Jet and Rocket Propulsion
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Lecture – 15

Good morning. So, we are back with our lectures on rocket and space craft propulsion. Today we are going to start a new topic which is arbitrary mechanics before starting that let us just recapitulate what we have discussed so far. We have discussed the history of rocket propulsion, then we had looked at the performance of a rocket vehicle particularly chemical rocket vehicle, chemical rocket and derived expression for the thrust and specific impulse, and after that we went on to discuss the flight mechanics or vehicle dynamics. We have seen that the most important parameter is the equivalent the velocity increment that we can achieve.

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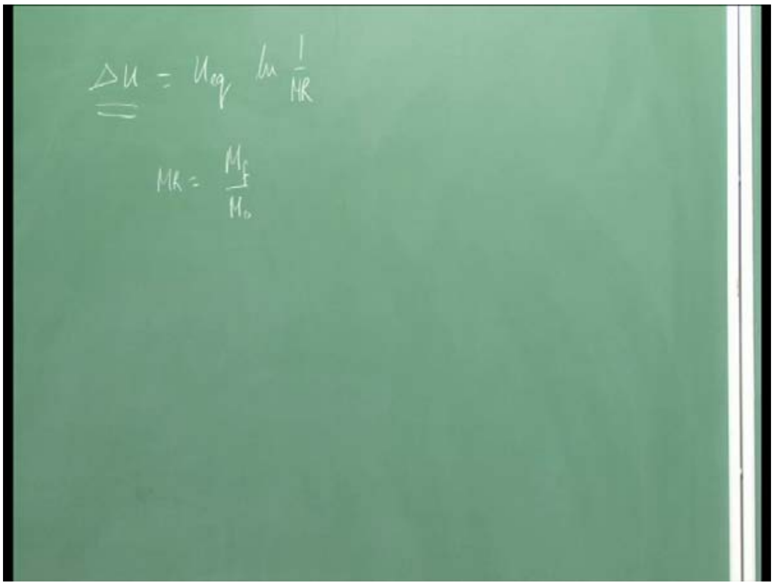

And we have seen that the velocity increment that can be achieved is essentially u equivalent l n 1 by MR, where u equivalent is the equivalent velocity, M R is the mass ratio which is equal to the final mass divided by initial mass of the rocket. And then after that we derive the dynamics equations for the single stage rocket. We estimated how far a rocket will go, we discussed the effect of the angle, the altitude that is provided for the rocket. After that we went to multi stage rockets, we discussed why having a multi stage configuration is beneficial over a single stage configuration, after that we discussed in detail the optimization of multi stage rockets. We have defined that the optimization the objective is to either get maximum velocity increment or to get maximum mass ratio, which means either we increase the lift the payload mass at the final mass as much as possible or we reduce the initial mass as much as possible. And we have discussed this in detail. Now one point that has came out again and again is the final aim of the entire vehicle dynamics analysis was delta u, how much velocity? We can get.

Now, the question arises that why do we need this delta u, and at one point of optimization we said our delta u value is given, and we tried to minimize the maximize the payload mass fraction. Now this given value of delta u where do we get it from? First of all why is it required? And that is where the orbital mechanics comes into picture. This is dictated by the machine requirement when we talk about rockets or space vehicles, we specify a machine for the space vehicle; that is the space vehicle has to deliver a payload which typically is a satellite to certain orbit. What do we win by orbit? Orbit is a path followed by the satellite for the present day scenario against earth around the earth, it follows the predefined well defined path, and it stays there. Now the question is how do we ensure that the vehicle, which is launched from earth reaches that orbit and stays there. And that is where delta u comes into picture, where the vehicle will stay and at what what how much time it will take to rotate around the earth, depends on how much velocity with which it is ejected into the orbit. So, that is where the orbital mechanics comes into picture.

