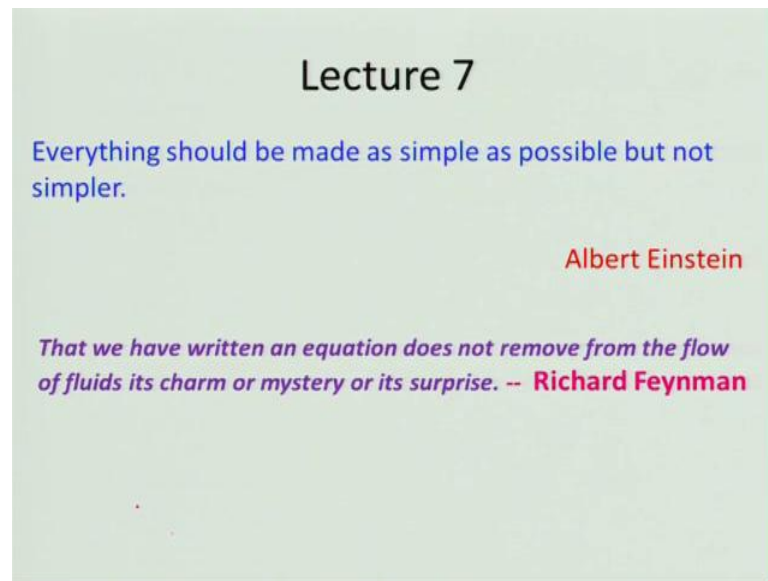


**Fundamentals of Aerospace Propulsion**  
**Prof. D. P. Mishra**  
**Department of Aerospace Engineering**  
**Indian Institute of Technology, Kanpur**

**Lecture - 07**

Let us start this lecture with a statement from great scientist Albert Einstein, who happens to be my favorite scientist.

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He says that everything should be made as simple as possible not simpler think about it you need to think about it may be we will discuss later on, but let us recall what we did in the last lecture can anybody tell me what did you learn because recalling is a very important aspect of learning you need not to look at notes rather recall from your mind. Let me you know tell that what we did we started this class with the concept of using the various coordinate systems whether flow will be 1-dimensional or 2-dimensional or 3-dimensional. And when we are talking about it which coordinate system we should use whether it will be rectilinear coordinate system, or cylindrical coordinate system, or the spherical and it is important to know that which one you should use where.

But in most of the aerospace applications particularly propulsive devices, we prefer to use cylindrical coordinate system because it is easier to you know analyze and write the equations. And then we move into how to derive the governing equations to carry out quantitative analysis. And the governing equations are best on the principle of

conservation of mass, energy and momentum and of course, we have not discuss about conservation of you know what you call the space. It is not space it is rather the mass being conserved, but spaces equation we did not discuss, but however we will be discussing it whenever we will be venturing into analyzing reacting flow in case of combustion.

And we also added one equation that is equation of state particularly for the an ideal gas in other words it is the ideal gas law which we have included in the governing equations. And all these equations if you look at it quiet complex and those equations has been derived since you know a long time, but if you look at particularly Navies root equations you know is not being solved in totality that is the big challenge right yet to be resolved. You can solve the Navies root equation or you can solve it you know by computational method which is a ad-hoc solution rather, but there is no identical solution for the Navies root equation complete navies root equation till now, but some simpler one you can get like paisley flow boundary layer flow you know Blasius equation I think you can solve.

That means if I look at a really this mathematics is not going at this moment it has stopped going since a long time back and some of you could solve then you know it will be a great thing as such. So therefore, another scientist who is Richard Feynman who is a very great person he says that, we have written an equation does not remove from the flow of fluids its charm or the mystery or its surprises. If you look at the way he has stated this you know this sentence he says what is the feeling for the fluid mechanics he was having.

And he is one of the greatest scientist in last century and if some of you might have read his book you know surely you are joking mister Feynman any of you have read right. It is a very wonderful book he has written another book also you are I am by Richard Feynman. And he is having a series of book the volume 1, volume 2, volume 3 on lecture notes on physics which some of you might have if not, then look at it is a wonderful book so and components also is there and fluid mechanics.

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**Basic Principles of Compressible Flow (Gas Dynamics)**

What is compressible flow ?

$$\text{Compressibility} = k = -\frac{1}{v} \frac{dv}{dP}$$

$$\Rightarrow k = \frac{1}{\rho} \frac{d\rho}{dP} \dots\dots (1)$$

$k_T = -\frac{1}{v} \left( \frac{\partial v}{\partial P} \right)_T$      $k_s = -\frac{1}{v} \left( \frac{\partial v}{\partial P} \right)_s$

**For water :  $k = 5 \times 10^{-10} \text{ m}^2/\text{N}$ ; at 0.1 MPa,  $T=298 \text{ K}$**

**For air :  $k = 5 \times 10^{-5} \text{ m}^2/\text{N}$ ; at 0.1 MPa,  $T=298 \text{ K}$**

**Air has 4 orders of magnitude compressibility than water.**

So, now we will be trying to look at the basic principles of compressible flow in some books you will find it is known as gas dynamics how it is different from the aerodynamics can anybody tell me or it is the same. Of course, one can say that flow related to the air you can call it as the you know aerodynamics, but then how the gas dynamic is different than that. Aerodynamic is the whole gamete of the fluid mechanics or fluid dynamics related to the air, but however in case of gas dynamics we will be confining to the compressible flow. That means question arises what is an what is a compressible flow or what is an incompressible flow.

