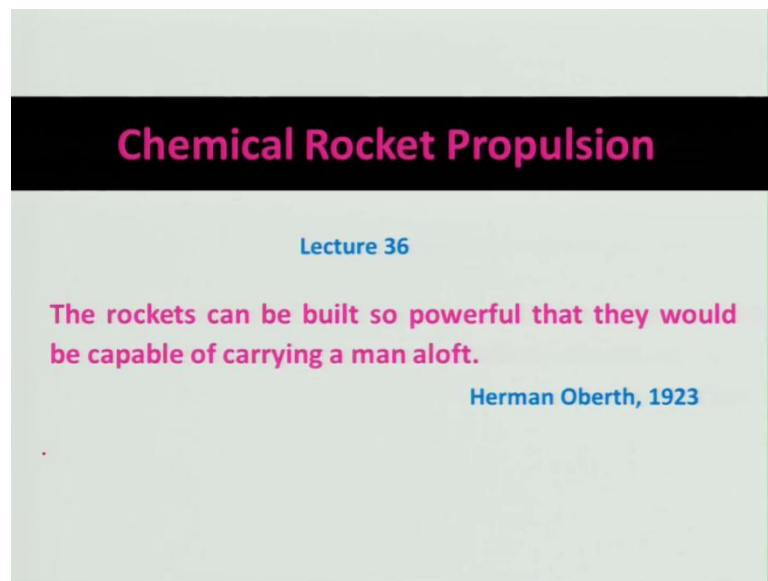


**Fundamental of Aerospace Propulsion**  
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**Lecture – 36**

Let us start this lecture thirty six with the hot passes from the rocket scientist Hermann Oberth.

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Who had given a statement in way back in 1923 he had told that the rockets can be build,so powerful that they would be capable of caring a man a lot ; that means, a lot means basically floating right. So, if you look at the time people were having the knowledge of rocket, but that was too tiny to really do anything and if you go back to the history Indians and Chinese's were basically good at making rocket and using for war phase also.

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And if you look at we are having a ambitions satellite launch vehicle program or space programme for which we have design several of launch vehicle. Of course, the latest one is GSLV that is Geosynchronous Satellite Launch Vehicle, if you look at this rocket vehicle what you can note here; that means, these are all rocket 1, 2, 3, 4 like at you know these are the ((Refer Time: 01:39)), these rocket are known a stop and rocket motor.

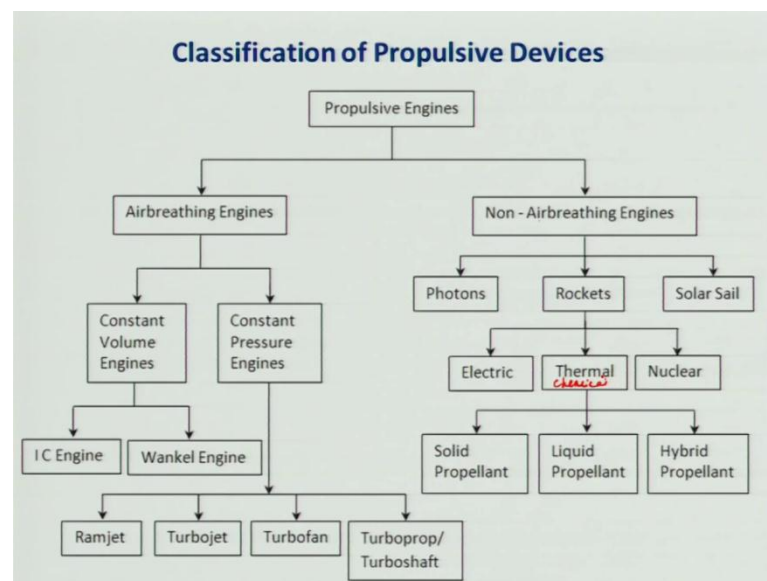
And this central one is having you know several stages like in the generally the initial stages will be the boosted and also this top one will be also for the boosted. Boosted means whenever you start from the ground you need to overcome lot of inertia right for that you need to give very high level lot of ((Refer Time: 02:09)) . So, that you can immersion and then after that of course, the staging in will be use which I would not be discussing; that means, you will first know you will give a very high thrust and it will move then this stoppen will be separated out and it will be detach from the vehicle.

So, also the some of the stages and you know main thing is, this is your pay load and as it goes on to get a very high velocity, you need to eject the you know first stage, second stage, third stage you know like generally GSLV will be having first three stages in a rocket vehicle and why those stages are required . As I told it is meant for to get a high velocity, so that you can put the satellite in certain high orbit right, because the velocity injection velocity will be very high.

And if you want escape from the earth gravitational pool ,then you will have to go to a very high velocity escape velocity, for that you need to have a you know very high velocity, which you cannot get with the whole rocket vehicle,sSo therefore, you need to reduce. And I would not be discussing about that part particularly what will be trajectory, what will be maximum, but from the point of view like type mechanics and also requirement of the engine design . So, what I will be doing, I will be looking at basically the various aspects of the combustion and then burning propellants and then nozzle and expansion as a thing both aspect will be looking at.

That means we know our this, what you call it requirements and then we are looking for it right. And we want to see what is the how to you know go about and what are the passes involved in combustions and all those thinks will be looking at. As I told you earlier that rocket engine you know is basically will be chemical rocket engine and it can be also solar, it can be electrical, it can be thermal and it can be nuclear.

