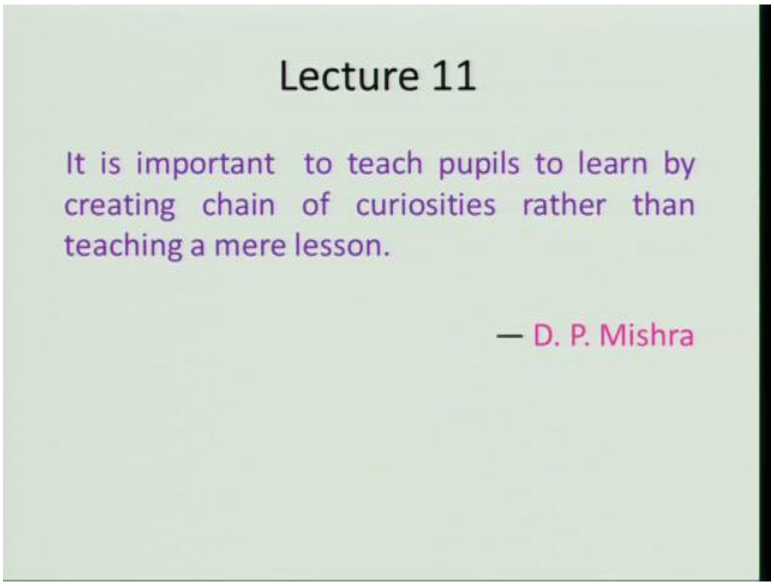
Fundamentals of Aerospace Engineering
Prof. D. P. Mishra
Department of Aerospace Engineering
Indian Institute of Technology, Kanpur

Lecture – 11

Will shall start this lecture 11 from this thought passes, which was lurking in my mind. It is important to teach pupils to learn by creating chain of curiosities rather than, teaching a mere lesson, which is very important according to me because, I believe each individual is a learner and teacher by default. So, let us recall that what really we learned in the last lecture.

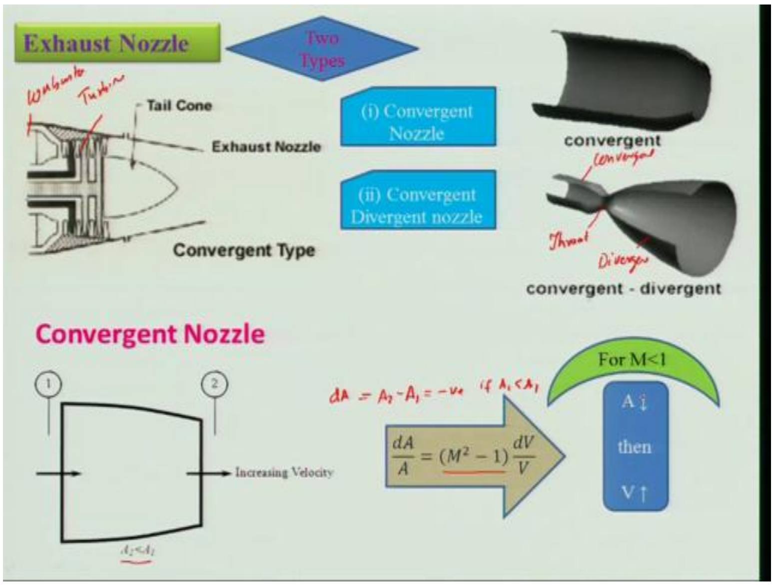
(Refer Slide Time: 00:23)



We started the discussion with a quasi-one-dimensional flow, quasi one dimensional flow means, where there will be change in area and you know very well that most of the propulsive dots particularly, nozzle and air intake will be having wearing area torque. Therefore, we need to look at it very carefully and learn how to analyze them. Not only the air intake and the nozzle, but, also the in case of a turbine and compressor we will be using some of these analysis as well and for that purpose what did we invoke, the conservation of mass momentum and energy equation along with equation of state for a perfect gas or a an ideal gas.

Then we combine and derive a relationship between the area ratio and Mach number, area ratio is any local area divided by critical area and which is a function of Mach number and the gamma that is c p by c v. We will use these equations and apply for what you call nozzle flow, because nozzle flow is quite important and are being used in the exact stream of a turbo jet engine and the rocket engines for producing thrust, which is the basic requirement of a propulsive device or an engine. In addition, if you look at the exact thrust which is broadly divided into two categories one is the convergent nozzle.