That means if compressible flow is there, there is another flow which is incompressible right, when you talk about it then naturally question arises that what is it flow will be incompressible or flow will be compressible or fluid will be compressible or fluid will be incompressible. So, what is the meaning of it how will charge whether it is compressible or incompressible if I take a solid one let us say if I take a chalk of course, if I apply a very little pressure what will happen there is no change in it say for size.

But if I apply a large amount of pressure then what will happen it will break may be right, but if I take a what you call a water right and I am trying to pressure it. For example, if I take in a syringe right and I have taken water here right and syringe what I am doing I am just trying to press it right is it possible I can press it I cannot why because if I am pressing it is it any change in the volume. There would not be change in

the volume, but if I take air, if I take air right and try to let us say it is here I am trying to press it, I am closed keep it mind that I have closed this inlet or the nozzle and I am pressing it I could press it.

Of course, after certain time it is difficult I need to pressure more put more force or the pressure right that means what it says, it says that the air is compressible and water is incompressible. That means the change in the pressure you know or change in the force what is being applied it will be changing change in the pressure or when I am applying pressure its volume is changing. And if volume is changing then I am calling it is as a compressible if it is not changing it is an incompressible right. We will take a may be a fluid element just to illustrate it further and this pressure is applied on this fluid block which is having let us say one unit block. This means in a 1 centimeter, 1 centimeter, 1 centimeter cube and is having certain volume and I am applying the pressure let us say  $p$ ,  $p$  all are there it is in equilibrium.

If I go on adding this pressure let us say  $dp$  here everywhere right all the places then what will happen that means it will be subjected to string provided it is a fluid right it is compressible. For example, if I will do then what will happen this volume has been change due to the application of pressure which was in equilibrium if I consider the fluid element. Now, what is happening the change in the volume is happening with the pressure so then if I want to judge quantitatively because this I am talking about qualitatively you know it is being decreased. But if I want to quantify it then I need to define a term right which I can call it as a compressibility which will indicate how far it is compressible.

Then what I need to do basically it is the same thing what I will do so change in specific volume per change in the pressure per unit specific volume, we call it as a compressibility  $K$ . And where this minus sign comes from where because always whenever you are applying some pressure or the force there will be change in volume. It will be decreasing volume this decrease, we call because of this because you know final volume minus the initial volume divided by the you know pressure change over the unit specific volume therefore, negative sign comes into picture.

Now, whenever you are saying this is it good enough to say this because we know if I take a cycle pump. You know may be earlier days I remember when I was a kid we used

to pressurize the or the in flat the tube of the cycle or the you know by using a piston cylinder pump or air pump piston cylinder air pump usually like we will have to do this. Then if I apply the pressure and go on in flattening my tube then what I will feel what is happening to the cylinder if it is metallic now it is of course, you know people are not using metal. So, you would not feel it whether it is hot or cold what it will be it will be hot right why it will be hot.

There will be two reasons one is that because the pressure you know you are applying this molecules will be compress you know then there will be increase in pressure in the cylinder and there will be temperature. Beside this you know a piston is moving so there will be the big action friction will be there so there will be heating something generated also, but those things we will be saying it neglected as compared to that further discussion however you cannot neglect it. So, naturally this compressibility what we talk about is not really good enough.

That means I need to define a term which will be either the that means temperature should be remaining constant that if I do it I call it basically isothermal compressibility right and how I will define it then this will be minus 1 over  $\nu$  and  $\frac{d\nu}{dp}$  at  $T$  and keep in mind that this is a what you call the partial derivative. And similarly, if there is no heat loss if I take that example of piston cylinder then I can define another compressibility right because there is no heat loss. So, it will be adiabatic if I assume it is isentropic that means there is no effect of friction in that type isentropic reversible. Then I can call it as a another what you call compressibility is known as  $\frac{1}{\kappa}$  I am saying IS isentropic.

And we know that specific volume can be related to density because  $V$  specific volume is  $\frac{1}{\rho}$  so if I write down this equation compressibility I can write down  $K$  is equal to  $\frac{1}{\rho} \frac{d\rho}{dp}$  and that is basically compressibility. Similarly, you can talk about isothermal compressibility and isentropic compressibility by the partial derivative keeping one of them constant.

