Fundamentals of Aerospace Propulsion
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Lecture – 12

So, let us start this lecture 12, with the thought process from Max Planck it goes like this, anybody who has been seriously engaged in scientific work of any kind realizes that over the entrance to the gates of temple of science are written with words that is 'you must have faith'.

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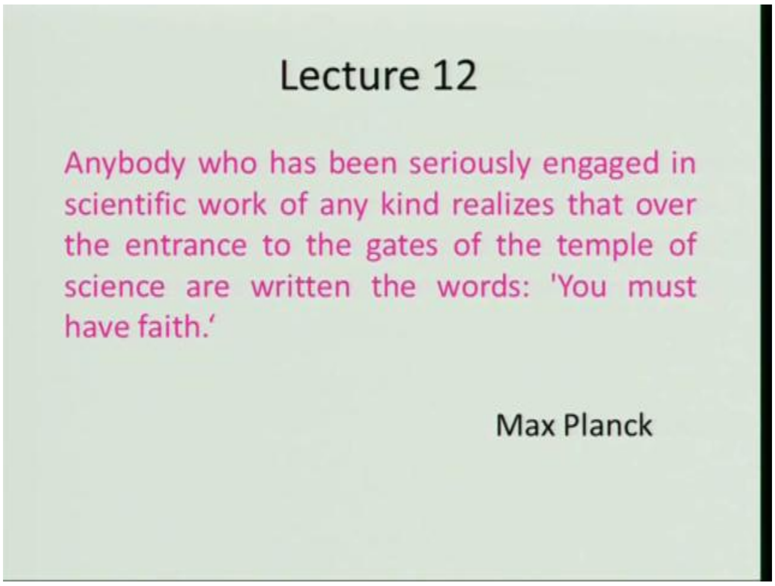

That is true for the life, if you want do to carry out a decent life you should have faith in yourself. When in yourself cannot be achieved unless you have faith in relationship. Faith in the heritage therefore, it is very important to look at the holistic approach to the life. So, that also Max Planck has talked about the science. So, let us recall what we learnt in the last lecture, we basically use the isentropic analysis to discuss about the processes which is involved in a nozzle. We discuss about basically 2 nozzle 1 is conversion nozzle and diversion nozzle. Then, I introduced a concept known as Choked condition or Choked flow.

Now, we also invoke the mass flow rate expression or mass flux expression and figure it out how this choke flow can be avoided in real situation. I had asked 1 question that whether we can use this Choked flow as it is disadvantages all the time, all the times for our advantages. Is there anybody really mailed over it, Let me give you a clue that when you were designing an experimental setup, can you use this concept to your advantage, because unless otherwise you get your answer to the question of yours, of course.

Then, you cannot get faith in yourself, so it is very important to think about it and get the answer. So, let us now look at what do we mean by shock, what you mean, we do all experience shock in our life. For example, I was travelling may be a month back in my cycle on the road and suddenly a motor bike bumped into me, I could not really do anything ,right?

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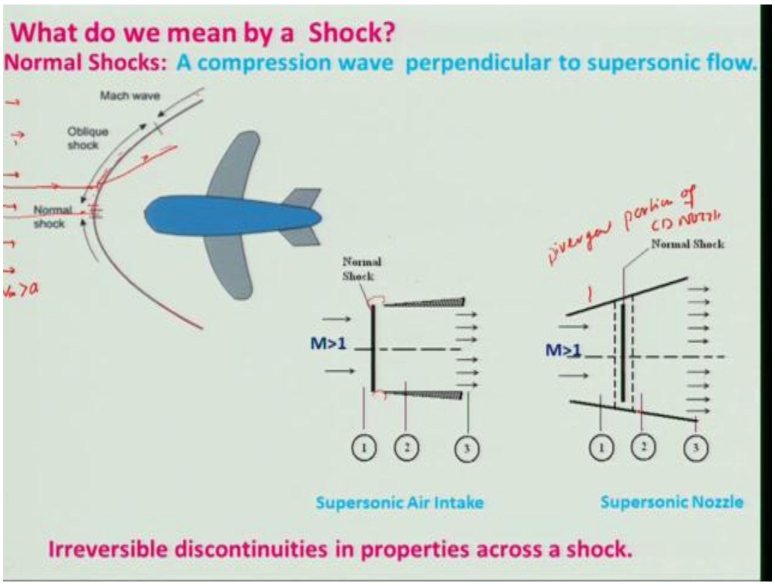

So, why I could not do anything, is because it came suddenly such that my mind was freeze. I could not really do anything to take, to have any action on it. So, that is 1 kind shock what we experience any sudden things which will give a shock that means, what is the meaning? it will be giving a kind of a freezing or kind of coming to an end. So, you cannot take any action. Similarly, what happens in the fluid, that when the fluid is flowing, right? You suddenly ask to stop, what will happen? I will give another example to understand, what you mean by shock.