(Refer Slide Time: 03:13)



In addition, other is your convergent divergent nozzle in convergent nozzle. What is happening here, what we have shown here is a decrease in area which we can be used only when the flow is sub sonic and we will develop on it and see why we cannot use it for supersonic flow. And there is a convergent divergent nozzle, if you look at this, it is your convergent version and this is your divergent version and this is known as the throat, that is the minimum area where the sonic flow will be sonic or unity. Mach number will be there all the time or under certain condition, it will be existing.

So, this convergent nozzle is being used in a jet engine, which is being shown in this figure and you can recognize this as a basically combustor which is nothing but a turbine. That means from the combustor the hot gas at high pressure passes through the turbine gets expanded and it is further expanded in a convergent nozzle.

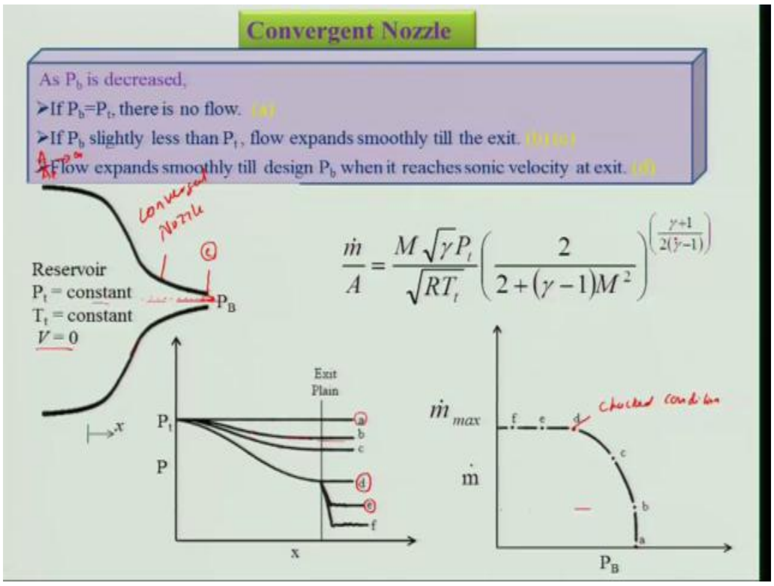
To produce the thrust in most of the jet engines, except may be turbo prop where the nozzle is being used to produce the thrust and so in a ramjet engine and in the rocket engines. However, can I use this convergent type nozzle for the ramjet engine rocket engines or not, we will see a little later on. Let us look at convergent nozzle and learn how to analyze, how to understand what is happening inside the convergent nozzle by invoking the analysis, which we discussed in the last lecture.

For that purpose let us consider convergent nozzle, keeping in mind that 2 is less than 1 which means, the area cross-section at station 2 is less than the area at cross-section 1 and the flow which is coming from upstream passes through this convergent nozzle. And certainly there will be an increase in pressure and increase in velocity. If I want to increase the pressure at the exit of a conversing dot, it is possible, provided the flow is supersonic. So let us discuss further by invoking relationship what we have derived in the last lecture, it is d a by a is equal to m square minus 1 in 2 d v by v.

That means, if there is a decrease in this cross-sectional area, which means a 2 is less than a 1 and that also means a 2 minus a 1 and that will be, if it is a 2 by a 2 less than a 1, which means this will be a negative, this is basically d a, d a will be negative provided if a 2 is less than a 1. If this is a negative quantity and if m is sub sonic, which means m is less than 1.

So, then this also will be a negative quantity and then d v v 2 should be greater than v 1 because left hand side is negative and hand side is also negative. So naturally v 2 should be greater than v 1. So, which means, if there is a decrease in area v will increase, but if it is other way round, it means the area is increasing. Which means it is a diverging dot, then velocity will decrease, it will act like a diffuser or an air intake where decrease in velocity and of course, increase static pressure.

(Refer Slide Time: 09:07)



So let us look at what is really happening by putting this a convergent nozzle, this is a convergent nozzle and to where reservoir, as I told reservoir means, it is a very big supplier of the air wick large size tank, where the velocity will be 0, which means that amount of air is so much such that if a small amount of air is passing through this nozzle, Then there is no change in the pressure and temperature in this reservoir, which means p t is constant and this means total pressure remains constant and total temperature also remains constant.

