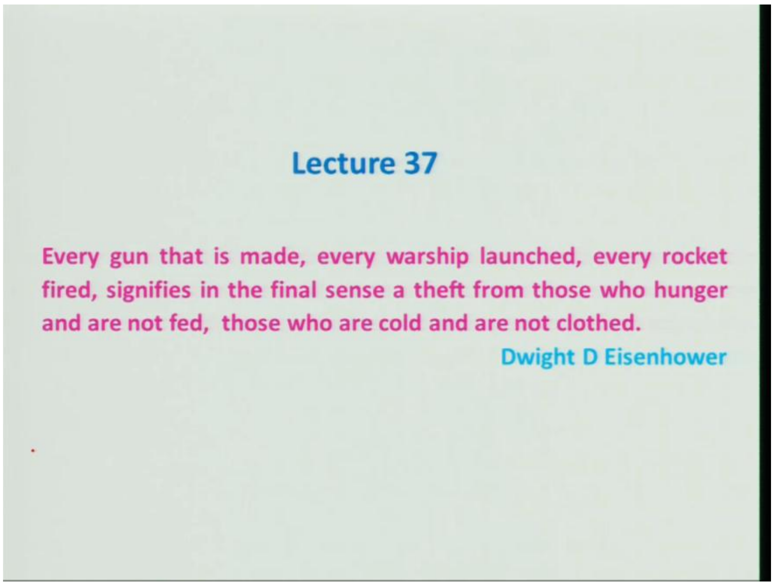
Fundamentals of Aerospace Propulsion
Prof. D.P. Mishra
Department of Aerospace Engineering
Indian Institute of Technology, Kanpur

Lecture - 37

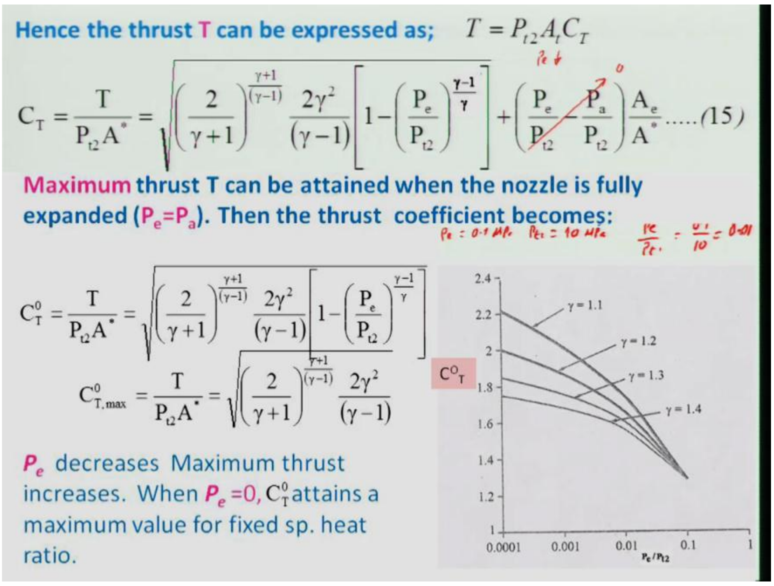
(Refer Slide Time: 00:23)



Let us start this lecture thirty-seven with a hot process from Dwight D Eisenhower. He was a, you know past president of US long time back around nineteen sixties kind of thing right. He had told every gun that is made, every warship launched, every rocket fired, signifies in the final sense a theft from those who hunger are not fed, and those who are cold and not clothed. When I am taking about this rocket propulsion engine, I must communicate this massage, which was similar to mine and told by a USA president who always try to you know USA try to have a war, and then you know all sorts of kind of thing. So, it is very important for us to understand these things, we should use the technology for the peaceful coexistence not for a war.

So, let us recall what we have learnt in the lecture, I was looking at basically the performance of rocket engine. Of course, before that had given you a brief introduction to the rockets, and its utility under that right. And when we discuss the performance of rocket engines, we basically made some assumption. Those are ideal you know assumptions, but however, it can be useful for the real analysis. As well because we have seen that it is not varied you know different than that of what is that that we talked about just coefficient right. And we had derived an expression for thrust coefficient. So, we will go on discussing that and by knowing this trust coefficient, we can also find out the trust or vice versa.