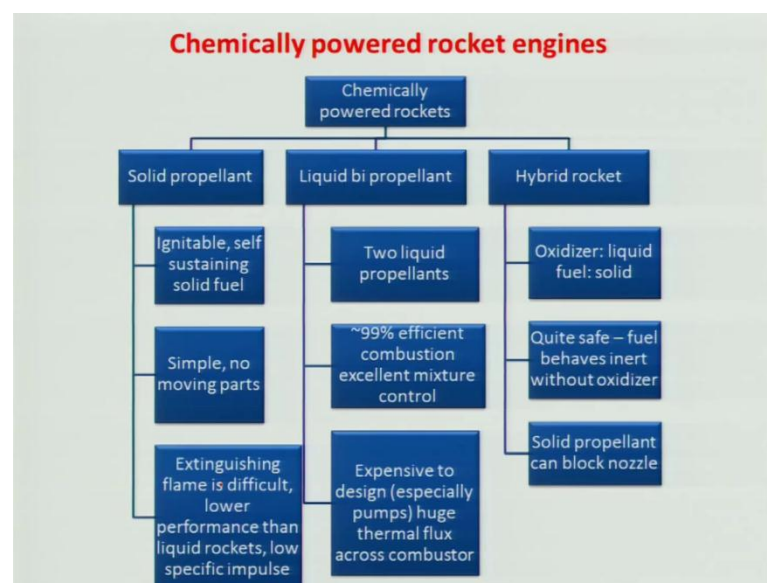
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And if you look at look at this we had visited this propel classification of propulsive devises earlier right and rocket engine comes under non air breathing engine. And in the earlier lecture we have discussed about air breathing right particularly this ramjet, turbojet, turboprop and turboprop engine all those things we are discussed in last several series of lecture and we had carried out cycle analysis .

But now we will be concentrating only on the thermal or I call it chemical you know rocket engines and this chemical rocket engines is divided into three categories, one is solid propellant rocket engines, and liquid propellant rocket engines, and hybrid propellant rocket engine. We will be trying to look at you know all these three aspect very briefly right, so therefore you know we will looking at detail about these things. And before really doing that we need to look at general the rocket engine and its performance some of the performance parameter we have already seen, but we will visit some of them.

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To start with we will look at the what to call chemical power rocket engines right and as i told that this is the solid propellant and just to distinguish how it is different and what are the advantages and disadvantages we would not be discussing now, but I will give the overall view like if you look at solid propellant, is basically it is ignitable and self sustaining well; that means, you can ignite

look atOnce you ignite you know you cannot really it is stop it is difficult in generally rocket solid rocket engine however; that means, it is difficult to extinguish the flam . And it will be have the lower performance than the liquid rocket engines and low specific impulse what is the specific impulse we will be defining and all those thing keep in mind that the simple

And it is quite simple and no moving parts like unlike our liquid propylene engine were there will be some moving parts we will see that. And it is a can be you know bi propellant, it can be mono propellant, bi problem one is liquid you know oxidizer and other is liquid fuel that is bi propellant and it can be both together you know that is known as mono propellant will be discussing little bit about those things. And if you look at look at it is you know two liquid propellant as I told it bi propellant and the high level of combustion efficiency order of 99 percent efficiency you can get, because it can mix the liquid and then vaporize them very easily like solid which is difficult to burn you know like so, but it is quite expensive to design especially the pumps and use thermal flux across the combustor.

when you will go liquid propellant engine you will see how complex it is, it is not as simple as what we had seen very beginning of lecture some schematic right that is not simple, but it is quite complex. And beside this the hybrid which is the basically the combination of solid and liquid; that means, the solid will be the fuel and liquid will be oxidizer like, but it can also vice versa for the first one is being preferred. So, that is known as the hybrid and which will as I told oxidizer will be liquid and the fuel will be solid and it is quite safe to use, because fuel bear bears it is like a inert.

Whenever you know it is divider oxidizer; that means, oxidizer is not their then it will be as inert as it was not any of the things. So, it is that is why it is quite safe whereas the, what you call liquid propellant and other things.

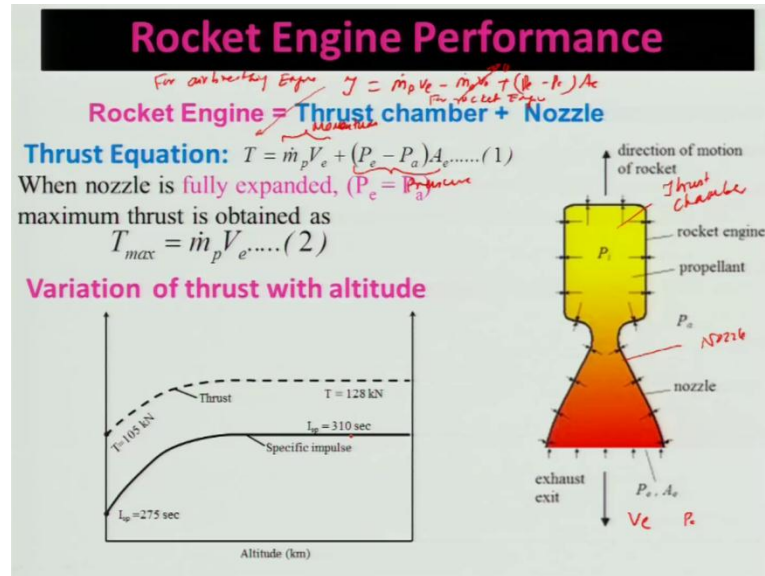
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Solid propellant will be looking at like you know there will be double based propellant and composite propellant all those thing we will be looking at... And solid propellant, but sometimes what will happen, the solid propellant can be broken of course, not in a regular way due to some mal functioning and then some portion it can go block the rocket nozzle ((Refer Time: 09:20)) nozzle. Then what will happen then pressure will built up then you know it will go it is sometimes it may lead to explosions as well.

