

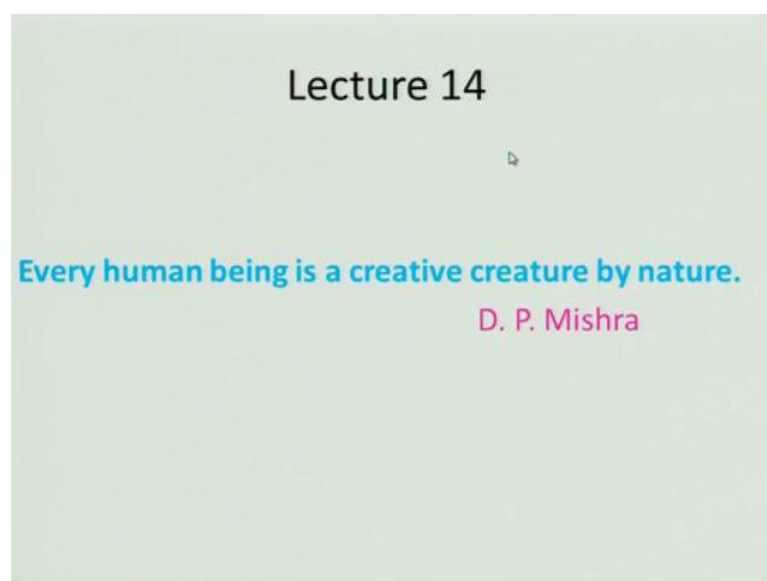
Fundamentals of Aerospace Propulsion
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Lecture - 14

In the last lecture, we had discussed about the oblique shock. If you recall, I also mentioned that oblique shock is like, it can be treated as a normal shock with the respect to the normal Mach components of the upstream velocity like why it is; so and how you can arrive, that conclusion is very important. You must, we look at it, we already derived, but I just want to mention that by invoking the conservation of momentum equation along the shock direction, we have shown that the velocity along the shock in the station one is equal to velocity tangential velocity along the shock station two. That we have done.

Then, we, you know, derived several expressions for various properties ratio P_2 by P_1 , T_2 by T_1 and also the total pressure, temperature ratio, density ratio. If you look at all, they will be in terms of what inlet Mach number and also the outflow Mach number. Finally, we derived into the inlet Mach number and for that, we also looked at a relationship between betas. Beta is which angle? If you look at your note, it is the angle of the shock and theta is the angle of the wedge and Mach number.

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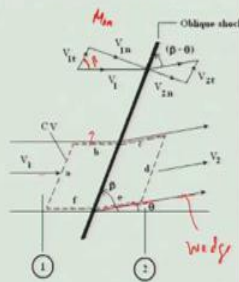
We have looked at that beta theta Mach number diagram. Either you can use a diagram or you can use an expression what we have already derived.

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Example 2.6 :- A steady supersonic flow with $M_1 = 3.0$, $P_1 = 100$ kPa and $T_1 = 298$ K enters into an air intake with a wedge of included angle of 40° . Determine (i) Shock angle β , (ii) Exit Mach number M_2 , (iii) Exit static and total temperature, and (iv) Exit static and total pressure.

Given:- $M_1 = 3.0$, $P_1 = 100$ kPa, $T_1 = 298$ K and $\theta = 20^\circ$.
To find :- β , M_2 , P_2 , P_{t2} , T_2 , T_{t2}

Solution:- From β - θ - M diagram, $\theta = 20^\circ$ and shock angle $\beta = 37.5^\circ$. Now we can estimate normal component of inlet Mach number as



Now, we will take an example just to see how we can use and how we can compute what you call properties for an oblique shock. This is the example that is a steady supersonic flow, Mach number M_1 of 3, Mach number is 3 that is quite high and pressure is P_1 , 100 kilo Pascal, T_1 is 298 Kelvin enters into an air intake with a wedge. Wedge means it is a two-dimensional basically of included angle of 40. We need to determine the shock angle that is beta exit Mach number M_2 and exit static total temperature and exit static and total pressure. We need to determine these things.

So, for that, we will look at, you know, shock as it is as we are seeing this figure keeping mind at what I did in the last lecture that I took. If you look at this, this is my solid surface. This is the solid surface and this line is coming and Eddie earring to the solid surface. It is making an angle theta. In this case, what will be this theta here? This theta will be 20 degree, not 40 degree. I have taken this control volume and how I choose in this control volume? Did you do yet? This is because this is my stream land. Keep in mind in that we are taking care of the in viscid flow.

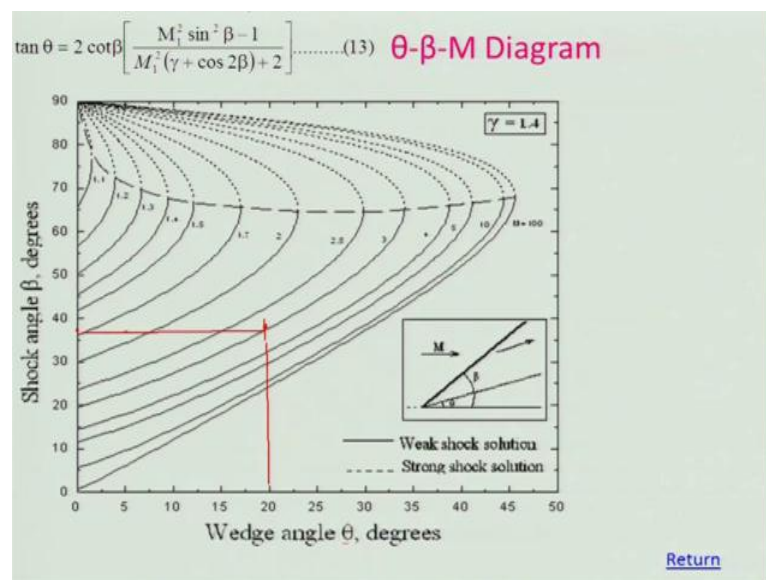
In other words, we are considering in viscid flow. There is no boundary layer although it is the solid surface. This is your wedge right of wedge other and then another fall what you call control, what you call this stream line I will take. Then, we know that the mass

will pass through this what you call stream line to stream line. Therefore, I can put this control surface along with the stream line. Similarly, there is a control surface over here. Then, I will put this control surface parallel to the shock. That is very important.