In the highway, all the cars are moving at a very terrific speed, right? May be hundred kilometer per hour, all of a sudden a police man says that stop by using hand, what will happen to the cars, which are moving at a terrific speed, What will happen? They will come. Let us say, at this place the police man is there, but the far away from the, this you call police man signal, nobody can see that he has asked to stop.

So, then what will happen to the car? Will it be coming here and at a certain distance it will be stopping or trying to stop. As a result they will be colliding and of course, after that they will again move. Of course, they may be in, I mean some refined out and they may move at a slower velocity. If they can manage to do, so that means what is happening is shock is been formed. Of course, this thing you need to imagine and then have a feel, question arises in this example, why it is happening and enough time to stop, that means the signal given by the police man or traffic policeman cannot be really followed by that already they have combined.

So, that means they are getting compressed nearby to the police post, right? Suppose this is a police post nearby they will be trying to get compressed. Let us look at whether, similar thing is happening here in an aircraft which is moving. This aircraft, let us say it is moving and air is still that means air is moving is the aircraft is moving, right? For the time being.

Then, air is, What is happening. Then, it is like a piston which is moving the fluid towards the left, towards this side. That means he would recall that, I had taken a piston cylinder examples. When we are discussing about the speed of sound that means this is creating a disturbance because of movement of the air of 1 aircraft in the steel fluid and that disturbances travels at what speed, it will be travelling that will be speed of sound.

Now, that means it is a disturbance created by the presence of an aircraft or any other body, let us now imagine that my aircraft is just stationary and fluid is moving at a very high-speed and my aircraft is test scenario which can happen in a wind tunnel kind of things. So, then if the flow is subsonic then what will happen? It is because, if the flow inlet fluid. Here, you know it is moving towards the aircraft at a subsonic speed.

Then, what will happen because the signal from the what you call this aircraft can be reached the fluid element. The fluid element will take care of it and such that it will be just gliding over the surfaces, but if the velocity here is quite high. If it is v infinity is greater than the speed of sound then what happens, It will be coming straight. For example, if I take this it will be coming straight, right? Then, it will be known that by certain distance, that of course depends upon several conditions like this is what you call an aircraft such that, then it will be clawless in these regions.

So, for the streamline is concern and after that will be going to slow at a slowdown and then travel. What whether it will be subsonic, Whether it will be supersonic, How it will happen?

All those things, we will be looking at, but if you look at as a result there is a theme region here which occurs. That region you know, is you can call it as a wave basically. It is a compression wave, keep in mind that, in this case it will be stationary, right? If you look at the these region, in this region just in the stagnation point over here, I can say that it is perpendicular to the velocity.

This region, we call it as a normal shock that means a compression wave perpendicular to the supersonic flow. Then, we call it as a comeback keep in mind that this is not really perpendicular except at the stagnation point or the line along the stagnation point, it is little bit curve, but we are assuming it to be a normal for simplification. However, in this region, it is an oblique shock because it is making an angle to the incoming fluid.

Therefore, we call it as an oblique shock and in this region we call it is Maxwell, what is that Maxwell? Can anybody tell me that is, Maxwell number is 1. That means when the speed v or velocity fluids, velocity is closed to speed of sound then there will be a compression where density of it will be quite less that is known as Max wave, will be invoking this concept little later on. If look at what will be the thickness of this waves, what will be order of can anybody guess 10 power To 5 what? 10 power to minus 5 meter is it?

So, it is really big or small, Actually it is 10 power to minus 5 centimeter in other words, it is 10 power to minus 7 m, m in other words it is 0, 1 microns, it is very thin region and let us look at where else we will be encountering so far the propulsive devices concerned. The normal shock, that is if you look at it like when an air intake basically supersonic air intake is being used.

There is the likelihood of that a shock will be formed near this call leave. It may be with touching or it may be away of course, those things which we may discuss sometime and there is a normal shock which we can think of, but keep in mind that what I have shown here did not be formed in real situation, particularly in this region it will be different. If it is so, if it is away from that it may be bow shock like this. Is a bow shock you know? As it is like a bow and arrow.

So, however we can approximate it is as a normal shock and besides this there is another place where we will be encountering normal shock. To the in a c d nozzle c d means conversion diversion nozzle. Do you remember? I was talking about it yesterday and what I have shown here is the diversion portion of c d nozzle and conversion portion is there and upstream which is not been shown in this diagram. Of course, there will be throughout conversion portion then throughout the diversion portion, which I have shown here.