I would like to point out here one thing that orbital mechanics is a sub part of space dynamics. Space dynamics in itself is a fast course, there are various issues in space dynamics. For example, when we launch a vehicle it directly does not go the required orbit, it is first launch into a transition or temporary orbit, and from there against a from there it is a pushed again to the final orbit. So, it is not a single step process, there are multi stage steps involved in putting a satellite into orbit, and not only that lets say if you have to for example, if I look at a space shuttle; space shuttle is to go to the orbit and then the orbit and come back. So, that is also a maneuver dictated by space dynamics, how it needs to be done and then when it is entering the earth s atmosphere what should be the velocity, because earth atmosphere is much denser than the outer space. So, the density increases the frictional forces increases, and the vehicle which is moving at very high speed when it enters it has to visit the aerodynamic hitting.

So, if it enters at a very steep angle, it will just burn out. So, it has to enter in a shallow angle. So, there are various aspects of space dynamics we will not go into to full detail, we will essentially see what are the conditions that must be there in order for a vehicle to take a orbit, and when I say orbit essentially it means that it is going round; orbit can be circular, it can be elliptical, but it is going round and round the heavenly body that is what an orbit is... So, therefore the orbits are conic sections, circle, ellipse, etcetera. The however, the path of the vehicle when it is flying may not always be ellipse or circle part of ellipse or circle it can be parabola, it can be hyperbola. So, depending on this path we will also show that what path is feasible to be transitions into a orbit, and what is not? All this things we will discuss, again I will have to point here that we will not go into the details of the mechanics.

We will consider that the mechanics is known, we just do the balance of forces, and also energy after a while and essentially do some geometrical manipulation along with the equations of motion to find out which orbit is possible, which orbit is not. So, the knowledge we will gain from this discussion is essentially to find out how much velocity we should provide to the vehicle if it has to take up a specified orbit, that is whole crux of this discussion. So, therefore with this little background, let us now talk about orbital mechanics.

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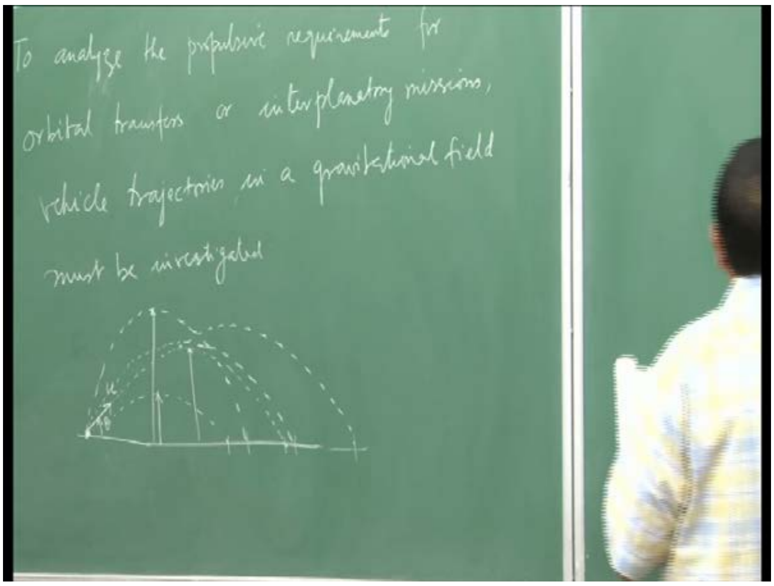

Now, what is the goal of the orbital mechanics? What do you want to find out from there? We want to analyze the propulsive requirements remember that in their rockets and space craft propulsion, we are talking about the proportion system only. This comes as an input to the rocket designer. So, you want to analyze propulsive requirement for orbital transfers, what do you mean by orbital transfer? As I said that when satellite is launched, it is not possible to take it to the desired orbit in one go.

So, first it goes into a transfer orbit, and from their again it goes to the final orbit. So, essentially there will be multiple orbit it is will take till it reaches the final designated orbit. So, there is then an orbital transfer maneuver that needs to be carried out, and all this maneuver essentially depends on how much velocity increment we are giving. The point is if it is one orbit in order to stay in that orbit, it has to move with certain delta u a certain u certain velocity, when you has to go to some other orbit at certain other distances, it has to move to move with some other velocity.

So, now, this velocity change needs to be provided. So, as the space craft designer we want to find out how much velocity change is required. So, that we can design our rocket and how do we design our rocket, again the optimization that will find out how much specific impulse should be there for the rockets or what is the mass ratios different mass how they are distributed. So, that we can achieve this velocity change from going from one orbit to another orbit in the most economical manner.