(Refer Slide Time: 02:20)



So, if you look at the thrust is basically, thrust is P T 2 that is nothing but a chamber pressure into A T, A t is the throughout area of the nozzle. And keep in mind that I will be using A star and A t is star for me it is same for the time being. And C T is your nothing but your thrust coefficient. If I know the thrust coefficient, and I know the chamber pressure, I know the throughout diameter, I can find out thrust or other way around. That means, if I could evaluate the thrust may be static thrust in a laboratory and then I know the chamber pressure, I know the throat area, I can get the thrust coefficient experimental. But what we are derived an expression that is meant for an ideal case. This expression C T which is basically derived for an ideal case ideal in the sense making ideal assumptions like not classes and other things we have not consider.

And if we look at this expression which I have shown here it is basically function of gamma, it is a function of P e exit pressure and also the chamber pressure we always talk in terms of ratios. And other one is your atmosphere pressure and the chamber pressure P t 2; chamber means basically combustion chamber of the rocket engine. And A e ratio A e by A star or I can say A e by A t. And keep in mind that when this P e if it is goes on decreasing like means P e will be reducing, goes on decreasing what will happen, this term will be coming closer to the P a, it may happen. And what happens to these term for the same chamber pressure P e is decreasing, this term will be smaller. If I take gamma is the something 1.2, and it is point you know like one point two point two that is basically five times into one point two this will be coming around is one by six. So, the six times in the you know then this is a very very small number yes or no, that means, these thrust is reducing, but these become a bigger quantities. Because one minus this, if it is one minus that and it will be thinking that will be going towards one basically point nine nine or something nine eight as the P e goes on decreasing.

So, maximum thrust can be obtained basically if look at when the nozzle is fully expanded this another way of looking at it like means if P e is equal to P a, these term will be zero. And then the thrust coefficient will be basically that is root of what 2 by gamma plus 1 gamma power to the gamma plus 1 divide by gamma minus 1 and into 2 gamma square divide by gamma minus 1 in the bracket 1 minus P by P t power to the gamma divide by gamma minus 1 and that is the maximum thrust one can think of. So, what you can see that it is only a function of P e and P t 2 and gamma.

Now if P e is as I told P e is goes on decreasing, what will be happening. It will be these become it will goes on increasing, yes or no. This term will be goes on increase C t naught will go on increasing. And what is the effect of gamma then I can vary this parameters you know gamma from 1.1 and to 1.4. If look at what is happening, I am plotting over P e by P t 2. And this P e can be same as that of the P t 2, the one situation yes or no? It can be worst case, then it will be one of course, it is we would not be considering. If we consider then what will happen this will be zero; that means, the thrust coefficient will be zero. And we would not get anything out of it. So, that is a useless to talk about it.

So, therefore, we will consider the P e by P t 2 around 0.1 to 0.001. When it is 0.001, when P e is very very small as compared to the P t 2. If I take the P t 2 as 10 megapascal, if I take P e is something 0.1 megapascal atmosphere pressure, what it would be then 0.001, 10 megapascal if you look at 0.01, kind of things here is not it. If I take P e as 0.1 megapascal, and P t 2 is 10 megapascal, and what will be P e by P t 2 will be 0.1 divide by 10 that will be 0.01 yes or no?

So, you will be here and as it is goes on like you know it can be 0.01 megapascal then it become 0.001, it will be for the same chamber pressure. And if I am saying 0.001 megapascal P, it will be these values. You see that this is C t star, and C t naught and it is basically what you call decreases with increasing P e by P t 2 that is one. And it is the gamma effect is very very predominant in the whenever the pressure is low; that means, exit pressure is very low as compared to the chamber pressure. But as the exit pressure is you know and the ratio of exit pressure and chamber pressure becomes higher then gamma effect is negligibly smaller all are coming to the almost similar values.

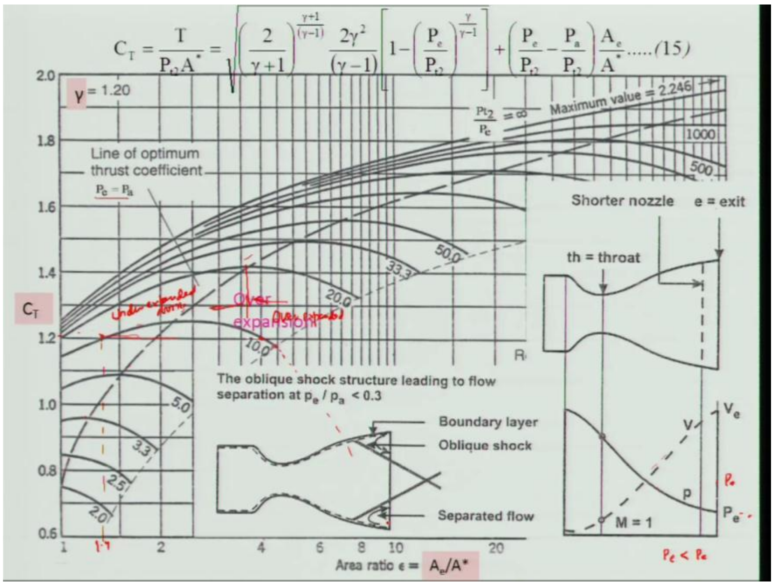
Now, you can look at a situation where this you know P e become zero. So, if it is zero, these term will be zero. So, we will get a maximum you know thrust coefficient and when it is nozzle is fully expanded that is a condition we are talking about.

