Fundamentals of Aerospace Propulsion
Prof. D. P. Mishra
Department of Aerospace Engineering
Indian Institute of Technology, Kanpur

Lecture - 06

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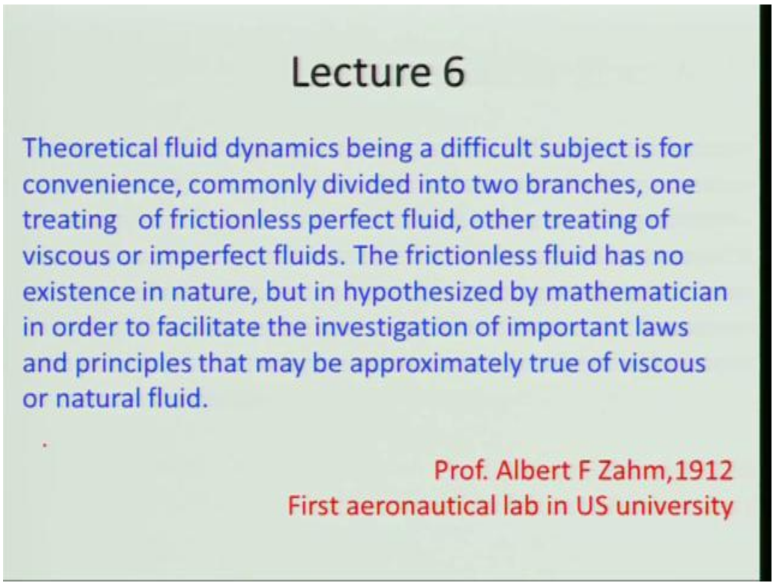

Let us start this lecture six with a statement given by Professor Albert F Zahm in 1912 who was the first person to establish an aeronautical lab in US. He states theoretical fluid dynamics being a difficult subject is for convenience commonly divided into two branches one treating of frictionless perfect fluid, other treating of viscous or imperfect fluids. The frictionless fluid has no existence in nature, but in is hypothesized by mathematician in order to facilitate the investigation of important laws and principles that may be approximately true of viscous or natural fluid.

If who consider this statement that the fluid is basically viscous in nature whether you like or dislike it why it is so why nature is so you know what you call kind to us to be or unkind to us to be viscous in fluid is it good or bad. Always you know aerospace engineer will strive to reduce the viscosity we have seen in the last lecture that whenever the viscosity flow is viscous in nature. Naturally what will happen the drag due to viscous effect will be higher right that is the another kind of drag as well, what is that drag that is due to pressure or the form, so body has to be stream line to have a reduction in form drag.

However, you are having a increase in viscous drag because surface area increases, but we always try to reduce it why it so can anybody tell me the viscosity is your problem, but it can also be a solution some extend. Otherwise if viscosity is not there can I really make it to fly an aircraft with this question we will move into several other aspects what we discuss just to recall that we discuss about the effect of viscosity and the fluid. There are 3 effects one is your what do call the drag other is the effect of this boundary layer, and other is what we call it as a aerodynamic heating these are the three effects what we talked about.

Then subsequently we discuss about steady and unsteady flow now I am having one question to you do really understand, what do you mean by steady and unsteady flow? I will ask this will try to answer this question today. And I will check whether you have understood really what do you mean by steady and unsteady, but before getting into that aspect let us look at how many ways we can analyze the flow.

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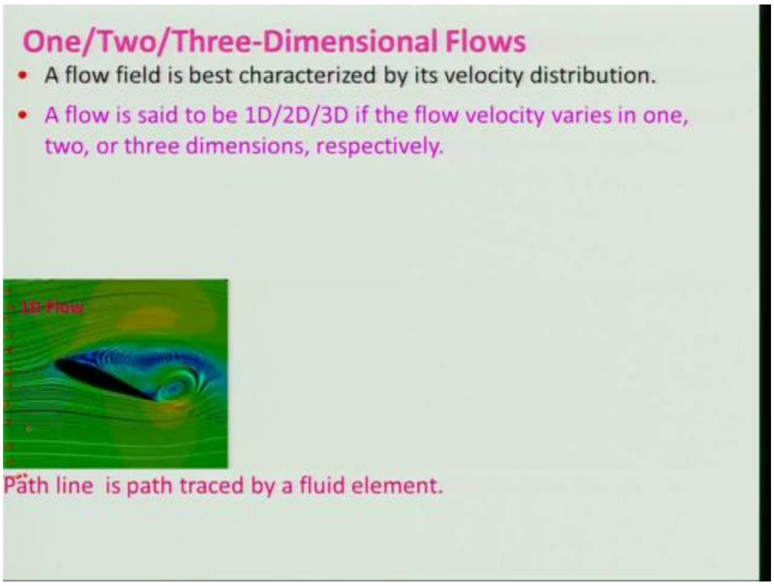

We can think of the flow to be you know 1-dimensional flow, 2-dimensional flow and 3-dimensional flow if look at all the natural flow will be 3-dimensional in nature. What is the meaning all 3-dimensional when you talk about dimension in a space if I go where I will talk about dimension? When I take this room I will be having certain dimensions like I can say along one you know from a corner of a room, I will say that it is one is x direction other is y direction. Other is vertical one is given you know z direction or vertical direction, but in the space what we will talk about.

