Fundamentals of Aerospace Propulsion
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Lecture - 15

We will start this lecture 15, with a thought process from Martin Luther King Junior, and which is quite true at this moment.

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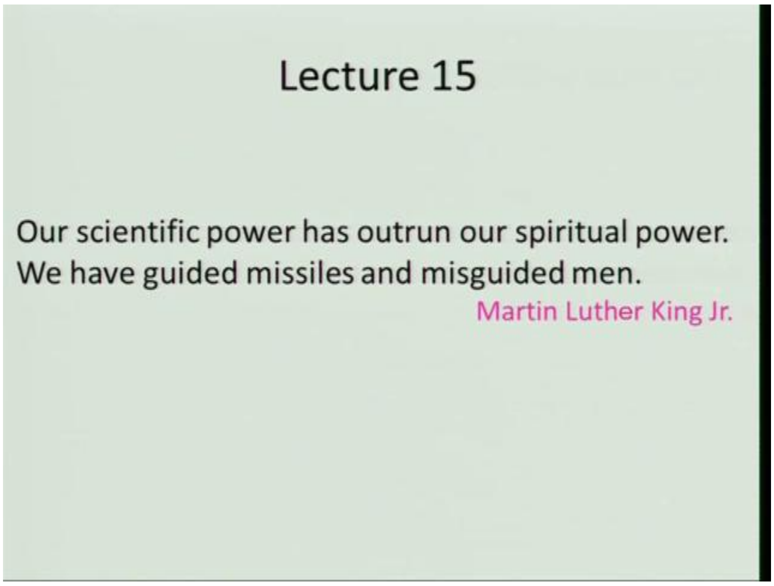

Our scientific power has outrun our spiritual power. We have guided missiles and misguided men. It is very appropriate, when we are discussing about propulsion, and missiles. Let us get into our usual things and we will have to recall, what we learnt in the last lecture. In the last lecture, I basically initiated discussion on the one dimensional flow with heat addition, and we have derived all the expression for the properties ratio across the heat addition zone.

Then we looked that those properties, for example, pressure ratio, temperature ratio, total pressure ratio, density ratio are function of gamma m 1 and m 2. Later on, I also gave a relationship between the Mach number, inlet Mach number and outlet Mach number and we found that it was quite difficult to handle. Then we devised ways of referring to a sonic condition.

For that, you need to re-express those expressions for the ratios, so that you can solve very easily. If you remember correctly that we took an example, where we use the reference conditions corresponding to sonic flow and then solve the problem. If you look at your notes you will find, what we learnt for the subsonic flow, that means that example was meant for subsonic flow.

What do we see is that the temperature at the station two, that means downstream of the heat addition will be greater than the upstream condition. But what happens to the pressure? Can you just look at your note? Pressure at the downstream condition of the heat will be decreasing. We have seen what happens to your density and what happens to the total pressure and total temperature. Now we will try to look at what happens when the heat addition, instead of supersonic flow and if it is subsonic.

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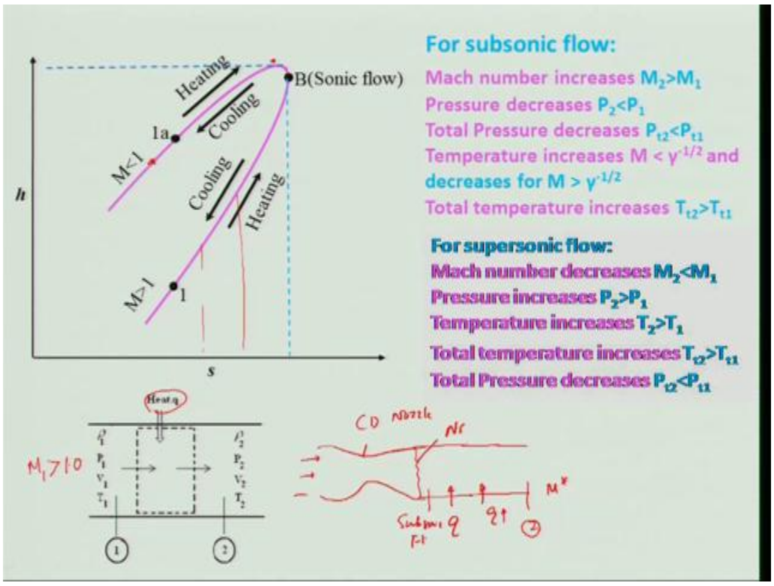

In other words, if the flow is subsonic, we have seen that something is happening. If the flow is supersonic what is happening is that just opposite of what is happening in the case of subsonic flow. That is what we had discussed in the last lecture by considering an example.

So, let us look at basically the processes for both the sonic. Supersonic flow is plotted in a Mollier chart. In the x, what we call in the vertical axis that is enthalpy, in the horizontal duct that is entropy. This is the curve which has zone for both supersonic and subsonic. Consider a supersonic flow, if I add heat that means if I consider this is the inlet condition, this is corresponding to the station one inlet condition, where Mach number 1 is greater than 1 that means it is supersonic flow. For example, if it is 2 or 3 or something like that.