Student: ((Refer Time: 10:00))

Professor: No, no p e is zero like you go to the vacuum right where p e is equal to p a right the vacuum is right, p e is equal to p a vacuum condition in the deep sea, I mean it cannot be zero, but you can assume it tending towards zero or something like that. So, then you will get an expression that is you know C t max is only function of gamma nothing else it is only c t max is a function of gamma; that means, you know we always go for a smaller values of gamma. So, that you can have a you know higher C t max kind of thing.

So, if you look at these you know you can interpret and even can what is the implication of that you will be designing for a certain ratio, of course, if where assuming it fully expanding. And what we will do now, we look at these expression the generalize expression for the thrust which is having you know what you call condition being place in the sense, the actual what we call thrust without any simplifications. And look at vary this parameter what are the things will you vary will you varying A e by A star and will be varying P e by P t and will be varying P a by P t 2, and we are taking gamma as a constant that is 1.2.

(Refer Slide Time: 11:36)



If we look at this is being plotted C t versus the A e by A star right, A by A star is your A e of ratio that is exit area by the throat area. And keep in mind that this is being logarithm like it is from one to hundred kind of things. So, what you can note from here that is if I take this pressure P t by P e as 2, it is C T is the lower than 1, and it is goes on decreasing. And of course, when you go of increasing this, you will find that there is a increase in the it goes on increasing with the area ratio and reach a big value here you know and then it is decreases. And these area ratio of the nozzle is corresponding to fully expanded nozzle that is the optimum thrust or the maximum thrust you are getting with the respective P e is equal to P a. When exit pressure is equal to P a, you will get fully expanded, so therefore, you will get a maximum.

So, if you look at this as you goes on what you call increasing these value right P t by 2 by P e what is the meaning for a particular chamber pressure, you are going on decreasing P exit presser or the exit pressure, p e or the exit pressure. You are go on decreasing that is why it is increasing for particular chamber pressure. And you will see that it is similar in nature that means, this is you know if I take a let say twenty, I will take twenty pressure ratio that is P t 2 by P e, if it is goes on increasing as the area ratio increases and reach a big value, and then it decreases. What it indicates this big value is the maximum what we call thrust coefficient or thrust one can get and corresponding to the fully expanded nozzle is that clear? That means, in these zone what I will call, I can call these zone is as an over expanded. And whatever this portion if I take, this portion from here, you know this portion here on wards this will be under expanded nozzle.

Now and one is fully expanded and you will get the big value, both the side the thrust coefficient is decreasing. Now, which one you will choose, which one will be more, you know better. Because if I assume that I am going to I am designing, let say I am taking about let say I will take for the time like, I am having same here and there is another here same. I am having same C t that is 1.2. If I take these values you now right one is this side is over expanded this is under expanded, which one I will choose area ratio and why. It is the very simple logic; I will talk about it. Because, in this case, if I choose this area ratio, you know if I choose this under expanded area in ratio, so they you know under expanded case for the same thrust coefficient of 1.2 for the pressure ratio of P e by P t 2 obtain. So you will get that area ratio will around may be 1.4 kind of the because this is 1.5, this is 2, so this will be 1.5. But if I take here it is around 4; that means, I am having a bigger nozzle A e by A star is 4; in one case, it is 1.4. This will be around 1.4.

So, if we look at for the same thrust coefficient I am getting; that means, length as label I will be getting for the same chamber pressure within same throat area, are you getting my point? So, therefore, here I will be using a smaller these things smaller nozzle divergence portion will be must smaller. So, my weight will be reduces and my cost will be reduce and payload I can carry more for the same engine same kind of thrust label. So, therefore, it is advisable to go for a under expanded nozzle than that of the over expanded, and from that point of view, but there might be several other point.