Under what condition a normal shock will be formed? It is not only the Choked, but also when the pressure, back pressure will be above the design pressure then only a normal shock is likely to form depending on the how far this back pressure is higher than the designed pressure. For which, this you know conversion dock diversion nozzle is designed is being with the location of the shock will be dependent on that. Keep in mind that in real situation this shock would not be what I have shown here like a normal shock because boundary layer will be there, when boundary layer is there, the fluid will be subsonic.

Similarly, in this region it will be quite complex in nature which I am not going to discuss now, maybe later on we will see in the major portion because, in the accelerating flow the thickness of boundary layer is very small. Therefore, in major region 1 can consider to be normal shock, right? Now, what we will do, we will be trying to analyze with this normal shock and see how we can predict the properties across the normal shock.

So, let us do that and keep in mind that this properties, right? Will be changing across a normal shock and there will be irreversible discontinuities in the properties across the normal shock. That means the processes will be irreversible in nature, but then how will do it because, what we have discussed now is only for anisotropic not for non-anisotropic?

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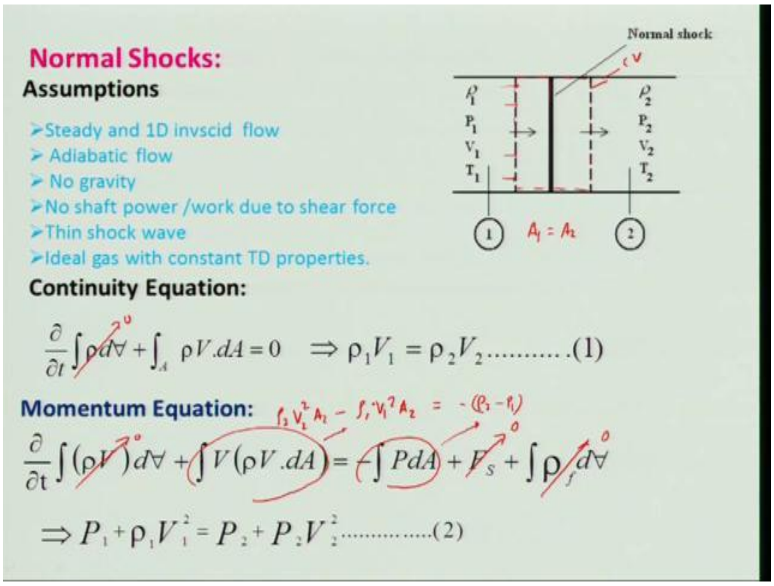

As I said that we will be using anisotropic relations all the time, but analyzing our, what to call propulsive devices and the phenomena like shock permissions and other things. So, let us look at normal shock which is occurring over here. In this region keep in mind, this is fixed, what to call normal shock and in which the fluid is coming over here at certain velocity. We want and it is living the normal shock at certain. You know, fluid properties like rho 2 V 2 P 2 and T 2 and in the inlet to this normal shock that is rho 1 P 1 V 1 T. To keep in mind that both are different and in this case we are taking that is, A 1 is equal to A 2, because it is a constant. They are the kind of things we are considering and we will include this normal shock and take this dash line as a c v control volume. That means my control volume will be like this.

Now, we will make certain assumptions, which we have already done just for the completeness. I will enumerate them that we will be considering it to be a steady one dimensional in viscid fluid. That means that is no dissipation effects, that means there would not be any friction effects in this case and dissipation. Of course, there can be different kinds, it can be mass diffusion. it can be. You know, like thermal diffusion and other things and there is no heat is going out of this control volume.

You are not really heating it nor there are any combustion taking place nor are you cooling it. So, we can assume to be an adiabatic. Of course, the fluid is will be at a higher velocity, right? Then, naturally we need not to worry about gravity and there is no work done here. So, no soft work and keep in mind that shock is thin, as I mentioned just now, I mean that the thickness of the, what you call the shock will be around 10 power T minus 7 mm. So, it is quite thin and we are using ideal gas equation with a constant property and thermodynamic properties.

That mean that, it is basically what calorically perfect gas, we will be using. We will be now considering, what you call rather, invoking all conservations loss like conservation of mass. That is continuity equation and if I take this control volume and also the steady flow condition this will be 0. Then, for this control volume, what it would be, It will be between station 1 and station 2, it will be rho 1 V 1 A 1 is equal to rho 2 V 2 A 2 as A 1 is equal to A 2. Then, rho 1 V 1 is equal to rho 2 V 2, which we have already seen. So, we need them to spend much time on it.

