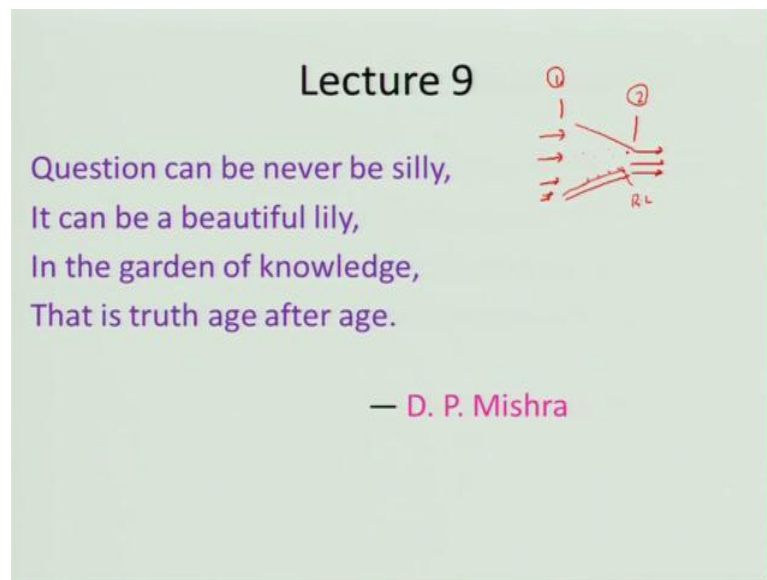


Fundamentals of Aerospace Propulsion
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Lecture – 9

Let us start this lecture nine with a thought process, I have taken from a poem known as question as if we ask a lot of question when we are particularly babies or small child, but as you go along we will start not asking questions. Maybe, it is because of social reflect or the society what we are living.

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The slide has a light green background. At the top center, the text "Lecture 9" is written in black. To the right of this text is a hand-drawn diagram in red ink showing two points labeled ① and ②. From point ①, three horizontal arrows point to the right. From point ②, three horizontal arrows point to the right, and a single arrow points from ② down to ①. Below the arrows, the letters "R.L." are written. On the left side of the slide, a poem is written in purple text: "Question can be never be silly,
It can be a beautiful lily,
In the garden of knowledge,
That is truth age after age." Below the poem, the name "— D. P. Mishra" is written in pink.

Therefore, it is very important to ask questions for pursuing the scientific and engineering work particularly or any other work. Therefore I want to say that this question can never be silly. It can be a beautiful lily, in the garden of knowledge that is the truth age after age. Whatever the science and technology what we see it is, it has come out of asking questions. Let us look at what really we studied in the last lecture, if you recall I ask a question at the end. That is why we look at the isentropic relationship although, it is very restrictive in nature.

Of course, before that we derived the Bernoulli's equation for compressible flow. Also, the incompressible flow right and generally the Bernoulli's equation for compressible flow is you know being used, but it cannot really be used for the compressible flow.

Then we moved into isentropic flow where there will be no dissipation, that means no change in entropy and also it will be adiabatic. So I guess you might have thought about the question I had ask in the last lecture, that why really we will have to look at isentropic flow, which is quite restrictive in nature.

Can anybody tell me why it is so? Let us consider a flow right in a converging duct, what we have seen right let us say there is a flow, which is coming over here. What will happen? It will increase because there is a change in what you call velocity between the station 1 and station 2. Now, what is happening, the flow is accelerating and where really this dissipation will be occurring? Dissipation means what there will be kind of friction. So, friction will be there, what you call in boundary layer here, somewhere. Unfortunately I have drawn a very thick one, but it is very thin.

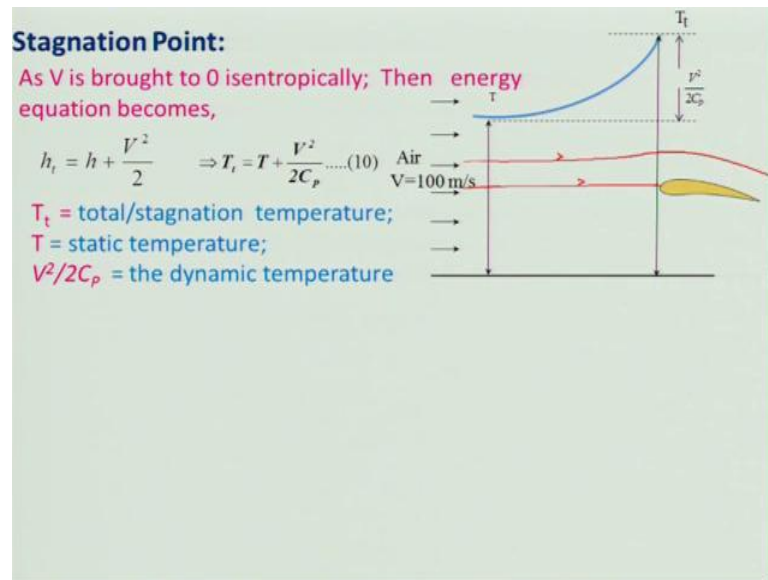
If I just remove it, we will see that it will be very thin and where there is a likelihood of friction that will be taking place a role. It is because of boundary layer, because of boundary layer, also on the upper surface, because it is circular. So, it is on the over the internal surface area. It will be there and whereas, inside this flow whatever inside. It will be not subjected to any what change entropy, because we are assuming the flow is to be uniform. Also, there is no gradient that is occurring for entropy to occur.