If I look at the other way around, you know can I not go beyond this, for example, if I am taking about this can I not go over like this yes or no, that is one question I would like to ask. Let us look at what we are observing P a here; that means, if I go on this increasing this P t 2 chamber pressure by exit pressure ratio, you know you will find that it is again increasing thing at getting a big values and then decreasing. And of course, it goes to A whenever you know it become a maximum value, you will get of C T that is 2.246, when P t 2 by P e is equal to infinity. That means, the P e is zero, exit pressure is zero as you told like it will be occur when it is in a vacuum condition of course, you would is fully expanded. Otherwise, you can have expansion, which is very very low.

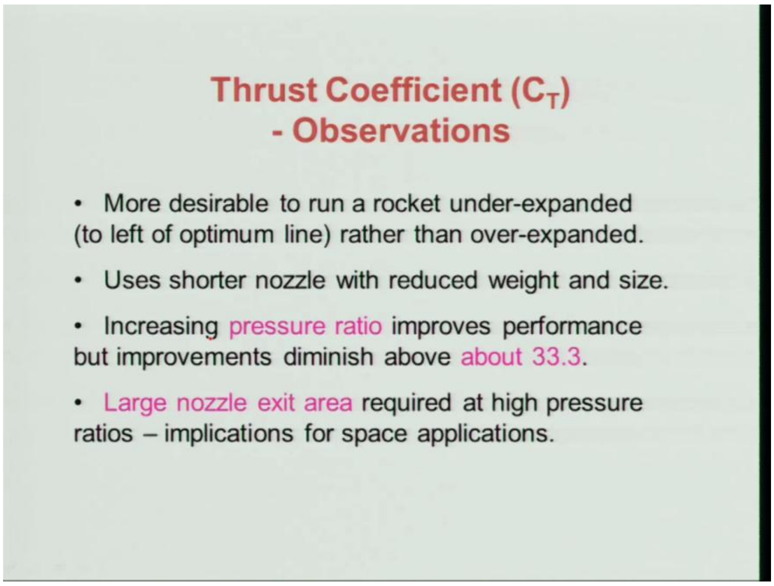
So, and if I will you know that is the and you will see that there is a value of 33.3, you know beyond that there is not really not much value here increasing because this is in case in then it is almost this maximum value is almost becoming like that. So, you are not really gaining more except that you are as I am going for higher pressure ratios then my nozzle size if look at it is become 40, 100 you know kind of things is very very big if I want to even operate at a maximum value. So, it is a very big. So, you are not really gaining much beyond the pressure ratio of 33.3 then a people to restrict to that. And if we look at if I join this big values and this line is known as line of the optimum thrust coefficient; that means, it is basically maximum thrust coefficient of the thrust coefficient corresponding to the maximum thrust. And that is a limit because if you use this line you know beyond this as I told then there will be also the flow will be separated.

Now, why flow will separate what is the reason because if it is over expanded then what will happened, let us a look at it. I mean what really will happen, let us say that this is a what you call a convergence-divergence nozzle. And this is a throat region, this is your exit and the pressure if you look at this is your you know throat where Mach number is occurring, the pressure is changing is decreasing and it become in a same as that exit pressure for the nozzle. And of course, the velocity increases as the flow of exit, you know expanded in the divergence portion of the nozzle.

So, then if we assume that you are having a sorted nozzle, what is happening you are having a pressure which is P, which is lower than the if I say this is you know P a expanded fully and then it is that means, this is higher. That means there is some more expansion can takes place right; that means, it is under expanded in this case. But if we consider that you know this another case where it is not P e is not equal to this thing but rather it is P a that means, it is expanded beyond this pressure, and then the P e is basically right is a smaller than P a. And if it is smaller, then what will happen, there will be formation of slag and particularly this slag can be formed, it can be oblique slag. It can be at a you know when it is very closed to that there will be a what we call the sag will be form as the exit.

But if it is P e is less than P a, and then slag you know the what you call are little bit, you know this thing then the flow separation will take place; that means, if this is occurring if P e is you know and P a ratio is less than 0.3 let kind of things. The naturally the shock the boundary layer will be separated, because the adverse pressure guide in right. So, then when it is separated then what will happen losses will in curve and then who one cannot really operate and in this condition. So, therefore, it is being you know like a this line is being consider that with there is you know flow is likely to separate. Of course, it may differ if when we are using various kinds of other nozzle like plug nozzle and then bell nozzles and the conical nozzle, generally you know conical in bell nozzle is being used kind of thing. So, these you should keep in mind and therefore, whenever you are designing will be restricting the operation, and generally prepare able that.

