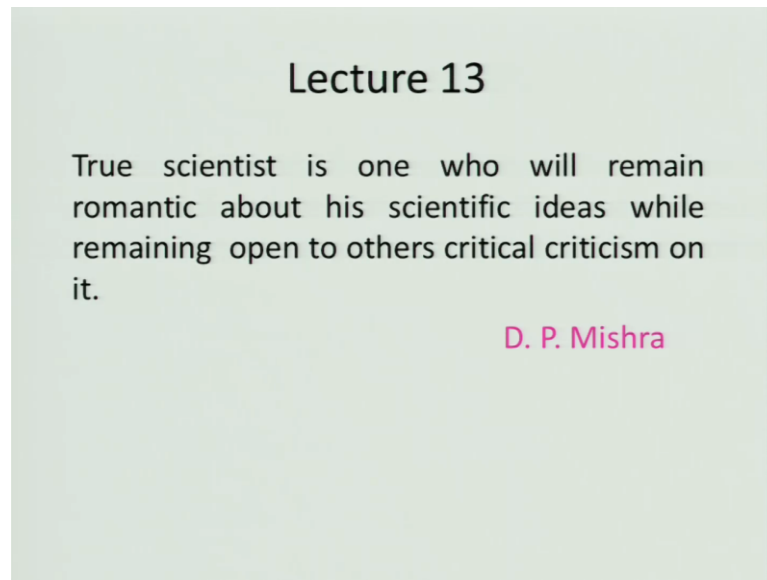


Fundamentals of Aerospace Propulsion
Prof. D.P. Mishra
Department of Aerospace Engineering
Indian Institute of Technology, Kanpur

Lecture - 13

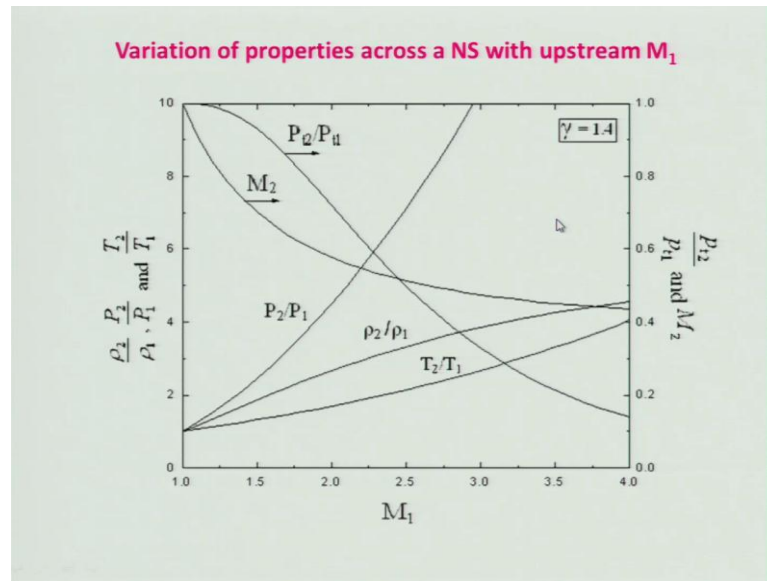
(Refer Slide Time: 00:19)



Let us start this lecture with a thought passes that is, true scientist is one will remain romantic about his scientific ideas. While remaining open to others critical criticism on it is very important to have an open mind to be, to become a scientist or any other work as matter of fact. So, let us, now recall what we learnt in the last lecture, we started discussion on the basic concepts of shock waves. Then we moved into the normal shock waves, of course the shocks can be defined divided into types, one is oblique and normal shockwave. Then by invoking the principals of conservation of mass, momentum and energy equation we looked at those equations, along with the equation of state for perfect gas.

We derived the expression for various properties ratio for example pressure ratio, temperature ratio and total pressure ratio, density ratio across the shock. We learned that those relationships for properties ratios will be dependent on inlet Mach number and also the specific heat ratio that is gamma.

(Refer Slide Time: 01:54)



Let us dual upon those relationship and vary the inlet Mach number and see how it is you know properties being varied, let us look at this is the pressure ratio like static pressure ratio and density ratios. Temperature ratios are plotted in the x axis that is the inlet Mach number and of course these from the right and vertical axis are. You know the total pressure ratio Mach number being plot you can note that when the inlet Mach number is increasing, from Mach number of 1, you see that the downstream Mach number of the shock, normal shock decreases.

If the upstream Mach number is greater than 1, then what happens is that become a subsonic. So, if you look at in this region it become subsonic and it goes on asymptotically decreases at Mach number at higher Mach number. When Mach number tends to their infinity, what will be the number you will get that will be 0.378 that is your downstream Mach number, that means, that Mach number downstream of the normal shock.

You see that there is a, as their Mach number increases total pressure decreases, however the static pressure goes on increasing at a very higher rate. You can see from here this plot and as compare to the static pressure ratio, density ratio and the temperature ratio decrease increases at a very slow rate. That means slope is small and of course this we have plotted for gamma is equal to 1.4, but in reacting flow and other places whenever you are talking about this cannot use. This gamma is 1.4 you need to take what you call

proper gamma value because of properties need not to be air, it will be others spaces and also you know constituents and different, so therefore you have to take proper value.