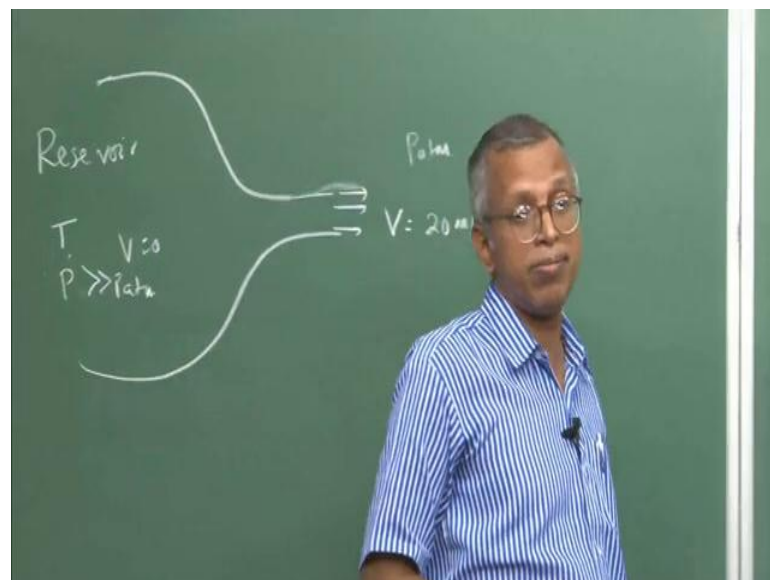
Now, if I talk about this compressibility right is it I am talking about fluid or I am talking about flow so far this compressibility is concerned. It can be fluid it can be flow as well right we will see later on let us consider this as a fluid. For example, if I take this compressibility  $K$  value for a water what it would be  $5 \times 10^{-10}$  meter

square per newton at 0.1 mega Pascal and 298 Kelvin because that will be dependent on that. And whereas, for air it will be  $5 \times 10^{-5}$  meter square per newton at 0.1 mega Pascal  $T$  is equal to 298 kelvin.

So, if you look at the what is the difference or if you take a ratio what it would be it will be four order of magnitude between the water and air right compressibility higher the compressibility, higher this compressibility factor what it will be they will be more compressible right am I right or wrong higher compressible. So, therefore you know like water is consider as to be incompressible fluid, but if I apply very high pressure. Let us say 1000 bar or more than 1000 bar can I call water as a compressible substance certainly no because pressure is quiet high. But if I am applying let us say you know may be 10 bar or 5 bar I can assume it to be water as an it compressible fluid.

You know 1000 bar is a very big amount of pressure right or may be 2000 bar it is a quiet big. So therefore, in practical sense the water is considered to be an incompressible fluid. Now, let us see that whether we can apply this for a flow that means if flow is taking place and you are having a stationery and you will pass through a nozzle.

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Suppose this is a very infinite reservoir and this is your nozzle where flow is taking place this is the reservoir at certain pressure of what you call  $T$  and  $P$  right here velocity will be 0 and some pressure will be there  $P$  atmospheric pressure keep in mind that this  $P$  is much higher that atmospheric pressure. So, you will find there is the velocity finite

velocity may be let us say 20 meter per second or may be it can be you know 20 meter per second let us say. Now, this why it is occurring the flow because there is a pressure gradient because when fluid is flowing why it is flowing pressure gradient we know the temperature also flows because of the temperature gradient, but why it is really flowing do you know. If you go back to the concept of equilibrium it wants to attain the equilibrium therefore, it is flowing.

Similarly, we do some you know kind of work right it is basically the all activities is because of gradient any activities which is occurring you know, but ultimate objective of thing is that you need to attain equilibrium. That is the beauty of all the natural passes and so also the human life we should attain equilibrium means peace and bliss. So, coming back to this that if fluid is flowing because of pressure gradient then there will be change in density.

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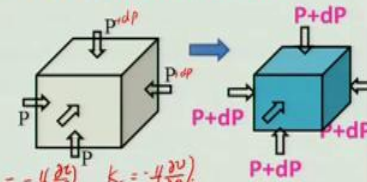
**Basic Principles of Compressible Flow (Gas Dynamics)**

**What is compressible flow ?**  
**Change in fluid density during flow due to pressure gradient.**

Compressibility  $= k = -\frac{1}{v} \frac{dv}{dP}$   
 $\Rightarrow k = \frac{1}{\rho} \frac{d\rho}{dP} \dots\dots(1)$

**For water :  $k = 5 \times 10^{-10} \text{ m}^2/\text{N}$ ; at 0.1 MPa,  $T=298 \text{ K}$**   
**For air :  $k = 5 \times 10^{-5} \text{ m}^2/\text{N}$ ; at 0.1 MPa,  $T=298 \text{ K}$**

Air has 4 orders of magnitude compressibility than water.  
 If the fluid is flowing due to  $dP$  then Eq. (1) becomes  
 $\Rightarrow d\rho = \rho k dP \dots\dots(2)$