If I put parallel to the shock rather perpendicular to the inlet velocity, then I will be landing in pabulum of handling equation. So, you please think about it little more and appreciate why we choose in this control volume and the control surface which is I will show here parallel to the shock oblique shock. Then, of course, let us look at what we know. What are those things are given in this example? M_1 is 3, Mach number, P_1 is given, T_1 is given, theta is 20 degree, and I already told you this is the half wedge angle. To find, if you look at what is to be found, if you look at this, I need to find out beta. I need to find out M_2 . I need to find P_{2p} , T_2 and T_2 and T_{t2} .

So, these are the other things to be found. Now, how will we go about this and how because I know inlet Mach number? I know the inlet Mach number. Then, how I will do? That means I need to find out what is the shock angle today. Why should I find out shock angle? This is because I want to know what the normal Mach number. That means M_1 is.

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Sorry, that is what you call M_1 . We need to find out for that because if you look at what is the angle beta, which angle it is, beta is this angle, and then am I right? Yes or no? Therefore, I need to find out the beta and what I will have to do I will have to use beta

theta m diagram or I can use an expression, but generally it is easier and less time consuming to use a beta theta diagram.

If I look at this diagram again and see that beta to be found out from this diagram, how I will do that? I know. What I know is the inlet Mach number and that line corresponding to what this is three. This is the three. So, this line is there. So, I know the theta angle, Theta angle is here right in this scale. So, I can find out what will be beta. Sometimes, you know like beta and thetas are known in a problem, but you need to find out what is the Mach number. So, you can find out that. For example, if I just take this kind of a line, I will have to go a vertical line here cross at this point.

If I take a horizontal line at this point right, then one has to very carefully draw it and you will get a line. This point will be the shock beta angle. That happens to be maybe 37.8.

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Example 2.6 :- A steady supersonic flow with $M_1 = 3.0$, $P_1 = 100$ kPa and $T_1 = 298$ K enters an air intake with a wedge of included angle of 40° . Determine (i) Shock angle β , (ii) Exit Mach number M_2 , (iii) Exit static and total pressure, and (iv) Exit static and total temperature.

Given:- $M_1 = 3.0$, $P_1 = 100$ kPa, $T_1 = 298$ K and $\theta = 20^\circ$.
To find: β , M_2 , P_2 , P_{t2} , T_2 , T_{t2}

Solution:- From β - θ - M diagram, $\theta = 20^\circ$ and shock angle $\beta = 37.5^\circ$. Now we can estimate normal component of inlet Mach number as

$$M_{1n} = M_1 \sin \beta = 3 \times \sin 37.5^\circ = 1.826$$

From Normal shock table for $M_{1n} = 1.83$, we can get,

$$\frac{T_2}{T_1} = 1.551; \frac{P_2}{P_1} = 3.723; M_{2n} = 0.6108 \text{ and } \frac{P_{t2}}{P_{t1}} = 0.801$$

Hence we can evaluate properties easily as;

$$P_2 = \frac{P_2}{P_1} \times P_1 = 3.723 \times 100 = 372.3 \text{ kPa} \quad P_2 > P_1$$

$$T_2 = \frac{T_2}{T_1} \times T_1 = 1.551 \times 298 = 462.2 \text{ K} \quad T_2 > T_1$$

$$M_2 = \frac{M_{2n}}{\sin(\beta - \theta)} = \frac{0.61}{\sin 17.5^\circ} = 2.03$$

From Isentropic table for $M_1 = 3.0$, we can get, $P_{t1}/P = 36.37$, $T_{t1}/T = 2.8$.

$$P_{t2} = \frac{P_{t2}}{P_{t1}} \times \frac{P_{t1}}{P_1} \times P_1 = 0.8 \times 36.37 \times 100 = 2942 \text{ kPa} \quad P_{t2} < P_{t1}$$

$$T_{t2} = T_{t1} \times \frac{T_{t2}}{T_{t1}} \times T = 2.8 \times 298 = 834.4 \text{ K}$$

We will see that from these which can find out to get that and from that, we are getting 37.5 degrees. This is the angle. What is M_{1n} ? That is this there will be you know Mach number perpendicular to the oblique shock M_{1n} . This is the vector velocity vector diagram and Mach number. You can have similar diagram. So, then it turns out to be 1.826. Keep in mind that although it is easier to use beta theta m diagrams b, but then it incurs certain air. We need to tolerate that and we will have to take care not to make any silly mistakes. Therefore, they will be little error in incorporating into our calculation.

So, from this, we have you know like wedge 1.826. I will take 1.83 and then invoke a normal shock table or use a relationship for the normal shock. If I know this inlet that is perpendicular to, you know, it is perpendicular to this one to the circle through these things, then what will be the downstream Mach number which will be also perpendicular to the shock? So, for this Mach number, this is concerned as M_{1n} and this is as if it is a normal shock. That means we can straight a normal shock with respect to the normal components of the Mach number, all the velocity that you must keep in mind.

So, what will be this downstream Mach number? That will be subsonic in nature. This is because always the Mach number downstream, the normal shock will be subsonic and more the upstream, then less will be the downstream Mach number. You must know by this time. Am I right? So, from the normal shock table which can gave these values, you know if you look at the M_{2n} is 0.6108. Sometimes, you will have to interpolate. You will have to carry out a linear interpolation between two points, whatever is given in your table.

Similarly, we can get T_2 by T_1 , P_2 by P_1 . These are all static temperature ratio, pressure ratio from the normal shock table and total pressure ratio across this one, oblique shock, but normal components, we consider treating it as a normal shock. So, by this way, we will get this value. Hence, we can evaluate properties easily. What are the properties? What will be the Mach number downstream of the oblique shock? Will it be subsonic or supersonic or will it be sonic? It can be anything. Am I right? What do you think? It will be, it can be depending on the things. It will be supersonic or will it be all the time supersonic? In the flow up stream, Mach number is supersonic.