(Refer Slide Time: 04:43)

Isentropic flow table for $\gamma = 1.4$ Return					
M	T_0/T	P_0/P	ρ_0/ρ	A/A^*	MFP
0.1	1.002	1.007	1.005	5.822	1.99
1	1.2	1.89	1.577	1.0	18.26
2	1.8	7.824	4.35	1.68	29.83
2.2	1.968	10.69	5.43	2.0	31.38

Normal shock table for $\gamma = 1.4$						
M_1	M_2	P_2/P_1	ρ_2/ρ_1	P_{t2}/P_{t1}	T_2/T_1	S_2-S_1
1.1	0.91177	1.245	1.169	0.99	1.065	0.147
1.5	0.701	2.458	1.862	0.929	1.32	9.13
2	0.577	4.5	2.67	0.72	1.687	40.908
2.2	0.547	5.48	2.951	0.628	1.8568	58.09

[Return](#)

So, let us, now take an example to see how we can really look at that before that let us look at a table like normal shock tables because this properties what I had shown they can be plotted in what you call in a plot. But, we need to use the table for a solving some problems and we can use also direct are what to call equations, for that you need to remember those things.

But, let us look at the table which is have shown some of the dates you can you can see that when Mach number 1.1 and the downstream Mach number. The shock is 0.91 and as the Mach number increases let us say around 2, then you see that downstream Mach number 0.75, that means it is decreasing as the inlet Mach number increases more.

The supersonic upstream flow becomes more supersonic and downstream flow will be more subsonic as a result what is happening is that total pressure losses will be all. So, you know quite high, but the high Mach number because the total pressure ratio is small the pressure ratio means what P_{t2}/P_{t1} minus P_{t2} that we call it as a pressure loss. So, therefore if this ratio is small than pressure losses will be higher and of course what is happening to entropy all.

So, it is increasing and similarly the static pressure ratio increases as the upstream Mach number increases, so all, so the density. So, also temperature, but you note that that pressure ratio increases at a higher rate as compare to the temperature ratio and density ratio.

(Refer Slide Time: 06:46)

Example -: A normal shock occurs in front of a supersonic air intake with an area ratio of 3 for a gas turbine engine powered air craft travelling at Mach number 2.0 at an altitude of 10 km. Determine the Mach number, static pressure of the flow leaving the diffuser.

Given:- $Alt = 10 \text{ km}, P_1 = 26.48 \text{ kPa}, M_1 = 2.0, A_3 / A_2 = 3$
 To find: M_3, P_3

Solution -: Assume steady 1D flow in an air intake. Isentropic flow between (2) and (3)

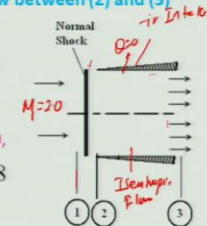
From NS table we can have properties ratios for $M_1 = 2.0$
 $M_1 = 2.0; M_2 = 0.5774 = 0.58; \frac{P_2}{P_1} = 4.5; \text{ and } \frac{\rho_2}{\rho_1} = 2.667$
 $\frac{T_2}{T_1} = 1.668; \text{ and } \frac{P_{t2}}{P_{t1}} = 0.7209$

From Isentropic flow table we can have properties ratio for $M_1 = 2.0$,
 $\frac{P_1}{P_{t1}} = 0.1278; \frac{T_1}{T_{t1}} = 0.5556; \text{ and } \frac{A}{A^*} = 1.688$

$$P_{t1} = \frac{P_{t1}}{P_1} P_1 = \frac{1}{0.1278} 26.48 = 210.02 \text{ kPa};$$

$$P_{t2} = (P_{t2} / P_{t1}) P_{t1} = 0.7209 \times 210.02 = 151.4 \text{ kPa}$$

As isentropic flow in air intake, total pressure remains constant between (2) & (3);
 $P_{t3} = P_{t2} = 151.4 \text{ kPa}$



So, let us take an example where we can will have to use this analysis, what we are done that example is normal shock occurs in front of a supersonic air intake with an area ratio of 3 for a gas turbine engine. Gas turbine powered engines like powered aircraft and travelling at Mach number 2 at an altitude of 10 kilometer, determine we need to determine the Mach number. Static pressure of the flow leaving the diffuser that means, let us look at a typical supersonic air intake where the Mach number, you known is inlet Mach number what is given as 2 with each flow is entering.

There is a normal shock keep in mind that it is closer to the leap of this curve of the diffuser, this is your air intake and this one is supersonic intake because of flow is supersonic. If it is detach than the flow will be quite complex and the shock maybe assume to be in front of that normal. But, however there will be some obliques of will be there and the shock is attack of course I am not showing, here there is a small gap. But, you can assumed to be a tape leap of the supersonic air, but if it is detach then can use there this will be steady or unsteady, it will may likely to be unsteady, but here it is almost attached.

So, if you look at this is almost attach kind of thing that assumption we are making and we are taking a station 1, as this one like a just upstream of the shock and downstream of the shock. We are taking the station 2 and at the exit of this what to call air intake, we are taking the station 3 and we are assuming the flow to be 1 dimensional because a normal shock is existing and it is other way around because of flow is 1 dimensional. Therefore, the shock will be normal shockwave, so far this right and what happens in the downstream of the shock, it will be definitely sub sonic, we have already seen and then there will be all.

So, boundary layer along with these and along with these surface, but however we will assume there is no boundary layer in inviscid flow that is on assumption that there is no heat loss over here right outside. So, then we can call it as an isentropic flow between station 2 and station 3, these are the assumption we are making and let us look at what is given, the given is altitude is given if altitude is given. That means from the altitude data table I can corresponding to 10 kilometer I can get the static pressure that is my P_1 and temperature also I can get I may not need.