If I add heat over here, then what happens at station two? That will be dependent on the extent of heat being added, that means it will be depending on the amount of heat which is being added to this. We will see that, if we go back to our expressions that at the exit of this heat addition, of the downstream of heat addition, there will be decrease in the Mach number. In the last example, when the flow is subsonic, what is happening when you add heat? The Mach number at the downstream is increasing as compared to the inlet condition. But in this case, it is just opposite. That is Mach number at the downstream is less than the inlet Mach number.

As you goes on adding the heats what will happen? If you look the entropy also is increased, that means if entropy is here, and then another place here, entropy is increasing. It goes on increasing this Mach number in case of supersonic flow, on heat addition, till it attains the maximum entropy value that is the sonic condition. What is the meaning of it? That means the flow is chocked, and it is thermally chocked. That means you cannot really go on giving the heat for that, to change the flow properties at the downstream of the heat addition zone.

Suppose I want this flow as attain the sonic condition and decrease the flow to subsonic condition, what I will have to do? I will have to cool it, so that it will go in this direction. Similarly, if I am at the subsonic flow conditions, and I am heating, what is happening? Mach number goes on increasing. If you look at Mach number goes on increasing, and so temperature is goes on increasing, however till this point.

After that, if you look at your enthalpy is goes on increasing till this point, after that it decreases. However the Mach number goes on increasing till it attains the sonic condition. And also the flow is thermally choked. Beyond that you cannot really add any further heat, unless you change the inlet condition. If you are coming from the subsonic and you want to go beyond the sonic and to the supersonic, then you will have to cool this, so that you can reach the supersonic condition.

Just to recall that what we have learnt, the Mach number increases in case of subsonic flow, that is M 2 is greater than M 1, and pressure decreases P 2 is less than P 1, this is for the subsonic. A total pressure decrease, P t2 is less than P t1 and temperature increases till M is less than 1 over these values. Then of course, after that further addition of heat, it will be decreasing. That means when M is greater than M gamma power to the minus 1 over 2.

Total temperature of course, goes on increasing because heat being added. So in case of supersonic flow, what will happen? Mach number actually decreases, that means M 2 is less than M 1. Pressure increases, P 2 is greater than P 1 and temperature increases T 2 is greater than T 1, and total temperature of course, increases all the time, and total pressure decreases both the cases, whether it is a subsonic flow or the supersonic flow.

Suppose, you added heat and if you are in the choked condition, it means you are in the supersonic flow, if you are in the thermally choked condition, and if you go on adding heat, you are at this point, I am not changing the inlet conditions, what will happen? How we will get a supersonic flow? For example, if I take a heating zone, and this is a subsonic, this is a C D nozzle and then this is been added heat. So, if I go on adding heat in this region, so that this region that is two, has reached the sonic condition.

If I go on adding further heat, then beyond this choked condition, what will happen? A normal shock will be formed, such that at the downstream, this will be subsonic flow. Then the condition has been changed. That means, you have gone to this region somewhere and then you can have, but if it is other where on the flow is subsonic and you go on adding heat, and when it is choked condition, what will happen? At that point, it will be sending some pressure waves, such that this condition, instead of at this point, it will to go over somewhere this point, such that the inlet condition will be changing.

Now, we will move into another topic, that is the constant area one-dimensional flow with friction. Of course, whenever constant area is there, there will be one-dimensional flow. And it will be a steady flow, and keep in mind that we have not considered the friction earlier, but now will be considering friction and there are no shock formations and others.

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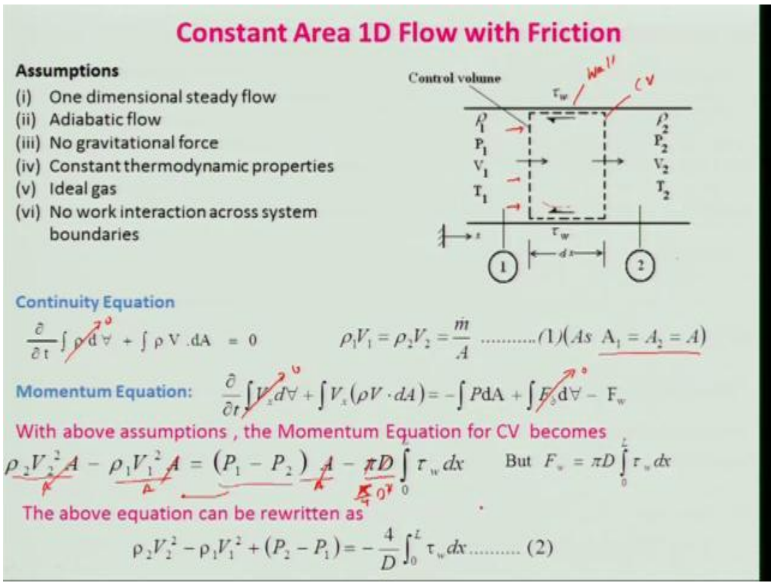

Let us look at a simple case of one-dimensional flow, where the flow is taking place over here, and having certain properties P 1, V 1, T 1. If you take a small control volume, this is small although it looks big, but it is a small one having a d x and it is having certain properties at the station two, we are considering there is a frictional effect on the wall, and this is the wall. There is a frictional force here, stress which is on the wall itself. What we will do? We will try to analyze and see what is happening particularly to the properties right at the station two, whenever there is a friction.