Let us look at momentum equation, that is steady process so this will be 0 and then we will have a what you call the surface, that means we are not considering the friction forces. Therefore, we need not worry about it, no gravitational or the body forces rather will be 0. So, this turns out to be what if you look at this term convective term that will be turning out to be rho V 1 square right or rho V 2 square minus rho V 1, this term if I write down rho 2 V 2 square A 2 minus rho 1 V 1 square A 2 is equal to minus P 2 minus P 1. Can I not write like this, because this term this is for this term is for this left hand side and this term is for right hand side.

So, when I simplify it, I will get basically P 1 plus rho 1 V 1 square is equal to rho 2 plus P 2 square V 2 square right and because A 1 A 2 will be basically cancel it out kind of things. So, we may not consider A 1 A 2 kind of things, so and keep in mind that A will be there here and this will be what you call this is A 1 is equal to A 2 is equal to A. Therefore, it cancel it out, right? If you look at this energy equation and similarly, we can do this.

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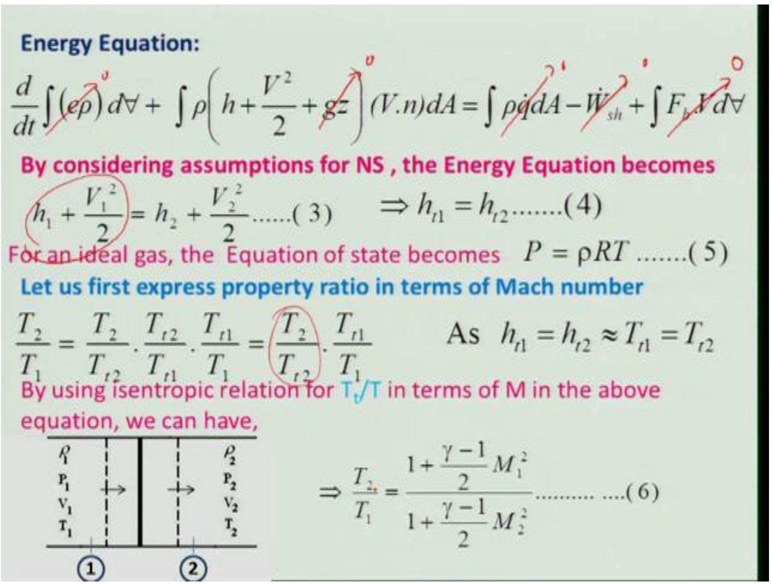

This will be 0, steady blow passes, there is no body force gravitational no heat losses. This will be 0 sat hoard. This will be 0, there is no body forces, so there will be 0, so that turns out to be something like which we are very familiar, h 1 plus V 1 square divided by 2 is equal to S 2 plus V 2 square divided by 2. If you look at this term, what is this term? This term is nothing but your total Enthalpy. That means total Enthalpy in the station 1 is equal to total Enthalpy in the station 2 that means, what the total Enthalpy remains constant across a shock.

For, an ideal gas the equation of the state becomes P is equal to rho R T that we know very well. So, let us first express the property ratio in terms of Mach number right because we have seen this equation 1, 2, 3, 4, 5, right? So, these equations we will be expressing now using this equation. Will be expressing property ratio across a shock in terms of magnum, so what will do then. Then, we will find out this is T 2 divided by T 1 is equal to T 2 divided by T , T 2 into T, T 2 divided by T 1. Can I not write like this, right into T, T1 divided by T 1 and right.

Similarly, we can write down T 1 T 2 into T, T 1 by T 1 and keep in mind that, what is saying that this is a basically station 1. If you look at it, I can take this as a station 2 station 1 and then you know, we can assume that s T 1 is equal to s T 2. That already we have seen by invoking equation 4 and if you take this calorically perfect gas or we can say that property is not changing.

Then, I can write down T t 1 is equal to T t 2. So, then by doing this thing we can really get this expression as I told and now what this expression? How can I express this in terms of Mach number, I can express this in terms of Mach number of M 2 by using isentropic relationship, but I have already told that this is not isentropic right or irreversible processes occur across the shock. Then, how can you use, this is something wrong am I right? Is it really wrong because there might be dissipation across the shock or is irreversibility across the shock, but however, so far station 2 is concerned or station 1 is concerned, it will be isentropic.

Therefore, we can relate that, if I will do that we know that T t 2 divided by T 2 will be what? 1 plus gamma minus 1 divide by 2 M 2 square. I can write down here and similarly, I can write down 1 plus gamma minus 1 divide by 2 M 1 square. So, what I am doing, I am just trying to relate this property as station 2 and station 1 in terms of Mach number. In one case it is Mach number 1 and Mach number 2.