(Refer Slide Time: 22:12)



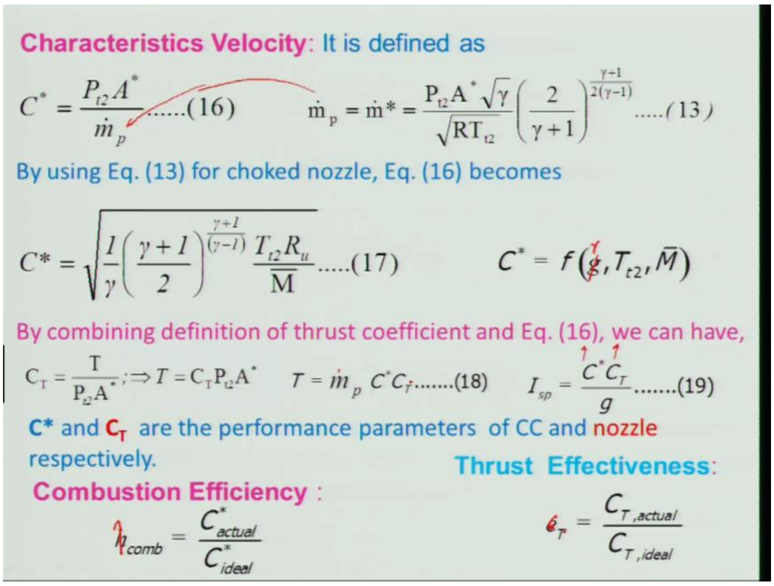
Let us summarize what we have learnt from this that more desirable to learn a rocket engine under-expanded condition that is left to the optimum values, what we are seen rather than the over-expanded because of you know area penalty. And also it will lead to the flow separations due to the formation of shock and there will be losses also due to the formation of shock. So, therefore, and you just shorter nozzle with reduce weight and size that I have already emphasize because and I increasing the pressure ratio improve the performance, but improvement diminishes above 33.3. We are not really because we are go on increasing the area ratio; that means, weight and size, but you are not getting that much of you know gain in the thrust coefficient are the thrust label.

So, therefore, that has to be restricted and large nozzle area required at high pressure ratio was implication for this space application, because we cannot avoid. Suppose we want to saying some you know deep space the naturally, I will have to expand;

otherwise, it will be not nice you know it will be you know like not giving with the desired thrust what you need, so that has to be kept in mind.

Now, if we look at whatever we have look at the thrust coefficient; keep in mind that it is talking about the performance of the nozzle. It is not function of the temperature of the combustion chamber, it is a function what pressure area ratio, gamma, exit pressure ambient pressure all those thing right that means. These parameter is taking about only the performance of the nozzle, but we need to understand how well the combustion is taking place for that we need to define a parameter right.

(Refer Slide Time: 24:08)



And let us define of another parameter what will be using is characteristics velocity that is basically C star is equal P t 2 A star divided by m dot p that is the mass flow rate of proper length through the nozzle that is the amount of mass flow rate. And we keep in mind that we always talk about you know condition known as choked condition; that means, the rocket nozzle is all the time or most of the time is choked. So, we will be using the term that choked mass flow rate, we know we already derive these thing is m dot is equal to P t 2 A star root over R T t 2. And of course, in the numerator root over gamma then multiplied by 2 by gamma plus 1 power to the gamma plus 1 divide by 2 gamma minus 1, this we have already derived and discuss several times.

If I just substitute over here in this equation, you know here, what I will get, you will look at that P t 2 by A star will cancel it out right. So, what you will get is the you know

C star is basically in terms of in a gamma that is one over gamma and the bracket gamma plus one divide by two power to the gamma plus one divide by gamma minus one T t 2 and R u. R u is the universal gas constant, m is the molecular weight of the gas, which will be expanded in the nozzle. And keep in mind that gamma is changing, when it is a going through the nozzle, but we are assuming to be constant, because that value is not really much changing.

And what it indicate this characteristic velocity, and how it is functionally. If we look at this expression of equation 17, you can see that C star is a function of the chamber temperature that is T t 2, and the molecular weight of the gas and also the gamma. These are the functions, but what it indicates it indicates the capability of the combustion chamber that can produce certain pressure like if you go to the basic definition chamber pressure per you know the mass flow rate for certain fixed value of mass flow rate of propellant per unit cross sectional area throat. For that how much pressure it can produced that it indicates that is the capacity which is indicates, but it is basically independent of if look at beauty of this is independent of the chamber pressure. Although it is definition which is there, but if I look at this expression equation seventeen it is independent of pressure it is dependent of temperature.