But, you can take those data and then Mach number is given that is M_1 is 2 and A_3 by A_2 , the area has given that means, this area at this point the area and this area is given as 3. But, here is not to be seen keep it mind that the diagram is not really proper right, so it could have been little different diagram.

(Refer Slide Time: 06:46)

Example :- A normal shock occurs in front of a supersonic air intake with an area ratio of 3 for a gas turbine engine powered air craft travelling at Mach number 2.0 at an altitude of 10 km. Determine the Mach number, static pressure of the flow leaving the diffuser.

Given :- $Alt = 10 \text{ km}$, $P_1 = 26.48 \text{ kPa}$, $M_1 = 2.0$, $A_3 / A_2 = 3$
To find :- M_3 , P_3

Solution :- Assume steady 1D flow in an air intake. Isentropic flow between (2) and (3)

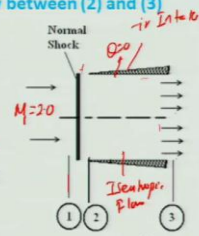
From NS table we can have properties ratios for $M_1 = 2.0$
 $M_1 = 2.0$; $M_2 = 0.5774 = 0.58$; $\frac{P_2}{P_1} = 4.5$; and $\frac{\rho_2}{\rho_1} = 2.667$
 $\frac{T_2}{T_1} = 1.668$; and $\frac{P_{t2}}{P_{t1}} = 0.7209$

From Isentropic flow table we can have properties ratio for $M_1 = 2.0$,
 $\frac{P_1}{P_{t1}} = 0.1278$; $\frac{T_1}{T_{t1}} = 0.5556$; and $\frac{A}{A^*} = 1.688$

$$P_{t1} = \frac{P_{t3}}{P_1} P_1 = \frac{1}{0.1278} \times 26.48 = 210.02 \text{ kPa};$$

$$P_{t2} = (P_{t2} / P_{t1}) P_{t1} = 0.7209 \times 210.02 = 151.4 \text{ kPa}$$

As isentropic flow in air intake, total pressure remains constant between (2) & (3);
 $P_{t3} = P_{t2} = 151.4 \text{ kPa}$



So, now we need to find out to find that M_3 that we will have to find out, we will have to find out P_3 as it is. So, this is the format how we can go about this for example like Mach number that is 1 given is, here if I assume it a normal shock and I can get directly what will be M_2 by what, by invoking a relationship are by using a shock table normal shock table right. So, I can get, I need to find out this P_3 M_3 , how I will get that can anybody tell me how to go about this problem, I already those made assumption that is flow in this region. You know it will be isotropy in the downstream region, it will be also isentropic that means between station to 1 3, when I can use some isentropic relationship because if I now the inlet Mach number.

That means the pressure ratio between station 2 and 3, I will be knowing the temperature ratio also, so also the total pressure ratio and, so also other properties. So, then how to go about because A_2 is not given, but whereas A_3 being asked, A_2 A_3 by A_2 is given, so how will go about it, I already given some things that will have to use isentropic relationship. You will have to use also the relationship for the normal shock and in some situation, you know if you do not want to use the relationship then you can use the table for data, yes that I will already told. But, how I will go from this P_2 to P_3 , but you know area how I will find out that also I already told you, no those things if you talk about those equations.

You know are the expression for pressure ratio or temperature ratio across the normal shock in you in a isentropic flow, those are all using continuity momentum energy. All those equations are bended into those equations, to the equations are derived by invoking those let us look at how we will do that. So, I already talked about that we are assuming steady 1 dimensional flow in an air intake because the shock is attached to the leap of the air intake and isentropic flow between station 2 and 3 that I already talked about it. From the normal corresponding to Mach number 2, if you look at I can get these values M_2 , I can get P_2 , 4.5 ρ_2 by ρ_1 , 2.67 and P_{t2} by P_{t1} , I can get all those things values.

We can take and those values or given over, here M_1 is 2, M_2 is 0.5774 and that is 0.58 and P_2 by P_1 , 4.5 and ρ_2 by ρ_1 , 2.667. Then of course the temperatures and other values been taken from the normal shock table, from isentropic table, from we can get properties for M_1 that we can get these values. Keep in mind that it is A by A^* 1.688 and similarly the pressure static pressure and total pressure ratio static temperature and total temperature ratio. So, what will do, we will basically find out the what is the total

pressure at station 1, here that is P_{t1} divided P_1 and P_1 I already getting from this altitude data and I will get that is 210.02 kilo Pascal.

It is just a multiplication nothing else and by knowing this, pressure ratio across the shock P_{t2} by P_{t1} , I can get P_{t2} because I know P_{t1} . So, that happens to be 151.4 kilo Pascal, so as the flow in is isentropic we do in the station 2 and 3 then total pressure remains constant between station 2 and 3, can I say that because flow is isentropic there is no losses is such. So, P_{t3} is equal to P_{t2} is equal to 151.4 kilo Pascal, is it clear this is a very important and critical assumption otherwise I cannot really find out P_3 , unless I know this P_{t3} how can I get that I cannot really get. You know in isentropic table relationship, we always relate to the total static pressure is related to the total pressure. So, therefore for I need to know the total pressure.