Keep in mind, that these frictional forces will be dependent on the properties of the wall, if it is rough, if it is a smooth and then it will be dependent on the length of the tube. So, let us make usual assumption that is one-dimensional steady flow, adiabatic flow, there is no heat being added, no gravitational force, ideal gas. We are considering with a constant thermodynamic properties. That means it is basically a calorifically perfect gas, and no work interaction across the system, that means if I take this control volume, there is no work interaction.

We will basically look at the continuity equation as we have seen, like these will be 0 because the flow is steady. For this one-dimensional flow it is basically rho 1 V 1 is equal to rho 2 V 2 as A 1 is equal to the A 2 and A that is constant area deck. Therefore, we will get this expression very easily. Momentum equation is given in integral form, this is the unsteady form, the convective form, pressure, this is a body force and this is the wall shear stress. Keep in mind that we are looking at in a very simplified way, and this body force is 0, and the steady is 0, so what is remaining is your convective terms and this is your pressure.

If we integrate it, the control volume will get, rho 2 V 2 square A minus rho 1 V 1 square A and P 1 minus P 2, because although there is a negative sign, it will become P 2 minus V 1, then it is being absorbed over here into area, and what is this F W is equal to basically pie D by tau w d x, because F w we have is nothing but a tau w, and keep in mind that these pie d is circumference, and then d x that we have put it.

If I will divide this expression by area over here, this will cancel it out, this also is cancel it out, and this will be pie by 4 D square. If I do that, I will get an expression basically like rho 2 V 2 square minus rho 1 V 1 square plus P 2 minus P 1 have taken to the left hand side to the right hand side, I have taken this portion to that side and then is equal to minus 4 by D, D D cancel it out and pie is cancel it out, 4 by D tau w d x. So, if you look at this is the extra term, which is coming so far momentum equation is concerned, as compared to the one-dimensional flow with heat addition. So, we will be looking at this term.

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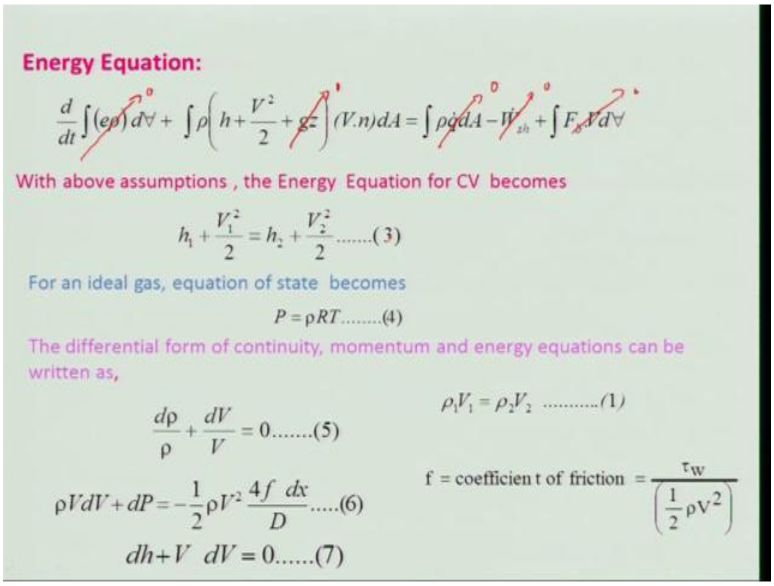

Let us now invoke the energy equations as having unsteady terms, convection terms, heat interactions, work, and body force, this is 0, this is also 0, there is no heat transfers, so this will be 0 and this is also coming because the same plane and is unsteady, so this is 0.

Then it will be a usual form that is h 1 plus V 1 square divided by 2 is equal to h 2 plus V 2 square. It indicate that the total enthalpy, along the direction of the flow are, be it in the station one, station two remains constant.

For an ideal gas equation will be using P is equal to rho R T, that is the usual expression we have used several times. Differential form of continuity equation is rho 1 V 1 is equal to rho 2 V 2. If we look at differential form, it will be basically d rho by rho plus d V by V is equal to 0. This is the form which will be getting. Similarly, we can look at the momentum equation, rho V d V plus d P is equal to half rho V square 4 f d x by D, but what is this f? f is coefficient of friction, tau w divided by half rho V square. This is the coefficient of friction, what we have included in this expression.

Instead of tau w we are putting basically f and taking that into consideration. Energy equation three becomes d h plus V d V is equal to 0, this we have seen earlier. We can see that from equation three we can get equation seven very easily. Now, we need to look at these expressions.