When we talk about temperature, it is basically how good is combustion is you know it in other words you can use this parameter as a performance parameter indicating how good the combustion is right. So, and as I told C star is a function of basically gamma T t 2 and molecular weight. And if you look at if I want to have high C star where I want to have high C star or I want to have low C star on the characteristic velocity. What I want how can you say that now, now what we need we need to have higher highest speed we need to have higher thrust you know these are the requirement. So, therefore, we need to look at those thing whether we can relayed this C star and thrust coefficient at the characteristic velocity of the thrust coefficient to the thrust are not or the higher sphere are not. Then only you can talk about is not it, but if I want to have C star to be higher naturally I should have a higher combustion chamber that combustion chamber temperature that is T t 2 and lower molecular weight you know because molecular weight; that means, lighter propellant I should be use.

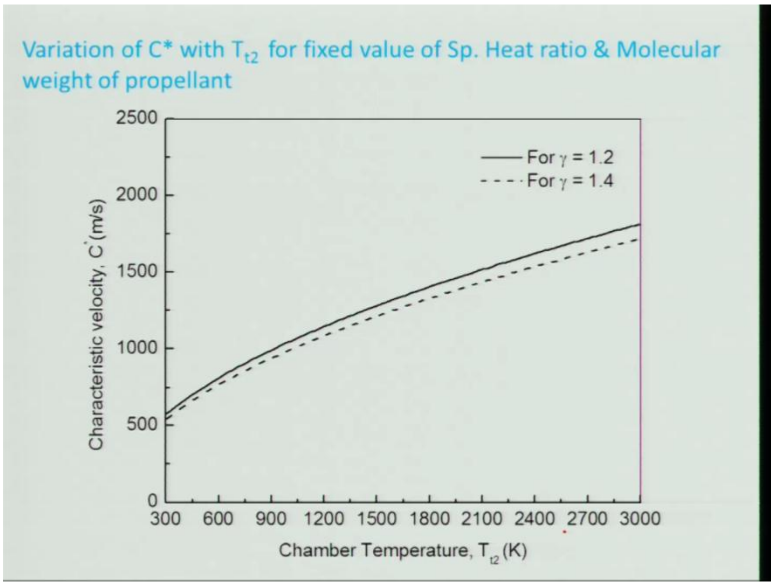
And of course, the gamma one can have a lower values you know, so that you can have higher these thing. So, from the combining the definition of thrust coefficient an equation seventeen, I can have C T is equal P t by P 2 A star then thrust is basically C T P t 2 A star. This one is basic definition; that means, thrust is basically you know thrust coefficient multiplied by chamber pressure there is P t 2 and A star is the throughout area, but if you look at definition of characteristic velocity and substitute is value for P t 2 by A star. We will get a basically thrust is equal to m dot p that is propellant mass flow rate C star and C T; that means, if I want to have a higher thrust the naturally I should have higher thrust coefficient and also the higher characteristic velocity. Of course, for a particular mass flow rate of propellant or if I want to there is a limitation in this quantities. So, naturally I will have to enhance the mass flow rate you know of the propellant.

So, highest people look at it is c star by c t this is by from the basic definition of the g; that means, if I want to have a higher what we call highest speed natural highlight enhance this and highlight enhance this. But there is limitation and thrust coefficient there will be also limitation of the C star you know that is why we cannot really beyond and certain higher speed as I had discuss in the last lecture. That is you know this something in the solid propellant we are having limitation of something 350 of isp, isp I mean is the c condition and that is what we call in the second in terms of second. So, if we look at the C star and C T that is at the performance parameter of the combustion chamber, C star mean it indicates how good the combustion chamber is working in converting the whole into the certain temperature or how good the combustion is an the c t is talking about the nozzle performance.

Now, taking this in mind we can define a combustion efficiency, this is basically you keep in mind this is eta combustion is equal to C star actual divide by C star ideal. I can get from this expression equation seventeen, C star, but then actual how I will get what I will do, I will have a tersely. Where I can measure the mass flow rate of the propellant which is passing through it like this if you look at the equation sixteen. Let the m dot p i can measure the chamber pressure, I know the throughout area i can get actual one right.