Only then we can call it as a reservoir, which means if you look at it, if I take this a e is the exit and if I take this area it will be ideally a by a e. If I say this, a by a e, if I say this is, my exit will be tending towards infinity, only then it is possible otherwise it is not possible. By simply applying the continuity equation one can think about it. Now let us say that, because here in the reservoir the total pressure which is remaining constant during the operation and if I will keep this pressure at the exit or may be little away from the exit then we call it as a back pressure. Same as that of the total pressure, what will happen if somehow by some means I am keeping the downstream of this nozzle, it is because this is the nozzle and what is this nozzle? This is the convergent nozzle and I am keeping the pressure p t at this point..

Same as that of the p t so what will happen, there would not be any flow and what will happen to the pressure here in this locations, along the x direction? If you look at this x direction, what will happen to in this direction? It will remain constant, which means, if I am talking about this a where the p v is same as that of the p t, then there would not be any flow because there is no pressure gradient and if this p v is smaller than the p t by a small amount or is less than the p t by a smaller amount then what will happen? Then there will be a flow over here.

Right but, what will happen to the pressure at this point? Each point pressure will be decreasing, which means pressure will be decreasing. If I decrease this p v. That is back pressure further then what will happen? The pressure will decrease and which is lower than the previous case, but, what happens to the velocity here? What happens to the mass flow at the exit of the nozzle? This is my nozzle exit so what happens to the nozzle mass flow rate as the nozzle exit will it remain same or it will be decreasing or it will be increasing. What will be as my back pressure that is decreasing, my mass flow rate will be increasing because my velocity will be increasing and my static pressure also will be decreasing.

If you look at, if I am here, there is no flow that is corresponding to condition where back pressure is same as that of the total pressure in the reservoir and when I go to the b that is back pressure which is lower than the reservoir pressure or the total pressure. Therefore, there is an increase in mass flow rate, if you look at mass flow rate here and if I further decrease to the point c there is an increase in mass flow rate and decrease the pressure along the x direction in the nozzle is clear now. If I will come to a point d where it will give me the velocity, certain amount of velocity, certain amount of mass flow rate and for which of course, this nozzle is being designed. You can call it as p v which is equal to the design pressure, but, if I decrease this pressure further; let us say at the point e what happens, there would not be any increase in mass flow rate or there would not be any increase in velocity.

Why is it happening and under what condition will it happen, can anybody tell me? That means if there is no increase in the mass flow rate, by decreasing the back pressure or the downstream pressure of the nozzle we can call it is almost choked or it is choked, which means you cannot really increase the mass flow rate of a the nozzle. Whatever the pressure you apply, why is it happening and the in what condition it can happen? If I further decrease it back pressure may be in anticipation that it may work for me, then till there will be no increase in mass flow rate. There will be no increase in mass flow rate, which means it is choked. Whatever I do, even if I go to the vacuum, the lowest pressure I can think of is absolute vacuum.

Then I cannot really increase the mass flow rate, which means it is choked. Why it is happening? Why because, at the exit it has reached the Mach number of 1. If it reaches then it cannot really sense the any change in the back pressure, which in turn will not sense, which means the fluid element which will be in the reservoir on the upstream of this exit would not sense. There is a change in the pressure, so that the fluid element suits. Let me move it, they cannot! Why it is so?

What I did I say earlier, it is because the speed of sound. There will be propagation, which means the disturbance at the downstream which is controlled by the back pressure is basically a disturbance. It moves at the speed of sound, therefore, it cannot really know it is happening because this fluids are at stationary. Therefore, it would not be and we call it as a choked condition that means this is my choked condition. This is the choked point and what pressure it will come at that will see little later.

I want to increase the mass flow rate, what are the ways and means of increasing the mass flow rate? I cannot! Is there any way I can do it? Is it a requirement in propulsive devices, where we need to increase the mass flow rate? For example, I am chasing you now with a fighter aircraft; so I will have to increase the mass flow rate, otherwise I will not be able to manage. Suppose the pilot wants to go at a very high speed and it is choked, then what you will do? You may say that he can add more fuel, is it possible? Why not, he can put more amount of full. Can I increase the thrust? I will be discussing later about it, but, as you all know that thrust will be professional to the exit velocity. I cannot increase exit velocity, it is choked.