So, that means the fluid if look at is basically 3-dimensional in nature, but however for our own you know simplicity or to analyze it we always talk about a 1-dimensional. What is that 1-dimensional flow is one in which the flow field will be not a remaining you know constant in two other 2-dimension only to leave varying in one direction. In other words how will characterize whether it is 1-dimensional flow or 2-dimensional flow we need to look at the velocity distribution. Generally, but you can talk about others so as temperature distribution and you know what do call the other properties like mass fraction, mole fractions in case of reacting flows.

So, a flow is said to be 1-dimensional, 2-dimensional or 3-dimensional if the fluid velocity is varies in either in one or two or 3-dimension respectively, then only we will be talking about it. And in case of a 1-dimensional flow which has shown here right if you look at this is a flow over a an aero foil where is the flow is 1-dimensional can I say the flow here is 1-dimensional can I say here some of you are saying yes how it is possible. If I say this is my x direction, this is my y direction right can I say flow here is 1-dimensional certainly no because that is a recirculation flow is being separated at this point.

Similarly, I mean little away from the leading h and in the dwelling h the flow is also re-circulated its separated and is re-circulating here. So, I cannot call it as a 1-dimensional flow however to creates this kind of images what do call flow visualization. There are various techniques we use queen tunnel or a watt tunnel right which is where you can have a control over the fluid flow velocity and also its distribution. In this case people have assume the flow to be 1-dimensional here that means this is the uniform velocity what you can see right is it you can observe this what are these lines if you look at these are the some lines what are these lines.

These are steam lines are you sure it is steam line what do you mean by steam line can anybody tell me please what do you mean by steam line.

Student: It is generally the path traced by the fluid particle.

Path trace by the fluid particles in a flow is known as steam line are you sure what do you are saying.

Student: velocity is perpendicular at every point on the steam line

Steam line that means both are same what you are saying am I right or wrong can anybody tell me what is the difference between this two or both are same. In this photograph is it steam line or path line because first definition what he told it is the path line, the second definition of what he says it is a steam line. In this case what kind of lines we are having because these lines are very important for visualizing the flow. So, coming back to this path line is basically path trace by a fluid element right what do you mean by this suppose I will take a fluid element here.

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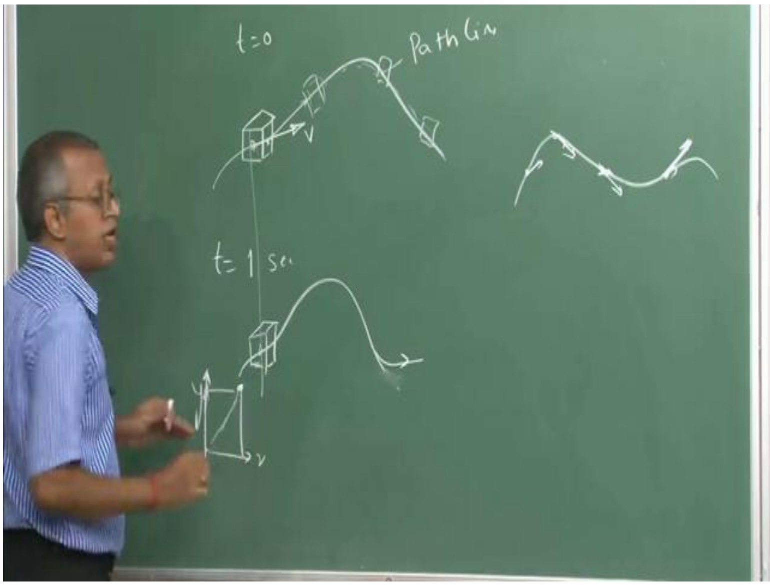

And a time t is equal to 0 let say it is moves like this that means at fluid is here and at certain time it will be going over there and other thing right. That means the time trace by this fluid element along this path will tell me the path line, it is here it will go over there, it will go over there, it will go over there. That is known as path line, but do then when will call it as a steam line that means it is when it is fluid, fluid is having a velocity right yes or no. That means this is the vector as you told that steam line will be basically a curve, whose tangent at any point is velocity vector at that point will you call it as a steam line. Then because I am showing the same thing like you know in the same line can I call it as.

Student: Sir, uniform flows immense and are same

Uniform flow.

Student: For steady flow.

I think the second answer what is given by him is the right is the steady flow that means what is the meaning how I will get a path line whenever I am conducting experiment how, that means that will be time exposure of a photograph or time exposure photograph of a fluid element. That means if I take a photograph and look at how this path is traced by a fluid element and for instance certain time then I call it as a what path element. But however if I take a what we call instantaneous photo right and then try to look at each point what is the velocity vector.

For example, let say this is the my you know flow is taking place here I can say that this velocity is here and this velocity here maybe this velocity is there and that point. That means this picture have taken at one frame of time maybe at instant right and what I need to do the fluid element will be there. And then it is some you know I am just taking a vector looking at velocity, velocity is the vector quantity its having magnitude it is having a direction.