So, let us look at how we can get it. We can get very easily because we know M_{2n} . The M_{2n} is 0.6108. If you look at, what this is, and then this is $\beta - \theta$, this angle. So, I can write down M_{2n} . Of course, here I am showing this velocity vector, but you can draw a Mach number wedge similar to that. It is almost similar, but it will not be the same. M_2 will be M_{2n} divided by $\sin \beta - \theta$. So, we know β because β we already find out 3.75. So, θ is known to be 20 degrees. So, it becomes $\sin 17$. So, you can get 2.03. So, if you look at this value, then is higher than the sonic value. M is equal to 1.

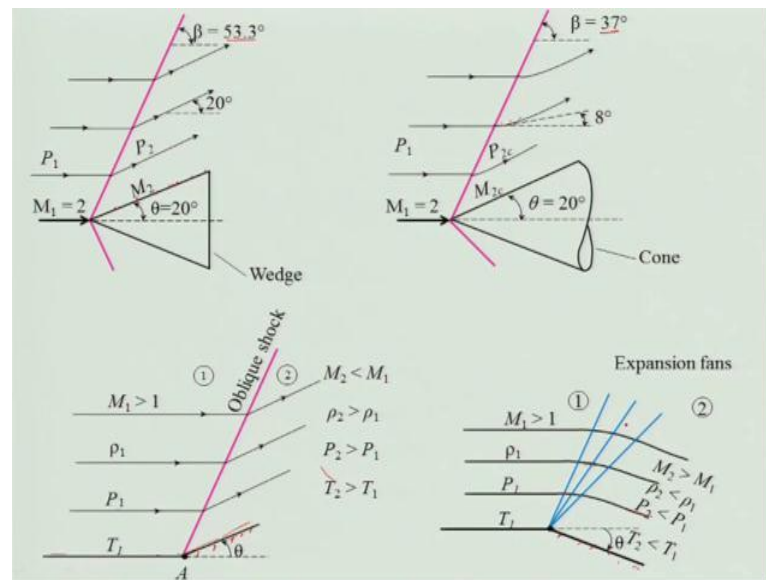
Therefore, it is flowing. So, it is supersonic. Of course, this is the property that we can get like, for example, I want to look at what is static pressure, P_2 , P_2 by P_1 . I know this value. So, I will just put P_1 because P_1 is given as 100 kilo Pascal. So, you will get 372.3 kg Pascal. So, what you can see from here? This result means the static pressure downstream of the oblique shock will be greater than the inlet pressure. That means upstream of the shock that is P_1 that means in this case P_2 is greater than P_1 . Similarly, we will see T_2 is equal to T_2 divided by T_1 into T_1 , T_1 .

So, this is 462.2 Kelvin. If you look at T_1 , it is 298 Kelvin. Therefore, T_1 is greater than T_1 . We need to know find out what you call the total pressure. So, how we will do that? How will we do? We need to invoke, you know, let us P_{t2} . So, what I will do? P_{t2} into P_t , P_{t2} by P_{t1} into P_{t1} , I will get P_{t2} . So, if you look at this, I may know P_{t2} is equal to P_{t2} divided by P_{t1} into P_{t1} into P_1 into P_1 . So, P_{t1} by P_1 , I can get with respect to this station corresponding to Mach number or from the isentropic table. I can get P_{t1} by P that is 36.37. Here, in this case, in the station, one the flow is isentropic. Yes, it is acceptable.

So, I can use the table and put that number here that is 36.37 and I know this ratio. Then, when you will put this values and get 2924 kilo Pascal, then what will be higher than the P_{t2} or it will be low here than P_t . It will be lower than the P_{t1} . That means P_{t2} is less than P_{t1} . Isn't the point is coming? If you look at this, if I take this out, then 36.7 that is basically 3637 kilo Pascal will be P_{t1} and P_{t2} is 0.8 of that that is 2942 kilo Pascal. So, similarly, you can find out T_2 , which is nothing but T_{t1} because we are considering P to be adiabatic and T_{t2} by T_{t1} .

Actually, there is a little mistake here. T_{t1} into T will be T_{t1} by T into 2988314 Kelvin. What you will see? T_{t2} is greater than T_{t1} or it is equal. It is actually equal because it is adiabatic. Now, we will see that some more aspects.

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Let us look at a wedge, which I have already shown here, which is having a wedge angle of 20 degrees and the Mach numbers are 2. If you look at this, it is this stream line downstream of the oblique shock is parallel to surface. These are the surface of the wedge. All are parallel making 20 degree and also the shock angle is 53.3 degree. You know this is a two dimensional body that means perpendicular to the surface will be having some length, which is quite large as compared to this. This is two dimensional. Now, in place of wedge, if I take a cone, then what will happen?

If I consider the same Mach number, then of course, there will be formation of shock like Mach number 2, some pressure, temperature inlet, all those properties will be there, but will the stream line go parallel to this? Yes or no, please tell me certainly. No, what will happen? This will be having tried the dimensional effect that means if the cone is there, then there is a more chance for the shock to be adjusted and because the mass can move into because this is occurring by mass being accumulated. Therefore, the pressure is to be increased and you will get a shock.

If you remember I have told you how the shock is being formed because the supersonic flow would not be know that there is a disturbance itself because the disturbance cannot propagate and reach to the fluid element to be getting adjusted up stream because the speed of sound is less than the incoming fluid. Therefore, in this case, there is a something what you call three-dimensional effect. So, it became a curve kind of thing.

If you look at this stream line, this becomes a curve and this will be changing as it goes away from that. Keep in mind that as a result, the shock angle is being reduced that is 37 degrees in this case, whereas the wing case of the wedge is 53.3 degrees. The state of the shock will be decreasing in case of a cone. If you look at it, then it is not making angle 20 degree right which is supposed to be changing. These slopes are changing. In this case, it is 8 degree, but as it goes, it may be increasing. If you look at this, so this is a very important aspect. That means for the same Mach number same inlet condition, the cone will be producing. We saw this. That is why we use that in the air inlet.