So, if you look at this combustion efficiency is very very you know closer to one; that means, it will be point nine eight nine nine sometimes even nine seven you know like you get. So, therefore, it is good enough to talk about even calculate ideal one while designing, but we generally use a value of particular kind of engine then look at while designing we have tested it we know we cannot really test. So, we take this combustion if we C T and c and using similarly we can also define thrust effectiveness that is a epsilon T that is C t actual divide by C t ideal, C t ideal. I can get from the equation fifteen and C t actual. I can conduct experiment and find it out while designing will be using this you know epsilon t values and we know the ideal one we can estimate the actual one and then carry out the design procedure. So, the beauty of this performance parameter have already discuss that you can separated it out you know and look at it and and then look at this efficiency this can be use as a design tool for the rocket engines.

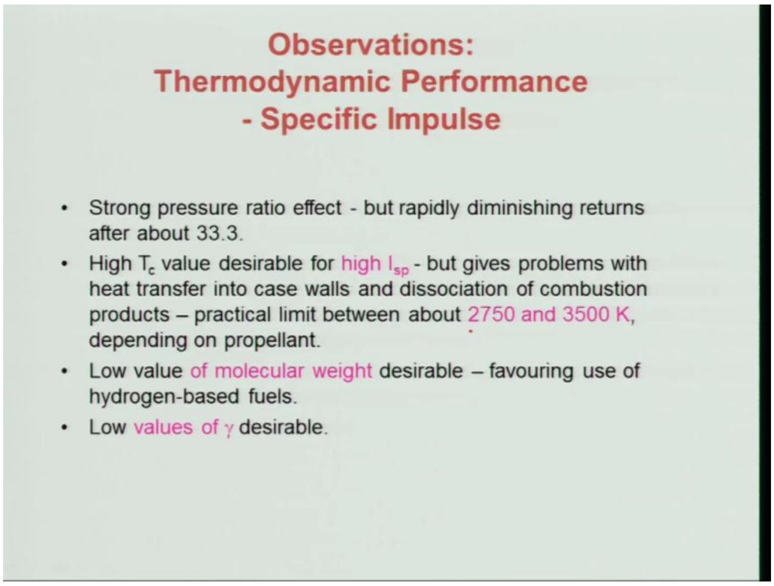
(Refer Slide Time: 32:52)



So, let us look at the variation I did not really discus there, but let us look at now the characteristics you know velocity how it is varying with temperature right. So, if we look at like, it is from the expression, it is very clear. And if I take two gamma values gamma 1.2 and gamma 1.4, you see this characteristics velocity is increasing with the chamber temperature are both the gamma values. And keep in mind that this gamma 1.2; that means, low gamma value is having higher characteristics velocity and higher temperature the difference is match higher as compare to the low temperature that is one thing and we can really go on increasing this.

Because if I want to have higher, we natural will have to go on increasing this characteristic velocity, but if I go beyond something three thousand you know five hundred kind of things the naturally what will happened requisition will happen then I cannot really go for that you know it will be do symptom raises. So, that is the restriction then only the people use something where 2500 to 3500 or sometimes we will say 2700 to 3500 kind of thing zone where they will be operating.

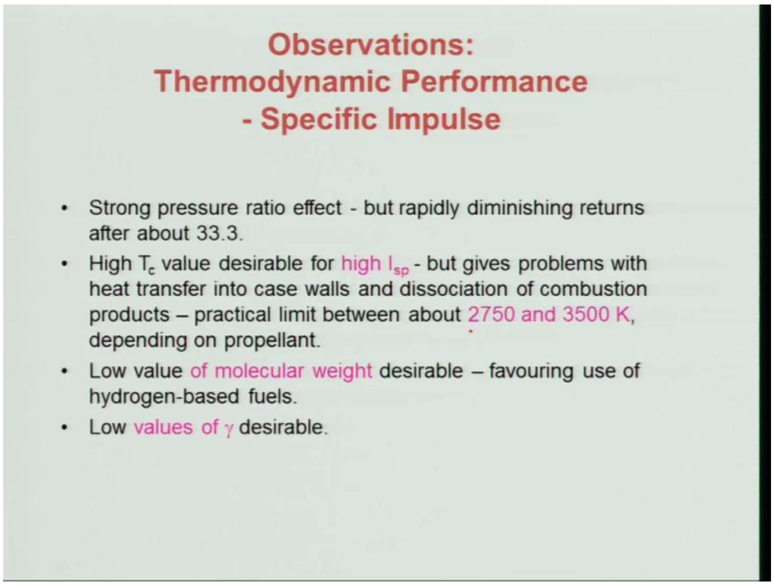
(Refer Slide Time: 34:12)



So, now let us summarize what we have learnt as, so for this characteristic velocity which is basically thermodynamic performance and strong pressure ratio effect as I told you. But rapidly diminishing returns about 33.3 that we have learnt in the from the thrust coefficient curve versus area ratio, for different pressure ratio and higher T c that is basically T t 2, T c is nothing but T t 2 chamber temperature value, desirable for higher ISP. But problem is that the more the higher the temperature then the heat transfer will be more that is the one problem, but again higher the temperature then dissociation will occur that means, you know it will be consumed in the passes of dissociation whatever being release. So, it will be again inefficient therefore, practical limit being use as i told something twenty seven fifty two you know thirty five point eight some people use 2500 to 3500 Kelvin kind of thing.