So, then I call it as and I can obtained if I will be getting what do call a single frame of a motion picture of the flow right do you know I can take a video at one instant of time I will get full a what you call view of the entire domain on which I have focus. That is basically a steam line, but when I take this time t is equal to let say 1 second this image you know this path may take a different path kind of things this can take a different path. So, then this will be what do call unsteady phenomenon that means at the same location same if I take this is x and this is y I am having certain distance here the distance is almost same, but I am plotting at different time step.

From this reference O you know it is having same dimensional let say 3 meter from one side and other point is 4 meter, x is equal to 3 meter, y is equal to 4 meter at this point I am measuring a you know point. So therefore, I am keeping my pro or looking at it at the same point, but I am measuring, but if this path is same as that of the a time t equal to 1 second a time is equal to 0 is same then both the path line and the steam line will be same.

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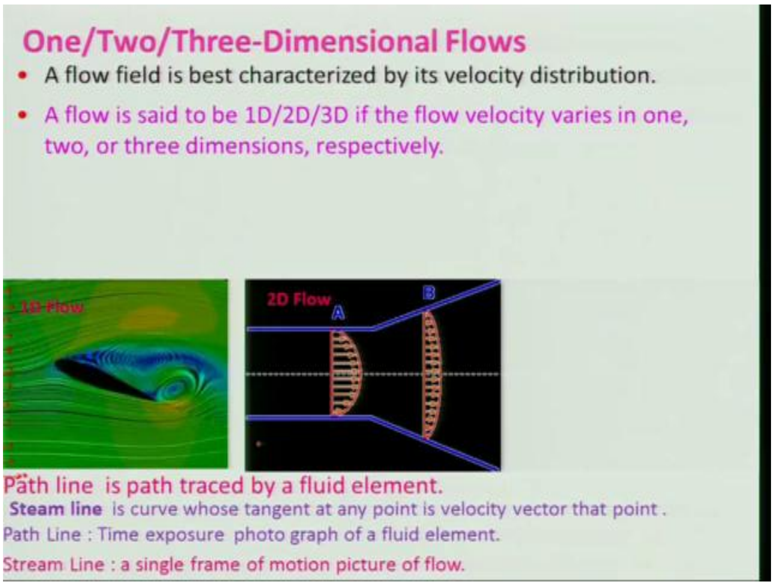

Generally, that means under steady state condition steam line or path line are same so we generally try to conduct experiment like this to what you call such that we will have a steady flow and steam line path line can be same. These are very important concept if some of you may not have a what I have seen it is you are getting confuse so it is very important to know about it.

Here, I can say the flow is 1-dimensional in nature and let us look at 2-dimensional in nature this is a duct it can be a circular duct it can be a squared duct, but we always like to have a circular duct kind of things. And it can be channel as well if it is a channel what it will be it will be 2-dimensional flow you understand what do you mean by channel.

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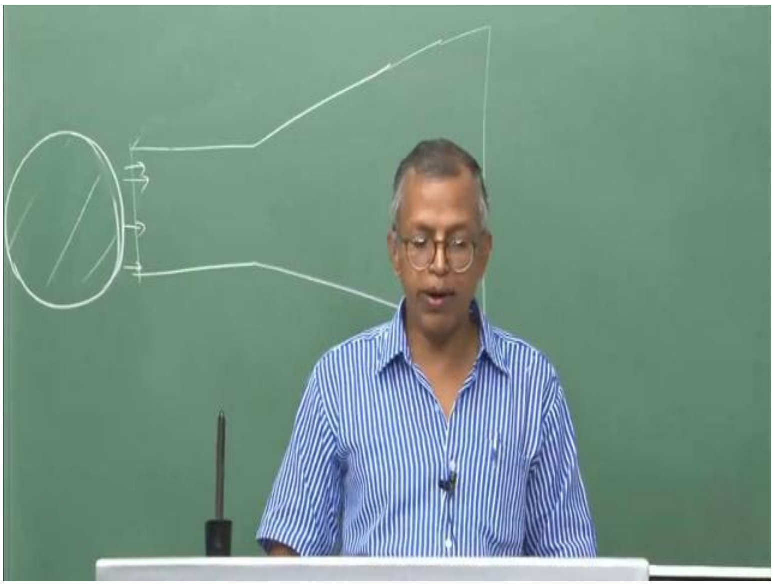

If I draw here I am just drawing in this figure right flow is coming over here will you call it as a 2-dimensional or channel flow or can I call it as a what we call a pipe flow can we call this. From this drawing I am talking about I am asking very simple question of drawing from this drawing can I say whether it is a channel flow channel means, what? That means along with this direction you know the flow is you know, it remains what you call uniform should be or need not to be what it should be.

That depends on what is the inlet condition if you are having uniform flow at the inlet it will be not changing because dimensions are quite big there is no boundary layer, if at both the this side and then this side you know kind of thing, but is it a pipe flow or not what kind of dimensional we talking about. Will you call it as a 2-dimensional flow or a 1-dimensional flow what I have shown here in what you call a velocity profile here at A. And there is another velocity profile at B will you call it as a 1-dimensional flow or 2-dimensional flow.