Of course, it will be having several other problems, but it is preferred to use the cone over the wedge, but in our case what you call calculation, what will be doing is we will be using because the analysis of the cone, the sub formation over a cone considering and cone will be quite complex. It is on the scope of the present lecture like course. However, some of you are interested to look at it and you can look at it in advanced books and gas dynamic. Whenever you look, then most of the problems will be looking which are the simplest to handle. Also, it is being used in the practical aid and take part several other considerations.

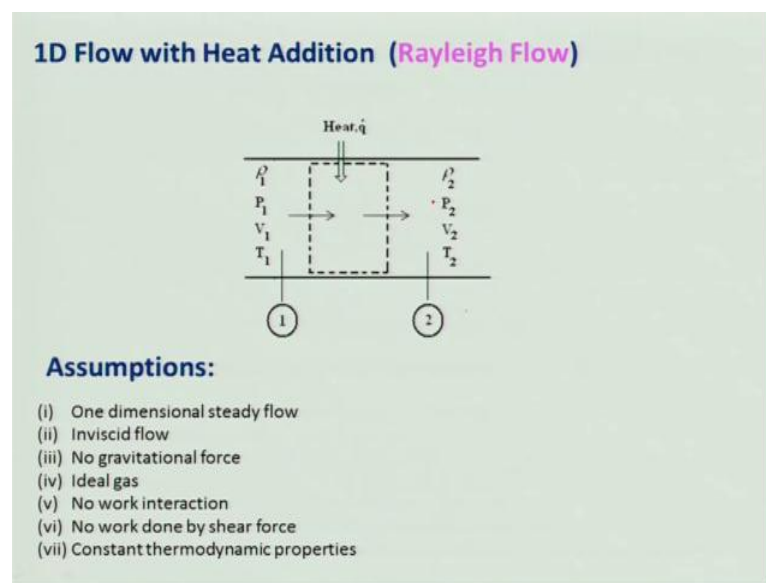
Now, let us you know discuss it little further. If you look at this, then there is a wedge over here and this is the wedge in an angle θ . This stream line is idea ring to this wedge. This is the two d wedge. Keep this in mind. Then, flow will be in our two d. We are seeing that M_2 is less than M_1 , ρ_2 is greater than ρ_1 , P_2 is greater than P_1 and T_2 is greater than T_1 . All those things we have seen, but in case of this, you know where. Now, you will say that it is going in the opposite direction. That is there is a solid body which is not turning convex with an angle of θ .

So, there is a 1T here. What will happen? The flow will be because it is a two d, again the flow will be parallel to this solid surface, but however it is going through a several expansion fans. That means it will be expanded. So, adjust it into that. If you look at this, then it will be just opposite that of an oblique shock because there it will be compressing and here it is expanding. So, it will be taking a smooth turn over here. I do that expansion, but how to handle those expansion and other things, I would not be covering. You may read little bit because we may need to understand this expansion passes and which can be invoked while discussing about over expansion usual, under expansion usual, kind of this.

I would dare you people read little bit more about expansion passes and particular this relationship and other things, but those relationships, we would not be using, but we will be talking about more quality ability considering what I recovered in the class. Now, let us move into another topic that is still.

Now, we have looked at that that the properties can be changed in case of you know formation of shock whether it is a normal shock or an oblique shock, but the properties can also be changed if there is heat addition. For example, in case of a combustor, in gas by combustor rocket combustor where we will be adding a large amount of heat for producing the task, in that case, what happened to the properties? Of course, considering a very simply field version of flow like one dimensional flow like what I have shown in here, I have shown here this is one dimensional.

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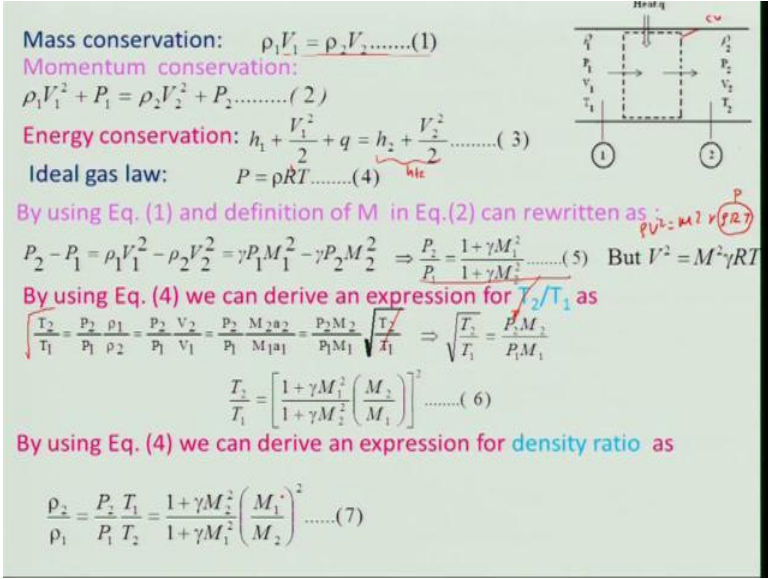
These are the properties P_1 , ρ_1 , V_1 , T_1 and there is a heat addition over here. What happened to the properties to the P_2 , ρ_2 , V_2 , and T_2 ? This flow we will be looking at now and learn how to handle this kind of picture because it is a having practical implication that is the flow is sub sonic. What happened if the flow is supersonic like you know supersonic vehicle likes scam and other vehicle scam? Then, we need to understand that is what really happened? So, that will be looking at. We will be looking at one dimensional flow with heat addition and that is being known as rally flow.

So, we will do standard assumption, which we have already seen that is one dimensional steady flow. We are considering steady flow in viscous flow that means no effect of our to call boundary layer or the friction, no gravitational force, ideal gas, no work interaction and no work done by the shear force.

You know there is pressure on the wall or any other place and constant thermodynamic properties as usual, we take for simplicity, but however, reality would not because there is a heat addition. The temperature will be changing. TP is the function of temperature. So, also you know that as combination is taking place, what you call very small, equal, under, the things will be getting changed in these kinds of things. So, we cannot really talk about it, but however, for simplicity, we are considering. So, let us look at the mass conservation equation for this. I have taken this as a control volume. So, invoking mass conservation, you will get $\rho_1 V_1$ is equal to $\rho_2 V_2$, which is very simple and which we have already done.