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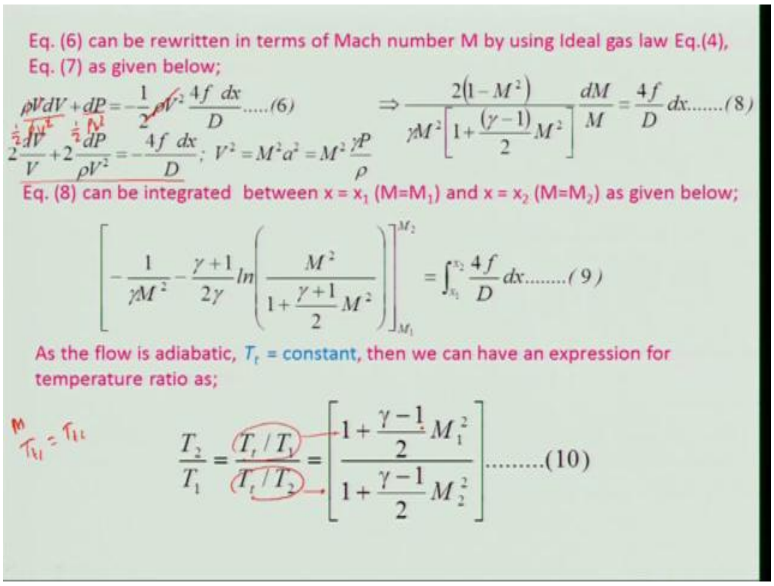

Equation six can be written in terms of Mach number using ideal gas law of equation four, as rho V d V plus d P is equal to half rho V square 4 f d x by D. I will just divide this expression by rho V square and 2 and similarly, I will divide this by half rho V square and this will be cancel it out, and if I simplify I will get basically 2V d V because this cancel it out, and then V cancel it out d V and two d P by rho V square and 4 f d x divided by D. I want each term put in terms of Mach number. So invoke this Mach number definition we know that V square is equal to M square a square, where a square is nothing but, gamma P by rho. So, V square is basically M square gamma P by rho.

If you differentiate these things V square, you will get 2 V d, and you will be getting all terms on the right-hand side. If you club this expression together, expand it and express in terms of Mach number which turns out to be, 2 1 minus M square divided by gamma M square into 1 plus gamma minus 1 divided by 2 M square dM by M is equal to 4f and D, d x. I would urge you people to derive yourself and let me know, if you find difficulties.

So equation eight, can be integrated between the x is equal to x 1 and x is equal to x 2, because we have taken a d x and between the Mach number M 1 and M 2 and we can express these as minus 1 over gamma M square minus gamma plus 1 divided by 2 gamma ln M square divided by 1 plus gamma plus 1 divided by 2 M square and this is between M 1 and M 2 and of course, the right-hand side is simply kept as it is that is, 4f by D d x x 1to x 2.

We will be looking at this expression very carefully and how to handle that because unless we know this M 2, we will be knowing M 1, we may be knowing the friction factor, and we may be knowing the x 1 and x 2 you need to find out M 2 which is not as easy to get. So, let us consider that the flow is adiabatic; temperature is remaining constant during this frictional being considered. Then we can have expression for temperature ratio. So, if we look at temperature ratio T 2 divided by T 1 is equal to T t divided by T 1 divided by T t divided by T 2.

I am saying T t1 is equal to T t2 from the energy equation. Therefore, I can write down this portion as, 1 plus 1 gamma minus 1 divided by 2 M 1 square, and this portion is nothing but the denominator of the equation ten, which is a very familiar form of expression, what we are used to. Now, we will be getting expression for various properties ratios.

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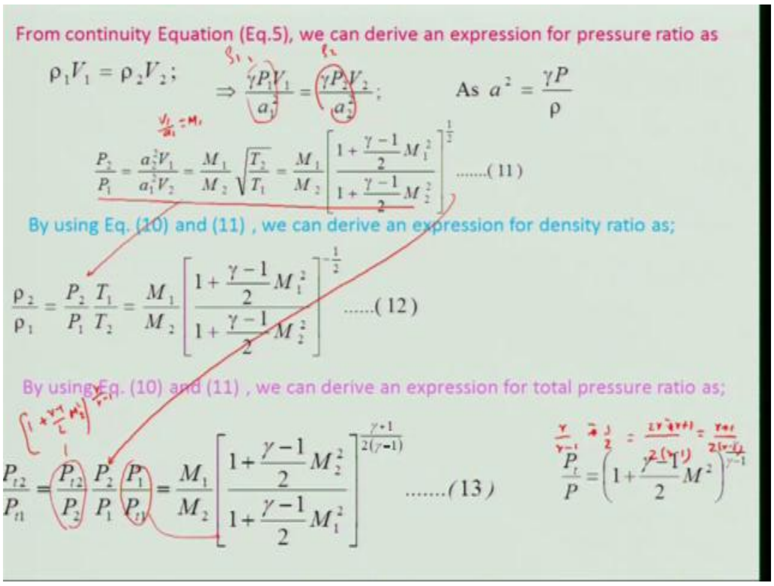

From the continuity equation, I can get rho 1 V 1 is equal to rho 2 V 2 as a square is equal to the gamma P by rho. That is from the definition of speed of sound, I can get that gamma P 1 V 1 divided by a 1square, that means this is the rho 1. Similarly, this is my rho 2. So, I will just put this expression in Mach numbers kind of things, and I will get P 2 by P 1 is nothing but, a 2 square V 1 divided by a 1 square V 2 and V 1 by a 1 is nothing but M 1. I can get M 1. Similarly, I will get V 2 by a 2 M 2 and what is remaining here is root over a 2. I can get root over T 2 and divided by root over T 1.