But generally I means that it is not been really occur where you properly design, but it just can happen any time you know. Therefore, one has to be careful about this while designing. So, if you look at this is the basically I have given some overall view of the

chemically power rocket engine which can be broadly divided into solid propellant engine, rocket engine, liquid propellant rocket engine and hybrid rocket engine.

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So, what will do now is basically look at all these three rocket engines that we will call chemical rocket engines and those things. Performance we need to look at and which we have already seen like the thrust and other things. At a little different perspective. So, if you look at the rocket engine, it will be having two components, what are those components? One is the thrust chamber, the other is your nozzle.

So, if you look at this portion, if you look at this portion, it is basically the thrust chamber and this is your thrust chamber and this is your nozzle, right, which are given as nozzle. Like there are two components: one is the thrust chamber. What do we call the combustion chamber and the other is your nozzle. Nozzle generally convergent-divergent nozzle is being used, unlike your toy rocket that we use in Diwali and another thing it will be having not any convergent-divergent, maybe simple, whole will be there you might have seen. When you know you might have played during your childhood about rocket, what we call your toy rocket for the fire crackers. So, but the generally the convergent-divergent nozzle is being used. Of course, several other kinds of the nozzle which have come up in recent time like plug nozzle and other everything which we would not be discussing, but. So, if you look at the thrust equation as

we have already derived for the air breathing engines and non air breathing what it will be having because no flow is coming in all this propellant is inside.

So, therefore, the momentum drag due to the propellant input or in will be zero. So, we will get an expression as thrust is equal to  $\dot{m} p$  that is the total what you call flow rate of propellant which we will be consisting of both oxidizer and fuel the  $V_e$  is the exit velocity this is your  $v_e$  exit velocity right and there is a pressure thrust. you know thrust be to the expansion in nozzle  $P_e$  minus  $P_a$  into  $A_e$  and this is what do you call the momentum thrust due to the momentum and this one is your patient. And what difference you could get, the different is that it is the drag right inlet momentum drag is zero in this case like, but whereas, in the air breathing engine the thrust will be basically for air breathing engine term, the thrust will be  $\dot{m} p v_e$  minus  $\dot{m} v_{naught}$  plus  $V_e$  minus  $A_e$  right this is basically 0 in case of rocket engine right.

So, therefore, that comes around here now when we thought about this thing we know very well that when nozzle is fully expanded that is  $P_e$  is equal to  $P_a$  we will get the maximum thrust right. So, that is the maximum thrust will be equal to  $\dot{m} p v_e$  and if you look at this thrust whether it is basically it is having the pressure component the pressure change in the nozzle or you know when it is fully expanded it will be dependent on what the thrust will be dependent on altitude right is it not. Because your  $V_e$  will be decided by what pressure it will be; that means, this  $P_a$  this is  $P_e$ , but  $P_a$  which is the ambient air it will be dependent on the altitude. As you go along the height along the altitude, so its pressure goes on decreases right is it not. So, when you go to the very deep space ideally it will be zero right exactly ideal.

So, therefore, if you look at the thrust will be varying with the altitude; that means, if I take the thrust it will be let us say with the 105 kilo Newton it will go on. And after that you know certain it will remain almost ((Refer Time: 15:04)) increasing that means, there will not be much change and it will be remaining almost constant right for a very high altitude. And of course, the  $I_{sp}$  will be similar because  $I_{sp}$  can be related to the thrust that will see in a movement right. So, now what will see we will look at certain definitions other things what will be using as formulas parameters.

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**Specific Impulse ( $I_{sp}$ ):**  
 $I_{sp}$  can be defined as the impulse per unit weight of the propellant.

$$I_{sp} = \frac{I}{M_p g} \dots (3)$$

$$I_{sp} = \frac{\int_0^{t_b} T dt}{g \int_0^{t_b} \dot{m}_p dt} \dots (4)$$

$$I = \int_0^{t_b} T dt = \int_0^{t_b} \dot{m}_p V_{eq} dt = M_p V_{eq} \dots (5)$$

$$I_{sp} = \frac{V_{eq}}{g} = \frac{T}{\dot{m}_p g} \dots (6)$$

$I = \text{total Impulse,}$   
 $M_p = \text{total propellant mass}$

$V_{eq}$  is assumed to be constant  
 $t_b = \text{burnout time}$

$I_{sp}$  does not depend on the flight velocity.

$I_{sp}$  indicates Performance of rocket engine.  
**Higher  $I_{sp}$  means better performance.**

Rocket Engines	$I_{sp}$ (s)
Solid	200-310
Liquid	300-460
Hybrid	300-500
Solar	400-700
Nuclear	600-1000
Electrical (Arc heating)	400-2000

So, the specific impulse if you look at can be define as the impulse for per unit weight of the problem; that means, so much impulse you can give whenever the certain amount of propellant is been you know ejected out from the nozzle right. So, by definition it will be  $I_{sp}$ ,  $I$  equal to  $I$  divided by the mass of the propellant into  $g$  like  $g$  is the gravity generally in the c level will be using. So, if you look at  $I_{sp}$ ,  $I$  can what is this impulse.