Similarly, for a momentum equation, we will get $\rho_1 V_1^2 + P_1$ is equal to $\rho_2 V_2^2 + P_2$. However, what are the assumptions we made? We can consider and get this expression. Similarly, energy equation is very obvious to you except that you know there is a heat addition.

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Mass conservation: $\rho_1 V_1 = \rho_2 V_2 \dots (1)$

Momentum conservation:
 $\rho_1 V_1^2 + P_1 = \rho_2 V_2^2 + P_2 \dots (2)$

Energy conservation: $h_1 + \frac{V_1^2}{2} + q = h_2 + \frac{V_2^2}{2} \dots (3)$

Ideal gas law: $P = \rho R T \dots (4)$

By using Eq. (1) and definition of M in Eq. (2) can be rewritten as:
 $P_2 - P_1 = \rho_1 V_1^2 - \rho_2 V_2^2 = \gamma P_1 M_1^2 - \gamma P_2 M_2^2 \Rightarrow \frac{P_2}{P_1} = \frac{1 + \gamma M_1^2}{1 + \gamma M_2^2} \dots (5)$ But $V^2 = M^2 \gamma R T$

By using Eq. (4) we can derive an expression for T_2/T_1 as
 $\sqrt{\frac{T_2}{T_1}} = \frac{P_2}{P_1} \frac{\rho_1}{\rho_2} = \frac{P_2}{P_1} \frac{V_2}{V_1} = \frac{P_2}{P_1} \frac{M_2}{M_1} \sqrt{\frac{T_1}{T_2}} \Rightarrow \sqrt{\frac{T_2}{T_1}} = \frac{P_2 M_2}{P_1 M_1}$

$\frac{T_2}{T_1} = \left[\frac{1 + \gamma M_1^2}{1 + \gamma M_2^2} \left(\frac{M_2}{M_1} \right) \right]^2 \dots (6)$

By using Eq. (4) we can derive an expression for density ratio as
 $\frac{\rho_2}{\rho_1} = \frac{P_2}{P_1} \frac{T_1}{T_2} = \frac{1 + \gamma M_2^2}{1 + \gamma M_1^2} \left(\frac{M_1}{M_2} \right)^2 \dots (7)$

That means if the total enthalpy, if we look at this, then this is the total enthalpy plus heat addition and total enthalpy at the downstream. That means whatever the enthalpy coming

in plus whatever the heat being added is equal to the total enthalpy to the downstream that is the one dimensional flow, which we have already seen.

So, ideal gas law as usual P is equal to ρRT and what you will do now that we will just take this momentum equation and rewrite P_2 minus P_1 is equal to $\rho_1 V_1^2$ minus $\rho_2 V_2^2$. What you will do? If you look at this, then we need to express this ρV in terms of γ by using the definition of M that is the Mach number.

We know that V^2 is equal to γRT . If I say that there is a ρ over here that means ρV^2 is equal to M^2 . If I can write down ρV^2 is equal to $M^2 \gamma \rho RT$, then this is nothing but P , some ideal gas equation. So, I can write down ρV^2 is equal to $\gamma M^2 P$ in this term for this station 1. Similarly, for the station 2, it is $\gamma P_2 M_2^2$. If I will simplify it further, I can get it is, P_2 by P_1 is equal to $1 + \gamma M_1^2$ divided by $1 + \gamma M_2^2$.

Could you recognize this equation? It is similar to your normal shock relations, the pressure ratio across the normal shock. Am I right? Can you look at here? Note that by using the equation 4, we can derive an expression T_2 by T_1 because I can look at this ideal gas law. Then, I can rewrite that as T_2 by T_1 is equal to P_2 by P_1 into ρ_1 by ρ_2 in place of ρ_1 by ρ_2 . I can invoke, write down from this conservation, mass conservation equation as P_2 by P_1 into V_2 by V_1 , V_2 by, sorry, V_1 is basically substituted for ρ_1 by ρ_2 . We can use this definition of Mach number and then write down that is V_1 is nothing but $M_1 a_1$.

Similarly, V_1 is $M_1 a_1$. In place of a_2 , I can write down γRT_2 . If I simplify, then I arrive at P_2 by P_1 , M_2 by M_1 root over T_2 by T_1 . If you look at the left hand side here, and if I cancel it out, then we will get a relationship that is root over T_2 by T_1 is nothing but P_2 by P_1 into M_2 by M_1 . So, I can write down this as root over T_2 by T_1 is equal to P_2 by P_1 M_2 by M_1 in place of P_2 by P_1 . I can use this expression right over here and square this expression equation. I will get T_2 by T_1 equals to $1 + \gamma M_1^2$ divided by $1 + \gamma M_2^2$ M_2 by M_1 .

Now, I square and get this equation. I am using equation and just substituting in this place. You will get this expression that is the temperature, static temperature ratio. Keep in mind that this both pressure ratio and the temperature ratio is a function of what is the

function of Mach number and gamma. By using equation four that is again ideal gas law, we can get this expression for density ratio which turns out to be pressure ratio and temperature ratio. We just substitute over here. You can simplify and get this expression.

Now, all these expressions are function of M1 and M2. We will see also the temperature, total temperature ratio, pressure ratio, but we need to find out how we can solve unless we know the M2. If you know M2, then this expression will be very easy to find out various properties, ratio across control volume in which will be added.

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Similarly total pressure ratio can be derived as;

$$\frac{P_{t2}}{P_{t1}} = \frac{P_2}{P_1} \cdot \frac{P_1}{P_{t1}} = \frac{1 + \gamma M_1^2}{1 + \gamma M_2^2} \left[\frac{1 + \frac{\gamma-1}{2} M_2^2}{1 + \frac{\gamma-1}{2} M_1^2} \right]^{\frac{\gamma}{\gamma-1}} \dots (8)$$

Similarly total temperature ratio can be derived as;

$$\frac{T_{t2}}{T_{t1}} = \frac{T_2}{T_1} \cdot \frac{T_1}{T_{t1}} = \frac{1 + \gamma M_1^2}{1 + \gamma M_2^2} \left[\frac{1 + \frac{\gamma-1}{2} M_2^2}{1 + \frac{\gamma-1}{2} M_1^2} \right] \dots (9)$$

Given q, T_{t2} can be obtained using Eq. (3) as;

$$T_{t2} = T_{t1} + \frac{q}{C_p}$$

If we know M₂ for any flow condition, we can easily determine properties ratios by using expressions for them. But it is quite cumbersome to get M₂ from Eq. (9) by using trial and error method.