Particularly if you look, there is no temperature gradient, we are not adding any heat or we are not having any diffusion mass diffusion, where the entropy can also change, because of mixings. Then naturally we are not adding it, so it will be adiabatic. So, there is no dissipation in the major region of the flow, then we can consider the flow to be isentropic in the major portion of the flow, except near in this boundary layer, which is very thin.

What will be the thickness of the boundary layer? It will be order of few m m. Of course, that depends upon inlet Reynolds numbers and up to talk about or the fluid velocities. As it is accelerating it will be very small as compared to a decelerating form. Therefore, as most of the propulsive devices are what you call can be considered, propulsive devices like rocket nozzles or a diffusers or a you know even turbine flow through the turbine compressor. Of course, some extent we can consider in it to be an isotropic and carry out analysis for that.

So, there is another concept what we would like to look at which is the stagnation point. What do you mean by this stagnation point? Let us consider a flow over an aero foil, I have shown aero foil here and flow is coming with a uniform velocity

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It will move away from the aero foil and keep in mind that air is going over this aero foil at a velocity inlet velocity or stream velocity of 100 meters per second. Now, if this fluid you know we will be taking, it will be going in a streamline kind of thing. If I take this fluid elasticity over here, if you look at the leading edge it will come to a; what you call stand still. So fluid element has brought to 0 velocity. If I take, of course somewhere here, then fluid will go over like this.

Similarly, on the lower region another thing and now when this is brought to the 0 velocity adiabatically and also reversibly, then we call it as an isentropic. That means if the fluid element is broad isentropically to the 0 velocity. Then we call it as a stagnation point or a stagnation properties. That means if I am considering as a pressure, it will be, fluid experience, like fluid will be at stagnation pressure.

If I apply under this condition the first law of thermodynamics, that is the energy equation that became h_t is equal to h plus v square by 2. What it indicates? It indicates the total enthalpy is equal to the static enthalpy plus v square by 2. That is your dynamic energy and if I assume a calorifically perfect gas. That means what the c_p is not a function of temperature. In this case it means h will be basically $c_p t$, which is your

calorifically perfect gas. Then I can write down that T is equal to t plus v square by $2 C_p$. What I am doing is, I am by definition saying that h is equal to $C_p T$.

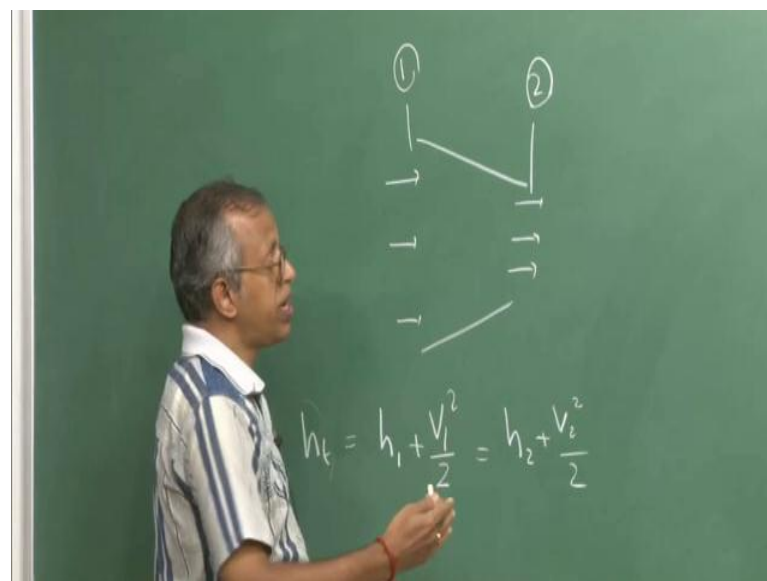
Then putting into here, that is the total and then here also it will, what you call $C_p T$. Then if I divide it across both the left and right hand side, I will get t , which is equal to t plus v square by $2 C_p$. So, what is this term, this we call it as a t this 1 we call it as, this term we call it as a static temperature.

This T we call it as a total or stagnation temperature v square by $2 C_p$, we call it as a dynamic temperature. What do you mean by this static temperature? Can anybody tell me? That means when the temperature experienced by the fluid element can be termed as a static temperature. In other words if I put a sensor on the top of a fluid element, which is moving at the local velocity, what will it be experiencing?

It will be experiencing the bombardment of the molecules on the top of it, which is basically the temperature, the kinetic energy what it will be experiencing. So, that is known as static temperature and as it is moving at a certain velocity. Therefore, that velocity will be nothing but v square by $2 C_p$ is basically the dynamic temperature.

Similarly, we can define also the total pressure, total you know density like that, are you getting my point? So, what is the meaning? This equation if you look at h_t is equal to h plus v square by 2 , what it is saying that means, if I take a station 1.