Impulse will be you know thrust integrated over the time that is from 0 to  $t_b$  is a bending type bending type of what, bending type of propellant right whatever it is right and then what is this  $M_p$   $M_p$  is the total propellant being burn during the time what you call during the time 0 to  $t_b$  as the duration of the bending right. So, then  $M_p$  will be  $\dot{m}_p$  integrated over the 0 to  $t_b$

So, if you look at  $I$  will be if  $I$  put this thrust expression without considering the ratio component that we are doing right or  $I$  can write down this thrust you know as  $\dot{m}_p V_e$  plus  $V_e$  plus  $P_e$  minus  $P_a$   $A_e$  is equal to  $\dot{m}_p V_e$  equivalent right this is equivalent velocity which has taken care of this term. So, if you put that things  $\dot{m}_p$  and then if  $I$  say that this  $V_e$  equivalent is not changing over the time is it really possible, it is not really possible when it is moving because your  $P_e$  and  $P_a$  will be changing, but you can assume variation expected the inertial case.

And rest of the time it is not varying as you have seen that thrust you know not changing along with the altitude of certain this thing right we have seen the last slide. So, if you



have assumed that then you can say it is not changing much, but in real situation it would be it be some finite change. So, therefore, I can write down  $\dot{m} \dot{d} t$  will be nothing but,  $M_p$  that is total propellant of mass right and then I can write down  $I_{sp}$  you know will be you know well when I substitute this value over the equation in this what I am getting instead of this I will get  $m_p v$  equivalent. So, this is cancel it out, so I will get  $I_{sp}$  is equivalent by  $g$  and if I put the definition of this you know thrust into  $\dot{m} p$  I will get  $I_{sp}$  is basically proportional to thrust and inversely proportional to the mass flow propellant through the nozzle.

And of course, the  $g$  is the gravity what we take in the sea level right, but when you take to the other place you can also do that. So, this is the one  $I_{sp}$ , but there is another way people would not consider  $g$  even and when you consider the  $g$  then you know the unit of the  $I_{sp}$  will be in second right.

And when you consider the  $g$  then you know the unit of this  $I_{sp}$  this will be in second yes or no right if you look at this is the thrust is the force Newton right Newton means  $kg$  and meter per second square right if you look at let me write down here this will be  $kg$  meter per second square and this is  $kg$  per second right and into this is meter per second square. So, this is will cancel it out  $kg$   $kg$  cancel it out and this is nothing but, your second right.

So, the  $I_{sp}$  is in second provided you are using  $g$  otherwise it is not right. So, in some book you may find it is not consider  $g$  because if go to the space what do the  $g$  consider that is the that is why some people use with certain unit, but mostly it is used as you know.

What is the...

Student: ((Refer Time: 20:19))

Equivalent equivalent what I am saying that this total  $m_p$ ; that means,  $v$  equivalent will be I can write down here  $v$  equivalent will be  $V_e$  plus  $P_e$  minus  $P_a$  equivalent divided by  $m$  that is my  $v$  equivalent right. So, I can say this is the equivalent and when  $P_e$  is equal to  $P_a$  is nothing but, exit velocity same as the equivalent right that is just from that expression right. So, of course, I have already talked about this equivalent is assumed to be constant and of course, during this burning period  $t_b$  is the burned I out

time burned out means whatever it has taken to burnt of course, it is been very much used in case of a solid propellant.

But, in liquid you can deal it you know you close the wall, it will be that or till you terminated the operation as  $I_{sp}$  keep it in mind that  $I_{sp}$  does not depend on the slide velocity because your thrust is not depending on the slide velocity. So, therefore,  $I_{sp}$  would not be depending on the slide velocity this is a very simple one. So, but  $I_{sp}$  indicate the performance of the rocket engine whenever you talk about performance of rocket engines we always use the  $I_{sp}$  and  $I_{sp}$  with  $g$ .

Of course, mostly, but now a day's people are you know using you know without  $g$  that is thrust divided by  $\dot{m}$  right which will be having the unit of what do you call that is you know like if  $I$  multiplied by these meter per second it will be that or you can say Newton per kg per second you can say.

Student: ((Refer Time: 22:16))

You see that what I am doing like  $I_{sp}$  if you look at I am using thus no if you look at what is the thrust? Thrust is equal to  $v$  equivalent basically what  $v$  equivalent the thrust is equal to  $\dot{m}$  into equivalent correct. So,  $v$  equivalent will be thrust divided by  $\dot{m}$ , so I put it  $p$  by  $\dot{m}$   $p$  is that clear. So, if you look at let us look at  $I_{sp}$  of various solid propellant, liquid propellant and hybrid propellant. Let  $I_{sp}$  will be two in the solid propellant rocket engine varying from 200 to 300 and whereas the liquid will be having higher value 3 And hybrid of course, 3 into 500, but generally people consider the  $I_{sp}$  of the hybrid engine is between the solid and liquid it rather lower than the , but some people are claiming beyond that that is the again not very clear, but generally the  $I_{sp}$  of the hybrid engine will be in the range of liquid but particularly is lower than the ((Refer Time: 23:41)) engine  $r$ , but higher than the other.