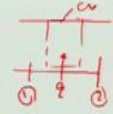
$$\frac{T_2}{T_1} = \left[\frac{1 + \gamma M_1^2}{1 + \gamma M_2^2} \left(\frac{M_1}{M_2} \right)^2 \right] \dots (6)$$

$$\frac{T}{T^*} = M^2 \left[\frac{(1 + \gamma)}{1 + \gamma M^2} \right] \dots (10)$$

$$\frac{\rho}{\rho^*} = \frac{1}{M^2} \left(\frac{1 + \gamma M^2}{1 + \gamma} \right) \dots (12)$$

$$\frac{P}{P^*} = \frac{1 + \gamma}{1 + \gamma M^2} \dots (11)$$

$$\frac{P_t}{P_t^*} = \frac{1 + \gamma}{1 + \gamma M^2} \left[\frac{2 + (\gamma - 1) M^2}{\gamma + 1} \right]^{\frac{\gamma}{\gamma-1}} \dots (13)$$

$$\frac{T_t}{T_t^*} = \frac{(\gamma + 1) M^2}{(1 + \gamma M^2)^2} [2 + (\gamma - 1) M^2] \dots (14)$$


Similarly, total pressure ratio, you can find out that is Pt2 by Pt1 is equal to Pt2 by P2 into P2 by P1 into P1 by Pt1. If you look at this expression, then this is nothing but this portion and Pt2 by P2 is nothing but what you call 1 plus gamma minus 1 by, sorry, 1 plus gamma minus 1 divided by 2 M2 square. Here, that will be Pt1 by P1 will be nothing but 1 minus gamma minus 1 divided by 2 into M1 square. Yes, you are right, but if I consider station two, there all my heat being added is over. I am not adding anything at that point. Therefore, I can consider this isentropic flow.

I think I need to look at what you call, may be I will draw over here like this. This is my control volume Cv. This is my station one and this is my station two. The heat is being added over here. So, when the station one is concerned, then I am not adding any heat here at this point. Am I right? So, I can consider this as a Pt1 divided by P1 isentropic relation for Pt1 divided by P1. Similarly, at the station 2, at this point, I can consider very

easily the same name because all heat being added by the time reaches the station two. So, I can happily use the isentropic relationship. Is that clear to you? I can do as it is a property because I am not doing anything. I have already added. That is the way you can do that.

Similarly, total temperature ratio can be derived T_{t2} divided by T_{t1} is equal to T_2/T_1 into T_1/T_2 . This is the similar expression. It looks be quite complex, but it is taken from earlier relationship and this is isentropic. If you look at this, then this is the heat addition relationship between the inlet and outlet of the control volume where it is in added. This relationship is for isentropic flow at the station one and station two. Now, we know that q . So, I can find out what will be T_{t2} because I know T_{t1} . If I know this T_{t1} , I know this q that is heat being added. I must know. Then, I can find out what is T_{t2} if I know this T_{t1} . That means we can find out T_{t2} . This ratio will be known.

Then, how I will find out? This is because this is known M_1 , but inlet condition should be known. Can I find out? This is a non-linear equation. You need to go by trial and error or some other numerical technique to get that. That means if you know this M_2 for any flow condition, we can very easily heat, but however, you find difficult to get a close form solution direct. As I told you, one can do by trial and error. We can have what you call like table and use the table, but to solve this thing, then what I will do? What I will do? I will use all these problems.

I will be taking using a characteristic Mach number that is M^* . That means all the inlet condition, for example, here one inlet condition, I will take to the sonic condition. Similarly, for the station 2, I will take to the sonic condition that is M is equal to 1. Then, I can come back and then look at it for this. I will be taking an example for this point. Let us look at it. We need to find properties with the respect to the sonic condition. I will take the equation six. If I consider that M_2 , then it becomes 1 because this is the condition for a particular M . Then, what will be this? This also will be 1.

Similarly, instead of M_1 , I can write down the M square. Similarly, this I can write down as M square because there is no M_1 , M_2 . It is only the Mach number to generalize this. M_2 is anyway going to the sonic condition. So, I can write only an expression. What am I getting here? I am getting here $1 + \gamma M_1^2$. What am I getting here? I am getting here $1 + \gamma M^2$. So, if I take this M square out, then I will get T by

$T_{star} = \frac{1 + \gamma}{2}$ divided by $1 + \gamma M^2$. Similarly, P by P_{star} is equal to $\frac{1}{1 + \gamma M^2}$ and ρ by ρ_{star} .

I can get some expression keeping that M_2 . M_2 is going to the M_{star} , M_1 is equal to 1, and M is generally one Mach number. It can be any Mach number. It can be subsonic. It can be supersonic kind of things. So, these are the relations we can get for that sonic condition P_t by $P_{t,star}$ and T by T_{star} . All these expressions we can get from the expression like equation nine, eight, six, and seven; you know, all those eight we can use and get this expression by just putting M_2 is equal to 1 and M_1 is equal to M . So, you can get this expression and it can tabulate in a table known as Rayleigh flow table, ρ or Rayleigh law and you can use that.

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Example:- The fuel-air mixture enters a gas turbine combustor with a velocity of 78 m/s at static pressure of 50 kPa and temperature of 400 K. During combustion, heat per unit mass of 1500 kJ/kg is released. Assuming the combustion chamber to be a constant area duct with negligible friction, estimate the properties such as T_2 , P_2 , and M_2 at its downstream. Take $\gamma = 1.33$, $C_p = 1.281$ kJ/kg K.