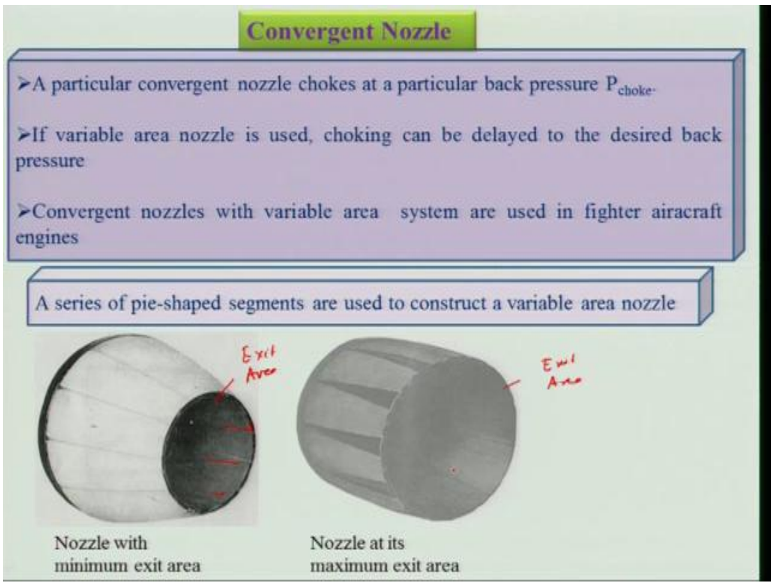
Therefore, I cannot manage. Actually it is not that way. This means that there are several ways of increasing the mass flow rate, also the velocity. Of course not, but, mass flow rate can, if it is choked. Like you know, how will I increase the mass flow rate then? That is one question and another question can I use this concept in some other places as well? This choked condition which is a problem, can you use this, as you know the solutions for some problems, because if you look at any problem that comes to you, you should take it positively and see that what really I can learn and how we can use it to my advantage, that is the very important, which is a concept in life.

So also in fluid mechanics, if you look at it, For that, let us look at the expression what we had derived in the last lecture, that is m dot a divided by a is equal to a m. This is the what you call Mach number root over gamma p t and root over r t and into in the bracket 2 divided by 2 plus gamma minus 1 m square power to the this gamma plus 1 divided by 2 gamma minus 1 this we have already seen. What it is indicating is that, it means this mass flux per unit area will be proportional p t and it is inversely proportional to root t.

And of course, the gamma and the Mach number, from these I can learn something looking at this expression, which means whatever the question I ask when I look at this expression, I can get the answer. What are those answers, can anybody tell me? I want to increase or change my mass flow rate rather increase the mass flow rate when it is choked.

So what are the ways to do it? Can you look at it and tell me. This expression means, increase the total pressure or what else I can do? Decrease the total temperature, there is another way but, Mach number is a problem because the exit Mach number becomes 1. It is choked; therefore, I cannot have a control over the exit Mach number. It is not flying; it is not an aircraft which is flying. We are talking about the engine in the nozzle, what else is there? Gamma is a fluid and if I am using air, I cannot change. In an aircraft can I change? Even if I will change, by what percentage? That is also a question. It is quite difficult and impractical, change the area; which means if I change my area I can do that.

(Refer Slide Time: 21:21)

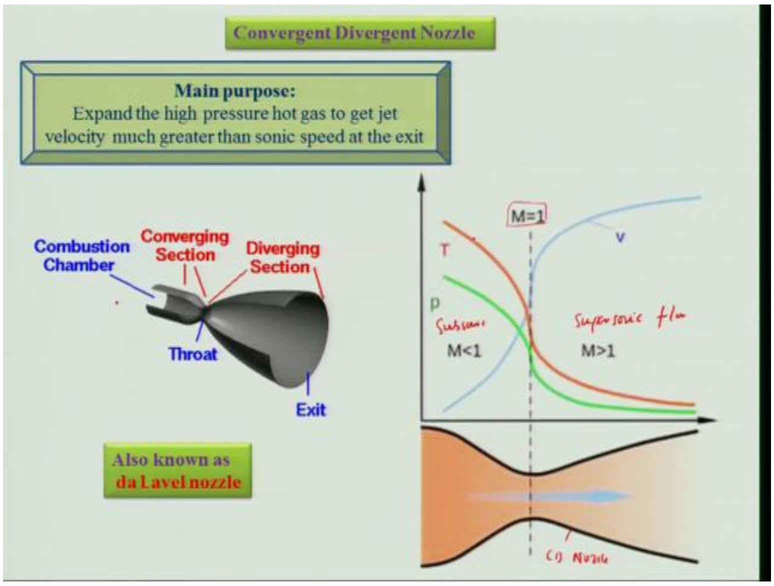


So let us look at whether the particular convergent nozzle chokes at a particular back pressure and at variable area nozzle is used. Choking can be delayed to a desired back pressure; convergent nozzles with the variable area system are used in fighter aircraft engines. It is to have a higher thrust particularly when you are using the after burner. So what is been used in real practical situation is that you vary the area..

