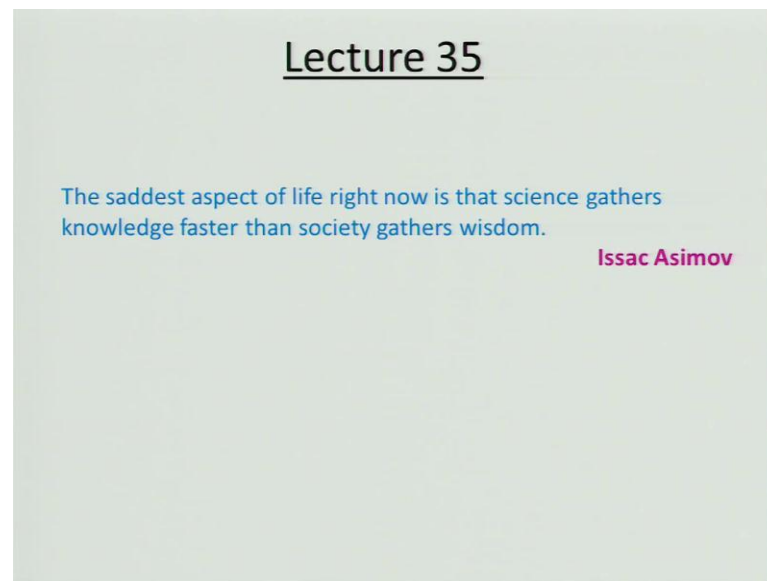


Fundamental of Aerospace Propulsion
Prof. D. P. Mishra
Department of Aerospace Engineering
Indian Institute of Technology, Kanpur

Lecture - 35

Let us start this lecture 35 with third passes from Issac Asimov.

(Refer Slide Time: 00:20)



Who says that the saddest aspect of life right now is that science gathers knowledge faster than society gathers wisdom. So, very important and also very critical, and let us speak all what we learnt in the last lecture, and last lecture basically we have looked at the analysis of turbo fan engines right. And then, we looked at the parametric you know characteristic of the turbo fan engines, and we have learnt that you know the bypass ratio and the fan pressure ratio, compression pressure ratio place very important role and by analysis the bypass ratio we can reduce that TSFC.

And very important thing that we have learnt that whenever we take care of the losses, the TSFC goes on in crease in a particular fix parameter right. And whereas, the specific thrust bring reduce and of course, the purposively efficiency gets analysis, but the thermal efficiency reduce. So, overall efficiency basically reduce whenever you take care of losses in the analysis.

(Refer Slide Time: 01:56)

Example 2 :

An turbofan is flying at flight Mach number $M_0 = 0.85$ at an altitude of 10 km. The pressure ratio across the compressor with polytropic efficiency of 85% is 10. In contrast, the pressure ratio across the fan is 2 and it has a polytropic efficiency of 82%. At the combustion chamber exit, the temperature is 1600 K. The burner has an efficiency of 94% and a total pressure ratio of 0.95. The bypass ratio of the turbofan is 5. Two separate convergent nozzle with same isentropic efficiency of 95% is used to produce thrust. The mechanical efficiency of this engine is assumed to be 95%. If the total pressure recovery factor in the air intake is 0.91, determine T_5 , TSFC, η_p , η_{th} and η_0 . Determine T_5 , TSFC under static condition at sealevel, if it consumes 150 kg/s of air. (Take $\Delta H_c = 43,000$ kJ/kg, $\gamma_g = 1.3$, $\gamma_a = 1.4$)

Given :

$Alt = 10$ km	$T_0 = 223.3$ K	$P_0 = 26.5$ kPa
$\pi_c = 10$	$\eta_{pF} = 0.82$	$\alpha = 5.0$

Now, we will take up an example as usual to illustrate how we can solve this problem without really resorting to this parametric analysis. That means, which will be carried down, we will be doing very systematic way and which can be you know basically done by hand, not by using computers. And for computers you will have to use the equations which we derive from the parametric analysis, turbo fan engine is flying at flight mach number of 0.85 at an altitude of 10 kilometer.

The pressure ratio across the compressor with the polytrophic efficiency of 85 percent is you know 10; that means, pressure ratio across the compressors of 10, but total pressure ration will be dependent on what is the fan. In this case the pressure ration across the fan is 2, so the total pressure due to the compressor will be 20 in this example, that you keep in mind. And this fan has polytrophic efficiency of 82 percent, at the combustion chamber exit temperature is 1600 Kelvin, the burner has an efficiency of 94 percent and a total pressure ratio 0.95.