(Refer Slide Time: 16:48)

For an isentropic flow in air intake, between (2) & (3), we can have,

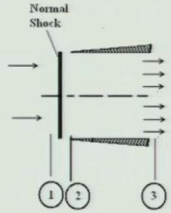
$$\frac{A_3}{A_3^*} = \left(\frac{A_3}{A_2}\right) \times \left(\frac{A_2}{A_2^*}\right) \times \frac{A_2^*}{A_3^*}$$

From isentropic flow table for $M_2 = 0.58$, $A_2/A_2^* = 1.21$,

$$\frac{A_3}{A_3^*} = \frac{A_3}{A_2} \times \frac{A_2}{A_2^*} = 3 \times 1.21 = 3.63; \text{ as } \frac{A_3}{A_2} = 3$$

For corresponding A_3/A_3^* from isentropic flow table we can get,

$$M_3 = 0.165; \frac{P_3}{P_{t3}} = 0.981$$

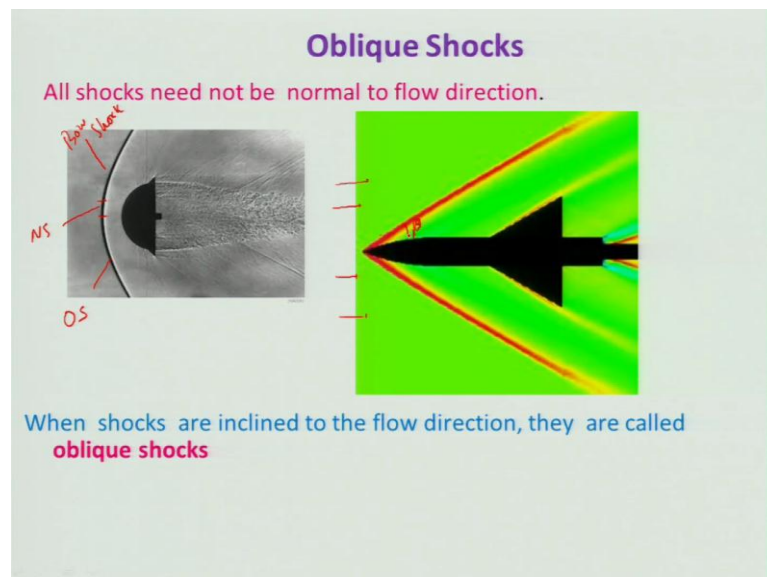
$$P_3 = 0.981 \times 149.4 = 146.56 \text{ kPa}$$


Now, for an a isentropic flow we can have A_3 by A_3^* because I need to know this relationship, because I can get from a isentropic star area with you know area ratio. That means any area with the respect to the critical area that is A_3 by A_3^* , I can write down as A_3 by A_2 into A_2 by A_2^* into A_2^* by A_3^* . So, if you look at that A_2^* and A_3^* ratio is given to you, but whereas A_3 by A_2 is given these values is given what is that ratio 3 and A_2 by A_2^* , I can get if I know the Mach number. That is that is one table if I know the Mach number, the downstream of the shock M_2 , so therefore I can get this from the isentropic table.

But, what about this A_2 by star by A_3 star, how I will get A_2 star by A_3 star that will be equal to 1 this is a very important, but you know although it is simple. But, it is very important concept that means what I am doing I am taking all these station 3 to the critical 2 to the critical and then relating, so this become 1. So, therefore I know need to now get this A_2 by A_2 star from the isentropic table corresponding to Mach number of points 5, 8 which you got from the normal shock table that means this is the Mach number at the downstream of normal shock. So, if I get these values I can substitute, here like A_3 by A_3 star A_3 by A_2 , I know this is given 3 and in A_2 by A_3 star 1.21 you will get 3.63.

Now, if I know these then I need to find out Mach number, what I will have to do I will have to invoke the isentropic table, so from corresponding into A_3 , sorry this will be A_2 star from an isentropic flow table A_3 by A_3 star because we need to find out this. That is from the isentropic table we can get a Mach number M_3 , 0.165 and pressure ratio P_3 by P_t 0.981, keep in mind that we need to interpolate the data. To get these values because A_3 by A_3 star happens to be 3.63, which may not be available in table date of for isentropic flow?

(Refer Slide Time: 20:56)



So, you need to interpolate assuming the data to be linear between two points that is very important point you should keep in mind and that means, now I got M_3 , 0.165 which is quite subsonic you know and P_3 by P_t 3 I will already know this P_t 3. So, I can get P_3

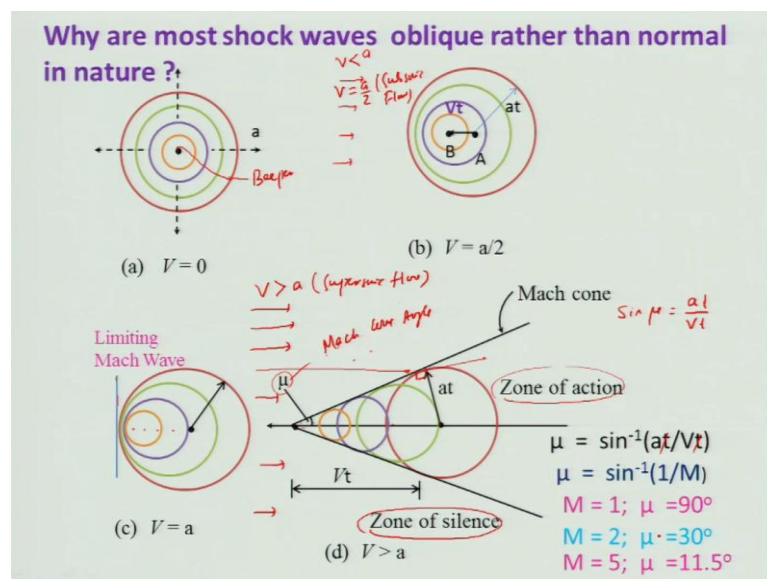
is equal to P_3 by P_1 , these values I will get 146.56 kilo Pascal, so we can solve the problem in the this manner, but those assumptions we need to keep in mind.