Then, I can get this, T 2 by T 1 and plug here from equation ten, in terms of Mach number. Thus we will get expression of pressure ratio in terms of M 1 and M 2, which is very straight forward. I am putting this T 2 by T 1 in this portion in terms of Mach. So, using equation ten and eleven, we can derive an expression for density ratio, that means rho 2 by rho 1 is equal to P 2 by P 1 into T 1 by T 2, and if you look at this expression I can put P 2 by P 1 here.

Similarly, I can use T 1 by T 2 from that, and I will get an expression that is M 1 divided by M 2 into, in the bracket 1 plus gamma minus 1 divided by 2 M 1 square divided by 1 plus gamma minus 1 divided by 2 M 2 square power to minus half. It will cancel out some of the things and then you will get this expression. So, by using this equation ten and eleven we can derive an expression for total pressure ratio that is P t2 by P t1 can be expressed as P t2 divided by P 2 P 2 by P 1 and P 1 by P t1.

I can use isentropic relationship for here, P t2 by P 1 which is nothing but1. I can use 1 plus gamma minus 1 2 M 2 square gamma, gamma minus 1 for that, this is two similarly, for these I can use, and from equation ten, I can use this expression over here. If I do that I will be getting an expression like this, M 1 M 2 we will be coming like this, and then I will be getting 1 plus gamma minus 1 divided 2 M 2 square, and from here it will be coming over there, 1 plus gamma minus 1 M 1 square power to the gamma gamma minus 1.

So, gamma, gamma minus 1 minus half, which will be 2 gamma plus 1, 2 gamma plus gamma minus 1, I will be getting gamma plus 1, 2 gamma minus 1. So, I will be getting these expressions. Therefore, I will get gamma plus 1 into 2 gamma minus 1. So all the expressions what we got, is basically in terms of M 1, M2 and gamma.

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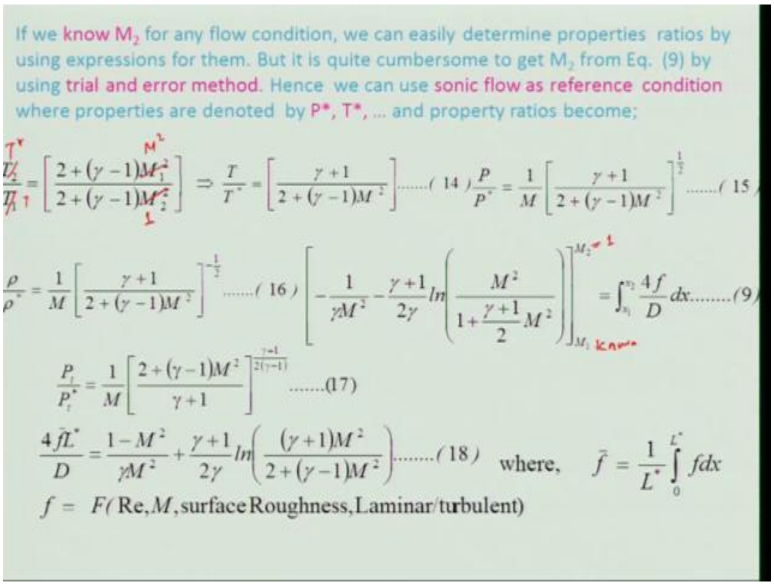

To find out these ratios, we need to know M 2, we will be knowing M 1, but to find out M 2 is quite difficult. For that we will be using sonic flow as reference conditions, where properties can be denoted by P star T star, so that we can use these for our calculations. Hence we need to derive the properties ratios for the reference conditions from what we have already derived.

So, if you look at T 2 by T 1 is the expression what we got 2 plus gamma minus1 M 1 square divided by 2 gamma minus 1 M 2 square. We can see that this condition M 2 is 1, that means this became 1, M will be M square, T 2 will be T star, and T 1 will be T. 2

gamma minus 1 becomes gamma plus 1, and this will be 2 gamma minus 1 M square. If I just inverse that, T by T star will be gamma plus 1, divided by 2 gamma minus 1 M square. That means, whatever we have derived earlier, we can get this expression with reference to the sonic condition.

Similarly, for the pressure ratio, we can get P by P star 1 by M into, in the bracket, gamma a plus 1 2 plus gamma minus 1 M square power to the half. rho by rho star 1 over M, I can get the similar expression with the pressure, only the sign is changing that is minus. We can also get the total pressure expression with reference to the sonic condition that is the P t star. As I told that, equation nine will be equal to 1, for the sonic condition. So, when it is 1, then you will get 4 f L star, because I am integrating for certain length that become L star divided by D is equal to1 minus M square divided by gamma M square.