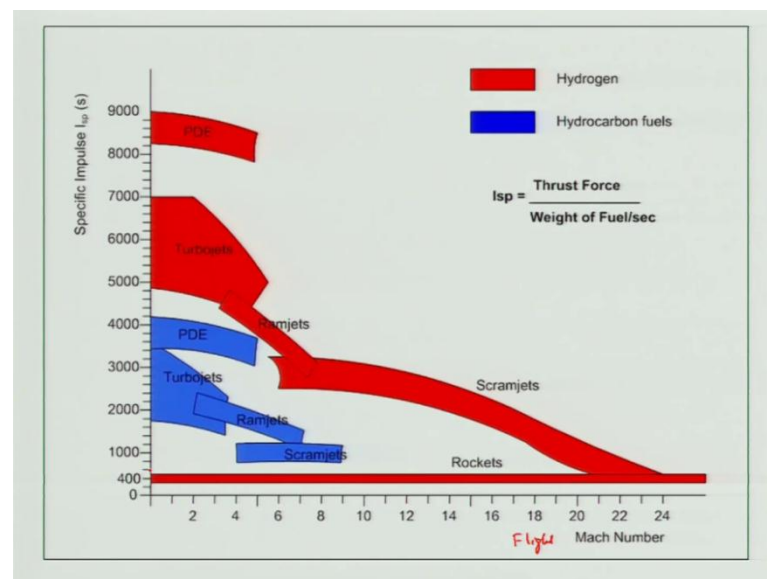
So, that is thing and solar of course, is very high values if you go for nuclear much higher elliptical it is much higher right, but keep in mind that this engines will be having very highest specific in path, but the thrust label is very, very low. You cannot really you know lift something from the this thing if it is something let say you are having a payload or something small satellite or something it is moving or you are sending a rope some other place you can use this kind of other thing.

But, you cannot really take from the earth you know it is very impossible even tiny thing we use this solar nuclear of course, nuclear may be, but at least electrical and solar all those thing are possible. So, if you look at I s p we have already seen how it is varying with the altitude, if you look at that is basically if you look at varying with this like; that means, I s p is goes on increasing and determent almost constant and higher value it is similar to the thrust you know which I have already seen.

So, if you look at higher I s p means what means is basically better performance of the rocket engine because it could more impulse to the vehicle or the space vehicle or the rocket engine of course, the rocket engine can be used for missile application as well which is not good, but it is being used right.

So, if you look at the specific impulse let us look at how it is varying with the altitude and as we have already seen that you know for you know altitude is not really you know changing at the ((Refer Time: 25:43)). But, what happens when the flied velocity changes because when you are you know having a rocket engines which will be moving from the very low velocity to the high velocity ((Refer Time: 25:55)) will be changing . And what happens to the I s p time what about the I s p of the other air breathing engines that will see.

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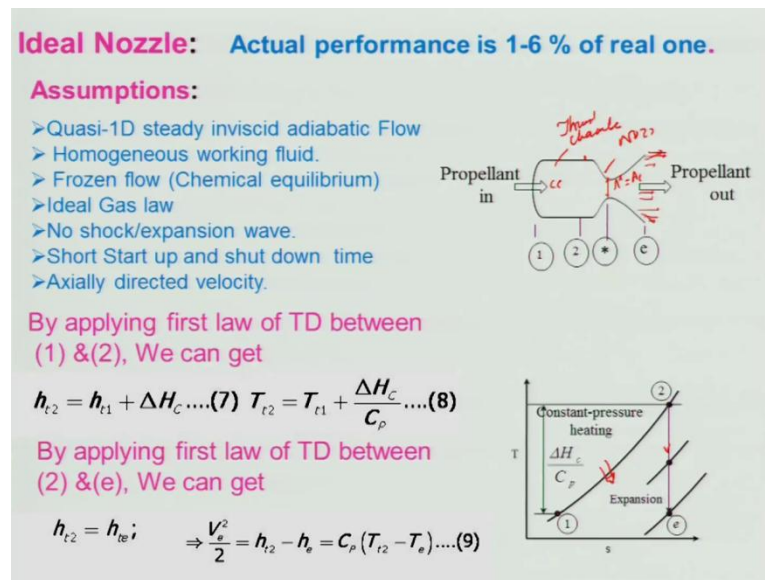
If you look at the air you know specific impulse is being plotted with the flight t number you know. So, this is basically flight mat number and you can see that these are the turbo

jet engine the blue two color is for hydro carbon well which is being used generally. And the red color is for hydrogen which is hypothetical I means no where it is being used as of now, but in future people may go for it because of course, in scam jet engine and you know people are using the thinking of using hydrogen.

So, if you look at it changing with respect to the flight mat number for the turbo jet and the ram jet is also change right from the very high I s p right and of course, the scram jet is having a much lower I s p as compared carbon jet, but again it is changing little bit you know, but where the beauty of this rocket engine it is independent of the flight mat number. So, that is the beauty; however, a beauty of the rocket engine, but; however, the its value is very, very low; that means, you cannot give a what to call a impulses or a take a thrust and other things is you know is very limited

So, therefore, you know turbojet people are thinking why not combine this right; that means, you start with may be a air breathing engines and then go to the rocket kind of think whenever you will pay. So, you can manage well right. So, that is the combine cycle what people are talking about there are several permutation combination of people are contemplating to use in future particularly whenever you are going for very high speed propulsion ((Refer Time: 28:11)). So, which we would not be discussing what I want to you know argue people to keep in mind that you know the limitation of the rocket engine and also those limitation how you can use and what is the good about considering the I s p as a performance parameter.

(Refer Slide Time: 28:33)



So, let us look at the nozzle component you know because to get the I s p or to get the thrust i s p ((Refer Time: 28:40)) you basically depend upon the nozzle and we will be considering ideal nozzle. And keep in mind that ideal nozzle performance which will be deferred from the axis actual one by 1 to 6 percent right. So, therefore, if you manage to look at ideal it may be good enough for your design calculation kind of things.