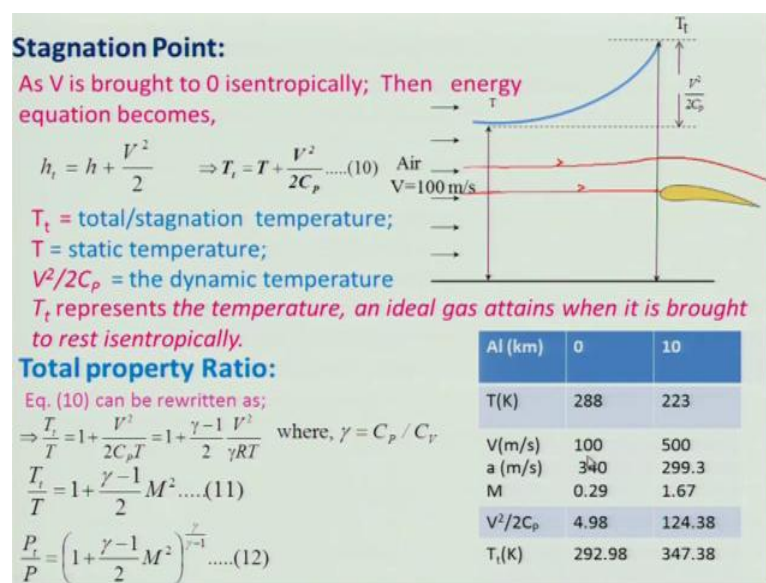
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I will take a particular station, let us say 1 and the station 2 then what I will get? For example, I am looking at a nozzle like kind of things. Because, I am taking this is 1 this is station 2 and there is a flow which is occurring, if I look at $h_1 + \frac{v_1^2}{2}$ is equal. I can write down $h_2 + \frac{v_2^2}{2}$, which means it is equal to h_1 . Can I say this, which means what I am saying is that the total enthalpy is remaining same between the station 1 and station 2. So, that means total enthalpy is remaining same, provided the flow is steady and whether you will be in viscous or not, it will be in viscous. So, for energy equation is concerned, we are not worried about whether it is in viscous or not. Then if it happens to be, that means total enthalpy along a streamline remain constant.

If it is uniform velocity at the upstream region, that means I can say that along with any streamline the total enthalpy remains constant, which means along the whole domain the total enthalpy, will be remaining constant, and provided the flow is steady okay. It need not to be 1 dimensional, because my streamline can be 2 dimensional as well. Therefore, it need not to be 1 this is the profound statement you may say. We have derived for 1 dimensional flow, now we are talking about, you can interpret that, but keep in mind that total temperature total pressure, is it really a quantity, which you can measure or what is so big about it. We are talking, that they are actually derived quantities.

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It is quite difficult to measure as such. Because, you need to bring the fluid isentropically therefore, but however you know at two location the flow need not to be isentropic, but however, we can use this relation locally, and that is a very important point. So, I have already talked about this, like T/T_0 represent the temperature and ideal gas attains, when it is brought to rest isentropically.

Having said this you know about this stagnation point properties. Let us look at the total property ratios and that is equation 10. What I will do, I will just divide this T/T_0 by the T/T_0 temperature and also the right hand side of this equation by the static temperature T , so what I will get? I will get T/T_0 by T , is equal to $1 + \frac{v^2}{2 C_p T}$ and for a calorific perfect gas. I can write down, that is c_p is equal to $\frac{\gamma R}{\gamma - 1}$.

I have done that here $\frac{\gamma R T}{\gamma - 1}$ divided by, I mean when I take this $\gamma - 1$ that becoming the numerator. It will be $\gamma - 1$, that becomes $1 + \frac{\gamma - 1}{2} \frac{v^2}{\gamma R T}$. So what is this $\gamma R T$? $\gamma R T$ is basically speed of sound square and where γ is C_p by C_v . Then I can write down T/T_0 divided by T is equal to $1 + \frac{\gamma - 1}{2} M^2$ square, what is this M ? M is basically Mach number, which is the ratio of velocity and speed of sound. Similarly, we can use the isentropic relationship for the pressure. Find out the relationship between the pressure ratios in terms of Mach number.

From the temperature if you look, at this is simple expression what we had derived in the last lecture P/P_0 by p is equal to T/T_0 by T power to the γ divided by $\gamma - 1$ in place of T/T_0 by T . I have put this expression that is $1 + \frac{\gamma - 1}{2} M^2$ square. So, if you look at this expression, you should keep in mind, however these isentropic relationships are available in the isentropic table.

So, what I am expecting is that, you should keep in mind and what it indicates? It indicates that this property ratios like temperature ratio and pressure ratio, will be dependent on the Mach number. That means it will be dependent on the flow and also it will be dependent on the kind of fluid. For example, γ will be depending will be dependent on the type of fluid is being used.

So, let us consider, what is the implication of this temperature. If I consider, let us say, an aircraft, an aircraft is at an; what you call altitude of 0. It is moving at velocity of 100

meters per second and temperature. You can take 288 Kelvin and the speed of sound will be 340 meters per second, and the Mach number will be point 28.