And this expression is same, rather with a positive sign, that is gamma plus 1 divided by 2 gamma l n, in the bracket gamma plus M 1 square divided by 2 gamma minus 1 M square. Here f is basically the friction factor, being integrated over 0 to the L star length, because it is going to the sonic conditions. So, this friction will be dependent on the Reynolds number, Mach number, whether the flow is laminar or turbulent, and surface roughness, that means there will be different surface roughness for the pipe, one can be smooth, it can be rough.

Particularly those people who are in IIT K, have already experienced winter season like we will be getting a very warm water than then, because our tubes which are quiet lengthy, number one, it is corroded and we are pumping at a very high velocity, dragged from a very lower height below the level, that means we are getting very low-level water. In other words, it is due to the friction, that we are getting this very hot, according to me.

Now the question arises, how to get actual value of f, because unless I know this values, I cannot really estimate. This is depending on several parameters Reynolds number, Mach number, surface roughness, laminar, turbulent and other things. For that, we need to use this chart, the friction factor diagram or moody diagram, which you might have studied. I guess, all of you will be familiar with this.

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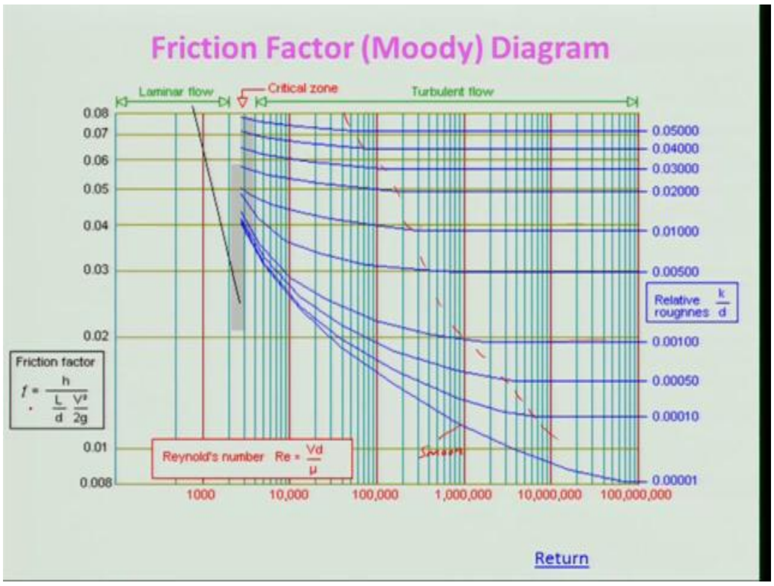

Friction factor is being plotted over this axis that is vertical axis, Reynolds number is being plotted in the horizontal axis and these numbers are relative roughness factor that means this is k divided by d of course, it is a non-dimensional number. What you could observe here? In this region where Reynolds number is very very small, you will see that the friction factor is decreasing, with the increase in Reynolds number.

There is of course, a critical Reynolds number the transition is occurring, and these Reynolds number is very smooth that means, this one is a smooth pipe, this is the smooth region, almost negligible. You can say there might be another place where it is smooth. So, if you look at this, all are converging. As the roughness increases, and Reynolds number is higher in these region, the friction factor is not changing. In these region, the friction factor remaining constant. That is the beauty of that.

The increase in roughness is independent of the Reynolds number in these regions. You know certain regions are there. As the friction factor increases, these regions are increasing. You can have a region, something like these, where it will be remaining constant. In this region, it is independent of the Reynolds number that means particular friction factor. That means if I look at one kind of pipe, friction factor will not be changing, it is no more dependent on the Reynolds number. So, getting these values, I can estimate f, and getting the f value I can do that.

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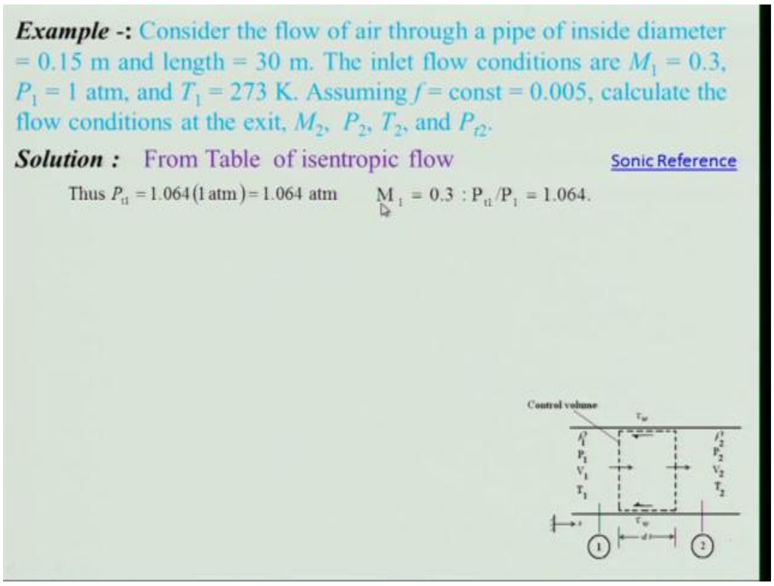