So, if I know Mach number 1 and Mach number 2, I can find out very easily the temperature ratio for across the shock, although the shock is irreversible. You know, the process is inside the shock will be irreversible in nature, but however, I am using isentropic elation that is the beauty of it. So, similarly, we will try to look at this express the properties across the shock that is P 2 by P 1 V 2 by V 1 rho 2 by rho 1 all those things will do.

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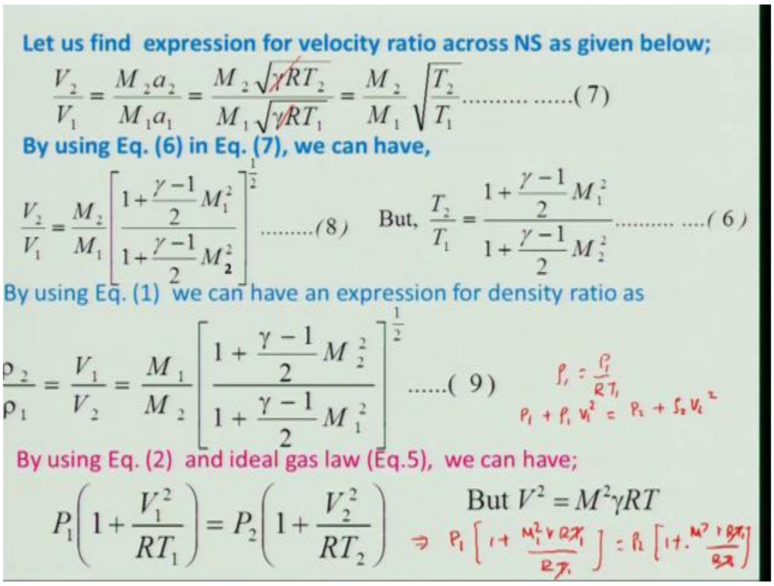

So, let us find out expression for velocity ratio across the normal shock as given below. How we will do that? We know, V 2 by V 1 is equal to what I need to express in terms of Mach number. So, naturally I will have to express in terms of what, Mach number natural. I need to relate those things to the speed of sound and also the Mach number by there from the basic definition, that is V 2 by V 1 is equal to M 2 A 2. By definition, V 1 is nothing but M 1 A 1 and where A 2 or A 1. That is the speed of sound will be nothing but equal to, that is root power R gamma R T 2, that is A 2.

Similarly, for a 1 and if you look at, I can cancel this term this gamma R. So, then that comes to be M 2 divided by T 1 and I have already derived the expression for T 2 by T 1. That is in equation 6. So, if I just put this equation in the 7, I will get V 2 divided by V 1 is equal to M 2 dived by M 1 and this term which is already power To the half. So, in the similar way I can get an expression for density ratio. Of course, for That we need to invoke the equation of what continuity or continuity equation that is rho 1 V 1 is equal to rho 2 V 2.

So, then I can write down rho 2 by rho 1 is equal to V 1 by V 2 right and V 1 by V 2, I can get from this equation and rewrite. If you look at this just opposite of that an equation 9 just opposite to of equation 8, so using equation 2 and ideal gas law can have momentum equation in this format, you know, what do. We are basically writing instead of density, what I am writing is here you know, this is this was rho 1 writing P 1 by R T P. I have taken out of this that means what I am writing rho 1 is equal to P 1 by R T 1.

So, earlier expression was P 1 plus rho 1 V 1 square is equal to rho 2 plus P 2 plus rho 2 V 2 square, so in plus of rho 1 am putting it P 1 R rho 1 t I am taking it back. So, I am getting this expression, if you know that this we square this nothing, but M square gamma R T. So, if I write down this equation, what it would be this will be P 1 plus M square gamma R T divided by R T 1 T 1. So, this I will cancel it out, this is equal to P 2 1 plus M square gamma R T 2 divided by R T 2, it means, what I am getting here P 1 plus gamma M 1 square and is equal to plus gamma M 2 square and I can rewrite this expression as P 2 by P 1 is equal to 1 plus gamma M 1 square divided by 1 plus gamma M 2 square, right?