Of course, there are several ways of doing it, this is one of them. A series of pie shaped segments are used to construct a variable area nozzle which is shown here. If you look at this case, this is all together. This is a lower exit area, if you look at the velocity here you know that it is coming out and this is the exit area. And when it is at maximum, like when you need we can increase this area. This is your exit area of the nozzle therefore, by just simple technology you can just increase it and decrease it. Collapsible as you know, it is a kind of collapsible like nozzle, you can increase it or decrease it like that mechanism is to be..

Cast to be done because similar thing cannot be really done for the convergent and divergent nozzle. Is it really true? Am I saying it or is there a technology available by which you can change it. We will think about it later, let us now look at convergent divergent nozzle and as I told you that in case of a convergent nozzle there may exist Mach number.

(Refer Slide Time: 24:00)



We cannot really go beyond 1 because it is choked, that means I cannot really use that convergent nozzle. When I want my exact velocity, it should be greater than 1. If I want that, then I cannot really use convergent nozzle, therefore, I need to have a convergent one.

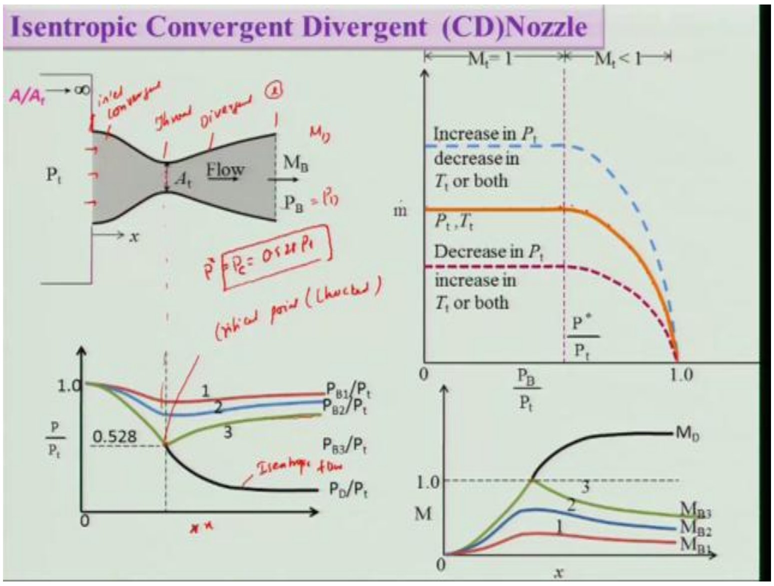
Divergent nozzle, as I already mentioned, its main purpose is to expand high pressure hot gas to have a high jet velocity much greater than the sonic speed at the exit. In case of a convergent nozzle we need to go for a convergent divergent nozzle. So if you look at a typical convergent divergent nozzle, it is shown here; which is having a convergent section, having a throat section and this is having a divergent section. Of course, the hot gas will be coming from the combustion chamber, particularly in case of a rocket engine.

Whereas this convergent divergent nozzle when I am using in case of a ramjet engine it will be also coming from a combustion chamber. But, when I am using it in a fighter aircraft of course,, then it will be coming from a turbine. The gas is hot gas and it will be coming from the turbine. Now this convergent divergent nozzle is known as alsde level nozzle because it is named after the person who really came out with this convergent divergent nozzle. Nozzle was fast you know whether this nozzle was first designed for an aircraft engine. What was it designed for any idea? It was really designed for the steam engine not related to it, but, it is profusely used in case of aero applications and other places as well.

Now coming back to this, what really happens is we will look at this isentropic analysis or the analysis carried out for a isentropic flow, which will be invoking and if you look at the flow that is taking place over here. And this is the throat where the Mach number 1 is occurring and if I look at the pressure along this c d nozzle [and this is a] c d nozzle, what is happening?

If you look at the pressure, it decreases because the fluid or the air will be increasing. If you look at the Mach number it increases here and then beyond this divergent portion it again increases further because the throat is already at the sonic condition. And this is a diverging portion, therefore, it increases further and this portion is the supersonic region, this portion is the supersonic region supersonic flow and this is sub sonic. In addition, keep in mind that temperature also decreases due to expansion of the gas.

(Refer Slide Time: 27:31)



So let us look at a convergent divergent nozzle particularly invoking the isentropic analysis and we will consider the nozzle being connected to a reservoir where the total pressure you know remains constant. In this case the by a t is tending towards infinity, which means area of the reservoir cross-sectional area of the reservoir is much larger than the throat area. This is your throat area and it is designed for certain you know Mach number, that is m d.