Let us now take an example. Consider the flow of air through the pipe inside diameter 0.15 and length 30 meter. It is a quite huge length, 30 meter is quite huge. In inlet flow condition M 1, 0.3 that is subsonic flow P 1, 1 at atmospheric pressure T 1, 273 kelvin and we are assuming the friction factor to be constant, that value is 0.005. If it is not given in the problem, you need to go back and do that calculation. Calculate the flow condition at the exit of the friction things like for example, in the station two you need to find out M 2, P 2, T 2 and P t2. We will form the isentropic table, P 1 is given, T 1 is given, also the Mach number M 1 is given, the Mach number M 2 is not given, we need to find out.

From the isentropic flow table corresponding to station one, corresponding to Mach number of 0.3, I can get P t1 divided by P 1, 1.064. I know the atmospheric pressure P 1, so I can find out what is the total pressure 1.064 atmosphere as it is the same. There is not much difference you could see, because the flow is subsonic, and very low subsonic. Now, we need to estimate the M 2. How we will do that? Because we know the length, we know the friction factor, but now we need to consider the sonic reference condition, the way we have done for the heat addition.

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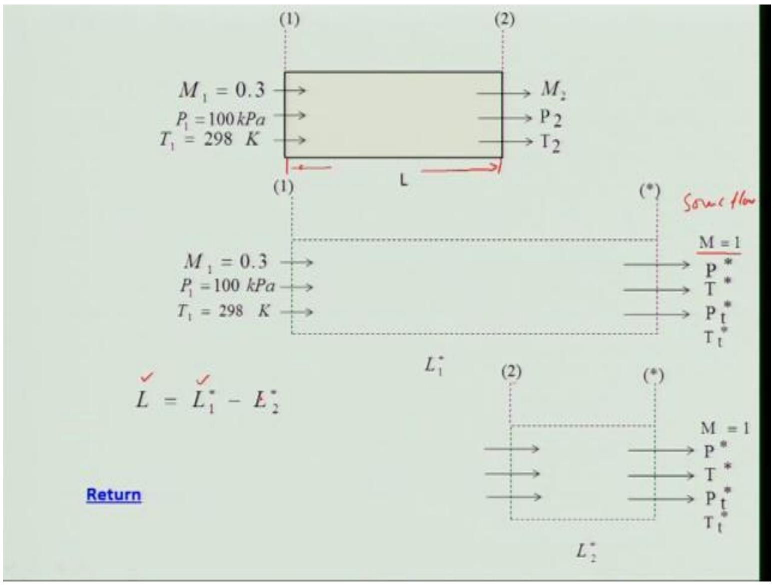

So, for example this is the flow, what we are considering M 1, 0.3, P 1, 100Kpa, T 1 this thing, and certain length, where the friction will be considering, is given that means, this length is from here to this region. In this case it is 30 meters. I will have to take this condition one, and go to where the flow will be sonic, that means M this is the sonic flow condition.

Then from that sonic condition, I will be coming back to the two to find out what is M 2, P 2, T 2 that means, here I will get what is this L 1 star. Similarly, when I will come from here to this region, I will get L 2 star. But I do not know. I know this L, this is given, I can find out what is L 1 star, and then I will find out what the L 2 star will be. This is by using the table.

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Table for Fanno flow, γ = 1.4

M	P/P*	T/T*	ρ/ρ*	Pt/Pt*	4fL*/D
0.3	3.619057	1.178782	3.070167	2.035065	5.299253
0.4	2.695819	1.162791	2.318405	1.59014	2.308493
0.5	2.13809	1.142857	1.870829	1.339844	1.06906
2	0.408248	0.666667	0.612372	1.6875	0.304997

Return

Let us look at Fanno table or we call it table of Fanno flow. These are the values what we can get. That means, for 0.3, P by P star is given. So, if you look a 0.3 I will be getting this P by P star, T by T star, rho by rho star, P t by P t star and 4f L star by D. I have given this inlet Mach number 0.3, 0.4, 0.5, the 2, this is supersonic condition, but in this example, we will be using these numbers. We will have two interpolate between 0.4 and 0.5. We will see how to do that.

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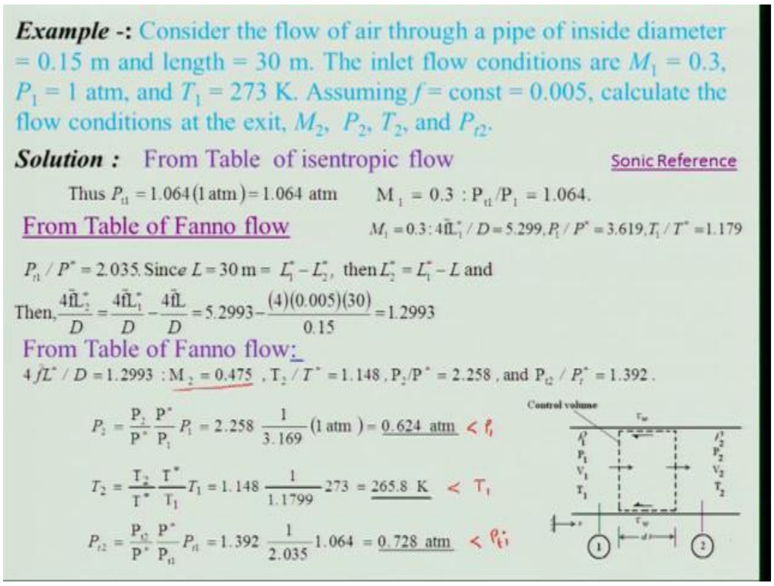

