho 2 u 2 d A.

So, we have now consider this area and that area, one more area remaining is this control surface area. Now, ((Refer Time: 25:23)) this is control surface here is a stream tube, what is the definition of the stream tube that the velocity vector is tangential, velocity vector is tangential at every point, that is how stream lines are defined. So, therefore, if the velocity is tangential velocity is like this everywhere, then v dot n, because n is normal. So, the dot product the angle between the velocity vector and normal is 90 degree, so therefore the dot product is v n cos theta, theta is 90 degrees, so this is 0.

So, therefore, considering this is a stream tube helps us in eliminating this term, because v dot n term for both this entire surface here is 0, so therefore what we are left with is only this plus this is equal to 0. Now, we can simplify this little bit as rho 1, now what we had assume is that rho u everything, rho is anyway constant u is uniform, so everywhere u is equal to u 1, similarly here everywhere u is equal to u 2. So, therefore, when we integrate it over this entire area A it is nothing, but rho 1 u 1 A 1, so therefore what I have is rho 1 u 1 A 1 is equal to rho 2 u 2 A 2.

This is my first equation let me call this equation 1; this is the continuity equation for a quasi 1 D steady ((Refer Time: 27:01)) flow. So, notice one thing here, what we have done is v dot n for the control surface is equal to 0, that is what we had done. Now, we had derived this equation which is the momentum equation, sorry the continuity equation or the conservation of mass equation. The next now will look the momentum equation, so the next topic of discussion is the conservation of the momentum.

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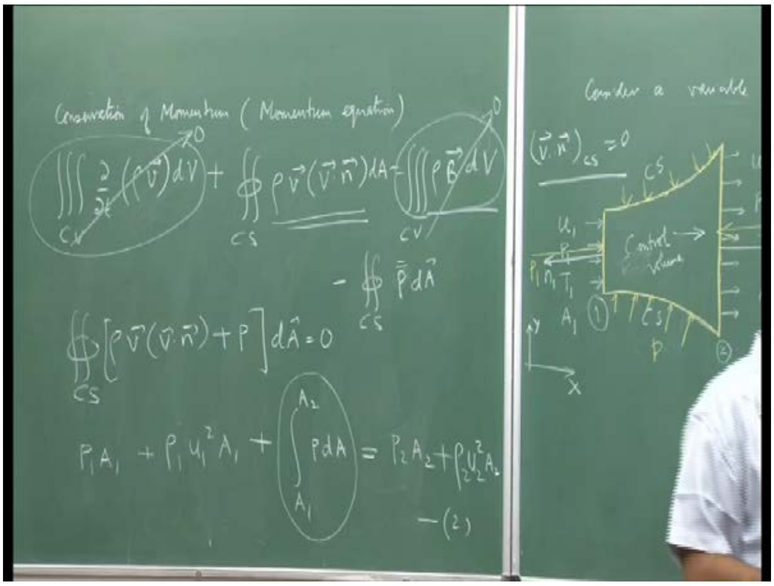


Next let us look at conservation of momentum or momentum equation, now in it is general form the momentum equation is written as integral over the control volume d d t of rho v d v plus integral over the control surface rho v v dot n d A equal to integral over the control volume rho B d V minus integral over the control surface P d A, this is the general form of momentum equation. Let us discuss the terms here, the first term here integral over the control surface d d t of rho v d v, how is momentum defined, momentum is mass times velocity. We have shown earlier when we discuss the conservation of mass that rho d v is my mass, so rho d v times v is the momentum, therefore d d t of that is rate of change of momentum over the entire control volume.

So, rho v d v is the instantaneous momentum of the entire control volume, integrated over the control volume gives us the momentum for the entire control volume at a given instead of time. When we take the time derivative of that, that gives first the rate of change of momentum, instantaneous momentum for the control volume. The second term here, again we have shown that rho v dot n d A is the mass flow rate, mass flow rate multiplied by velocity is the momentum flow rate. So, this is the momentum flow rate across the control surface or the momentum flux across the control surface.

So, this term is the momentum flux across the control surface, coming to the right hand side first of all the momentum equation is a representation of Newton's second law of motion. According to Newton’s second law of motion, the external forces acting on a particular body is equal to the rate of change of momentum of that body. So, the left hand side in this, our body essentially the control volume the flow through this control volume. Left hand side of this equation represents the rate of change of momentum of this control volume, which is the instantaneous rate of change and the flux through the control surface, momentum flux through the control surface.

Now, the right hand side here represents the external forces, here this vector B are the body forces, it can be the gravitational forces, it can be electromagnetic forces etcetera, these are the body forces which acts on the entire bulk. So, therefore, this is body force per unit mass multiplied by mass rho d v is the mass, so therefore this is the body force acting on this small element within this control volume, when we integrate over the entire control volume this gives us the total body force acting on the fluid inside this control volume.