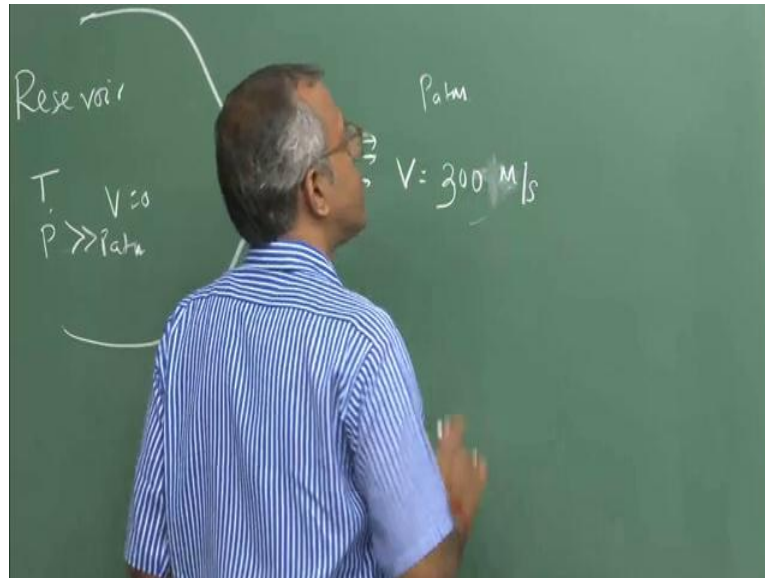


The diagram illustrates a fluid element (a cube) moving from a state of pressure  $P$  and density  $\rho$  to a state of pressure  $P+dP$  and density  $\rho+d\rho$ . The fluid element is shown in two positions, with arrows indicating the flow direction. The pressure and density are labeled on the faces of the cube. The diagram also shows the fluid element's volume  $v$  and the change in volume  $dv$  due to the pressure change  $dP$ . The compressibility  $k$  is defined as  $k = -\frac{1}{v} \frac{dv}{dP}$ . The diagram also shows the fluid element's volume  $v$  and the change in volume  $dv$  due to the pressure change  $dP$ . The compressibility  $k$  is defined as  $k = -\frac{1}{v} \frac{dv}{dP}$ .

If I rewrite this equation you know 1 then what I will get, I will get that  $d\rho$  by  $\rho k dP$  right what is saying, the change in density will be occurring which will be proportional to the change in a pressure and also it will be proportional to the compressibility sometimes you call compressibility factor. So, if you look at if it is a what you call a higher value right for example, 5 into 10 power minus 5 meter square per newton for air then naturally change in the density will be higher. For the same pressure change if I

consider water and air and if this pressure will be higher then what happens for the same air.

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For example, instead of 20 meter if I say it is something 300 meter per second or 200 meter per second as well as because I need to have a much higher pressure between this nozzle. Then naturally my change in density will be higher where I cannot neglect, but if it is 20 meter per second this velocity and correspondingly the pressure gradient which is occurring across the nozzle. The naturally I can neglect it the change in density because the pressure gradient across the nozzle is small.

So therefore, that we can say that the flow will be incompressible almost if the pressure gradient across a flow is very, very small leading to the you know change in velocity will be also small. That means this compressibility can be related to not only the pressure for the to the velocity as well. Now, how you chase it whether it will be compressible or incompressible for that we need to look at certain better quantities.



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**Basic Principles of Compressible Flow (Gas Dynamics)**

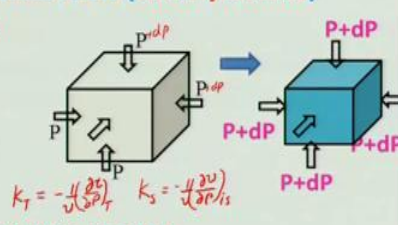
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Air has 4 orders of magnitude compressibility than water.  
 If the fluid is flowing due to  $dP$  then Eq. (1) becomes  
 $\Rightarrow d\rho = \rho k dP \dots\dots(2)$

**For practical Purpose:**  
**Compressible flow :** change in density  $\geq 5 \%$   
**Incompressible flow :** change in density  $< 5 \%$



For the practical purposes what we do that we say compressible flow wherever the change in density is greater than equal to 5 percent you know then you call it the compressible flow. If it is incompressible flow then the change in density you know must be less than 5 percent right and this is a little roughly we are talking about from practical point of view, but however we will again revisit it. But let us understand how we can really say whether the flow is compressible or not as I told.

That means velocity is the one quantity we can talk about because I told the velocity when it is 20 meter per second we can call it as a consider it is an incompressible flow although you know it is air flow is I mean we are considering the air flow. And whether it will be really compressible or not or it is just a assumption that we will see it later on, but as I told you that it is related to the velocity. And when you talk about velocity we need to also define a also look at a sound that means speed of the sound, we need to understand.

But let us first ask a question what do you mean by a sound can anybody tell me for example, I am taking right you could hear it that means I am producing some sound yes or no. For example, you are around may be 2, 3 meters away from me sitting and you could here if I am speaking in a audible voice normal audible voice right, but if you are sitting let us say 200 meter away from me can you hear this sound you cannot really. If

you are in between may be 15 meters or a 20 meters you may feebly hear try to you know.