Knowing this thing M 1 from the table of Fanno flow, I can get 4f L 1 star by D, 5.299, and P 1 by P star; I will get these values directly from the table. Also P t by P star and since L is 30 m, I can get L 1 star, I already got this, because I know D, I know f, I can find out the L. So, I will find out L 2 star. Then I will estimate these values, 4f L star kind of things, and if you go to this table, you will find these values is lying between these like, 0.5 and 0.4 because my values is lying between this, L 2 star. Then I will have to interpolate to find out what is the Mach number it corresponding. So, if you do that then I will get this Mach number is 0.475. Keep in mind that this is M 2.

Now, we got the M 2 here, which is higher than the inlet Mach number that is 0.3. Also we will get all this properties T 2, T star P 2, P star and P t2 divided by P t star. After that is very easy, P 2 is equal to P 2 by P star into P star by P 1 and P 1. This is known, you know this ratio already, we know P 2 by P star, and when just substitute, will get this value. Keep in mind that, P 2 is less than P 1.

Similarly, T 2, T 2 by T star and T star by T 1 when just substitute these values, we will get that, this is less than T 1. P t2, when substitute these values, you will get 0.728 and of course, it is less than P t1. That means there will be some total pressure losses. Now, we have learned how to really solve this problem, by referring to the sonic conditions and then work on it.

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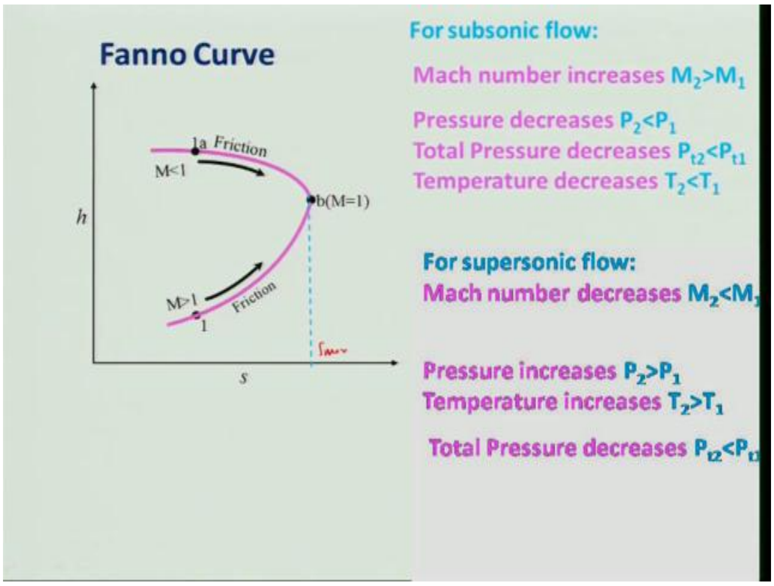

So, let us look at what happens to the Fanno curve, and how it looks, whether it is similar to the Rayleigh flow curve, which we have seen. Using the Mollier diagram, you can see the curve shown here. If I consider this a subsonic flow at the station one, what is happening? With increase in the friction, that means, if I increase the length of the tube, then Mach number increases, we have seen in the last example. If I go on increasing length of the pipe, for the same friction f values, then you we will reach a condition, where it is a sonic condition, that means where the entropy will be the maximum value, this is S maximum.

Similarly, when it is supersonic flow, then what will happen? Then Mach number will be decreasing, and it will be coming to the sonic conditions, we call it as a choked flow. One is frictional chock, one is thermal chock, and also aerodynamically choke, that means you cannot really go beyond that. The flow is choked; you cannot increase the mass flow rate. What is the meaning that? You cannot really change that one.

In this case is this the mass flow being choked or is this the friction, I cannot have frictionally change anything. If I want to change, I will have to change this inlet conditions, and particularly if you go on doing that, in case of supersonic flow, then flow condition will change, and then you can have some sub formations like the way, it is being done in case of a Rayleigh flow, and then you will get a sub sonic condition.

So, let us summarize what we have learned. For the sub sonic flow, Mach number increases with the addition of friction, M 2 is greater than M 1, pressure decreases, total pressure decreases of course, and temperature decreases. For supersonic flow, Mach number decreases, that means M 2 is less than M 1, pressure increases, P 2 is greater than p 1, temperature increases, and total pressure decreases in the both the cases. So, with this I will stop over. If you are having any questions, we can discuss.