In this case if I take my x direction here you know and if I take this is my y direction and if you look at my velocity profile is changing along with the y direction because here this is the higher velocity this is a smaller velocity. And if I moving towards this you know my velocity changing that means if I am at A location and B location then my velocity profile here it is different at the center line this is a higher this become a smaller.

Therefore, I am having 2-dimensional flow am I right or wrong all agreeing, but if I am considering only in one domain then what will happen I will consider only in these region what will happen or I will consider in this region what will happen. In this region means in the entrance region where there is no divergence it is just not changing diameter, remaining same of the pipe or the sorry channel I am talking about channel and then there is a change in the channel.

And if I am looking in this domain what will be flow will it be 1-dimensional or 2-dimensional. So, this is very important to appreciate or recognize whether the flow is 1-dimensional or 2-dimensional because the analysis what will be carrying out considering in propulsive devices, mostly either it is a quasi 1-dimensional or 2-dimensional. We need to understand what do you mean by quasi 1-dimensional and 2-dimensional and how we will handle that little later on, but you might be aware, but you think about what it would be.

So, this is what I told it is a channel kind of things because this dimension is not telling me because if it is a pipe flow what it could have if it is a pipe flow. I could have a line here you know isn't it or I could have a taken a cross section, which is looking at you know then only I can say it is a pipe. So, let us look at flow through a pipe.

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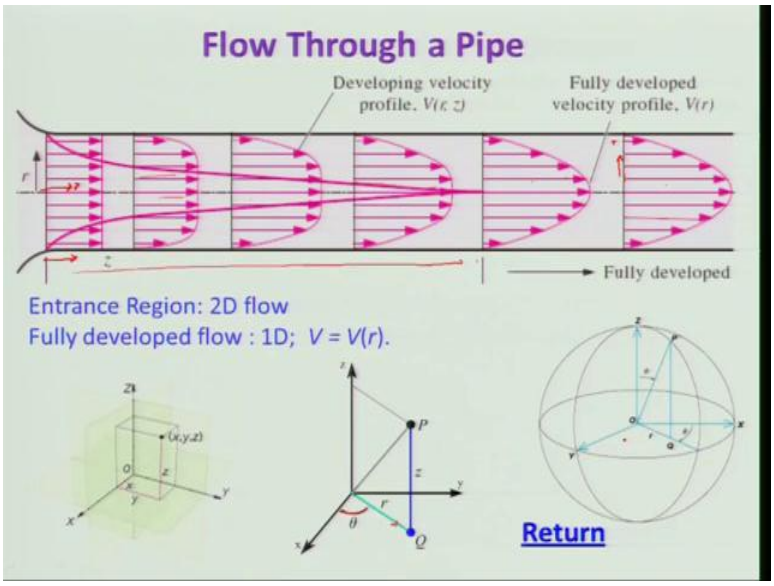

So, if you look at this is a flow through a pipe I have shown here you look at here at the entrance it is having what do call a uniform velocity profile, and keep in mind that this is your what do call z direction and this is your or I can say this is your z direction this is your r direction. And this is a what do call a another coordinate system, what we will talk about it that is known as r theta z coordinate or the cylindrical coordinates.

In this case what we are considering in the z direction keep in mind that in this figure z direction in this is a from left hand side to right hand side, and this is a r direction and theta direction is here it is moving. So, you take a this thing and move around a theta then you will get a pipe kind of things right. So, here we are considering basically a z and r that means velocity is changing along the z direction. If you look at started with uniform velocity what happens to this velocity profile it is different that the beginning of the uniform velocity profile it goes on changing along the z direction yes or no right.

And also it is changing a particularly here along the radial direction yes or no right in this point it is all changing along with this. Whereas, at the entrance there is no change in velocity along the radial direction, but what happens to theta is it changing or not in this figure what I have drawn or we can assume here that it is only the velocity particularly it changing along with the r and z direction. That means it is the 2-dimensional flow can I call it 2-dimensional flow or I will have to tell 2-dimensional axis symmetric flow, or in simpler axis symmetric you know flow kind of thing. Because along the theta direction I am saying it is symmetric. So, I need not to consider it is uniform duct.

So, you need to understand what is the difference between pure 2-dimensional flow and axis symmetric flow, it is very important because we will be dealing with axis symmetric flow most of the times because all our aerospace ducts and other thing will be cylindrical in nature. So, cylindrical coordinate system is more natural to be used and what happens to the velocity you look at here like it becomes you know changing and when you go to here there is a increase in this and then it becomes also become changing. And what is happening here the velocity profile is almost similar to that and in this, what is happening if I compare this two velocity profile here and here, what is happening can anybody tell me.

Student: Starting from.

One d flow how you are saying.

Student: it is not changing.

It is not changing along with the z direction however the change in the r direction right, this is r direction it is changing so therefore, we can call it as a 1-dimensional flow, but why it is happening why means why it is fully developed you call how it is happening. It is because the this line what I have shown here indicates the boundary layer which is grown from the inlet of the pipe and it merges after that. There is nothing really happening with the boundary layer because boundary layer two boundary layer upper boundary layer, lower boundary layer. Of course, I cannot say upper and lower, but it is a pipe right if it is channel I can say upper and lower boundary layer in a pipe what I will say upper and lower it is along with the theta you know like whole circumference is being merge.