These are all design you know thumb rule kind of thing one depending on the propellant because it will be depend the temperature what you can get will be dependent on the type of propellant you are using right. So, the low value of molecular weight is desirable as i mentioned that you know molecular weight will be lower because favoring particularly the hydrogen based fuels and therefore, you can get high rise because characteristic velocity will be higher even thrust coefficient if you look at one can also get that. So, now, we will I mean of course, the low values of gamma is desirable as the let demonstrated through you know characteristic velocity plot right. So, therefore, these are the things you should keep in mind and we can learn by just carrying out simple analysis about the performance of the rocket engines.

(Refer Slide Time: 36:10)

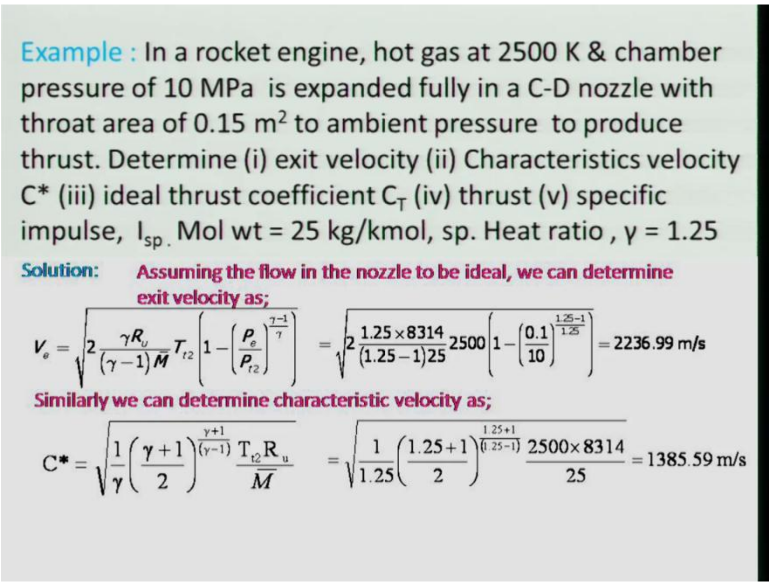


So, I will take an example in a rocket engine hot gases at 2500 Kelvin and chamber pressure ten megapascal is expanded fully in a C-D nozzle with the throat area of 0.15 meter square to the ambient pressure are producing thrust. And we need to determine exit velocity, characteristic velocity, ideal thrust coefficient, thrust and specific impulse. And molecular weight you can take 25 kg per kilo mol and specific heat ratio gamma 1.25, we have taken in this example. So, if we look at assuming flow in the nozzle to be ideal, we can determine these velocity and keep in mind this is gamma like 2 gamma R u gamma minus 1, molecular rate T t 2, and this is P e by P t 2 power to the gamma divide by gamma minus one.

We will substitute, because all these things we know, and gamma is 1.25 and the R u is the universal gas 8314, and molecular weight 25 and temperature is given 2500 and the pressure P e is 0.1 megapascal that is same as that of the exit pressure or the ambient pressure. We are assuming it to be fully expanded nozzle that is again and you know kind of things and which is given in the problem that is fully expanded right. So, and ten atmosphere is the chamber pressure. So, when you substitute, values you will get 2883.4

meter per second. Similarly, we can determine characteristic velocity is by just substituting values, which we have talked about and all are given. So, you will get 1385.95 meter per second.

(Refer Slide Time: 37:57)



So, ideal thrust optimum coefficient is and when you substitute this values because this is fully nozzle expanded. So, we can substitute, we will get, we are getting something 2.08 you know kind of thing which is quite good. And the specific impulse can be determine as C star and C t, the g and you will get you know these way and you will get 293.85 second, you are getting right which is you know solid propellant kind of engines you can think of. Similarly, we can determine the thrust produce by this rocket engine as basically you know C T star, C T and P t 2 and A star, we know all those things, we will substitute it is something 3.12 meganewton is the quite high power, you know high thrust engines right. So, we will with this, we will stop over and will continue.