So, then what do you mean by this sound how it is what is the mechanism of this sound because if I am making generating the sound here you could hear from a distance. So, naturally it is moving right it is propagating if it is propagating how it will be propagating what is the mechanism how we can quantify it whether it will be dependent on the medium or not. For example, instead of now it is the air between let us say I am in water like in a swimming or in a you know sea deep you know like a swimming. Now, it is deep water swimming people are enjoying today you know with a modern technology.

So, that whether you want to make a sound whether you could hear or not or if I go to the vacuum in the space and I will be talking whether you could hear or not right. So, these questions we need to understand a bit and for that you know for example, we know that the speed of sound is will be less than the speed of light. How do you know that do you have a an experience yes certainly yes if you observe you know now a days like your rainy season where thunder bolt is likely to occur and you see the lighting, but after long time or may be certain time you could hear the thundering sound, but already you know the sparkle has occurred.

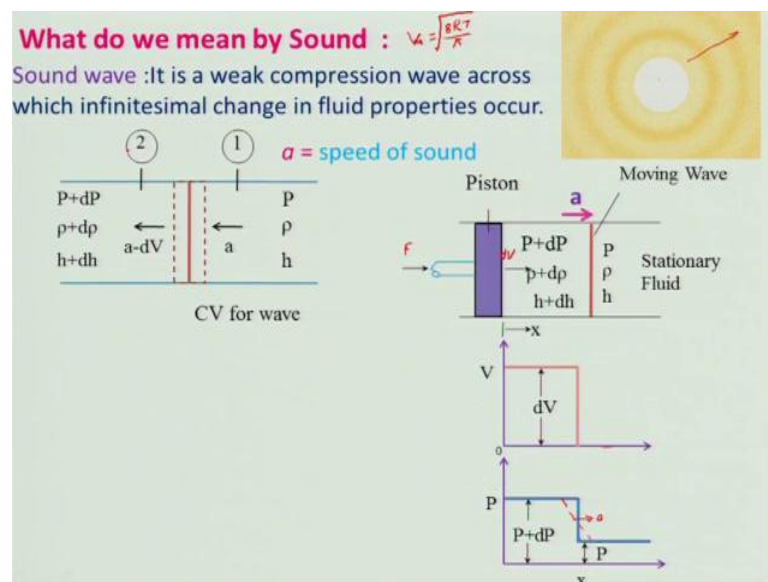
So therefore, you could you know see that the speed of sound will be lower than the speed of light, but however what is the mechanism of sound I will take a example of fire cracker. If a firecracker will be off or it will be busted out then what happens there you produce a detonation how do you produce the detonation you basically initiate the combustion reaction. That means some amount of chemical energy is converted into a pressure energy or a so that it will be released and adjacent molecule will be get affected by this.

The molecules will be moving at a higher velocity although all the time molecules will be in a random in nature moving, but this will be colliding with the other molecules and then it will be as it is you know colliding adjacent molecules it will be propagating. And then you could hear in your ear drum, ear drum will be vibrating and then you could hear the sound. If you look at what is really happening whenever this you know molecular interaction takes place or molecular motion takes place what really happens there will be change in pressure. If there is a change in pressure there will be change in density

because we are talking about an air medium or the gas kind of thing there will be change in temperature as well.

If there is a change in pressure change in temperature we have seen that is a that means it will be moved like you know and creating a molecular motion right and we can you call it as a wave because some people say sound wave kind of things. And wave what kind of wave whether it will be a longitudinal wave or a transverse wave right what it would be the transverse wave I will let me understand what do you mean by transverse. Suppose you know I will put a stone in the still water right what happens you know there will be repels. Have you seen observed that when you were a kid particularly like there is a here and then repels will be there.

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If I take these things let us say there is a some sound generated at the center then it will be moving if you look at these particles are being compressed over here and expanded and compressed and it goes on so it will be moving in this direction. Therefore, this is known as a you know because with the length along with the radial direction it is moving in gas medium or the liquid medium, it will be moving in a what you call longitudinal wave. But whereas, in the case of solid through which sound can also move it can move both the longitudinal and the transverse direction or transverse waves.

Therefore, sound whenever you say it is a both combination of it, but it moves in the longitudinal if you want to see this how it is to have a feel it you can go to the Wikipedia

and then you see this animation I have taken this picture from there. So therefore, we can define what you call a sound wave is a weak compression wave across which infinitely, infinitesimal change in fluid properties occurs. When it is occurring the fluid properties change then you know we call it as it will be moving with certain velocities and we will be interested in speed of sound.

And as I told you the while discussing about mechanism of sound that molecules will be colliding and moving. That means there will be average you know molecular velocity whenever there is a sound being you know imposed on it. Now, if you look at that means there will be increase in average velocities of the molecule. So, if I can invoke the molecular theory of gas you know right or kinetic theory of gas I can get the velocity I can relate the speed of sound to the molecular average molecular velocity.