When I calculate v^2 by $2 C_p$, it will be 4 point 98. Roughly you know, 5 Kelvin and T_t becomes 290, 2.98 Kelvin if you look at it is 288. Then add this temperature you get this. Whereas, if I go for this altitude 10 where the temperature is lower down, that is 2 to 3 Kelvin. Your vehicle is moving at a higher velocity that is 500 meters per second.

The speed of sound is reduced, that is 299.3 meters per second and the Mach number is 1.67, but what is happening to this v^2 by $2 c_p t$, it has increased. As a result the total temperature T_t is 347.38 meters per second 347.38 Kelvin. Whereas, the actual temperature, static temperature, this is the static temperature t_k . The first rho that is 223, what can we know from this example can anybody tell me?

It is because this is at the data; I have just put some data in front of you. I want to see what you could observe and tell me, so I mean, what you can look at it is actually 5 Kelvin. Here v^2 by c_p , which means the temperature difference between total temperature and static temperature is only 5 Kelvin.

Now, what will be the percentage, if I neglect, say that I will, I am not interested in the total temperature. It will be almost equal to the static temperature. I need not worry about compressibility, if at all you can say that, or the change in the temperature. However, in this case why I am neglecting it is that the percentage of error I will incur by not considering. You know the flow or the dynamic air that will be around something may be 2 or 3 percent.

Whereas, here if you look at, in the other case, that is where the static temperature 2 to 3 and the total temperature 347, then what will be the effect? I cannot neglect it, because it will be almost around 56 percentage error, if I will neglect that. Therefore, it is important to consider, you know these kinds of things particularly at the high velocity. Also, the high altitude because the speed of sound is changing. Suppose in case, the case and the same attitudes at the sea level, that is 0 Kilo meter. You are considering the speed of sound could have been 340 meters per second.

So, this term could have been like reduced, because this, it could have been not 1.67 with the 500 meters per second. It could have been, may be little lower and it goes by

squaring this expression. Therefore, one has to look at it and be careful while doing. Keep in mind that this is a stagnation property or you know, it is a very important concept, which will be useful when you are analyzing.

As I told that even though flow is non-isentropic, we can still use it. Because, we can evaluate the total temperature locally and can use it. I will take some, may be example and show that it will be useful. So, let us take an example, air is flowing through the duct with an inlet velocity of 200 meters per second, pressure 0.1 mega Pascal and temperature of 298 Kelvin.

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Example 1: Air is flowing through a duct with inlet velocity of 200 m/s, pressure 0.1 MPa and temperature 298 K. Determine (i) stagnation pressure and temperature, and (ii) inlet Mach number.

Given – : $V = 200 \text{ m/s}$, $P = 0.1 \text{ MPa}$, $T = 298 \text{ K}$.

Solution:

$$T_t = T + \frac{V^2}{2C_p}$$

$$C_p = \frac{\gamma R}{\gamma - 1} = \frac{1.4 \times 287}{0.4} = 1005 \text{ J/kg K}$$

$$T_t = 298 + \frac{200^2}{2 \times 1005} = 318 \text{ K}$$

$$P_t = \left(\frac{P_t}{P} \right) \times P = \left(\frac{T_t}{T} \right)^{\frac{\gamma}{\gamma-1}} = \left(\frac{318}{298} \right)^{3.5} \times 0.1 = 0.126 \text{ MPa}$$

$$M = \frac{V}{a} = \frac{V}{\sqrt{\gamma R T}} = \frac{200}{\sqrt{1.4 \times 287 \times 298}} = 0.57$$

We need to determine the stagnation pressure temperature and inlet Mach number, which is the very simple questions, but I just want to say that you need to use, whatever equation we have derived. Then velocity if you look at given 200 meters per second pressure is 0.1 mega Pascal and temperature is 298 Kelvin.

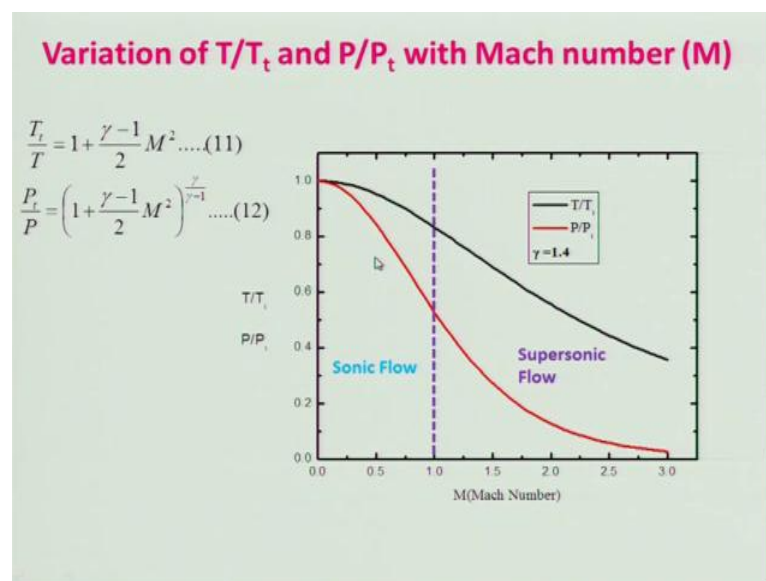
We need to find out total or the stagnation temperature. We know that T_t is equal to t plus v square by 2, c_p for air. We know, we can evaluate the c_p as γr , keep in mind that this is the specific gas constant r , which is equal to R_u divided by molecular weight of the particular gas, particularly for air it is being considered. Therefore, it turns out to be 105 Joule per kilo gram Kelvin. Sometimes, we will find 1.05 Kilo Joules per kilo gram Kelvin. When you substitute these values, you will get, 3, you know 318