So, that is the final m. So, for orbital transfer or inter planetary missions, let us say we want to go from earth to mars. So, that mission requires certain well defined maneuvers when it has go to one orbit to the another orbit to another orbit like that. For example, in the history part I talked about international space station; international space station is moving in its own orbit, and I have said that the international space station can be used as a launching pad for going to mars.

So, now if a vehicle has to go from earth, first it has to dock in international space station, it will orbit with that then it will take off from there go to some other orbit, other trajectory, let us say then from that trajectory it will go to a temporary orbit for mars and then go to the final orbit for mars, like that it is the mission is designed decided. So, therefore for inter planetary mission also it is important to know how the vehicle has to move? Then vehicle trajectories in a gravitational field this must be investigated, what do we mean by vehicle trajectory? We have talked about the vehicle dynamics.

Therefore part of the dynamics is the path vehicle is taking, that is called vehicle trajectory. Again the path it will take depends on how much velocity initial velocity we have provided, to give a very simple example. Let us say there is a basic mechanics example, we have lets a body here, and we eject the body by some means with a velocity u and angle theta. Now at the effect of gravity it will take a path, where it will reach a maximum point and then come down here. So, this the range that it is attained. Now if I shoot it with a higher velocity, it will go further and go to larger range, right. Other hand if I such as this angle theta the initial angle theta, if it is reduced - it will go to a shorter length, if it is increased - it will go to a larger length up to a point beyond that it will come down to a lower orbit. So, therefore, what we can see here is that the point at which this projectile is going to hit the ground depends on the velocity with which it is launched and the angle at which it is launched.

So, these are the two most important parameter that will dictate how far will it go. One more point, we will see here it is not only how far it is going, even what is the maximum height that the projectile is attaining also depends on this too the initial velocity, and the angle at which it is launched. So, therefore the vehicle trajectory and all these variation essentially depends on the gravitational field at which it is operating, this is for earth's gravitational field can we go outside the gravitational field has to be defined there separately. So, therefore what we see is that the vehicle trajectory depends on the initial velocity and the angle. So, for the for all this missions then this vehicle trajectory under the gravitational forces must be investigated in order to find out how much velocity change we need to provide. So, before we start designing; first thing is to do this, mission is defined and then we have to find out what will it take to attend this machine.

So, first part is that. So, first this is this will define the mission requirement then from orbital mechanics analysis, we will find out what kind of velocity will it take to attend this. After we do that we go back to the discussion that we had so far on vehicle dynamics from their will find out. What should be the mass distribution to attend this velocity, which these guys are saying this is predicting. Once we get that then we go to the actual rocket design, once we have the mass distribution, we known how much propellant mass is there, how much every other mass is then we go to the vehicle rocket engine design, then we design the because after that once delta u is fixed, we now know how much equivalent velocity is needed to get the equivalent velocity. Now once we know the required equivalent velocity, we design our rocket motor, which is the combustor and nozzle to provide that equivalent velocity. So, that is the full I would say algorithm for the design of a rocket vehicle, therefore orbital velocity mechanics is a starting point, where once the mission is defined we find out the mission requirement. So, coming back to this them.

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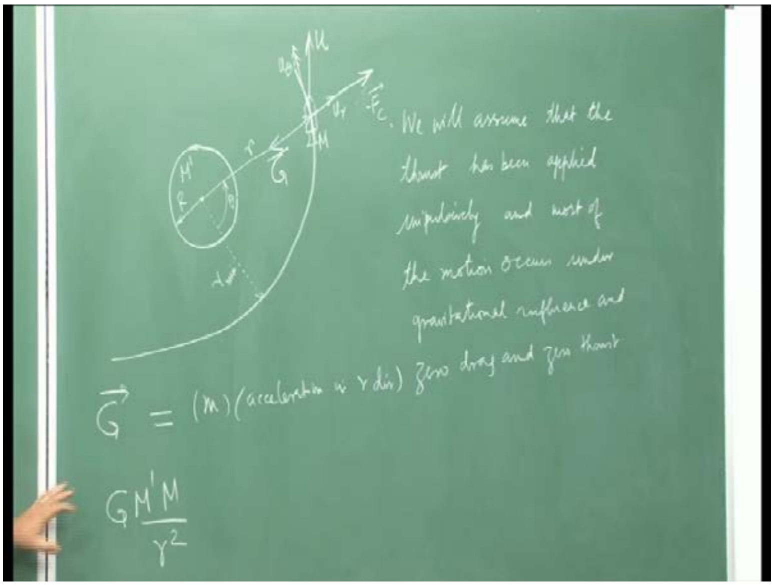