If you look at I mean some of you might have studied that in your plus two or may be in engineering we know that this what you call the molecular velocity, average molecular velocity  $V$  will be equal to  $\sqrt{\frac{8RT}{\pi M}}$ . This is the molecular velocity, average molecular that means it says that it will be dependent on temperature and you can roughly get back the speed of sound is basically 3-quarter of this molecular velocity. You can look at it from the kinetic theory of gases, but however we would not be resorting to that we will be not interested to look at microscopic effect rather macroscopic effects kind of things.

Now, one idea is crossing my mind I want to cross check you know whenever this pressure or the wave is concerning there is a change in the pressure. What could be the change in pressure what will be the order of magnitude right we will get to that. Let us understand that there is a you know there is the piston cylinder adjustment I have taken where you are just pushing this pressure you know with the piston with certain force what is happening it will be creating some kind of a change in velocity. When it is changing the adjacent molecules will be having a certain pressure  $P + dp$ .

Whereas, away from that you know at the stationery where the fluid will be stationery you know it will be  $p$  and density  $\rho$  and enthalpy  $h$ , whereas here there is a change because of piston is there you are creating a small disturbances. And as a result there is a wave which will be moving because it will be compressing the way it is moving in this direction. So, what is that with what it will be moving, it will be moving with a velocity

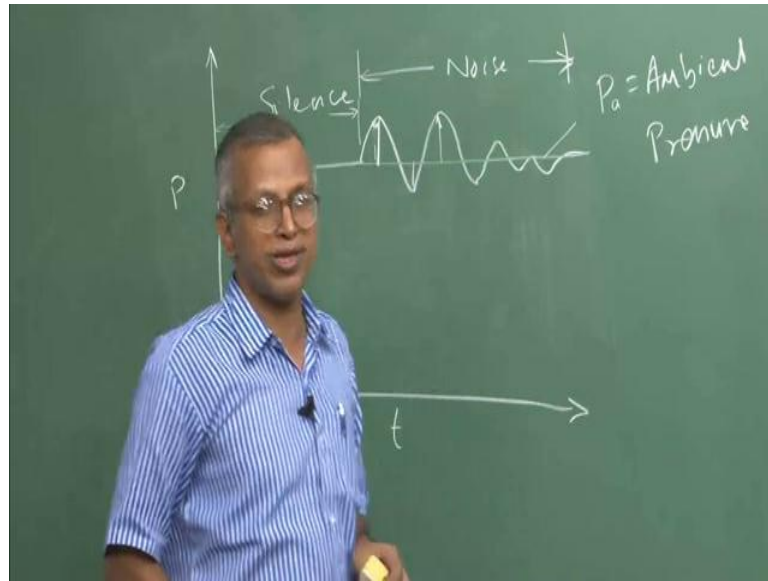
a. Let us say this is a wave which is moving with certain velocity we calling  $a$ ,  $a$  is basically we are calling speed of sound because these disturbances is creating the sound.

Now, coming back to the what will be there, there will be change in the velocity and here we are saying it is a you know certain velocity there is a change of course, this is a 0 here right it has come to 0 velocity stationery to start with. And similarly, there will be change jump in pressure and keep in mind this is the model, but actual system will be like this in reality, but here we are calling it is a step function you please keep this in mind which will be moving at certain velocity of  $a$ , this will be moving.

Now, if I want to make it stationery what I will have to do you will have to sit in this way which is propagating this in the  $x$  direction then what will happen you feel as if the fluid is coming from this side and you are stationery. This concept is very important right then we will get as if the fluid is coming from this side and with a certain velocity of air and there is a change in velocity because  $d p$  is coming opposite to that. So, then the pressure  $p$   $\rho$  and  $d h$  and it is getting compress  $p$  plus  $d p$   $\rho$  plus  $d \rho$  and  $h$  plus  $d h$  if you look at this is the one and this is the two right is that clear.

This is the very important concept which will keep in mind, but let me try to answer that question what I told you that what will be the pressure. Whenever suppose I am making a sound right there is atmosphere silence nothing is there what will be the pressure atmospheric pressure right that is 101325 Pascal's, but that depends up on height as it I told you depends upon the temperature depends several thing, but atmospheric pressure will be there we can roughly say it is one bar, but if I make a noise or a sound what will be the change in pressure. It will decrease certainly no it will be increasing or it can also decrease it is not that it would not decrease only it can increase and decrease what it would be if I you know.

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If I plot this pressure versus time whenever the sound is created what it will be it will be something like that you know depending upon it need not to be so regular I have drawn, but if I say this is the pressure which is average pressure you know this is my ambient pressure. This is ambient pressure and there will be you know or change in the pressure peak values and there is a decrease in pressure what I will zone this will call this will be silence. There is no, there is no change in pressure, but however there will be this zone is noise, but what will be change in this pressure that is my question.

Of course, it will be having amplitude what I have shown here and it is having certain frequencies we know that we can hear human being from 20 Hertz to 20000 kilo or 20 kilo Hertz, but actually it is 18 kilo Hertz, but we will just to remember we say that. This is the frequent, but what will be that value I want to hear from you will it be you know that this pressure will be what it will be approximately Pascal right 10 power to 5 Pascal right yes or no.