And when it is merge that is known as fully developed flow and for that you know equations are there Poisson equations. We can solve that you might have expose in your fluid mechanics. Of course, but now what is happening to this velocity in this zone what I call this velocities and this velocities and this velocities what it is right this is known as the core region in your z flow you must be knowing we will be discussing little later on, z flow where the core region is there what we call it as a core region, I will leave that question.

Now, let us get into at what distance I should get a fully developed flow will it be dependent on what, will it be dependent on inlet velocity profile will it dependent on the how this flow is entering basically I am saying the same thing in a different work. Then what else will it be dependent on laminar or turbulent nature of the flow at the inlet will it depend on the friction or the roughness of the pipe and the material of the pipe. If I want to determine where I can apply the 2-dimensional equation and 1-dimensional equation for analysis.

That means in this region under developed flow what is known as or developing flow you can say at this region from here to here this region it is basically call as a developing flow or undeveloped flow whatever, you call right I think developing flow is a positive word like we are developing country. So, there what you need to consider whether the flow will be 2-dimensional or 1-dimensional it will be 2-dimension that means you need to consider the governing equations that. So, how I will go about it, it is basically L by D if I take this diameter of the pipe for a fully developed flow will be dependent on Reynolds number right Reynolds number.

And for a critical flow that means where it will go from laminar turbulent l by d for a fully developed flow will be around what around 125 this is rough number for a thumb rule you can say. And in case of turbulent flow it will take more length to be fully developed for a particular diameter or not or less.

Student: Less.

It will be taking less distance and these are very important for analyzing and developing or designing anything. So, I will stop over here and then we will be moving of course, before getting into that where we are discussing there is a another a coordinate system which is being use is known as spherical coordinate system. This is basically r and phi and theta direction we would not be using very much however you might have expose to some other, but we will be mostly using this is known as a cylindrical coordinate system. So, let us get back to our this thing like a 3-dimensional flow.

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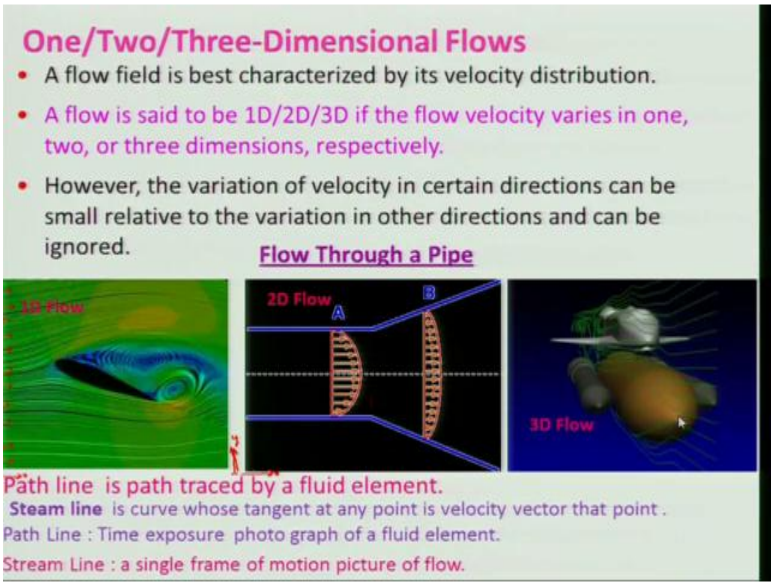

So, if you look at flow over a I have taken a space craft here and you can see these are the lines which indicates it is you know you can say this as a path line and if it is a steady flow as I told it can be steam line as well if it is a flow is steady. So, these lines are quite you know 3-dimensional in nature therefore, you can call it as a three dimensional flow.

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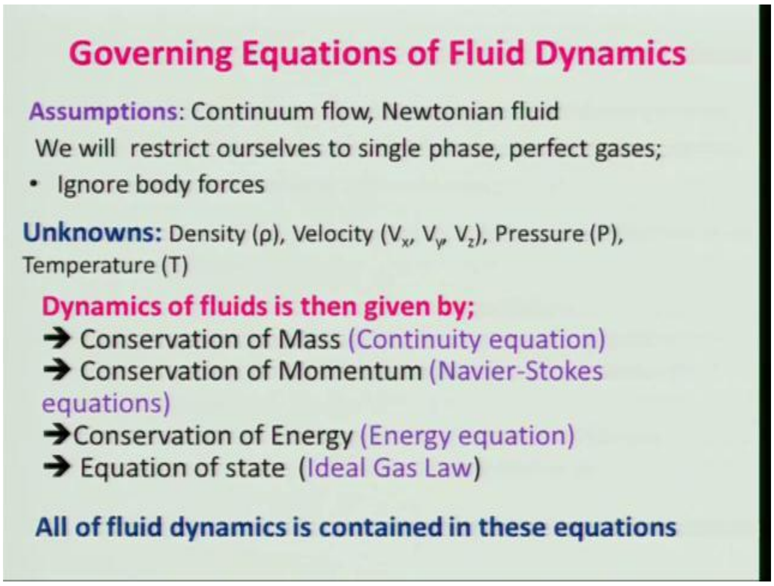

So, now we have talked about to very what we call qualitatively about the flow, but we need to now deal with the flow quantitative because we will be using this you know for designing certain thing. That means we need to quantify look at various properties for that we will have to consider the governing equations that will a basically dictate how the fluid flow will be there. That means I can really understand or simulate the fluid flow by using certain governing equations.