Now, we will move into oblique shocks and if you know that all shocks need not be normal to the flow direction in other words there shocks can be oblique nature that will make an angle with the respect to the flow. For example, if I take a bullet which is travelling at a supersonic speed, then what will happen, a shock will be appearing like a bow in front of what you call the bullet and this is known as a bow shock.

So, this is known as a bow shock and if you look at it is not really normal, but as I told last lecture we can consider these to be normal shock. But, in this case will have to consider as an oblique shocks, but if I take a missile which is having shockwave, you can see that these shocks are being formed with which is attached to the nose of this missile. It is making an angle of course at that downstream you can see that there is a another similar things like blah, but question arises why it is.

So, will you call it as a shock are will you call this as a shock and this also has a shock, now we need to think about and then see that what will be. But, what we are interested at this moment to talk about this oblique shock and see how we can analyze those things that means, when the shocks are inclined to the flow direction because if the flow is coming over here like that it is making an angle. So, that means these angle you know something angle is making, so then we call it an oblique shock.

(Refer Slide Time: 23:20)



So, question arises why are most shockwave oblique rather than normal shock in nature right why it is, so if I give a statement look most of the shocks are you know nature oblique. Rather than normal shock of course, that there certain situation where the shock is likely to be normal in nature like a bow shock are a shock in the downstream of, or in the divergent portion of a nozzle, or near the inlet in the front of the intake.

Certain places it can be provided is the attached to that, for that what you will do you will go to the basics and look at for example let us say that the flow is stagnant and that is a disturbance. That means disturbance will be for forgetting at the speed of sound disturbances means for example if the stagnant like there is no fluid movements and there is what you call disturbances, I can call a sound source I can call it as a beeper. Beeper you know which will be beep like a making sound and it will be troubling all the directions and of course is stagnant flow, let us consider another situation where this beeper will be moving at certain velocity V .

This is my sound which is moving at a speed of sound, this is my, what to call disturbance or a beeper like which is and also it is travelling with have a velocity V . This velocity V with which the sound producing device or the beeper is moving is half of the speed of sound and if it is travelling for a time t . So, what will happen it will travel a distance like a into t , the distance travelled by the sound wave will be much higher as compared to the beeper could have moved from A to B that is V . Therefore, if you see that it is not really you know would not be what to call knowing of course this is in the stagnation.

But, you can imagine assumed the fluid is coming over here and my disturbances fixed at a , I can do just opposite. So, then this is basically moving at A , you know speed of sound or I can say that V is equal to $a/2$ this is a sub sonic flow, I can call it as a sub sonic flow I can think of. So, let us take another example where this beeper is moving at the same speed of sound that means, the beeper is moving over here, here, here and then you know it is moving in certain direction towards the left. It is the speed of same speed of sound then what will happen all this waves will be coming and you know you could see that it will be creating a Mach wave, here in this region.

Let us take a another situation where these beeper will be moving at a velocity which is higher than the speed of sound, then if it is moving let say there is a beeper here and a

sound producer sound it is a moving, you know with a speed of sound for a time t . It will go again another place it will go and move, so that and what will happen and it goes a long with that. So, then if you take this circle because is moving then we when you join this tangent all those things what will you see, you will see that you cannot really hear this region the sound.

In this region this will be the would not be getting any sound if and only see because sound is travelling whereas the beeper as travel let they longer distance for the same time. If you look at for the same time the beeper as travel $V t$, this distance right from here to here, whereas the speed of sound as travel $a t$ the distance, so therefore this portion is known as zone of silence, where this is known as zone of action are sound at this disturbances sound and disturbances I am saying it is same. Therefore, if you look at this stagnant, but you can consider this as you know fluid is which is moving at certain velocity V which is greater than the speed of sound, that is supersonic flow, supersonic flow.

That means if you look at the fluid, it will be just travelling over here and then only you know that look there is a disturbances, here then it will be moving and take a save along with that you know. So, therefore this if I join this tangents of this point then I call this as a Mach zone because this zone of silence and zone of action is demarked by this region. Therefore, like this will be a Mach wave we can say that is the Mach wave will be always it and certain angle with respect to the flow. So, therefore we get an oblique shock or a mac, now if it is of course if it is a stronger one, you will get a oblique shock.

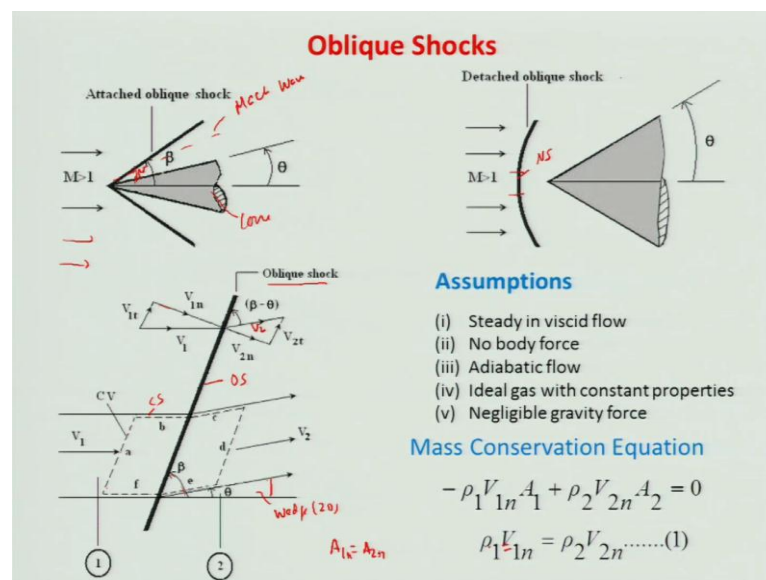
So, now we need to, now look at this angle what you call them Mach cone angle that is μ , this is known as Mach cone angle I can determine that angle what it would be, what is that if you look at this one this makes a 90 degree. If that makes a 90 degree, the sign μ will be what sign μ will be a t at by $V t$, so I can write down the μ will be equal to sign inverse you know $a t$ by $V t$. So, if the t , t is cancel out what I will get, I will get V by a , V by a is nothing but your Mach number 1 over Mach number that means, μ is equal to sign inverse 1 , 1 over Mach number.