Let me just draw as schematic, let us say that we have a heavenly body, which is quite massive, let say mass of this body is M dash, and around it we have an orbit on which artificial body, which is our satellite let us say is moving. Now, first what we do is we will put the nominication in place. First what we will find out is that lets say the radius of this heavenly body is r, first from the center of this body we find out what is the minimum distance of this trajectory or this path. So, let me say that this the minimum distance of this path, I call it r min minimum radius. Now let us say that the vehicle, the rocket vehicle is somewhere here, sitting like this and moving with speed u.

So, you find out the centre of this centre of mass of this vehicle, and now let us define some parameters. The distance between the centre of mass of the moving vehicle artificial vehicle, and the heavenly body is called the radius r. And the angle that this radius makes or the this distance makes with the minimum radius is theta, the angle theta. Now when this vehicle is moving, let us say we consider a planar motion it is moving in a plane around it. In that case it is moving with this velocity u, it actually has two components; one is along this radius let us say given by u r, and other is tangential to this path given by u theta, therefore the resulted velocity is u.

So, this is the nomenclature that we have. Now let us also consider that the mass of this small artificial body is M capital M. So, for this vehicle now we want to find out the vehicle dynamics, now I would like to point out let us say if I look at natural satellites, let us say moon; moon is a natural satellite of earth. Moon is at a certain distance from the earth, and it has certain rotational speed or period with which it is rotating around the earth. Now there is gravitational pull between earth and moon, because of this gravitational pull if why does not moon fall back into earth, because mars is much heavier than moon right. Why does not moon fall back into earth?

The reason is this the rotation that it is going through, because of this rotation it has some pre defined path or centripetal force, which balances the pull gravitational pull, because of that it moves. And if the path was different or if the period was different, moon would have fallen into earth or it would have gone out, it cannot be the national satellite. And that is exactly we are going to talk about in this topic, that what will be the condition under which a vehicle becomes a satellite, what will be the conditions where it cannot attain the satellites position it will just move away. So, for example the mitosis, these are not satellites asteroids, these are not satellites, but they also have some speed defined path. Only thing is that their speed is such or their trajectory is such that they cannot fastly became a satellite. So, therefore we will discuss those issues here. So, now coming back to this diagram. So, this is our let us say earth or any heavenly body, this is the artificial satellite, which is moving with a velocity u at certain angle.

First what we will do is we will assume that the thrust has been applied impulsively and most of the motion occurs under gravitational influence, and 0 drag and 0 thrust. What does it mean? We will consider that for a very short period of time we provided the thrust, and then we have cut off the engine, because its short impulsive thrust that has been provided there is a change in the velocity of the vehicle given by delta u which we have discussed. After that the thrust has been cut off and let us say the vehicle is moving in outer space, so there is no drag. Then once this velocity increment has occurred, now the gravitational force will act on it and then try to stabilize it into a give orbit.

So that is what we said that the thrust is applied impulsively, we do not have continuous thrust operating on it, only for a short duration. It has occurred if I look at the example of a satellite once again, the satellite launch we are putting from with a multi stage rocket and one after another the rocket stages are burning out till the final stage is burned out after the burn out of the final stage the satellite is thrown away right. And then it separates from the rocket, during this entire process. Now the satellite has attained a give velocity, now it moves with that velocity under the influence of gravity right, and because of that it attends sudden specified trajectory, it is not arbitrary the specified trajectory is dictated by the loss of gravity.