So, for that we will make some assumptions I have already talked about what is the continuum flow that means we will be considering continuum flow in our analysis all the time except. Of course, whenever you go to the space at a very what we call low pressure there the various kinds of flow will be coming into picture as I told sleep no sleep flow and then pre molecular flow kind of things, but we are not bother about that. And as I told we will be considering basically about the Newtonian fluid right, so we will restrict our discuss to a single phase fluid. And as we are dealing with the mostly the gases then we will be considering as a perfect gas what do mean by perfect gas.

Student: No, reaction takes place.

No, reaction takes place we call it as a perfect gas can anybody tell me please what do mean by a perfect gas can anybody tell me please what do you mean by a perfect gas

Student: Inter molecular forces are considered negligible.

That means inter molecular forces between or among them in a gas will be very very small. When it will occur, when it will occur this inter molecular forces will be less it can occur only when the pressure will be low what kind of low which can tell us that whether the flow is or the fluid is perfect or not or gas is the perfect or not is it the only condition or there is some another condition which will tell you whether the gas is perfect or not. The volume occupied by individual particles or the molecules will be quite less as compare to the volume of the fluid which will be considering then only if both this conditions satisfied then only one can call it as a perfect gas.

And where ideal gas law P v is equal to n r t will be applicable, yes it will be, but then that connotation will be little different perfect liquid will be there when you call about you know kind of gas things. So, here when you are saying perfect actually this perfect is not a good what as such right because you know we are saying it is be satisfying a that is simplified form of equations state what is known as ideal gas law.

Then we are calling it is a perfect gas actually I will say that is rather this is an in perfect gas according to my version, but people are saying we will have to go by that work. Naturally, but the question is that like a what will call at what pressure what temperature we can talk about whether the flow is perfect or not or whether it will follow the ideal gas or not, law or not is it. If I take ambient pressure in air and in this room let say at ambient temperature can we call it is an a perfect gas. And then apply the ideal gas law yes or no, but I want answer you know all those things, this you are learning from your class I think maybe tenth on words.

Ideal gas law is being thought I think right and you are being engineering already so you are you know all these things. Then how I will talk about it then you will have to go and look at it compressibility factor and let me tell you that you can really consider the gas to be an ideal gas. Even at a pressure atmospheric pressure not only that even at a very high pressure even up to the 100 atmospheric pressure at ambient temperature, you can you know consider it and if we increase the temperature naturally you go for even consider an ideal gas law.

You can apply the ideal gas law even at higher pressure because at the at higher temperature sorry at the higher temperature even at pressure molecules will be faraway right. So therefore, you can consider and some of you who were not having any idea I would ask you people to look at in thermodynamics book about compressibility effect. And we will be dealing with compressibility compressible flow from a little different perspective as well.

So, and we will be ignoring the body forces right and what will be the unknown in case of generally I am talking about. Of course, we will be more interested in density whenever we talk about compressible flow and velocity, pressure when you talk about velocity as I told in am considering 3-dimensional flow naturally it will be flow along the x that is V x V y and V z. But mostly we will be dealing with as I told quasi 1-dimensional flow or 2-dimensional flow at the most because it will be difficult to solve the equation in 3-dimensional nature.

And pressure and temperature kind of things and for deriving this governing equation we will be what we call applying the conservation of mass what we call it as a continuity equation I call it is a law of common sense. That is what it states that whatever mass accumulated or depleted you know is equal to mass in minus mass out. And I always considered mass can be compared with the money, money is coming in and coming out and whatever it is accumulation or it will be reducing or depleted.

So, conservation of momentum which we have already you know talked about it and whenever it will be applied, what we call this thing? Basically we have just initiated discussion by in working the second law of motion. Newton second law of motion and when it applied to the fluid flow it is known as Navier-Stokes equation, you will see may be today itself that it is quite complex in nature and it is quite difficult to solve this equations as well.

So, conservation of energy which we will already discuss about it by invoking the first law optimal dynamics both for the control mass system and control volume systems. Particularly, for control volume will be using and this is known as energy equation I have told you equation of state. And we will be using ideal gas there are several equation of states I will be we will be using ideal gas law.

So, this you know is the basis for deriving the governing equation and all fluid dynamics is contain in these you know conservation principle along with the equation of the state. Whether it is a Newtonian fluid non Newtonian fluid of course, the continuative equation will be what do call different. But however it will be similar so far this principle as concerned and keep in mind that you I have told you that we will be talking about single phase flow, but in our aerospace application we need to look at also two phase flow.