And the last term here is P d A, P external forces can be the body forces and surface forces, surface forces can be two types pressure forces and frictional forces, since we are assuming the flow to be inviscid there are no frictional forces, therefore the only such forces are pressure forces. So, therefore, this term represents the pressure forces and pressure is a tenser, which is force per unit area when I multiplied with the smaller elemental area this gives me the force, so this is the force acting on a small area on the control surface, when I integrate over the entire control surface this gives me the total pressure forces.

This negative sign comes, because again the normal vector and the pressure are directed in a particular manner for example, actually normal vector is always directed outward, pressure is always directed inward that is we get this negative sign here to take care of proper direction. Because, momentum equation is a vector equation, so directions are important, now coming back to this momentum equation, now we will simplify it for the problem we are discussing. I have already discussed about the inviscid part, because we are considering the flow to be inviscid, so therefore there is only one surface force which is the pressure force.

Next is let us consider the steady flow as in the conservation of mass we have discussed, if the flow is steady then the time derivative term is going to be 0, so therefore this term is going to be 0. Next let us add one more assumption to this list which is no body forces, this is a very standard assumption used in fluid mechanics, we will use that here the validity of this assumption you can read up any fluid mechanics book that why this assumption is valid.

So, if we consider no body forces, then this term involving the body forces is also 0, so now what you can see is that we are left with only two terms rho v v dot n d A equal to minus integral over the control surface P d A, this is one point. Second point you can notice is that, the integral for both this terms are same over the control surface, so what I can do is I can take it to one side. So, if I do that what I have is integral over the control surface rho v v dot n plus P d A equal to 0, so I have taken this to the left hand side and see again the limit of the integral were same, so I can write it like this.

Now, let me come to the actual problem, here when we are integrating it first of all what we can write it is that, first let us look at the intersection section 1. For section 1 we have pressure forces acting like this ((Refer Time: 34:51)) and the pressure direction of the pressure force is along this; and we have considered this to be our positive direction, so therefore the forces are directing in our positive direction. So, the magnitude of this force is P 1 times this area, because the pressure forces considered to be uniform, so we have P 1 A 1 is the pressure force acting at this section; then we have the momentum flux terms.

Once again the momentum flux is in this direction as I consider, so therefore this will be rho u 1 square A 1, so plus rho 1 u 1 square A 1, because this becomes rho u 1 A 1, so this is the entry to this section. Then we have this control surface contribution, the control surface here once again the flux term is 0 through the control surface, because v dot n is 0. So, therefore, the only contribution the control surface will give is the pressure term, now here the pressure will be varying, because my area is changing, area is changing with x, so therefore the pressure is going to have some contribution.

Because, pressure as you can see now the direction is changing, pressure direction is changing, so it will have some contribution at this point pressure has no contribution to the x momentum, but when we come to a curved section pressure will have a component in the x direction. And here we are talking about the x direction, because we are talking about a 1 D flow, so therefore I will have a contribution from pressure forces for the control surface given as area variation A 1 to A 2 P d A and this is equal to the flux term going out.

So, other at the contribution for the section 2, so section 2 we had the pressure forces acting equal to P 2 times A 2, now that is why am taking it to right hand side, because now my direction is like this. So, this P 2 A 2 plus the momentum term rho 2 u 2 square A 2, so therefore this equation is the one dimensional momentum equation as applied to the control volume with this assumptions, we have listed the assumptions that are there, so let me call this equation 2.

Now, I would like to point out here, that unlike the continuity equation which was an algebraic equation, ((Refer Time: 38:01)) this equation is not an algebraic equation, because of the presence of this term, integral A 1 to A 2 P d A. Because, this area is a function of x, so therefore this is not an algebraic equation, in the continuity equation if we had known the inlet area and the exit area that was enough to get the property relationship. But now we need to know the variation of A with x also only then we can solve for this term and without that then this equation is not solvable, so therefore this is not an algebraic equation.

This term here represents the pressure forces acting on the sides of the control force, so let us retain this term and go forward the next conservation that we look at is conservation of energy, so next let us look at the conservation of energy.

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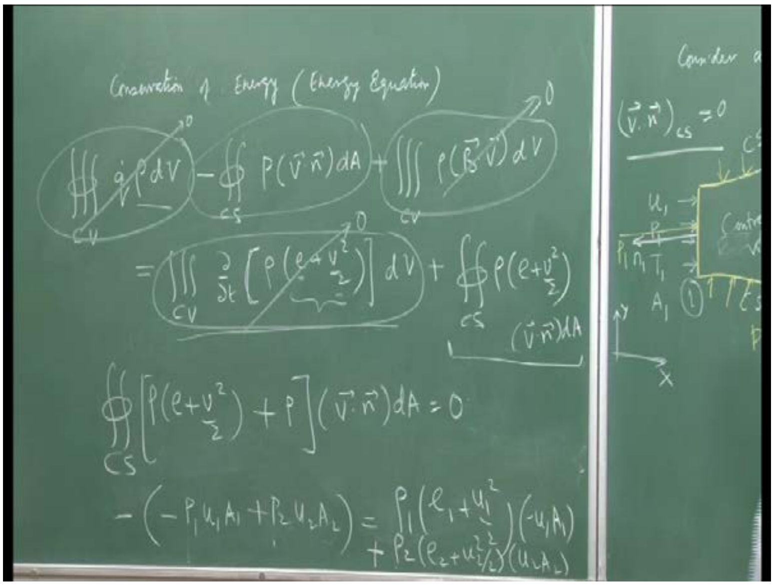


So, conservation of energy is also called energy equation, so once again we will start with the most general form integral over the control volume q dot rho d v minus integral over the control surface P v dot n d A I will explain this term, first let me write it down this is the general form of energy equation. Now, first of all what is energy equation, energy is representation of first law of thermodynamics, we says that the total change in energy of a system is the sum of the heat transfer and work done, heat transferred to the system and work done by the system.