Given:- $V_1 = 78$ m/s, $T_1 = 400$ K, $P_1 = 50$ kPa and $q = 1500$ kJ/kg, $\gamma = 1.33$.

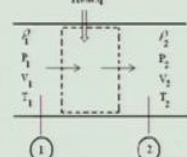
Solution:- $M_1 = \frac{V_1}{a_1} = \frac{78}{\sqrt{1.33 \times 287 \times 400}} = 0.2$

Form isentropic relation we can determine T_{t1} and P_{t1} as

$$T_{t1} = T_1 \left(1 + \frac{\gamma-1}{2} M_1^2 \right) = 400 \times \left(1 + 0.165 \times (0.2)^2 \right) = 402.6 \text{ K} \quad T_{t1} = T_t$$

$$P_{t1} = P_1 \times \left(\frac{T_{t1}}{T_1} \right)^{\frac{\gamma}{\gamma-1}} = 50 \times \left(\frac{402.6}{400} \right)^{4.0} = 51.33 \text{ kPa}$$

Let us estimate T_{t2} at exist as

$$T_{t2} = T_{t1} + \frac{q}{C_p} = 402.6 + \frac{1500 \times 10^3}{1281} = 1573.6 \text{ K}$$


Now, let us consider an example right that is the fuel air mixture enters a gas turbine combustor with a velocity of 78 meter per second and static pressure of 50 kilo Pascal right. If you look at this, the velocity is quite low. This is 78 meter per second and pressure is 50 kilo Pascal, temperature is 400 k. During combustion, heat per unit mass that is 1500 kilo joules per kg is being released. You know this really is combustion. Assuming the combustion to be a constant area, this is an assumption with not the case most of the combustible, but do use for the simplicity and with negligible friction. There is no friction. That is real situation where friction will be considered and how to handle friction in the next lecture.

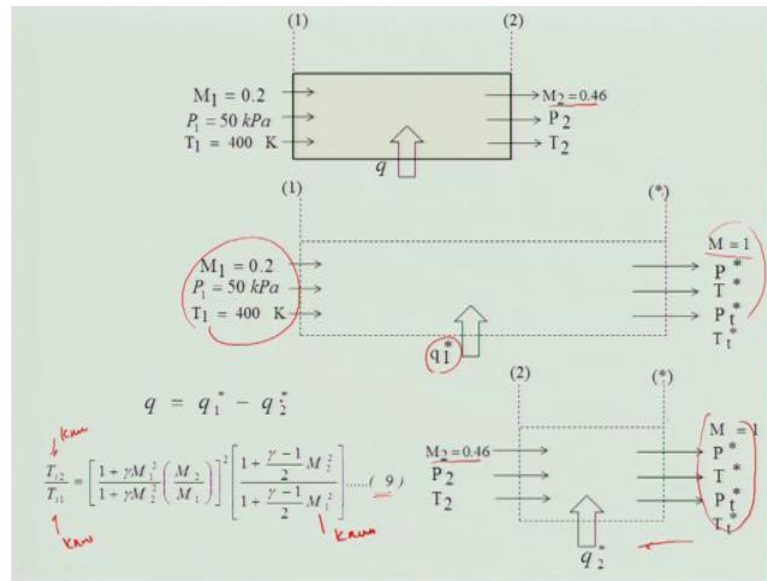
We need to determine the properties such as T_2 , P_2 , and M_2 at its downstream and keep in mind that it is not air. Therefore, we will be taking the γ 1.33. Of course, γ 1.2, you can also take depending on what kind of fuel and air you are using. C_p is needed not to be 1.005 kJ/kg K. We need to take proper C_p . All though C_p is you know, it can be and you can also use different C_p at station 1, station 2, we can do that, but in this problem, we will use, you know we will not be using. So, given that if you look at V_1 is given, T_1 is given 400 K, V_1 is given 78 K, P_1 is given, q is given and γ , all these properties are given, but we need to find out the properties, you know like at the downstream inlet P_2 , V_2 , you know, M_2 , T_2 kind of things.

So, what we will do? We will have to first find out the Mach number that is V_1 divided by a_1 and this is $\gamma R T$. This is γ and this is a . This is T . Again, the R will be different because we are not using air. You are using some different value. You should take care, but unfortunately, I have taken the air value here, but the actual value will be little different. So, 78 divided by that, it happens to be 0.2, which is a very subsonic case because you can consider the flow to be incompressible always, but in some situations, it will be little higher subsonic.

We have taken this example and form the isentropic relationship. We can determine what will be T_{t1} because I know T_1 . I know the Mach number. So, I can find out what it would be. In this case, it will not be very different and that we can determine by T_{t1} is equal to $T_1 \left(1 + \frac{\gamma - 1}{2} M_1^2 \right)$. If you substitute the values, then you will get. That means that in this case, T_{t1} is approximately equal to T_1 . There is not much difference. What is the difference? It is 402.6 K. It is not really much, but however, for completeness, we are doing it.

Similarly, P_{t1} , you can get this is a very low value that is 51.3 and the P_{t1} is 50. So, it is not really very much difference. So, let us now estimate T_{t2} at the edges that means at the station two. How we will do it? It is a very simple one because we know T_{t1} , T_{t2} is equal to $T_{t1} + \frac{q}{C_p}$. C_p is given here and q is given. This is given. This is given. This is given. You will just substitute those values. You will get 1573.6 K. Now, of course, inlet is 400. That means you have added enough of it, two half, 1573.6 K. If you look at it typically, then you know temperature one and you can get it.

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Now, this of course, higher temperature is being used at the exit of combustion for expansion in the particular gate. Now, we have already seen that this is known this is known. We need to find out and this is known. M_1 is known. So, you need to find out M_2 . What will we do? It is quite difficult to do by trial and error. We need to use this characteristic condition for the inlet. For that let us consider that this is our problem. This is having a station M_1 is given, P_1 is given, T_1 is given and M_2 is 0.46. That we have already determined. We have determined this. That we will know. We have not determined and we need to find out.