Kelvin. This is by just substituting this equation, we can find out instead of P/T by P . You can write down T/T by T power to the gamma minus 1 into the p .

Then you will find or substitute this T/T by T , which we have already evaluated. T/T is 318 Kelvin, that is static temperature 298 multiplied by 0.1 you will get 0.16 Mega Pascal. Of course, the Mach number you can evaluate very easily, because you know the velocity 200 meters per second and that the static temperature already known to you. So, you can find out by substituting these values v divided by gamma $R T$.

Then it turns out to be 0.57, which is the Mach number. With these things you can evaluate very quickly using isentropic table, however I would ask you to keep this small formulas in mind. So, that it will be helpful for you to visualize. Also, one has problem in the exams whenever it comes, let us look at the variation of T/T and p/p with the Mach number.

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We will be using this expression, which we have already derived and for the temperature ratio and pressure ratio. When you put this value the Mach number per gamma is equal to 1.4. I will get a plot like this, that means if you look at this line corresponding to the T/T and this the lower one will be corresponding to p/p and with the Mach number. What you could observe from this curve, which we have just used, this equation 11 and 12 substitute the value of gamma, which we have taken 1.4 for air and Mach numbers.

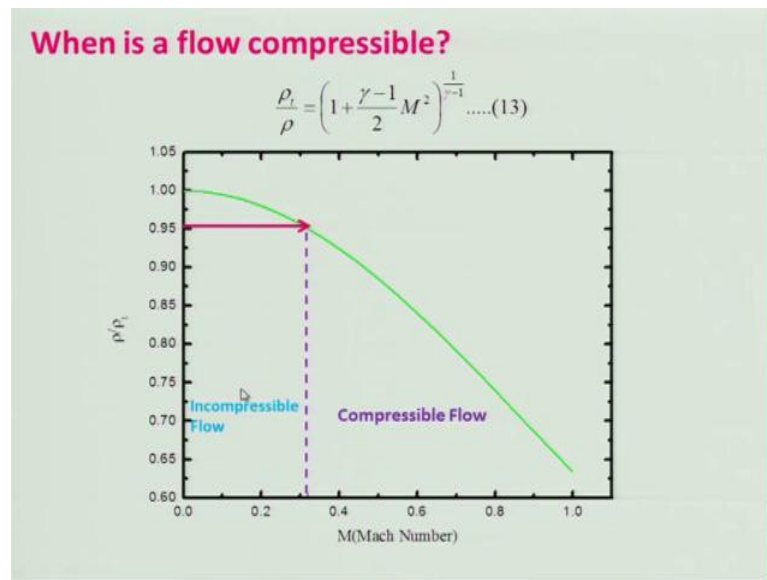
We have varied, we have created this plot, but we just said that it is changing. It is decreasing with the Mach number, which means what I am saying is that when the Mach number is equal to 0 that becomes equal to 1. It decreases very less in the near region. When the Mach number is something 0 to 0.5. Of course, at the Mach number 1 the change is quite drastic, because the slope changes.

Of course, the slope in case of the pressure ratio is much steeper than that of the temperature ratio. So, if I take this Mach number which is equal to 1 line and this will be basically sonic flow and this will be supersonic flow. We have earlier discussed about transonic flow, where it will be between 0.8 to 1.2 Mach number. We call it transonic flow, which we are not indicating, but what it indicates is that, it means that in certain ways.

Particularly, near to the Mach what you call low Mach number, it is not changing at all. So, whether we call it as a compressible flow or not, we need to look at it. So what is the criteria we have used. In other words when is a flow compressible or when we can we label a flow to be compressible? How I will go about it is, if you look at your definitions that we discussed, may be 2 to 3 lectures.

That we have to come to a conclusion or we have come to conclusion that there will be change in density, which will be very small. If there is a change in density, if it is small then it is incompressible and if it is more than the 5 percent, then we call it as a compressible. Therefore, we need to look at the expression for a density ratio that is ρ_t divided by ρ , which is equal to $1 + \frac{\gamma - 1}{2} M^2$ power to the whole 1 over $\gamma - 1$.