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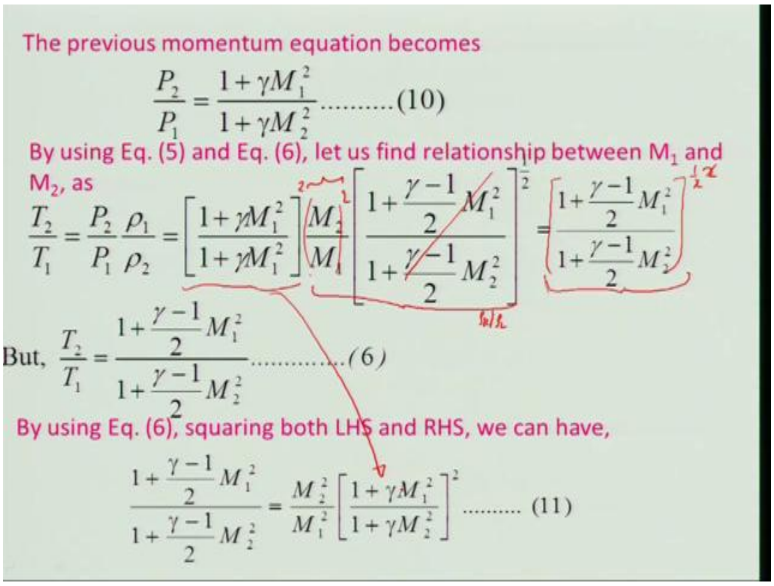

This is the pressure ratio relationship between you know, Mach number and gamma. So, in a similar way by using this equation 5 and equation 6 we find relationship between M 1 and M 2 because, what we done really basically express all the ratios like P 2 P 1 rho 2 rho 1 T 2 T 1 m in terms of inlet Mach number. That is M 1 and M 2, but I am may be knowing the M 1, but do I really know M 2 if I do not know then naturally I cannot use this expression.

These expressions will be useless. For example, if I know this M 1, I cannot evaluate P 2 by P 1 although I know gamma, let us say for air, it is 1.4, but I need 2 M 2 that means I need to find out a relationship between M 1 and M 2. If I know that, then I can substitute expire this P 2 by P 1 in terms of M 1 right inlet Mach number. Then I can do what we will do, we will know that by invoking this equation 5 and 6 to find a relationship between M 1 and M 2.

It is quite important, so what we will do is that we have already seen that T 2 by T 1 is nothing by P 2 by P 1 rho 1 by rho 2 and P 2 by P 1, it will be nothing but this portion, that is 1 plus gamma M 1 square divided by 1 plus gamma M 2 square, which has 1 here right at this region and rho 1 rho 2 you can use that expression. Then, put this equation here for the rho 1 by rho 2, but already we know that T 2 is equal to 1 plus gamma minus 1 divided by 2 M 1 square divided by 1 plus gamma minus 1 2 M 2 square, that means this T 2 expression must be same as that am I right yes or no.

So, that means I can write down this portion is same as that if I cancel it out, this is because if you look at this term, it is same as that if I cancel it out that, what I will get half over here? It is because I am cancelling now this is not there. If I divide this portion and if I take and divide it here it will go half right 1 up square root. So, if I will then square this portion that means I am squaring here also, this squaring here and if I do square this will cancel it out. Then, I will get an expression that is 1 plus gamma minus 1 divided by 2 M 1.

Square this, this is the portion because it will cancel it out. I am taking this portion over here in this place is equal to M 2 square divided by M 1 square. This portion, I am taking over here and then this portion am taking square. If you look at this portion am taking this and if you simplify these expressions and also to solve this equation you will get an expression.

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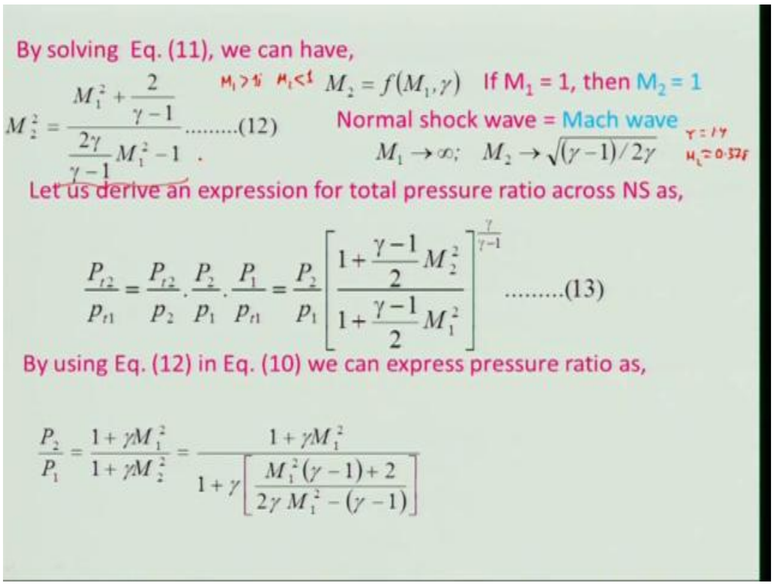

Like this, M 2 square is equal to M 1 square plus 2 divided by gamma minus 1 is divided by 2 gamma power divided by gamma minus 1 M 1 square minus 1, keep in mind that, this is first solution for whatever the equation. We have seen, right? Now, if you look at this equation 11 we have G 1 through a very adverse path, but it can be used, but it can be derived very easily by using the concept of specialize. What we call energy equation and then you will reach a pantry relations. Which means all these equations you need to all this point, we need to go to the characteristics point or the characteristics Mach number, which I am not doing, that I will give an assignment to look at it because this is little adversely.