Similarly, we need to find out P_2 and we need temperature T_2 . We need to determine this portion at the station two. How I will go about this? This is because using this expression, it is quite difficult to get the solution directly and close from solution, we will have to do trial and error or some numerical method or something like that. So, what you will do? Now, I will take, I will assume that heating is being done from the station, this condition that it is going to the sonic condition that is M is equal to 1 is virtual or what you are just doing is not actually happening. You are just extrapolating to the sonic condition. This is q_1^* . This q_1^* is not same as the q . Keep in mind that if I know this condition and I know this property, then I can find out.

Similarly, I will look at another condition which is M_2 condition. This condition is same as of that. Then, I will come back and get this condition. That means I am looking from

station one to the star. I am going from here and I am coming to the star. Then, I am coming back here. You are trying to do in a little easier way. So, you look at q is equal to q1 star minus q2 star. So, we will see how you are doing. How can you solve this problem?

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For $M_1=0.2$, by using Eqs. (10-14) we can determine property ratios w.r.t to sonic condition as

$$\frac{T_1}{T^*} = M^2 \left[\frac{(1+\gamma)}{1+\gamma M^2} \right]^2 = 0.2^2 \left[\frac{(1+1.33)}{1+1.33 \times 0.2^2} \right]^2 = 0.196$$

$$\frac{P_1}{P^*} = \frac{1+\gamma}{1+\gamma M^2} = \frac{1+1.33}{1+1.33 \times 0.2^2} = 2.21$$

$$\frac{P_{t1}}{P_t^*} = \frac{1+\gamma}{1+\gamma M^2} \left[\frac{2+(\gamma-1)M^2}{\gamma+1} \right]^{\frac{\gamma}{\gamma-1}} = \frac{1+1.33}{1+1.33 \times 0.2^2} \left[\frac{2+(1.33-1)0.2^2}{1.33+1} \right]^{\frac{1.33}{1.33-1}} = 1.23$$

$$\frac{T_{t1}}{T_t^*} = \frac{(1.33+1)0.2^2}{(1+1.33 \times 0.2^2)^2} \left[2-(1.33-1)0.2^2 \right] = 0.17$$

We can determine T_{t2}/T_t^* as: $\frac{T_{t2}}{T_t^*} = \frac{T_{t2}}{T_{t1}} \frac{T_{t1}}{T_t^*} = \frac{1573.6}{402.6} \times 0.17 = 0.66$

For Rayleigh flow table, for T_{t2}/T_t^* : $M_2 = 0.48$, $T_2/T^* = 0.7587$, $P_2/P^* = 1.851$, $P_{t2}/P_t^* = 1.22$. Then we can determine T_2 and P_2 as

$$T_2 = \frac{T_{t2}}{T_t^*} \frac{T_t^*}{T_1} T_1 = 0.7587 \frac{1}{0.17} \times 400 = 1785.2 \text{ K}$$

$$P_2 = \frac{P_{t2}}{P_t^*} \frac{P_t^*}{P_1} P_1 = 1.851 \frac{1}{2.21} \times 50 = 41.06 \text{ kPa}$$

$$P_{t2} = \frac{P_{t2}}{P_t^*} \frac{P_t^*}{P_{t1}} P_{t1} = 1.22 \frac{1}{1.23} \times 51.33 = 50.91 \text{ kPa}$$

$$\rho_2 = \frac{P_2}{RT_2} = \frac{41.06 \times 10^3}{287 \times 1785.2} = 0.08 \frac{\text{kg}}{\text{m}^3}$$

So, for $M_1, 0.2$ by using equation, I already told you, by equations 10 to 14, you can determine properties ratio with the respect to sonic condition that is M is equal to 1. We have already derived those expressions like equations 10 to 14. We can use those things or you can use a table. I have taken from here. I am using directly that expression. When you directly use, you would not incur any error.

So, if I will put this value that is 0.2, if I look here, I get it as T_1 by T^* is 0.196 because I know gamma. Gamma is 1.33 and M , Mach number; I know that I will get this. Similarly, other properties, I will be getting P_1 by P^* . You will get 2.21 and P_{t1} by P_t^* . You will substitute this value and you will get 1.23. Similarly, T_{t1} by T_t^* , you will substitute this value and get 0.17. We can determine T_2 by that. What we have done? We have looked at what you call these things from this is 1 and this is star condition, q_1 star.

This is the one condition that is Mach number is equal to 0.2, but in actual situation, it will be something different. We have taken too fixed here of the star condition. Now, we will have to do to the inlet because T_2 is known and I will have to find out T_2 by T_t^* star. What I will do for back like T_t^* star? T_t^* star is nothing but T_{t2} divided by T_{t1} into TT_1 by

Tt star because if you look at this, then that is 0.17. That is Tt1 by Tt star. Of course, there will be star here, 0.17. This is known as Tt2. I can find out. We already found out. So, I will get this one. If I know this value from Rayleigh flow table, which will be there or you can use this expression, then the similar is pressure. You can find out M2 is 0.48.

What you are observing? You are observing that the inlet Mach number was 0.2, but when you add heat or when heat is being added, then it has been increased Mach number. T2 by T star, we can get these values, P2 by P star and Pt2 by P star, all from the table or from the equation. This is because once I know M2, then it is very easy to get those things. Then, we can determine T2 and P2 because I have already got everything with respect to the star conditions or the sonic condition.

So, T2 is equal to T2 by T star into Tt star by Tt1 into T. This already you got the values. This is equal to 0.7587 and T1 by T star. Already, we have got 0.195 that is 1 over of that and you can get this as 400. You can get this 1785.2. Similarly, P2, you can get this value and substitute because this is known already from here. This you can get from here and P1 by P star. We have already determined that is 1 over 2.121 and 50. You see that the P2 is, in this case, P2 is less than P1 because P1 is 50 and this is 41.06. It is very less because inlet Mach number is quite small. Similarly, T2 of course, always will be greater than T1 because you are adding heat.

Similarly, total pressure, if you look at it, then it is 50.91 and this Pt2 is less than Pt1 because this is Pt1. Similarly, density, if you look at it, then you can estimate it to be 0.08 kilogram per meter cube. So, I will stop over here. We will see in the next class like what the properties are, how it is changing and what will be implication of those things in the next class. We will stop.