So, let us look at a typical rocket engines and it is irrespective of the propellant being used in generally you know rocket engine the station one the propellant is in and then of course, the two it is the sustainable end of the thrust chamber are the combustion chamber. This is basically chamber you can say it is a thrust chamber and this portion is the nozzle right the nozzle will starts from here somewhere right and it is having a throat throat I am using as a sometime I will be using as a t a t right it can be i am using as a air star or which is same as a t I am using right in some book people use only star and exit

Now what we will do we will basically looking at the assumption will make what are those assumption we are saying, we are saying this is quasi one dimensional invicid and adiabatic flow. Actually if you look at the flow will be in this directions you know from the nozzle it will be like that, but what we are assuming the flow is basically in this direction there will be some component which will be lost which we are not considering keep in mind. And homogenous working which is not really possible real situation,

because you are liquid, you are using solid and then there will be you know some of the thing which will be remaining un burnt and it cannot be homogenous type

And we are also assuming the flows and; that means, it is all chemical equilibrium it is been achieved, it is a the good enough, but it is not really the reality, reality will be there some of the thing will be accruing even in the nozzle. And of course, ideal gas law we can say it will be valued although the pressure is high, but temperature is equally high. So, therefore, we can assume and there is no shock formation in the nozzle or in the expansion vim, which will be depending on whether it is over expanded under expanded kind of nozzle remind which one as to undergo and nozzle as to undergo when it is moving, but we are not assuming.

And start of this is star sought start of; that means, it is taking a very less time to start, but in rocket propellant engine it takes a lot of time you know much higher as compare to other . So, therefore, it is an assumption and as I told in the very beginning actual directed velocity direction; that means, all velocity coming from axial, but in real situation it is not from the exit of the nozzle.

So, therefore, with these we will just apply the law of dynamic between station one and station two we will get  $s t 2$  is equal to  $s t 1$  plus  $\Delta s t z$  seat of combustion .And we are assuming the change in kinetic energy change in potential energy ((Refer Time: 32:13)) all those thing no work being done right. So, those assumption we have already made, so it will be coming to that. So, if you divide this thing and by  $c_p$  you know because I will assume that  $c_p$  is not changing with temperature.

But, which it is not regularly good assumptions know that  $c_p$  will be varying if it is you know like a cold kind of thing and it is a hot and the temperature is very much, but in case of the solid propellant. So, if you look at steady state it will wall equally hard right. So, therefore, one can assume that you know not vary, so that is an assumption. So, if I look at if I look at this process as a  $T-s$  diagram it will look at like here and he is being added right the  $c_p$  and as the result the temperature will goes up and this ((Refer Time: 33:08))  $\Delta s$  divide by  $c_p$ .

Keep in mind that here we are assuming combustion efficiency would be 100 percent which is not true real situation. So, then it will be expanded right in a nozzle from  $t 2$  to  $t e$  and you will get the third stage right. So, therefore, for by applying the first law

thermodynamic between the exit you know  $e$  and the station two we will get that  $h_e$  is equal to  $s t e$ . Because no heat is added nothing is happen and we are assuming the flow to be adiabatic in case of nozzle, but which we need not to be true and it can really not need because you know like the heat loss particular in cold region will be too much higher as compared to the other places right

So, but we are doing that and keep in mind that this is basically 3 square divided by two equals to  $s t$  minus  $h_e$ . So, that it will be  $c_p t_2$  minus  $t_e$  right again the same assumption we are making.

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For an isentropic flow, Eq.(9) becomes

$$V_e = \sqrt{2C_p(T_{t2} - T_e)} = \sqrt{2C_p T_{t2} \left[ 1 - \left( \frac{P_e}{P_{t2}} \right)^{\frac{\gamma-1}{\gamma}} \right]} \dots (10)$$

$$V_e = \sqrt{2 \frac{\gamma R_u}{(\gamma-1)M} T_{t2} \left[ 1 - \left( \frac{P_e}{P_{t2}} \right)^{\frac{\gamma-1}{\gamma}} \right]} \dots (11)$$

For given  $T_{t2}$ , Mol. weight must be lower to enhance  $V_e$ .

For air  $\gamma = 1.4$ ,  $\frac{\gamma}{\gamma-1} = 3.5$ ; For rocket exhaust,  $\gamma = 1.2$ ,  $\frac{\gamma}{\gamma-1} = 6$

For  $\frac{P_{t2}}{P_e} = \infty$ ,  $V_e$  attains maximum value, Eq. (11) becomes

$$V_{e,max} = \sqrt{2 \frac{\gamma R_u}{(\gamma-1)M} T_{t2}} \dots (12)$$

So, for an isotropic you know relationship I can get  $v_e$  will be basically  $2 c_p t_2$  minus  $t_e$  and you will find this is basically you know this one and I can write down here this is  $\gamma g$  and  $\gamma g$ . So, we will look at this  $p_e$  by  $p_2$  power to the  $\gamma g$  and  $\gamma g$  minus 1. So, this we already seen and in place of  $c_p$  I can write down  $\gamma g$ ,  $\gamma g$  minus one and this is nothing but, your molecular weight and where  $t_2$  is the nozzle is the what to call temperature this is chamber temperature  $T_{t2}$  and the  $a$  m this is basically molecular weight.