For certain Mach number this area ratio if you look at, this is my exit area and this is my inlet and throat area. This one is the design for certain, what you call design Mach number velocities and if this back pressure is same as that of the p t, there would not be any flow as we have already discussed in case of convergent nozzle because there would not be any flow as such. It is because there is no pressure gradient, now if this back pressure is lower than the p t then there will be flow. If there is a flow then what is happening here? Along from the inlet, this is your inlet and from the flow it is taking place, then what is happening to the pressure along the x direction or other center line. You can say center line of the nozzle, what happens to the pressure? It will decrease, but, then when it comes over here in the divergent portion, this is your divergent portion; what happens to the pressure?

That means the pressure will decrease in the convergent portion. This is your convergent portion and it decreases till this and then again it will increase because if the flow is sub sonic in this region which means of course,, at the reservoir p by p t will be 1. This is at the reservoir then after that it decreases and with certain value at the throat if you look at this, this is the throat kind of things and then of course,, it again increases depending upon how far the p v less than the p.

There we are seeing, it is very small as compared to the what you call a change from the p t by in finite decimal is small. Then if it decreases further, this back pressure, back pressure decreases it further what happens? It again follows the same path and that means pressure decreases and again increases further in the divergent portion and if that means at this point throat, what is happening to the velocity or the Mach number that is increasing?.

What happens to the mass flow rate? Then mass flow rate also increases at the exit of nozzle, but, what happens to mass flow rate at the throat? What happened to mass flow rate at the exit inlet? Is it same or different? Naturally it will be same and then it is all increasing when decreasing the back pressure. So if I will go and decrease this back pressure and reach a condition where the Mach number at the throat will be equal to 1.That means the pressure is decreasing in the convergent portion and then it is again increasing. So this condition, which means, what is happening, the throat is choked, throat is choked. If I decrease the back pressure further what happens? What happens to the mass flow rate? We will see what will happen to mass flow rate, it will, it will remain constant..

It cannot really increase beyond that because if you look at the Mach number as reach 1 here I mean this is the plot, Mach number versus x keep in mind that this is the also the same x along the x direction. So that means this point is changing, this is the throat region basically and then if I decrease it further, what happens any idea? Suppose I am here, something you know, some other region; what will happen? P b 3 instead of p b 3 and p b for ride because this is basically over here. What will happen? We will discuss, it cannot because if I decrease there is the mass flow. There would not be any increase in mass flow, mass flow will be remaining constant, for example, we may discuss little later. But if it is designed for certain Mach number that means it will be having certain exit pressure of the back pressure.

If I am applying the same back pressure here, which means p v is equal to p d, what happens, it comes over that means the pressure decreases and reaches the design value. And if you look at this p by p t, what is this relation? This is meant for only isentropic flow, which means if my back pressure happens to be same as the design pressure for which it is designed, it means that the same exit Mach number. Then the pressure will be like that, but, in between what happens if I decrease in between this pressure, what happens? There will be a shock formation, why it is so? Is a thing to be ponder about and we will be discussing about shocks little later on.

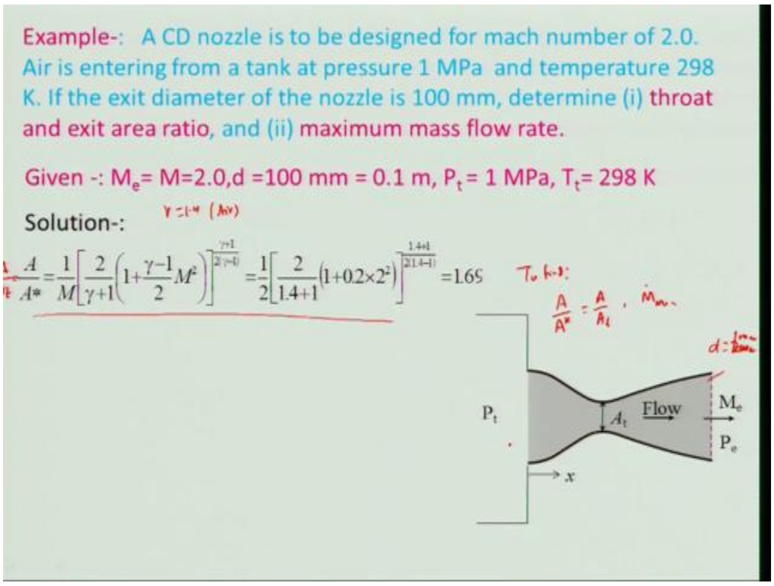
Now what happens to the Mach number here, which means it is also reaching the design Mach number whenever the back pressure is same as that of the design pressure at the exit of nozzle. But as I told you, what is happening in the mass flow rate if I go for this line. If it goes on decreasing the back pressure, the mass flow rate will be increasing till it reaches the Mach number 1 at the throat and mass flow rate will be remaining constant. Although it will keep decreasing further, that means in this region mass flow rate will be remaining constant.