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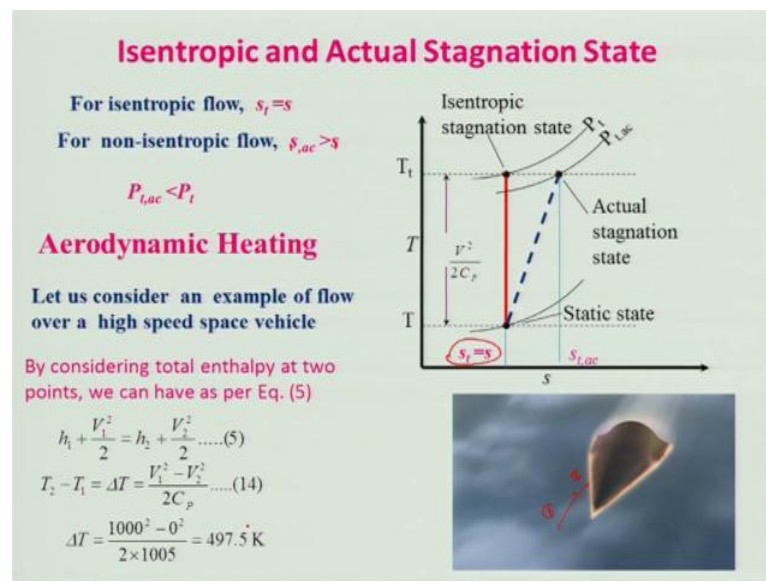
So, what we will do now, we will take this gamma 1.4 for air and vary this Mach number and see what we get. We are getting rho by rho, t is changing with respect to Mach number. If we use that criteria, that change in density should be less than the 5 percent for flow to be incompressible. Let us say that density changes by 5 percent, then that will be 0.95. So, if I take this 0.95, this density ratio, I will get a point. Then draw this point from a line from over here down, just to look at what is the Mach number. This Mach number will be around 3 to exactly to say, it exactly will be around 0.32.

That means we can call this portion as a compressible flow. This portion we can call it as a incompressible flow from the practical point of view, but if you look at the name incompressible. Then when will I get the flow, to be incompressible Mach number must be equal to 0. That means change in density is equal to 0. Is it really possible to have a incompressible flow?

Actually, incompressible flow is a myth, if you look at the strict definition of incompressible flow, change in density will be equal to 0. Therefore, if you look at that, it means at the Mach number 0, I will get rho by rho t, which is 0. That means change in density is 0. So, there is no flow. So, there is no incompressible flow as such, but in practical purposes we will be using in this regime, that when the Mach number is 0 to 0.3 or 0.32, then we can call it as a incompressible flow and use the governing equations.

You know for incompressible flow, which is quite simpler or simple as compared to the compressive flow. We can consider as a you know incompressible in this regime. Because, the whatever the error we are making it is within something 5 percent. Then that is good enough for us to make the analysis simpler. That we will be using these definitions or the basically it is a thumb rule. There is nothing sacrosanct about it, but we use it for a practical purposes. So, let us look at it.

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This isotropic and actual stagnation state that we have already discussed, that this is basically for, like a temperature. We have seen that T_t is nothing but T that is this point, that is static temperature plus v square divided by $2 C_p$. Then the flow is static, flow is stagnation, flow is isentropic. It means that the total temperature is equal to static temperature plus v square by t . This line that we have, shown here is basically static pressure line and this is your static total pressure line. This curve what I have shown this is the curve, now for this the entropy, that is the total entropy remains same as that of the static entropy.

So, if you look at this total entropy, it is same as that of the static entropy, but in the actual stagnation point it means, it can be adiabatic. However, entropy will be different, that means if I consider this point, this is my actual stagnation point. However, if I consider a calorifically perfect gas that will, say that T_t at this same point is same as the T_t at that point.

In other words what I am saying is, I am saying that the total enthalpy is remains the same, regardless of whether it is stagnation at isentropic stagnation or it is a non-isentropic stagnation point but in this case the entropy of actual is greater than the entropy of what you call, the isentropic flow. For the non-isentropic flow the entropy will be, you can say this is basically T and then T.

Here, if you take the total, than it will be greater because of what? It is because of dissipation and let us say that there is a boundary layer, but what happens to pressure will be actual total pressure that will be lower than the isentropic total pressure. It is because of dissipation, there will be deduction in the pressure. Therefore, what I want to you to know, as a conclusion of this, is that these stagnation properties can be used locally, even for a non-isentropic flow. Although, there is some kind of entropy change in there, we can still use it very easily.

Even, if there is a heat addition, we can use it, locally it is correct. Therefore, it is a defined quantity. It gives a very good idea about the flow without really getting into complexities involved. That is the beauty of this stagnation, what you call properties. So, let us look at another effect, which is the aerodynamic effect, which we have already looked at in one example. Let us consider an example, flow over a high speed space vehicle, if you look at your high speed vehicles. You can see these regions, which is quite, burning kind of things.

Here, every color you know will be kind of burning, why because, it is coming at a very high velocities. Mach number will be order of more than five. Particularly, the reentry vehicles or the intercontinental ballistic missile, the Mach number will be order of 25. So, in that case the heat in the dynamic heat, which is converted into the temperature, at this point, because these are the stagnation point, where the fluid cannot penetrate into the body.

So, by considering this enthalpy equation at 2 points, we can now determine like, let us say there is a point over here. Then you are going to find it, let us say this is 1 and this is 2 you are moving like this, then $h_1 + \frac{v_1^2}{2}$ is equal to $h_2 + \frac{v_2^2}{2}$.