Where the two phase flow will be coming into pictures particularly when we are working with the liquid fuels and solid fuels like in rocket engines or gas turbine in liquid fuel. So, also in the rocket engine will be using liquid fuel as well, but rest of the things will be but, how we will handle. We will not handle those things in a traditional way of two phase flow, but in single phase flow as we go along we will be discussing how to handle from the engineering prospective without complexing or without really involving complex mathematics which is un tractable.

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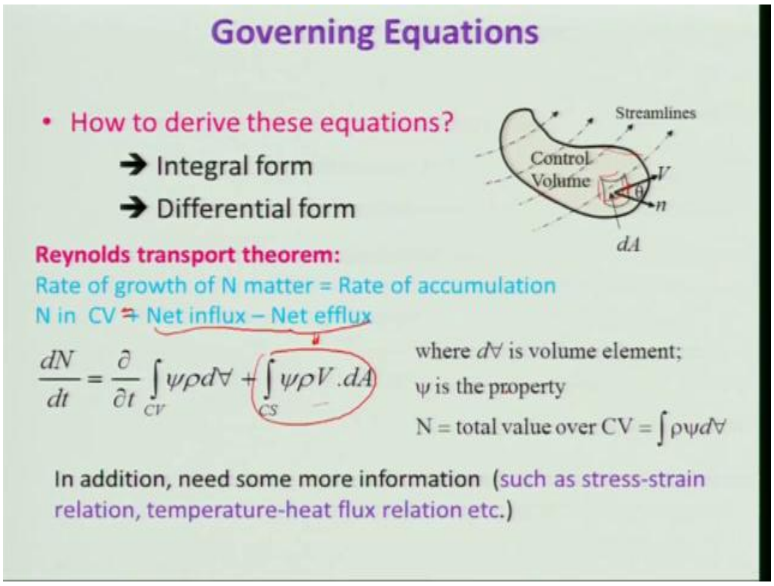

So, we need to look at this how to derive this equation which will be helpful right. There are two approach in that one is what do call the integral form other is your differential form. That means I will take a small elements and then look at it this balances mass or the momentum or the energy and then look at it where this delta x or other or the scale will be 10 into 12, 0 then I will get a differential equation. But however when I take a bigger volumes control volumes and look at as if the flow is taking place throughout, and looking at that inlet and outlet then we consider as a initiative form.

For example, if I take a arbitrary control surface here a control volume having a control surface. And if I take you know this stem line as shown right which is entering through going out some inlet and outlet. And if I consider is small you know surface area here as an element which is having dA and of course, it is having a volume dV and it is having a vector V. And you keep in mind this is a unit vectors that gives you, you know that is perpendicular to the you know to this surface, always this unit vector is perpendicular at that point where you are considering. And the angle is theta between the velocity and the normal or the unit vector.

So, and then what we need to do we need to use basically theorem which is very important particularly when I am talking about from differential to the integral form that is known as Reynolds transport theorem. We state that rate of growth of N matter that means you know it is a continuum flow so there will be a lot of matter will be there and we call it as a N matter and what is that N, I will talk about it. That is basically total value of a property over a control volume what you call rho psi and d v and where d v is basically the volume element of this small what I have taken.

Now, that rate of growth of N matrix is basically rate of accumulation in CV plus actually there will be equal sign here right is equal to net flux minus the sorry, sorry that is a net flux minus net. That means d N by d t will be is the rate of accumulation of in the control volume if I say dou by dou t psi rho d v plus this is the term which is basically net flux influx minus net flux that means this is equal to that. So therefore, that will give you the Reynolds transport theorem and keep in mind that for a whenever you are talking about continuity equation this psi will be equal to 1. Whereas, for the momentum and other things we will have to consider V basically velocity and then for other thing it will be energy the psi can be any property as I multiplier.

So, in addition to this like we need to concern about various other equations or consider equations for a relationships such as stress strain relationship, temperature heat flux relationships or a diffusion relationships and other thing. So, keep in mind that this Reynolds transport theorem is very important, particularly when you talk about this converting this into differential form to integral form and the vice versa.

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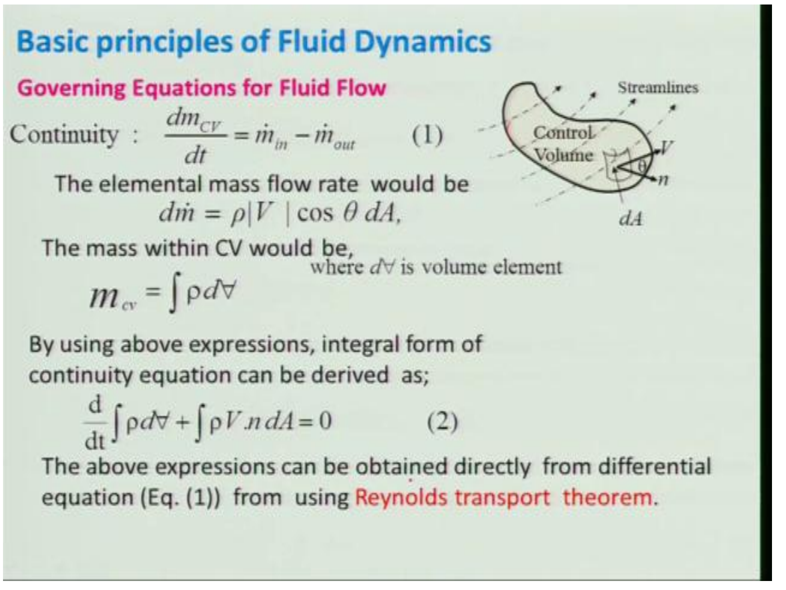