So, this is what we are trying to find out? We have some give some initial velocity after that there is no more additional force acting expect the gravitational force. And the satellite or the vehicle is reacting to this gravitational force. So, essentially if I look at this vehicle what is the direction of gravitational force which will be acting? It will be acting towards the bigger body right bigger body will pull it. So, the gravitational force let us say G will be acting in this direction, and the vehicle is moving around the certain period right it is turning around. So, there is a certain angular velocity with which with which this is moving, and since it is moving in a close path at every point of time depending on the angular velocity, there in centrifugal force that acts. And the centrifugal force will be acting away from it right, now coming back to basic mechanics this is my free body diagram now right that this is my vehicle, gravitational force is pulling it towards the centre of this bigger body, and the centrifugal force is pushing it outward when these two are balance only then it will move in this path, otherwise it will not. If this is greater than this - it will be pulled back, if this is less than this - it will move away.

So, therefore, the balance of this two the gravitational force, and let us say centrifugal force we will be write it as a FC, these are the two things that gives the vehicle in its intended trajectory or intended path. So, then from this free body diagram now, we can write the force balance; the force balance is that the gravitational forces is equal to the mass times acceleration in r direction right, because the direction that we are looking at is the r direction. Now, if we consider that this is positive direction, radial direction away is the positive directions then the gravitational force will be equal to from Newton's law of gravity G M dash M by r square, where G is the universal gravitational constant.

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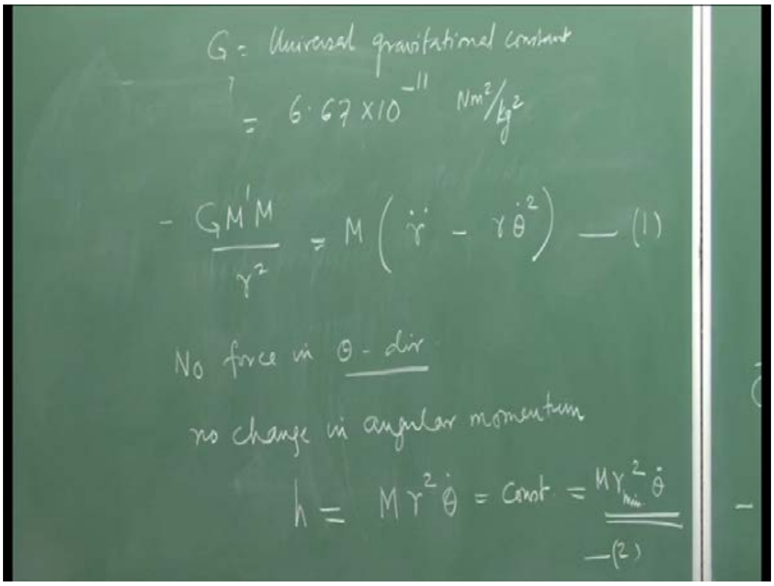

G is universal gravitational constant thus the universal constants value is given as is equal to 6.67 into 10 to the power of minus 11 Newton meter square per k G square, this is the unit of G. Then this is the gravitational force acting we will take the minus sign here, this is equal to mass times acceleration. So, let me put it mass, the mass of this small vehicle is M or our artificial satellite is M, and the acceleration; the acceleration will have two components - one is r, other is theta right. So, because actually one because of the this acceleration of there is because of this motion. So, there will be two components in acceleration. So, one is because of the radius radial location del square r del t square, and other is because of this rotational motion, which is given as r d theta d t square. So, this is the basic force balance for this vehicle.

So, this is the expression we write in orbital mechanics or in mechanics we will write it little more little differently. What we see is that the first term is the second derivative in time, right. So, we can write it as r double dot, here we have first derivative in theta square. So, this equation if I rewrite it, I can write it as minus G M bar M by r square equal to M r r double dot minus r theta dot square. Let me call this equation one, this is coming from the force balance in the art direction.

Now we do not have any force in the theta direction right. So, there is no force in theta direction, as we have seen that the force is all in the r direction. So, there are no forces in the theta direction. So, there is no acceleration in the theta direction also, if there no force there is no acceleration from Newton's laws, if there is no acceleration. Then the momentum is conserved right according to either Newton's law, force is rate of change of momentum, and force is mass times acceleration. So, if there is no acceleration, there is no force which means there is no change in momentum. So, therefore in theta direction there is no change in its net momentum, and the theta direction momentum is called angular momentum. So, therefore there is no change in angular momentum of this body. So, first of all let me get what is angular momentum? Angular momentum is given as mass times radius square times rotational speed theta dot is equal to essentially omega. Essentially what it is given as angular momentum let me write it as tau equal to mass times u times r, where u is the speed with which it is moving in theta direction or the u theta, and what is u theta equal to r omega right.