I mean, let us say flow is coming over like this and if I take an calorifically perfect gas, I can write down $t_2 - t_1$, which is equal to Δt . That means change in temperature

will be equal to v_1^2 minus v_2^2 divided by $2 c_p$. If I substitute these values, let us say v_2 will be 0 and v_1 may be 1000 meters per second. It is not a very big Mach number, but however you can see that the change in temperature is quite high, 497.5 Kelvin. So, it is quite high to be considering the effect of aerodynamics, but what it is happening, the change in kinetic energy has been converted into the enthalpy change, in enthalpy. As a result, there is an increase in temperature.

So, what we have looked at is that basically stagnation properties. Then how it affects and other things, but what we are discussing is compressible flow. In case of compressible flow; sonic flow is a very important concept, which we must look at, what is that? It is the flow at which the velocity of the flow is equal to the speed of sound. So, let us consider a fluid at a point, is accelerated to the speed of sound adiabatically.

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Sonic Flow

Let us consider fluid at a point is accelerated to speed of sound adiabatically. Then $V = a$. What will be its properties ?

$M = 1$ (sonic)

$$\frac{T_t}{T} = 1 + \frac{\gamma-1}{2} M^2 \dots (11)$$

$$\frac{P_t}{P} = \left(1 + \frac{\gamma-1}{2} M^2\right)^{\frac{\gamma}{\gamma-1}} \dots (12)$$

$$\frac{\rho_t}{\rho} = \left(1 + \frac{\gamma-1}{2} M^2\right)^{\frac{1}{\gamma-1}} \dots (13)$$

$$\frac{T^*}{T_t} = 0.832; \quad \frac{P^*}{P_t} = 0.528; \quad \frac{\rho^*}{\rho_t} = 0.634$$

$$\frac{T^*}{T_t} = \frac{2}{\gamma+1} \dots (14)$$

$$\frac{P^*}{P_t} = \left(\frac{2}{\gamma+1}\right)^{\frac{\gamma}{\gamma-1}} \dots (15)$$

$$\frac{\rho^*}{\rho_t} = \left(\frac{2}{\gamma+1}\right)^{\frac{1}{\gamma-1}} \dots (16)$$

All properties at sonic condition are denoted by *

Of course, we are considering that there is no heat inside or no heat is going out, whenever you are considering as a streamline. So, then v is equal to a , but question that arises is, what will be property? It is because we will be more interested about the properties. Properties mean pressure, temperature, density and other things, so what are those things we need to understand? For that we will start, whatever we have derived or stagnation temperature and static temperature ratio.

That is $1 + \frac{\gamma-1}{2} M^2$. This we have already derived, if I put this v equal to a , v is equal to a , which means that, Mach number is equal to 1. That

is, you sonic. So, if I substitute these values over here, what will I get? That means this m is equal to 1, which means this becomes 1. Then this term will be basically $\gamma + 1$ divided by 2. This becomes $\gamma + 1$ divided by 2. Am I right? Because, $2 + \gamma - 1$ becomes $\gamma + 1$ divided by 2.

So, then, I get an expression T by T is nothing but 2 by $\gamma + 1$. Similarly, I can get p by p ratio. I will also get, p plus 2 divided by $\gamma + 1$ power to the $\gamma - 1$. Similarly, density ratio is ρ^* divided by ρ by $\gamma + 1$ power to the $\gamma - 1$ and keep this in mind. This star that I am using, corresponding to the sonic conditions, which means all properties at the sonic conditions is denoted by star.

So, if I substitute, you know a γ value for air, which is γ , is equal to 1.4, I will get T^* divided by T is nothing but 0.832 and p^* by p is 0.528, ρ^* by ρ is 0.634, keep in mind. The highest is this value for temperature, then of course, density, then comes the pressure, because these are the coefficients.

So, similarly we can define a term known as characteristic Mach number M^* . That is basically the velocity and the a^* is the speed of sound. It might look to be hard, is always the speed of sound, but why it is, that question might come into picture. For example, there is a fluid which is subsonic. If I take this fluid to the sonic condition, that means I need to accelerate the flow.

Then corresponding to the local speed of sound I need to consider. Then that is the sonic flow velocity. Similarly, if it is the fluid that is supersonic, then I can decelerate the flow by using a diffuser to the sonic condition. Then I will talk about that, but that means it is fictitious. Local speed of sound is the speed of sound at sonic flow condition.

Then of course; a^* will be equal to $\sqrt{\gamma R T^*}$, which means corresponding to this value. So, that has to be considered, a^* is not actually the local speed of sound. It is the kind of imaginary, corresponding to a particular velocity. So, this is a very important concept, which you should try to assimilate and have a feel for it. As you go along we will see more about it.