So, let us look at the basic you know governing equation for fluid flow we will take this control volume as I have shown here and for the continuity if you look at a differential form that will be d m by d t. That means the amount of mass changing in the control volume with respect to time is equal to mass is entering into the control volume and mass is going out of the control volume. So, if I consider this is a basically differential form and if I consider this as my element, and then we will have to look at how this elemental mass flow rate would be.

Basically, d m into I will have to consider rho and velocity and if I look at this vector that means along this will be cos theta and dA, dA is area that means mass is entering you know mass is going through this area. And if I mass within this total control volume because in control volume I am taking this element. Now, I am considering the total control volume it will be basically m dot rho d v because this will be a volume.

And where d v is the volume elements and be by using above expression integral form of continue equation can be derived as that d by d rho d v plus rho v dot n dA is equal to 0. And this is the continuity equation and for the integral form, and this is the differential form above equation expression can be obtained directly from the differential equation using the Reynolds transport theorem.

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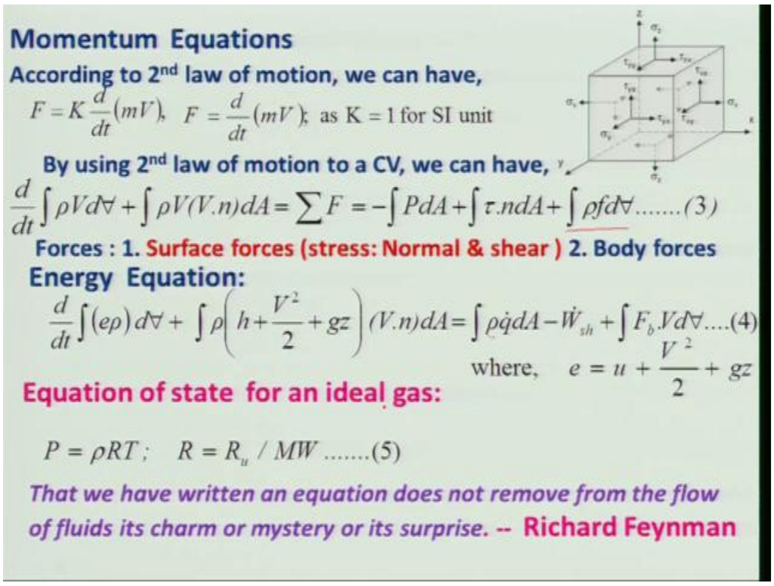

And we will be looking at momentum equation according to the second law of motion we can have F is equal to K into d by d t m V, this is your second law of motion. That means change in momentum with respect to time is nothing but your force is or force is proportional to change in momentum time and k is your proportionality constant and in case of SI unit K is equal to 1. Now, we will have to apply this what you call the second law of motion for a fluid flow. So, when we will do that what we will do, we will have to basically look at take that you know control volume what I have taken.

And we can see that we will be this is your what you call convection terms and this is your unsteady terms. And which is nothing but the summation of force which is acting on the fluid element and what are these forces, these forces will be having two kind of things one is your what we call pressure due to the because the fluid will be moving due to the pressure gradient. That means, that will be acting like a normal stress if you look at here in this case it is a fluid element, and this delta or sigma x which is corresponding to the pressure which will be acting.

And this is portion is basically the see here which will be acting like for example, tau x y. And in this case the plane in this case is basically if you look at this is your what you call in x, this is your z and y plane and this is in the what you call x and y, that means it is acting in this direction. So, in similarly other places it will be having several up shear stress which will be acting. So, if you look at it is basically a tenser right which you can see from this figure and this has to be integrated over the area and there is also a body force. Which we consider generally gravity for the completeness sake I have taken here and this is a with respect to volume keep in mind this is the force is basically with the area surfaces is being acting.

So therefore, this is known as surface forces that means it is having normal and shear and there will be a body force which will be due to gravity and other effects like electromagnetics and other things. So, if you look at this is basically the equation of what we call the momentum equation and similarly, energy equation you can write. Which we have already derived which already derived and then we will be looking at unsteady part. And this is your what you call total energy and this is the heat being transfer or the heat interaction with the system and a volume there is a shaft work. And there is also a what you call body force due to that some work will be there.

So, for an ideal equation as I told you P is equal to rho RT and where R is equal to R u divided by molecular weight. This is the universal gas constant the molecular weight and what we have seen these are the equation we need to consider, but what Richard Feynman has told. You may derive equation that is not good enough what is he says that we have written an equation does not remove from the flow of fluid its charm or a mystery or it is surprises. With this I will stop over and I must tell you that these equations you can derive of your own and having derived that does not mean we are having solutions for that. So, with this we will have to stop over and we will discuss in the next lecture.