So, u theta equal to r omega and omega is nothing but theta dot rate of change of angle or rotational speed, therefore this becomes equal to M r square theta dot. So, the this was just a small detail where we have shown that the angular momentum is given as mass time radius squares time theta dot theta dot, this is the angular speed. So, here I designate because this is the typically the designation of angular momentum in flight mechanics, sorry space dynamics. So, h is the angular momentum is M r square theta dot. Now, this is not changing, therefore this is equal to our constant, this is important to know, and then if the angular momentum is constant, it is constant at every point on this at every point on this trajectory. Therefore, its value at minimum r is equal to its value at r. So, I can write this as M r minimum square theta dot. This is what the angular momentum is which is a constant value. Now let we call this equation two, if we now combine equation 1 and 2, combining equation 1 and 2 we get an expression here for r.

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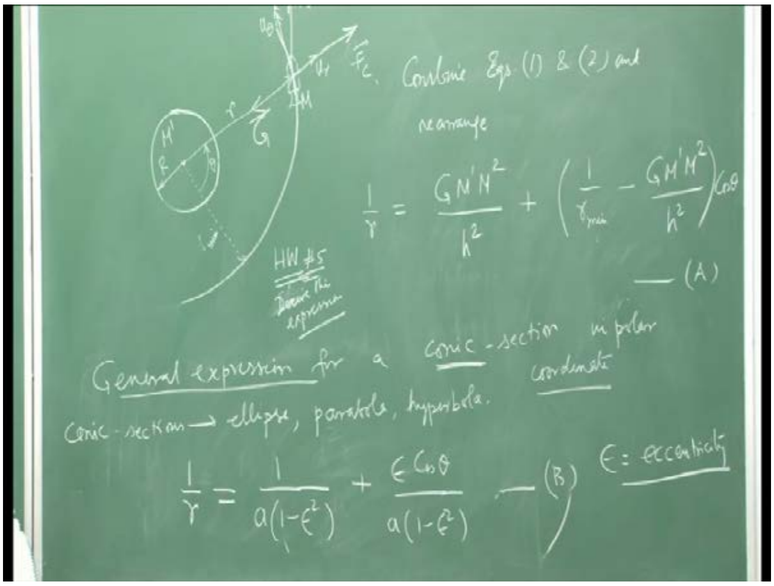

Let us next combine equations 1 and 2, of course it will require some algebra after we combined we and rearrange, we get an expression for r? We simplify for r here. So, the expression for r will be that is one thing, my h is constant right, my h is constant. So, from here solving from this h, I can get an expression for theta dot in terms of minimum r and mass which is an own quantity, we can put it back into this equation. Now this theta dot is a function of h M r minimum square. So, then it comes here like this. So, now what we will have is a differential equation in r, because this is second derivative in time, now we integrate that over the required time period, and we get the final expression for r. So, by doing that we will get 1 by r equal to G M prime by r square h square plus 1 upon r minimum minus G M prime M square by h square cos theta. Let me call this equation A, once again I have been giving some home works. So, this another homework, I think it will be home work number 5 to show this to show this expression, derive this expression.

So, now what we have is an expression for r in terms of a universal gas constant, the conserved angular momentum, the mass of the bigger body, heavenly body, the mass of the satellite, the minimum distance of the satellite from the body, and at a particular instant of time the angle that this makes at this body, we have got this expression. Now this is the general equation for orbit, the same expression or very similar expression if I look at the general expression for a conical section.