When Mach number is equal to 1 what will happen to the μ , μ will be 90 degree like this you know and of course if they it is a stronger one you will get a normal shock. So, if you look at normal shock or a normal Mach wave is a spatial class often oblique shock

or oblique Mach wave. If I go on increasing this Mach number what happens to μ , μ is 30 degree that means, it will be making an angle 30 degree to the flow direction of course right from here 30 degree that will be other way around. So, if I go on increasing the Mach number, this μ that is the Mach cone angle is 11.5 that means I am increasing Mach number 4, inlet Mach number.

Then my shock is coming closer to what close to the angle they became smaller that Mach angle became a smaller, so in order to appreciate this point we need to look at little further. That means we can say that it is quite natural to have a oblique shock rather than the normal shock because either flow is, when flow is supersonic and it is Mach, but however we will see that under segment situation will get a normal shock.

(Refer Slide Time: 32:18)



So, let us look at the oblique shocks as I told you that this is what to call a cone, this is a cone in the flow, and flow is supersonic M is greater than 1. So, this cone is basically creating disturbances what I was talking about beeper as a sound do you know because these cone is there. Therefore, it will be creating it disturbance that will be propagating at the speed of sound and it will make an angle β , then what will happen to the Mach wave.

The Mach wave will be somewhere here which is making an angle, you know μ which is always smaller than the β because this is a vanishingly small. Then only you know in finite decimally wave is there a very weak compression wave, if you look at this is

your Mach wave and similarly if I go on, you know look at this theta is little higher. So, I will get a detached where you know if theta is becoming higher will see that how it is, we can take this as a normal shock this portion.

So, now let us analyze this oblique shock and see that how we can handle an oblique shock, so we make these assumptions as we have already stated several times steady in visit flow and nobody forces adiabatic flow. Ideal gas with constant properties there is no gravitational effect and gravity force as such I am, that is quite negligible for that what you will do. We will consider you know a kind of, you know there is, this is basically which wedge is there that means this is my wedge, this is a making anhetal and we are saying a streamline will be coming over there.

Here, the streamline is just making an angle and this is a wedge means basically 2 d, saying 2 d, whereas if I talk about this cone with 3 d in nature, 3 d means access semantic approach. But, there is the difference between access semantic and 2 d, so let us say that t we are considering this control volume I will take a control volume which is embedding this oblique shock, this is my oblique shock. This if you look at this control surface that a is parallel to this shock, so also the d whereas this b the control surface this control surface is parallel to the streamline and I need to analyze these things.

What I will do, I will have to basically talk about this conservation of mass equation momentum equations and then energy equation and all those things. So, if I consider this you know what I will be doing I can resolve these velocity V_1 into 2, 1 is perpendicular to the perpendicular to what perpendicular to the oblique shock and other is parallel to the oblique shock can I, now do that. If I take this equation because steady flow process one $\rho V \cdot d a$ is equal to 0 for the continuity equation I have taken.

So, if I take these surfaces I will find that you know that surfaces this is $V \cdot d a$ parallel, therefore this term will be 0 and b, m, c, f and d only the terms which will be there for the control surface a and that terms out to be $\rho_1 V_1$. That is the normal component I am talking about because what the shock what this control surface will be looking at be $V_1 n$, similarly it will be looking at $V_2 n$ and keep in mind that this V_2 that is making an angle beta minus theta because the beta is here and this theta.

So, it is making an angle that, so then I will get $\rho_1 V_1 n A$ plus $\rho_2 V_2 n A$ is equal to 0. So, I can write down as $\rho_1 V_1 n$ is equal to $\rho_2 V_2 n$ because area what

we are considering is same that means we are saying that A_1 is equal to A_2 . In other words, you can say $A_1 \cdot A_2$ is equal to like because this is perpendicular to that you know that we are saying.