Now, what it would be what is the change I will get suppose I am speaking now there is a jet noise jet in the sense air craft is moving jet engine is moving what will be noise it will be how I will express this sound in reality. You know you use you might be knowing decibel we use right the change in pressure will be what order for a audible you know it will be generally 40 to 60 decibel audible. Whatever the way we talk unless you

But what will be the pressure than amplitude will it be you know 1000 Pascal will it be 10000 Pascal or will it be 5 Pascal or what will be what I am giving you. See you have all studied sound a lot you know in physics you have completed I am asking very simple question, I am giving what will be the sound I am talking. Now, what will be order I am not saying exactly you tell me. Certainly no you will be surprise it will be very, very low it is in micro Pascal's.

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## What do we mean by Sound :

$v_s = \sqrt{\frac{\gamma R T}{M}}$

**Sound wave :** It is a weak compression wave across which infinitesimal change in fluid properties occur.

$a = \text{speed of sound}$

**Mass conservation:**

$$\dot{m}_1 = \dot{m}_2$$

$$\rho a A = (\rho + d\rho)(a - dV)A$$

$$\rho a A = \rho a A - \rho dV A + d\rho a A - d\rho dV A$$

$$\Rightarrow a d\rho - \rho dV = 0 \dots (3)$$

Then what will be then mass conservation equation that is  $m \dot{1}$  is equal to  $m \dot{2}$  there is no accumulation because this is a steady. So, I can write out  $m \dot{1}$  is equal to  $m \dot{2}$  so then if I write down here what is the density, density is  $\rho$  here and  $A$  and this area cross section area I am taking as  $A$  that is the cross sectional area. Let me this is the mass flow rate into  $\rho$  plus  $d\rho$  into  $A$  minus  $dV$  the velocity into area I can cancel this out. And then expand it if I will do that what I will get I will get in the left hand side  $\rho a A$ ,  $a$  is the keep in mind small  $a$  is the speed of sound,  $A$  is the cross sectional area you can cancel it could have been more simpler maybe I can cancel right now here so nothing to worry about much.

And then what will get  $\rho a$  and this will be  $\rho a$  minus  $\rho dV$  and  $d\rho a$  minus  $d\rho dV$  and if you look at these terms will be higher order terms because this change in the pressure I have told you what will be the atmospheric pressure let us say and this pressure change is very, very small. Therefore, I am saying it is infinitesimally change it is really neglected, but it is having a effective. So, if it is order then I can neglect this term right and similarly, so what are terms remain that is  $\rho dV$  is equal to  $a d\rho$ . So, I can write down the  $a d\rho$  this term minus  $\rho dV$  is equal to 0 that is equation three, where three comes fine alright.

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**Energy conservation:**

$$h + \frac{a^2}{2} = h + dh + \frac{(a - dV)^2}{2} \Rightarrow dh - a dV = 0 \dots (4)$$

**Considering propagation of sound wave to be isentropic, we can have**

$$T ds = dh - v dP \Rightarrow dh = v dP = dP / \rho \dots (5)$$

**Combining Eqs. (3), (4) and (5), we can get;**  $a d\rho - \rho dV = 0 \dots (3)$

$$a = \rho \frac{dV}{d\rho} = \rho \frac{dP}{\rho a d\rho} \Rightarrow a^2 = dP / d\rho \dots (6)$$

**$a$  = speed of sound.**

**By using isentropic relationship for an ideal gas, we can get,**

$$P \rho^\gamma = C \Rightarrow P = C \rho^\gamma$$

$$a^2 = \left( \frac{dP}{d\rho} \right)_{isen} = \frac{\gamma P}{\rho} = \gamma R T \dots (7)$$

So, the energy equation I can look at the same way  $h$  plus  $a$  square divided by 2 is equal to  $h$  plus  $d h$  plus  $a$  minus  $dV$  whole square divided by 2 what are the assumptions it is



basically adiabatic process we are assuming and then steady flow these are the assumptions we are making. Now, if I just you know I can cancel here and expand it and higher order terms I can neglect then I will get  $dh - v dp$  is equal to 0 that is the equation number four.

Now, we will considering the propagation of sound wave to be isentropic that means not only the adiabatic, but there is no you know change in entropy across the thing. Then we can have that  $Tds = dh - v dp$  you know this equation. This is known as Gibbs first equation it is a combination of first law of thermodynamics and second law of thermodynamics. If it is isentropic flow so then I can say this is 0, but then this equation could have been  $dh = v dp$  and keep in mind that I can write down instead of  $v$   $1/\rho$  so I can write down  $dp/\rho$ .