Although the back pressure is much lower than the critical back pressure, if you look at this critical back pressure, it is basically equal to this. I call this critical point or choked point and this is equal to p, which is equal to 0.528 p t, that is critical and sometimes we use it as a star. You know p star, this is because of choked condition were the sonic flow; I mean that I will tell you where it is choked. If my p pressure at the throat is equal to the 0.528 p t then this is possible only for the for an isentropic flow. A non-isentropic will occur due to heat addition or due to other dissipative effects like boundary layers and other things, these would not hold good.

So now if I want to increase or decrease this mass flow rate what I will have to do? If I want to will increase the mass flow rate from this label to that label, then I will have to increase the p t of the reservoir of the total pressure or decrease the total temperature or both. It means 1 will be increasing and others will be decreasing, which is a combination of both. Similarly, I can decrease this, what you call total pressure and total what you call will increase the total temperature or combination of both for decreasing the mass flow rate in a nozzle. So this is the thing which we need to keep in mind and the question what I had asked; how to really increase this area in case of a convergent divergent nozzle is a design challenge its quite challenging.

How will we do that mechanically? Of course, people have done it and you can look at it as you go along, we will be discussing that if I will get some time.

(Refer Slide Time: 38:22)



Now let us look at an example, a convergent divergent nozzle is to be designed for Mach number 2, air is entering from a tank that is basically reservoir at pressure of 1 mega Pascal, temperature of 298Kelvin, if exit diameter of the nozzle is hundred m then we will have to determine the throat area and exit area ratio. That means throat and exit area ratio a e by a t we need to find out maximum mass flow rate it can handle. So let us look at our diagrams, what we have, it will be seen. We need to find out that are the given you know me exit is equal to two that is Mach number d. Here at the exit d basically is given as 200 m an sorry hundred m and p t is given as 1 mega Pascal t is given a 298 Kelvin, so we need to find out basically throat exit area that is a.

By a star or is equal to a by a t this is throat area and we will have to find out m dot maximum, to find that we will have to do very simple step because we know the Mach number here. If I know the Mach number I can and it is an air, so gamma is equal to 1.4. I can use for air so if I will invoke this expression and which we know a by a star equal to 1 by m into 2 by gamma plus 1 into 1 plus gamma minus 1 divided by to m square power to the gamma plus 1 divided by 2 gamma minus 1.

See gamma is known m is known so you can evaluate here, what it would be this is basically equal to a by a t that means this is basically a by a t and if you substitute these values you will get 1.69 that means it is a very direct to on. But you need to remember this formula. There is a another way we can, you now get these values from a table because we can really use isentropic table. Let us look at a typical table, for gamma it is 1.4.

(Refer Slide Time: 41:12)

Isentropic flow table for $\gamma = 1.4$ Return

M	T_0/T	P_0/P	ρ_0/ρ	A/A^*	MFP
0.1	1.002	1.007	1.005	5.822	1.99
1	1.2	1.89	1.577	1.0	18.26
2	1.8	7.824	4.35	1.68	29.83
2.2	1.968	10.69	5.43	2.0	31.38