(Refer Slide Time: 37:39)

Momentum Equation

$$\int (\rho V \cdot dA) V = - \int P dA$$

Momentum Equation along normal direction becomes

$$-\rho_1 V_{1n}^2 + \rho_2 V_{2n}^2 = -(P_1 + P_2) \dots (2)$$

Momentum Equation along tangential direction becomes

$$-\rho_1 V_{1t} V_{1t} + \rho_2 V_{2t} V_{2t} = 0 \Rightarrow V_{1t} = V_{2t} \dots (3)$$

Oblique shock = Normal shock w.r.t coordinate system moving with V_{1t}

$$M_{1n} = M_1 \sin \beta \dots (4)$$

The property ratios across shock waves become;

$$\frac{P_2}{P_1} = 1 + \frac{2\gamma}{\gamma+1} (M_{1n}^2 - 1) \dots (5)$$

$$\frac{\rho_2}{\rho_1} = \frac{(\gamma+1) M_{1n}^2}{(\gamma+1) M_{1n}^2 + 2} \dots (6)$$

$$\frac{T_2}{T_1} = \frac{P_2}{P_1} \cdot \frac{\rho_1}{\rho_2} \dots (7)$$

$$M_{2n}^2 = \frac{M_{1n}^2 + \frac{2}{\gamma-1}}{2\gamma/(\gamma-1) M_{1n}^2 - 1} \dots (8)$$

Similarly, momentum equation when we will do we will get that is $\rho_1 V_1^2$, we are considering only the normal component plus $\rho_2 V_2^2$ is equal to minus P_2 minus P_1 . If you look at momentum equation along the tangential direction what it would be it will be $\rho_1 V_1$ and $V_2 V_1$ is equal to $\rho_2 V_2$ and V_2 is 0, but we have seen that $\rho_1 V_1$ and $\rho_2 V_2$ is same. So, therefore I can write down V_1 is equal to V_2 that means these velocity V_1 is same as that of V_2 this is a very important statement, what it indicate it indicate that I can consider this oblique shock as a normal shock.

So, for this normal velocity is concern that means I can consider if this is a shock here and then I consider if turn it angle with the beta and theta it become normal and flow is coming V_{1n} . So, therefore I can consider, so oblique shock we can a normal shock with respect to coordinate system moving with V_{1t} , this is a very important statement. Therefore, for an oblique shock we can use their relationship what we had derived for the normal shock wedge that means V_{1n} will became what is this, sorry Mach number M_{1n} . There is a normal component and it will be $M_1 \sin \beta$ because this angle is beta I can

do that very easily and the property ratio across shock wave becomes like P_2 by P_1 is equal to $1 + \frac{\gamma}{2} M_{1n}^2$ minus 1.

If you recall this relationship for the normal shock, but in this case we are using is a normal component that means this portion I can put it here and then this P_2 by P_1 can be express in terms of inlet Mach number or upstream Mach number for an oblique shock. Similarly, ρ_2 by ρ_1 can be expressed in terms of these from the normal shock wave in only you just remember that this we have putting M_{1n} , this is not M_1 , M_1 means it is the upstream Mach number. But, M_{1n} means the normal components of the upstream Mach number that is very important because what this shock will be seeing when you consider as a normal only the M_{1n} .

So, similarly I can get an expiration were T_2 by T_1 is equal to P_2 by P_1 into ρ_1 by ρ_2 is I know this expiration why happen see, so I can get an expiration can T_2 by T_1 very easily and how I will get this M_2 then M_2 . Again I will use the normal fact relationship and, here in the left hand side the M_{1n}^2 and then I will get M_2 , from this is basically, for the equation 8 is basically we have derived that for normal shock. But, we are using for now oblique shock keep in mind that this is we are using the normal components of the which is a component of the stream velocity which is perpendicular to the oblique shock.

(Refer Slide Time: 41:56)

By using geometry, we can determine M_2 as;

$$M_2 = \frac{M_{2n}}{\sin(\beta - \theta)} \dots\dots(9)$$

θ is uniquely related to M_1 and β . Let us derive a relationship among θ , M_1 and β .

$$\tan \beta = \frac{V_{1n}}{V_{1t}} \quad \tan(\beta - \theta) = \frac{V_{2n}}{V_{2t}} \quad \text{As } V_{1t} = V_{2t} \dots\dots(3)$$

$$\frac{\tan(\beta - \theta)}{\tan \beta} = \frac{V_{2n}}{V_{1n}} \dots\dots(10)$$

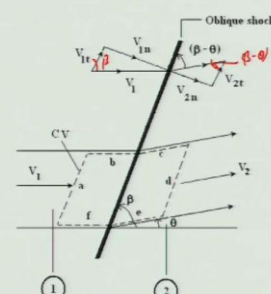
From continuity, we can get

$$\frac{V_{2n}}{V_{1n}} = \frac{\rho_1}{\rho_2} \dots\dots(11) \quad \frac{\rho_2}{\rho_1} = \frac{(\gamma + 1) M_{1n}^2}{(\gamma + 1) M_{1n}^2 + 2} \dots\dots(6)$$

By using Eq. (11), Eq. (10), Eq. (6) and (4), we can get,

$$\frac{\tan(\beta - \theta)}{\tan \beta} = \frac{(\gamma - 1) M_1^2 \sin^2 \beta + 2}{(\gamma + 1) M_1^2 \sin^2 \beta} \dots\dots(12)$$

By trigonometric manipulation, Eq. (12) becomes.

$$\tan \theta = 2 \cot \beta \left[\frac{M_1^2 \sin^2 \beta - 1}{M_1^2 (\gamma + \cos 2\beta) + 2} \right] \dots\dots(13) \quad \theta - \beta - M \text{ Relationship}$$