So, that is the basic definition of or at the statement of first law of thermodynamics, so if I look at the right hand side of this equation this represents my total change in energy. Now, the total energy of a system has multiple components it has internal energy, it has kinetic energy, it is potential energy. For the fluid systems typically involving gases we neglect the potential energy, so here we are saying potential energy neglected there of total energy is combination of internal energy and kinetic energy.

So, this term here e represents the specific internal energy, which is internal energy per unit mass and v square by 2 represents the kinetic energy per unit mass. So, therefore, this term here e plus v square by 2, which is pairing both comes here is the total energy.

So, this is the total energy per unit mass multiplied by rho d v, which is the mass of a small element gives me the total energy of the small element, then when I integrate it over the entire control volume this gives us the instantaneous total energy of the control volume. And the time derivative gives us rate of change of total energy, instantaneous total energy of the control volume, therefore this term here represents the instantaneous rate of change of total energy of the control volume.

Coming to the second term here, once again e plus v square by 2 is the total energy and rho v dot n d A is the mass flow rate. So, therefore, the product of this two gives us how much mass energy is convicted in or out through this control volume, through the control surfaces. So, when I integrate it this gives us the total energy flux to our control volume, so this term is the total energy flux through the control volume through the control surfaces, so therefore, the right hand side of this equation is the change in the total energy of the system.

Coming to the left hand side as we can see left hand side has two terms, first law says, first law of thermodynamics says the total energy is the sum of heat transfer and work done. The first term here is q dot rho d v, rho d v as I have said is the small mass, mass of a small element, q dot it heat transfer rate of change of heat transfer or the heat transfer to the or the rate of change of heat a thermal energy for this control volume of the small system. So, whether I multiplied with mass this gives me the total rate of change of heat for the small element, integrate over the entire control volume is a total heat transfer or the rate of change of heat, not heat transfer rate of change of heat of the control volume.

So, therefore, this term represents the heat term, then we had the work done, now again the work done can be because of surface forces and body forces, the third term here represents the work done by the body forces. Now, what is work done, work done is force times displacement, so if I take force times velocity is rate of change of what and that is what is done here, rate of change of work, so this represents the rate of change of work, because of the body forces.

And this is again with the same analogy this is rate of change of work what the surface forces, which in the present case since we are assuming the flow to be inviscid is only the pressure forces. So, now this is the general form of energy equation, let us now start using the assumptions that we have, first assumption is steady; if I have steady ((Refer Time: 45:07)) this term goes to 0. Next inviscid, inviscid assumption we have already built in here, since we are considering only pressure forces as the surface forces, so this already inbuilt adiabatic, adiabatic is there is no heat transfer, so therefore this term is 0, q dot is 0 since there is no heat transfer.

Then we have quasi 1 D that we will come back no body forces, so therefore this term is also 0, so what we are left with is again two terms this and this, and as you can see they look very similar. So, once again what I can do is, I can integral over the control surface rho e plus v square by 2 plus P v dot n d A equal to 0, so this is the simplified form of energy equation. What are you do now is take a little step further, so I will simplify it further more and before doing that let me just continue from here only little bit, I simplify it little later.

So, now, this is my conservation of energy equation, let me now write it for this specific control volume, so once again we have a contribution here ((Refer Time: 46:47)), we have a contribution here, v dot n term is 0, so we have low contribution from the control surfaces. So, what we have now is only the contribution for the inlet and exit, if I write it now expand it we will have minus times, minus P 1 u 1 A 1 plus P 2 u 2 A 2 that is this term, as we conceive we have a minus sign here.

And P 1 v dot n is minus u 1 and d A here when we come here it is P 2 u 2 A 2, this is equal to this term here, which is rho 1 e 1 plus u 1 square by 2 minus u 1 A 1 that is the contribution of a section 1 to this term plus contribution of section 2, which will be u 2 A 2. So, once again in this case, since v dot n is 0 there is no contribution from the control surfaces here, so therefore this is my energy equation. So far what we have done today we have derived the continuity equation, momentum equation and energy equation for a quasi 1 D flow with this assumptions, that is the flow is steady, inviscid, adiabatic, no body forces, no potential energy, will stop here for this lecture.

In the next lecture we will simplify this equations further, so that we can get relevant equations. Finally what we have to do is, if this properties are given at one, you have to find out what all the properties will change at two for a given area of variation, so I will stop here today.

Thank you.