I thought, I would do that and there is a very simpler part which give people need to look at, it is because those concepts will be using in our what to call later on while solving certain problems, which is easily what it indicates here. It indicates that aim to a function of M 1 and gamma right, it is saying that M 2 is a functionary M for a particular. That means M 1, you know, will be controlling what will be the M 2.

For example, if I say that M 1 is equal to 1 that means, what the velocity is equal to speed of sound. If M 1 or the, what to call free steam velocity in this case, it is a shock, which is equal to the speed of sound. Then, M 2 will be, if you put this 1 what will be, if I substitute this is equal to 1 M 1 square is 1 what will be returns out to be this portion. Will it be equal to 1 and when M 1 is equal to 1. Then, M 2 all so equal to 1 right that means that is known as a normal shockwave, which is basically a normal shockwave and we looking at this is very singly and is very small.

Therefore, it is now as Mach wave right, it strength will be very small now, if M 1 is greater than 1, what will happen if you look at this term, What will happen? You will see that M 2 will be less than 1. If M 1 is greater than M 2, it will be less than 1 that means, if the supersonic of the upstream flow is supersonic. Then the downstream of the shock will be always subsonic, but if M 1 is tending to increase that means it is very high. Let us say 40 30 Mach number, I mean very high than what will happen to the M 2. It can be 0, is it?

So, it will go to certain values, when you take the hospital rule apply to the equation 12, you will get the relation that is root over gamma minus 1 divided by 2 gamma. If I take gamma is equal to 1 point 4 under these conditions, what will be M 2? That means M 2 Mach number tending towards this will be something 0.378 behind, this you cannot go vocalized noise kind of thing. It will be tending towards that value right when it is very high.

So, therefore it cannot be 1, it is very simply small and if it is 1000 Mach number, then it will be 1 over, that is not that way it is, not that small, so it is tending to a value which is behind that, we cannot so. Let us derive the expression for total pressure ratio across the normal shock which is very important P T 2 divided by P T 1 which is equal to. We will be using similar relationship even in the cycle analysis.

Here, I can rewrite P T 2 divided by P 2 into P T 2 P 2 divided by P 1 into P 1 divided by P T 1 right and then can put this expression, although things can get this right equations very easily right. Now, what will we do, We will put here the expression for M 2 in this place and because to evaluate this P T to by P T 1. We need to evaluate this P 2 by P 1. So, what we will do now is P 2 by P 1, which is equal to 1 plus gamma M 1 square divided by 1 plus gamma M 2 square. So, in place of M 2 I will put this expression 12 and I will get this expression if you simplify it right.

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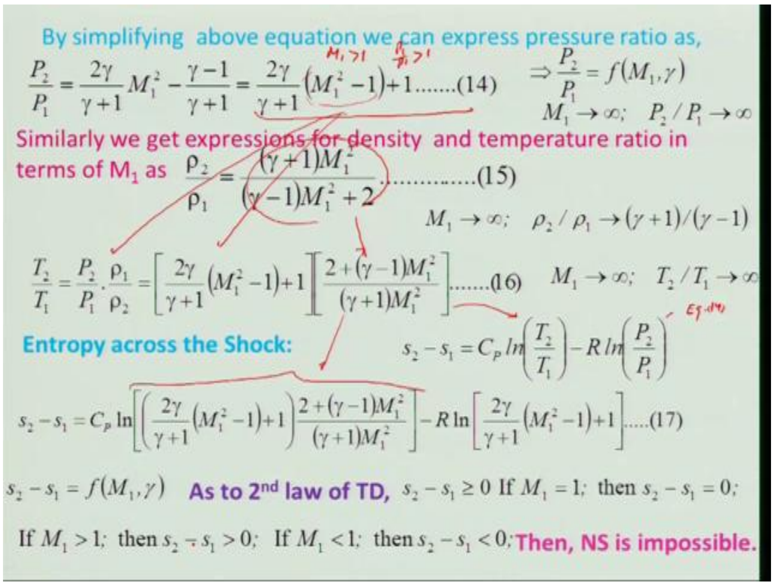