So, from this what you can you know learn is that if I want to have a higher  $v_e$  what I will have to do, I will have to basically lower the molecular weight of the gas which will be expanded in a nozzle and enhance the temperature  $T_{t2}$  by using proper propellant. And of course, the these portion you know is which will tell me  $p_e$  by  $P_{t2}$  will tell me

how far it is; that means, when  $P_e$  is becoming smaller right  $P_e$  is decreasing what will happen this term goes on decreasing it is and for a particular chamber pressure of course, right. So, these term will go on decreasing, so that more component from these you will get right. So, for if you look at this is  $\gamma$  is 1.4 and you will find this is  $\gamma$  divide by  $\gamma$  in this 3.5 that is for a if look at this is the term what where are using right, but however, the rocket engines we use  $\gamma$  as 1.2. So, that will be basically 6, so if you look at this component you know this portion is 3.5 per air now if you  $\gamma$  1.0 will become 6. Of course, it is having  $\alpha$  component here it is just suppose you know like same as that so; that means, you know it is goes on and it becomes a smaller.

So, if you look at it is very important to choose the proper you know  $\gamma$  for the rocket engine, because you can never really use a air unless otherwise you are having air rocket can I have air rocket I can have you know is it not I can put a pressure right at a very high pressure and then pass through a nozzle and I can get a right, but that will be for temporary kind of thing, so bending upon what capacity. So, if you look at this is  $p_t$  by  $p_e$  and that is the ratio you know and for a certain values you know  $v$  attains maximum value; that means, if this will be infinity right when  $p_e$  is equal to 0 then  $P_t^2$  by  $P_e$  will be infinity; that means, other around if you look at  $P_e$  by  $P_t^2$  will be 0 it is it not.

So, if it is 0 these term will be zero right and only one will be there. So, then  $V_e$  attain a maximum value that is  $V_e^2$   $V_e$  maximum will get root over  $g r_u$  this is this is  $\gamma$  and  $\gamma$  minus 1 molecular ((Refer Time: 39:06)). So, what it indicates it indicate that maximum exit velocity will be dependent on what the temperature it will be dependent on of course, this  $\gamma$  value which is particular for a propellant system it will be same is it not you can take average. Keep in mind it is not really the same when it is expanding in a nozzle, but we are assuming it to be constant right, when the gas is expanded for the same propellant right the  $\gamma$  will be changing because the temperature is changing right. So, therefore, that changes will be something may be 4 to 6 percentage kind of thing.

So, therefore, if we assume this  $\gamma$  will be same the naturally it will be dependent on chamber temperature or the what you call thrust chamber temperature  $T_{ch}$  and higher it will be  $T_{ch}^2$  will be higher and  $V_e$  will be higher and if the  $V_e$  is higher than actually I



s p will be higher for a same kind of you know system. And if the molecular weight it is lower then what happen V e is also goes up right and for that will be using lighten you know propylene particularly per hydrogen, oxygen you know like hydrogen considered as call it much lighter.

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**Thrust coefficient:**

Let us recall the thrust equation for rocket engine:

$$T = \dot{m}_p V_e + (P_e - P_a) A_e \dots (1)$$

In rocket engine, nozzle is choked for most of time. The expression for choked mass flow rate can be written as

$$\dot{m}_p = \dot{m}^* = \frac{P_{t2} A^* \sqrt{\gamma}}{\sqrt{RT_{t2}}} \left( \frac{2}{\gamma+1} \right)^{\frac{\gamma+1}{2(\gamma-1)}} \dots (13) \quad V_e = \sqrt{2 \frac{\gamma R_u}{(\gamma-1) \bar{M}} T_{t2} \left[ 1 - \left( \frac{P_e}{P_{t2}} \right)^{\frac{\gamma-1}{\gamma}} \right]} \dots (11)$$

By substituting Eqs. (11) and (13) in Eq. (1), we can get,

$$T = \frac{P_{t2} A^* \sqrt{\gamma}}{\sqrt{RT_{t2}}} \left( \frac{2}{\gamma+1} \right)^{\frac{\gamma+1}{2(\gamma-1)}} \sqrt{2 \frac{\gamma R_u T_{t2}}{(\gamma-1) \bar{M}} \left[ 1 - \left( \frac{P_e}{P_{t2}} \right)^{\frac{\gamma-1}{\gamma}} \right]} + (P_e - P_a) A_e$$

$\frac{P_u}{\bar{M}} = R$

So, now having you know done these things we need to now look at the which is effecting the thrust right, we will be defining a thrust co efficient right. Let us recall the thrust equation which we have done already look at it m dot v e plus p e minus p a ((Refer Time: 41:02)). And in rocket engine generally the nozzle is shown most of the time except what except may be in the beginning right it cannot be choked and after some certain time it become choked and you want to choked.

Then you know you cannot really not change unless change the pressure right you cannot change the mass pressure of the propellant which is passing through the nozzle right unless you change the pressure that means, combustion chamber or the thrust chamber pressure. So, the expression for the choked mass flow rate it can be written as m dot p is equal to m dot star that I am saying in basically corresponding to the chock condition, star is chock condition. And P d two a star root over gamma divided by R T t 2 into 2 plus gamma plus 1 power to the gamma plus 1 divide by 2 gamma this we had derived in very beginning of the class in the first you know few lecture particularly when we are discussing about compressing form now we will be using.