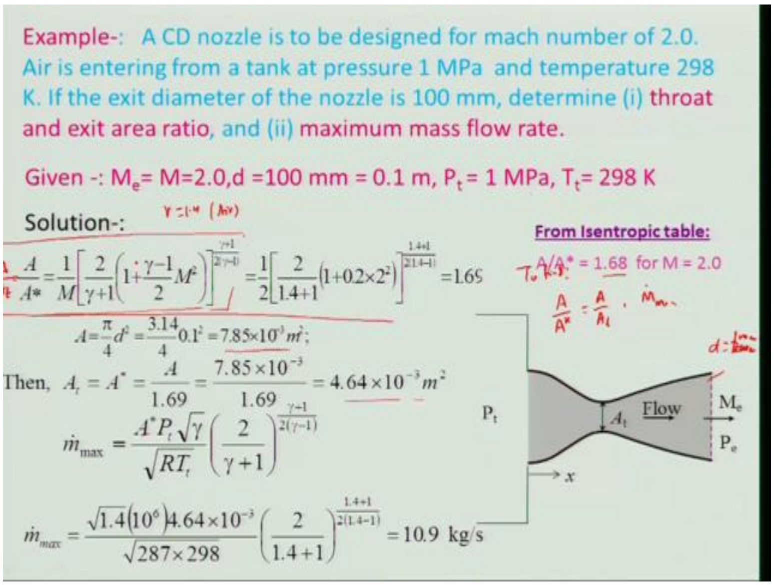
What will be given is your Mach number t by t and this t is total temperature divided by t for which already we know the formula t by t is equal to 1 plus gamma minus 1 divided by 2 m square. And p t by p you already know that you can use directly or you can use this table rho t by rho and a by a star. Already the expression that I have used to evaluate; in that example, this is mass flow parameters which already we have discussed, that is 1.9 and what you can observe from this table?

Can you observe something from this table when the Mach number is increasing that means 0.1 you will see[the]temperature ratio, you know t by t is almost equal to 1, but, if Mach number becomes 1 what happens to the t by t 1.2, You cannot really neglect the effect of Mach number when the Mach number is very high, that which we have already discussed. Similarly, p t by p is 1.89 and this is 1.07. Similarly, for the density what happens to area? It becomes 1 at sonic conditions, Mach number 1 is equal to 1 basically sonic so area ratio will be minimum.

That is a by a t or a star is equal to 1, of course,, the mass flux becomes more kind of things and of course,, when the Mach number becomes 2 it is t by t which becomes higher. So other quantity and also the a by a star becomes higher than 1. If you look at the Mach number which is less than 1, that is 0.1, it is very high here. But, let us take another data which has been shown here, that is 2.2 and others data like t by t p t by p rho t by rho and other things are given in this table now.

Suppose in your example it is given that your Mach number is 2.1 that means these data is given. I want to evaluate using the table what I will have to do is, do a linear interpolation between the data, that means you are assuming it to be linear and the relationships are linear. Certainly now you have seen those expressions that it is non-linear in nature, but, however we do it, for the simplicity it incurs certain errors and those errors are very less. Can do you know tolerable provided the intervals between 2 data's are small, so that means if the 2; therefore, the area ratio is 1.68, we will use this thing and this area ratio becomes 1.68. If you look at 1.69 and 1.6.

(Refer Slide Time: 44:37)



But of course,, it can be tolerable, you can use either the table or the equations, but, equations will be correct 1 or it will be closer to their or it is better to use equations when you want to have a error free data. However for the quickness you can use the table now.

The area I can calculate, that is pie by d and then you know we will have to get this, a small area like which is having, will just substitute values and get that. So than a t is nothing but, that is equal to a star and you can get this, which is a quite small as compared to the a exit area because this a is same as that of the exit area. It can be any other place as well depending upon the Mach number, that means this expression, it can be applied evenly cross-section and then in the mass flow rate m dot max, if you look at is equal to a star p t root over gamma divided by root over r t 2 divided by gamma plus 1, power to the gamma plus 1 into 2 power to the gamma minus 1. I will just substitute these values already and I evaluate a t you might be wondering why.

Why I was doing that is basically I want to find out maximum mass flow rates to the nozzle therefore, I need to evaluate this a star or a t throat area and p t. Already I know this is 1 mega paschal and t I know, so I will substitute these values and get that expression m dot max which is equal to root over. You can see that it happens to be 10.9 kilo gram per second so basically, we are using the isentropic relationship to get various values kind of thing, now the question that arises is why we need to go for the table. If you look at this expression, suppose in case I know the Mach number or sorry, let us say I do not know the Mach number, but I know the area ratio. How can I find out Mach number, is it possible? So for example, in this there is another example or another problem where a by a t is given, but I need to find out Mach number so how I will get it.

Yes, numerically you can get or there are an approximate solutions or approximate equations that will be there. Expression which you can get, that means I can express this Mach number in terms of a by a t, that is again an approximation. Therefore, we need to go for a and table where I can use those values because I can put various Mach numbers and then a by a t and then other p by p t and other things and then I can use it. So that is one way of you now solving this otherwise.

Of course, computer is not at your hand in earlier days particularly where computation was not that easy to do. So people were using this isentropic therefore, there is a importance for using the isentropic table or looking at the isentropic table. With this I will stopover and then we will see in the next lecture about the shock and how to handle normal and oblique shock.

Thank you.