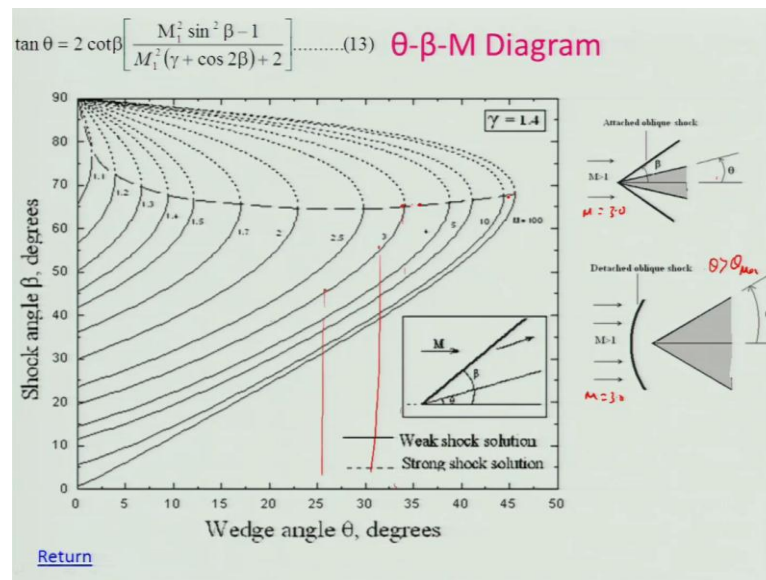
So, by using geometry we can determine M_2 , so if you look at if I know this M_2 is a of course I have drawn here velocity that I can draw the similar triangle for the number if I get this is basically β minus θ . So, M_2 become M_2 divided by $\sin \beta$ minus θ , I can get basically here you know this angle if I can take, I can find out what it would be. So, by if you look at this expiration θ is uniquely related to the M_1 and β why because M_2 is related to M_1 through the normal circulation, see and M_1 is related to the M_1 right in the terms of β angle.

So, we can have to derive that let us say that $\tan \beta$ what we have know this angle is basically V_1 divided by V_1 and $\tan \beta$ minus θ is nothing but V_2 divided by V_2 because this angle. If you look at this is β minus θ and as we know that we want t is equal to V_2 . So, then we can write down $\tan \beta$ minus θ divided by $\tan \beta$ is equal to V_2 by V_1 because it will cancel it as we know this one. So, from continuity we will get V_2 by V_1 is equal to ρ_1 by ρ_2 that we have already derived and let us, now club this all equations, equation 11, equation 10 and equation 4 that we had earlier.

Also equation 6, equation 6 we have seen ρ_2 divided by ρ_1 that we have already expiration terms of M_1 and when you combine all those things it involve little bit algebra. You will get an expiration $\tan \beta$ minus θ divided by $\tan \beta$ is equal to γ minus 1, $M_1^2 \sin^2 \beta$ plus to divided by γ plus 1, $M_1^2 \sin^2 \beta$ I will get, you people derive it if you find difficulty let me know. Then by trigonometric manipulation we can really express this $\tan \theta$ is equal to $2 \cos \beta$ in the bracket $M_1^2 \sin^2 \beta$ minus 1 divided by $M_1^2 \sin^2 \beta$ plus $\cos^2 \beta$ in the bracket plus.

So, if you look at this expiration is a very important expiration which is also known as β θ M you know relationship is that means it is relating to the β θ and inlet Mach number M_1 . So, what you will do we can this Mach number taking various θ and β and plot and see that how you know it looks and you can use this table, sorry use this chart for solving the problem, let us look at when we do that.

(Refer Slide Time: 45:27)



So, as I told that theta we have already you know talked about it let us plot this theta with a diagram in the left hand side, the shock angle beta is a plotted with the wedge angle theta in degrees. This curves or fog particular Mach number for example this curves is 1.1, this is for 1.2, this is for 1.3 like that it goes what you can observe from here that if I take a wedge angle of 20 let us say I will take a vertical line. You can see that if I take this kind of a here you know 3 Mach number let us say inlet Mach number is coming as A 3. Let us say the Mach number inlet is coming angle c theta I have taken, sorry this theta I have taken is equal to 20 degree, what I will get that there are two solutions, one is here the another is if I look at here.

That means, if I take, if I take a vertical line it will be something around 38 degree kinds of things, here it is also two solutions what is highest, which one I will choose. If you look at that is solid line whatever I shown here and the lower portion of this curve known as the weak shock solution. Here, this dash line is known as the strong shock solution in nature generally the weak shock solution or weak shock will be existing. So, therefore we need to take only in this region that means weak shock solution, so therefore it will be coming over here and most of the cases unless otherwise status. You know we will be considering only the weak shock solution and what else you can observe from here for example if I fix this what you call Mach number 3.

What I was telling that if I will take this Mach number let us 3 constant like Mach number is equal to 3, then if I goes and increasing theta, if I goes and increasing this theta what happened that means I will have to go in this I am here at 20. Then I will go here will see that the 25 it goes over there and then I will reach a point where there is, you know it is almost getting certain of value and if I go on increasing this theta some, here then I would not be any solution. That means beyond this point if it will you know theta is increased that means the wedge angle theta is increasing what will happen the shock will be detach.

That means, here the theta is greater than is theta Mach for this Mach number if I say this Mach number 3 for this case, it is having something 30 may be 33 or 35 or 36 degree. So, therefore it will become Mach number that the will be attach detach the attach becomes detach and you will get a detach oblique shock and when if you look at it is same for the all the Mach number you know like similar situation. So, if you join this of course that gives a curve, so we need to see there is another thing once would really a little concern about that if I go on increasing in this Mach number what happened through the theta maximum it goes on increasing.

But, when become very high let that Mach number 100, so it is having something 45 points, some degree beyond that it cannot really have that. So, keep in mind that these are there, it is when theta maximum you may get it strong wave in such region the solution, but that is resume is very small, to this I will stop over, I will continue in the next class.