So, if you look at we have already derived this is gamma you know kind of things like gamma and we have already derived here gamma minus 1 we have already derived V e right. So, what we will do, we will particularly use this equation 13 and 11 and in the equation 1 club all those together. So, if you look at what I am doing basically this m dot v that is under chock condition keep in mind the chock condition it is right and this portion is from here. And of course, this is your pressure; that means, the thrust can be expressed you know in terms of the right and keep in mind that this we are doing for you know for both under expanded, over expanded or fully expanded because we not made any you know assumption there only assumption we have made till now that nozzle he choked right.

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By simplifying the previous Eq., we can get

$$\frac{T}{\rho_2 A^*} = \frac{P_{t2}}{\rho_2 A^*} \sqrt{\left(\frac{2}{\gamma+1}\right)^{\frac{\gamma+1}{\gamma-1}} \frac{2\gamma^2}{(\gamma-1)} \left[1 - \left(\frac{P_e}{P_{t2}}\right)^{\frac{\gamma}{\gamma-1}}\right]} + (P_e - P_a) A_e \dots (14)$$

By dividing Eq. (14) by  $P_{t2} A^*$ , we can get expression for thrust coefficient as

$$C_T = \frac{T}{P_{t2} A^*} = \sqrt{\left(\frac{2}{\gamma+1}\right)^{\frac{\gamma+1}{\gamma-1}} \frac{2\gamma^2}{(\gamma-1)} \left[1 - \left(\frac{P_e}{P_{t2}}\right)^{\frac{\gamma}{\gamma-1}}\right]} + \left(\frac{P_e}{P_{t2}} - \frac{P_a}{P_{t2}}\right) \frac{A_e}{A^*} \dots (15)$$

$C_T$  indicates the amplification of  $T$  due to expansion in nozzle as compared to the thrust if  $P_{t2}$  acts on throat area.

$C_T$  varies from 0.8 to 1.9

In practice, actual /real  $C_T$  is around 0.98 ideal  $C_T$

Now, look at this that is basically by we will have to simplify these equations right and what will do, we will have taken a lot of terms inside right. If you look at this thing we have taken some of this term 2 gamma plus one inside you know if I look at these portion will be canceled out here right can I not cancel it root over R T t and I can cancel it out and T t a star and this portion right I can take inside . And keep in mind that this is also cancel because R u by m dot is nothing but, R u by m star is r simply .So, therefore, I will cancel it out and then I will you know simplify and then put it then next slide I will do. So, if you look at I am getting p is equal to P t two A star s is nothing but, your chock a d on the chock conditions . Of course, need not be same all the time, but we are 2 by gamma plus 1 power to the gamma plus 1 divide by gamma minus 1 and this 2 gamma

square it has come inside divide by gamma minus 1 into  $1 - p_e$  by  $p_t^2$  power to the gamma divide by gamma .

Actually in this case there is no gamma g as such right it is gamma, why because we are not using any cold all are hot right and we are using only 1 gamma in case of nozzle although it is changing right that assumption we have already discuss right I have already stated that. Although the gamma is changing across nozzle that change very very minimum we are not considering. So, by dividing this equation by ((Refer Time: 45:38)) you know this what will do I will just divide this thing a star and this  $T_t^2$  by  $A^*$  and this I will cancel it out what I will get is known as the thrust coefficient this is  $c_p$  right.

By definition it is like that thrust coefficient right that is  $c_t$  what we are trying to say, if we look at this is basically thrust will be produce if the  $p_t^2$  pressure will acting on the nozzle right is equal to this portion and also this of course, come from momentum and this portion from pressure. So, what it indicates, it indicates that the  $c_p$  is basically indicates amplification of thrust due to expansion of gas in a nozzle as compare what is that compare that if the total  $P_t^2$  will to be acting on the throat area right for example, if there is convergent nozzle right. So, what will be the thrust then it is be simply  $P_t^2$  into  $A_t$  if that is. So, these because that is a divergent portion, so therefore, the expansion is coming right and you are getting the thrust and this is the portion right this is the portion is basically amplification part as compare to the thrust if  $P_t^2$  acts on the throat area. Keep in mind that  $c_p$  that is vary that thrust coefficient vary from 0.8 to 1 and if it is fully expanded you will get what will you get one or something different.

I will leave that thing and keep in mind in practice actual to real  $c_t$ ; that means, thrust coefficient is around 0.9 of the ideal one right. So, if you look at actual other real thrust coefficient will be around of you know 0.98 of the ideal; that means, this is ideal one this is this is what this is ideal right, how I will get I will have to conduct experiment and then do that and then get the thrust values right by talk you know then that will be the actual one right this is the ideal and it is not very different from that. So, therefore, it is good enough for you to look at it and we will see define something and keep in mind that this thrust coefficient is function of what it is the function of  $P_e$  by  $P_t^2$  right and it is  $P_a$  by  $P_t^2$  it is the function of  $A_e$  by this area you know throat area are the choke condition right and it is the function of all.

So, the gamma; that means, the thrust coefficient can be you know varied by playing around this parameters like when you are designing you will have to decision certain things at your disposal a exit area and a b it will be there and gamma you can think of what you call will be depend on the kind propellant you are using. And interestingly this thrust coefficient is not dependent on the total temperature of the combustion chamber or the thrust chamber right is it there the term is not there right.

So, which will be finding out you know why not and what really it makes, but; however, it is dependent on the total you know pressure like you know we have already  $P_t$  it coming over here. So, with this I will stop over and we will discuss about the how this is varying and what are the you know, how you it is varying with the parameter gamma  $P_a$  by  $P_t$ ,  $P_e$  by  $P_t$  and any other issue in the next class then we will discuss about other aspect of solid propellant and other things.