You will get an expression, P 2 divided by P 1, which is equal to 2 gamma divided by gamma plus 1 M 1 square minus gamma minus 1 divided by gamma plus 1. If you simplify further you will get to know, we can take it T 2 gamma divided by gamma plus 1 into M 1 square minus 1 plus 1 and this little algebra you can do. I will leave as it is and what it indicates that P 2 by P 1 is function of inlet Mach number. That is M 1 and gamma, right? So, if I at M 1, which is equal to 1. Then, what happens P 2 by P 1 will be 1 do not be any changes in this study equation, but if M 1 is greater than 1 what happens to the pressure P 2 by P 1 it will be greater than 1, because M 1 is greater than 1.

These term will be positive or the greater Than 1. You know, M 1 square, so little positive therefore, and it will be more than 1, this term P 2 by P 1 will be greater than 1 it as it goes this terms will be P 2 by P 1 will be goes on increasing what it indicate, but it indicates downstream pressures will be higher than the of stream pressure that means start equation alright. So, and if M 1 is going towards infinity that becomes P 2 divided by P 1 lead towards infinity. Similarly, you get expression density and temperature ratios like rho 2 by rho 1 is equal to gamma plus 1 M 1 square divided by gamma minus 1 M 1 square plus 2 and in the similar way it is a function of Mach number 1 and gamma and when gamma M 1 is equal to 0.

So, what will be that, than means the term will be 0 and these also will be 0 right, it will be having certain values whereas when M is greater than 1 that rho 2 by rho 1 will be greater than 1. If M 1 is greater than 1 right, so when M 1 tending towards infinity the rho 1 divided by rho 2, divided by rho 1 tending towards gamma plus 1 divided by gamma minus 1. Similarly, we can have P 2 by P 1 in terms of Mach number and gamma we can derived this expression by just putting these values over here means, I can take these values over a here .You know, basically these values over here and this portion, I can take over here and get this expression and when m is greater.

You know, like than 1 P 2 by P 1 will be greater than 1 right, all less than 1, what it would be if M 1 greater than 1 vocalized noise right, it will be greater than 1 right and if M 1 is tending, you know, like tending towards infinity, T 2 divided by T 1 will have tending towards infinity as usual. So, let us look at to 1 concept. You know, which is very important whatever we are derived this expressions will you valid even if the of steam smack number is greater than 1 right, but is it is physically possible or not how we will determine because we are never told that Mach number cannot, well less than 1 right, with the shock will be formed right, we started with the equation.

Now, where is, we are told that look number cannot be equal whatever equation. We have derived till now, then we need to involve the second law of thermodynamics to judge whether the normal shock at the shock will be formed or not for. That we need to look at and entropic across the shock and that is s 2 minus s 1 is equal to C p l n T 2 by T1 minus R l N P 2 by P 1 will be using this expression.

You know, T 2 by T 1 equation 6 over here and similarly, substitute these values. You know, P 2 by P 1 from equation, you know, 14 case I will substitute these values, I will get, if you know recognized these term is nothing from here. Similarly, these terms is from the upper want like here now what is saying it saying that entropy change in a function of M 1 that is inlet Mach number and gamma. If there is according to the second law of thermodynamics, that is entropy always will be greater.

Then, it is equal to 0 that is the condition right and, if we can never will less than 0 I may right, so let us see that where it is been violated. If M 1 is equal to 1 then what will happen to 2 minus 1 is equal to 0, that means entropy change is 0 which is possible that means which is corresponding to a Mach where and, if M 1 greater than 1 s 2 s 1 s 2 minus s 1 it becomes these term will be greater than 0,yes or not ? If I put it M 1 greater than 0, these term will be you know, these term and this zone it will be larger.

So, that this term also will be and together it will be greater than 0, but if M 1 is less than 1 than entropy change across the shock wave will be than 0, which is not really possible. Therefore, if Mach number inlet Mach number is subsonic then shock cannot be formed. So, normal shock cannot be formed, but question arises why the entropy will be increasing across a shock when the inlet Mach number greater than, why? What is the physical phenomenon that is really occurring for entropy to increase across. Why is it so? You revise, but how it is occurring and why? I mean what are the causes, yes that means, if you look at the shock is very thin as I told 10 power minus 7.

So, the gradient of temperature, pressure and density are quite high to cause dissipation. Dissipation in case of what viscosity, I mean conduction and also the others, right? Therefore, it is an irreversible process, right? Across that and entropy change. You know, occurs across the shock, so with this we will stop over we will see in the next class.