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Special Forms of Energy Equation

For Adiabatic Steady 1D Flow:
The 1st law of TD for CV becomes a_t^2

$$C_p T_1 + \frac{V_1^2}{2} = C_p T_2 + \frac{V_2^2}{2}; \Rightarrow \frac{\gamma R T_1}{\gamma - 1} + \frac{V_1^2}{2} = \frac{\gamma R T_2}{\gamma - 1} + \frac{V_2^2}{2}; \Rightarrow \frac{a_1^2}{\gamma - 1} + \frac{V_1^2}{2} = \frac{a_2^2}{\gamma - 1} + \frac{V_2^2}{2}$$

Let consider station (2) as stagnation point ($V_2=0$)

$$\frac{a_1^2}{\gamma - 1} + \frac{V_1^2}{2} = \frac{a_t^2}{\gamma - 1} \dots (17)$$

a_t is the total speed of sound associated with the same point.

If we consider two stations along stream line, Eq. (17) becomes

$$\frac{a_1^2}{\gamma - 1} + \frac{V_1^2}{2} = \frac{a_2^2}{\gamma - 1} + \frac{V_2^2}{2} = \frac{a_t^2}{\gamma - 1} = \text{constant} \dots (18)$$

Let consider station (2) as sonic condition ($V=a^*$), then Eq. (18) becomes

$$\frac{a_1^2}{\gamma - 1} + \frac{V_1^2}{2} = \frac{a^{*2}}{\gamma - 1} + \frac{a^{*2}}{2} = \frac{\gamma + 1}{2(\gamma - 1)} a^{*2} \Rightarrow \frac{a_1^2}{\gamma - 1} + \frac{V_1^2}{2} = \frac{\gamma + 1}{2(\gamma - 1)} a^{*2} \dots (19)$$

$$\frac{a_1^2}{\gamma - 1} + \frac{V_1^2}{2} = \frac{a_2^2}{\gamma - 1} + \frac{V_2^2}{2} = \frac{\gamma + 1}{2(\gamma - 1)} a^{*2} \dots (20)$$

By comparing Eq. (18) and Eq. (20) we can get

$$\frac{a_t^2}{\gamma - 1} = \frac{\gamma + 1}{2(\gamma - 1)} a^{*2} \dots (21)$$

a_t and a^* remains constant along stream line.

So, let us look at the special forms of energy equation and we will revisit this again adiabatic for adiabatic steady 1 dimensional flow. If we invoke this 1st law of thermodynamic for a control volume, it becomes $C_p T_1 + \frac{V_1^2}{2} = C_p T_2 + \frac{V_2^2}{2}$. What will I do, if I put this for a calorific value? You know for a calorifically perfect gas, C_p of course, I have taken. Because, this equation is valid only for calorifically perfect gas. So, if I take the values of C_p . You know like in terms of gamma and r it becomes $\gamma R T_1$ divided by gamma minus 1.

Similarly, for the right hand side station 2 γr_2 , what is this? This is basically your speed of sound of 1 square. Similarly, I can write down this is as, 2 square. Then I can look at it as a 1 square divided by gamma minus 1 plus gamma 1 square divided by 2, which is equal to 2 square, divided by gamma minus 1 plus V_2^2 divided by 2. Let us consider a station 2, let us say it has come to a stagnation point. That means V_2 is equal to 0. If I will do that, then I will get this expression, a_1^2 square divided by gamma minus 1 plus V_1^2 square divided by 2, which is equal to a_t^2 square divided by gamma minus 1. Because, this term is equal to 0.

So, a_t is the total speed of sound associated with the same point. It is related to the same point, so if it is so, I can consider that the 2 stations along a streamline. Then equation 17

becomes a 1 square divided by $\gamma - 1$ plus v^2 square divided by 2 , which is equal to a 2 square divided by $\gamma - 1$ plus v^2 square divided by 2 . This is equal to a T square divided by $\gamma - 1$ is constant, what it indicates is that, this term remains constant along a streamline between the station 1. 1 and 2 in this expression will remain constant along a streamline.

As I told you earlier, that if the streamlines are emanated from a uniform flow at the inlet. Then naturally along the whole domain, this T will remain constant. So, let us consider the station 2 as a sonic condition, v is equal to a^* then equation 8 becomes like a 1 square divided by $\gamma - 1$ plus v^2 square divided by 2 . I am saying that, this v is equal to a^* , so if it is a^* , then this becomes a^* . If I just add this together, I will get $\gamma + 1$ divided by 2 $\gamma - 1$ a^{*2} , which is equal to basically constant. Therefore, I can generalize and say that a square divided by $\gamma - 1$ plus v^2 square divided by 2 is equal to $\gamma + 1$ divided by 2 $\gamma - 1$ a^{*2} .

Of course, between 2 stations, I can write down this equation 9 as the same. If I compare this equation 18 and 20, you would know that this equation. I can say that T^2 square divided by $\gamma - 1$ is equal to $\gamma + 1$ divided by 2 $\gamma - 1$ a^{*2} square. I am comparing this, because like equation eighteen, this portion is same. Therefore, this is the same, which means this portion is same as that portion.

So, what it indicates is that T and a^* remain constant along a streamline. It will be remaining constant, so this is very important, what you call concept, which we will be using as you go along and I will stop over here. In the next class we will be looking at the sonic flow, continued from here and see what we get. Then we will be moving into quasi 1 dimensional flow. Is there any question? Do you people want to ask any doubt? If there is no doubt, then we will stop over here.