So, conic section not conical section, the general expression for a conic section, what is a conic section? A conic section is like ellipse hyperbola, parabola, etcetera. So, the conic sections are ellipse, then parabola, then hyperbola; these are the general conic sections. See if I now look at the general expression for a conic section in polar coordinate system, what is the polar coordinate system? it is r theta z, right. So, we are having here r theta and since we are talking about a planar motion. So, z is not a appearing here. So, essentially if I look at a general expression for a conic section in a polar coordinate system, just looks very similar to that that expression is 1 by r equal to 1 upon a 1 minus epsilon square plus epsilon cos theta divided by a 1 minus epsilon square.

Let me call this b this comes from geometry. So, this is an general expression for a conic section, notice the similarity between these two, they look very very similar in form this term here can be equated to this, this term here epsilon upon a 1 minus epsilon square can be equated to this, because we have cos theta appearing here. So, if we consider this path taken is a conic section, we now have a similarity between the general definition of a conic section, and the orbital mechanics. So, what we can do from here is very interesting, if we define the path then all this geometrical parameter I will explain what this geometrical parameters are fixed. So, now I know let us say a value of a epsilon etcetera, I can just equate this to this, I know this value, I know this value, I know this value, I can find out h.

Similarly, I can find out the minimum r. So, once I have that I can then calculate that the velocity, because now the orbital requirement is specified. So, therefore that is the advantage that our general equation of the orbit which we derived from force balance is very similar to the general equation for a conic section. So, in this case epsilon is called as eccentricity, now I think we are almost reaching the end of this hour.

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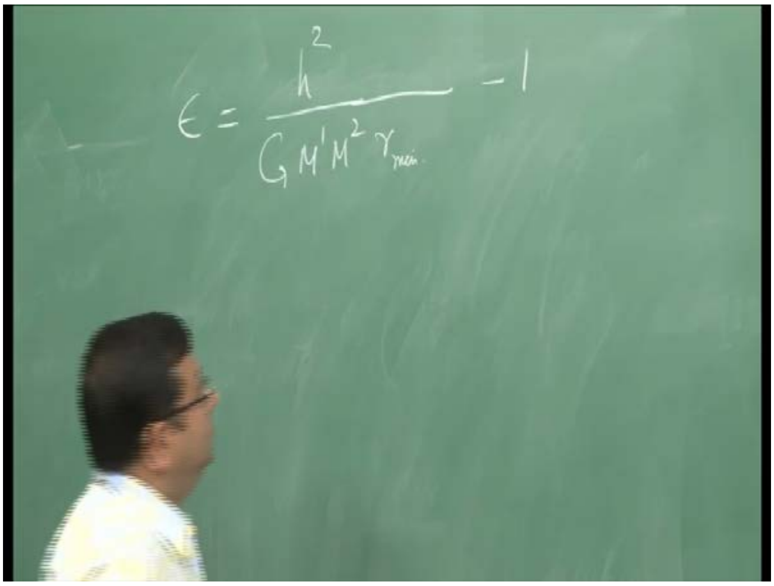

So, I would like to point out here that the eccentricity first of all relates to this value here, we can simplify this little bit and get an expression for eccentricity, which is epsilon is equal to it will be then given as function of the angular momentum h square G, which is universal gravitational constant M dash M square upon r minimum minus minus 1.

So this is their expression for eccentricity, and the equation for the conic section is this. Now what happens in practical scenario? This is what I specified, the path is specified. So, this is the equation for the path of the vehicle or the trajectory of the vehicle. So, this is specified to us we just now find out from this what solving this equation along with this, what are the values of a and epsilon? And then first a and epsilon value first we can solve from here, then we can put it back into this equation and get the values of h and r minimum, that is the cracks of the entire thing.

So, I will stop here now, and the next thing we discuss now is go into a details of conic sections I have. So far defined eccentricity which is a parameter, but I have not defined it geometrically I would not define a. So, now we will go into the discussion of conic sections and define this parameter. So, that we will see that once the path is specified out the trajectory is specified, what should be the values of this and then from their I will show how to estimate the velocity increment required to attend the given mission, which is to put the vehicle into a predetermined orbit or trajectory. So, that is what we are going to do next. So, next force a lecture we are going to focus on the geometry of conic sections, you have any question. So, what I will do is I will stop here for today, and in the next lecture we will start from the geometry of conic section, which is essentially discussing this equation equation b in more detail with respect to ellipse, parabola, hyperbola, etcetera.

Thank you very much.