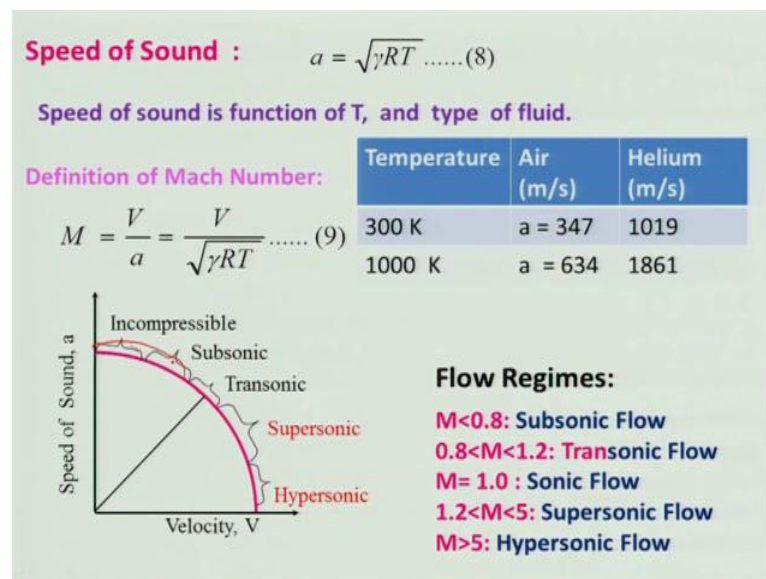
Now, what we will do we will combine this mass conservation energy conservation and this expression and look at what we are getting. So, what we can there we can start with this equation three  $a dv + v da = dp/\rho$  and then  $a$  is equal to  $dp/\rho dv$  by  $\rho$  can I write down this way is equal to write instead of  $dv$ . This  $dv$  if you look at I know from this equation four what is this  $dv$ ,  $dv = dh/a$ , but what is this  $dh$ ,  $dh$  is basically  $dp/\rho$ .

So, I can write down  $dp/\rho$  and  $a$  right and in place of  $dv$  I am writing  $dp/a$  and  $\rho$  so I can cancel this  $\rho$  can I not cancel. Then what will happen then this will be  $a^2 = dp/d\rho$  that is a square is equal to  $dp/d\rho$ . So, that means what is saying is saying the speed of sound square of speed of sound is basically change in pressure with respect to change in density or it is other way around. You know  $1/\rho$  the change in density with respect to change in pressure which is very simple and it is valid only for an isentropic flow.

If it is isothermal flow this equation will be valid or not it would not be do not say yes it is it would not be it will be changing and  $a$  is the speed of sound as I told. By using this isentropic relationship like that we know  $p/\rho^\gamma = \text{constant}$  to an ideal gas that is  $p/\rho^\gamma = \text{constant}$  and what we will do now we will try to differentiate this and you know put into this equation. That is of course, we write down  $P$  is equal to  $C \rho^\gamma$  because  $v$  is equal to  $1/\rho$ .

So, I can write down  $P$  is equal to constant into  $\rho$  gamma then I can put this equation you know I can differentiate it and I can simplify that thing and then write down  $dP$  by  $d\rho$  is equal to gamma  $P$  by  $\rho$ . And if I use ideal gas law that is  $p$  is equal to  $\rho R T$  then I can write down  $P$  divided by  $\rho$  is equal to gamma  $RT$ . So, if you look at this we can now speed of sound square is related to gamma  $RT$  right you remember that I told you the average molecular velocity is basically also in similar in nature.

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Now, if I take the root of that you know then this is  $a$  is nothing but your gamma  $RT$  and from this what you can learn it is that means speed of sound is dependent on the gamma, gamma is what specific heat ratio will it be dependent on the fluid also. Like suppose if I will take helium if I take air what will happen it will be different and it will be dependent on the temperature also. So, let us take this you know example like if I take 300 Kelvin for the air gamma is basically 1.4 and this speed of sound will be 340 meter per second if I increase the temperature to 100 Kelvin then the speed of sound is equal to 634.

Whereas, for the helium the gamma being different right it is basically the 1.66 right am I right for helium right. So, then the speed of sound in helium will be 1019 meter per second and at 300 Kelvin at 1000 Kelvin it is 1861 meter per second. So, all this thing we are doing to basically talking about Mach number which is nothing but the ratio of the velocity and speed of sound. So, that is basically Mach number is equal to  $V$  divided by  $a$  and  $V$  root over gamma  $RT$  and depending on this Mach number we can you know

talk about various regime of flow one is Mach number when it is less than 0.8 this will be subsonic flow I mean in this region.

Here I have plotted speed of sound versus velocity in this subsonic you know flow of course, there will be some incompressible flow as such of course, all this thing from here to here it is subsonic because less than you know 0.8. Of course, by definition we call whenever is Mach number less than 1 you know we call subsonic it is not true. And when the Mach number is varying from 0.8 to 1.2 in this regime is known as transonic flow Mach number is equal to 1 it is sonic flow.

And Mach number you know varies from one point of course, I could have told like it will be from 1 to 5 sorry this is 1.2 so Mach number 1.2 to 5 is supersonic flow and the Mach number greater than 5 is hypersonic flow. So, what we will do in the next class that is we will be talking about little bit about various regime of flow again and then we will be moving at how we can use this term to talk about compressibility and some other things we will take out in the next class. With this I will stop over if you are having any doubt we can have a discussion may be for couple of minutes. So, then we will stop over here.