That means, high percent losses will be there in pressure total pressure in the combustion chamber. The bypass ratio of the turbofan is 5 we have taken in the examples, the two separate convergent nozzle with the same isentropic efficiency of 95 percent is used to produce thrust. You know here it is giving the same isentropic efficiency for the both nozzle, but need not to be and it is not mixed together the turbo fan engine, where it will be mixed and expanded in a single nozzle right, which are not taken in your this thing.

But; however, it may be required to solve in a look at this things for a from your side, mechanical efficiency of this engine is assumed to be 95 percent, total pressure recovery factor in the air intake is 0.91 with need to determine the specific thrust, thrust specific fuel consumption, purposive efficiency, thermal efficiency, overall efficiency of course, you know one can find out that the static condition what it to be the values right if it consume 150 k g of second.

I think the first part will looked at it, and the second part I will leave as an assignment of the first part means whenever it is the flight mach number 0.8 will be considering that. So, of course, those values of gamma g and gamma a is given keep in mind is gamma g what will be using is 1.333 or you know it is not 1.3 right what I am mentioned here. At these are the conditions which I want to skip basically, and if you look at bypass ratio 0.5 and altitude 10 kilometer T naught is 223.3 Kelvin, P naught is 26.5 pi c is 10.

(Refer Slide Time: 05:13)

$\eta_{pc} = 0.85$	$\eta_{pt} = 0.88$	$\eta_b = 0.94$
$\pi_b = 0.88$	$\eta_n = 0.95$	$\eta_m = 0.95$
$T_{t4} = 1600 \text{ K}$	$M_0 = 0.85$	$\Delta H_c = 43,000 \text{ kJ/kg}$
$PRF = 0.91$	$\gamma_g = 1.333$	$\gamma_a = 1.4$
$C_{p,a} = 1.005 \text{ kJ/kg.K}$	$C_{p,g} = 1.148 \text{ kJ/kg.K}$	

Solution: $T = \dot{m}_c (V_9 - V_0) + A_9 (P_9 - P_0) + \dot{m}_F (V_{19} - V_0) + A_{19} (P_{19} - P_0)$

The flight velocity V_0 is

$$V_0 = \sqrt{\gamma R T_0} M_0 = \sqrt{1.4 \times 287 \times 223.3 \times 0.85} = \underline{254.6 \text{ m/s}}$$

For isentropic flow, we know that

$$T_{t2} = T_{t0} = \left(1 + \frac{\gamma_a - 1}{2} M_0^2\right) T_0 = 255.6 \text{ K}$$

Again, from isentropic relation, we have

$$\frac{P_{t0}}{P_0} = \left(\frac{T_{t0}}{T_0}\right)^{\frac{\gamma_a}{\gamma_a - 1}} = 1.6$$

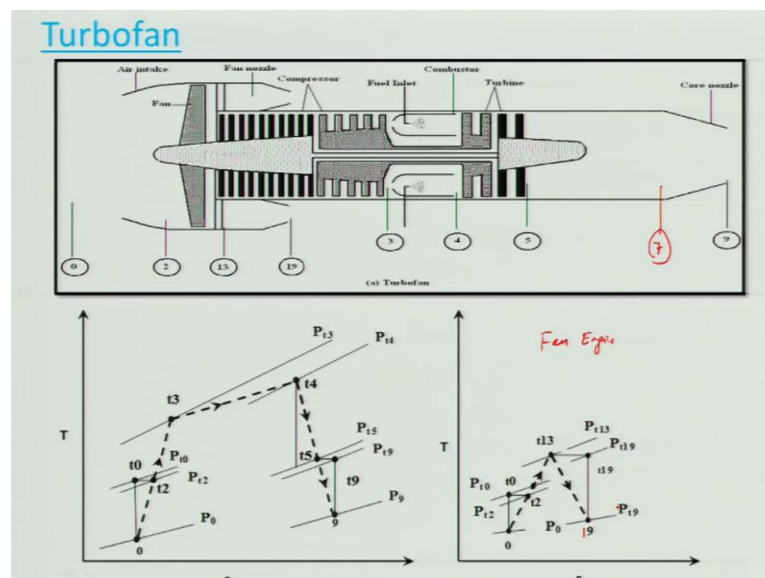
And these are the values which here given I have just jointed down, and what we will do basically we will have to find that total thrust right. Total thrust if you look at it is m dot c that is the mass flow rate Clorox engine which is passing through V 9 minus V naught plus A 9 multiplied by P 9 minus P naught the pressure different across the plus that is a for the fan, this is again m dot F into V 9 minus V naught plus A 19 into P 19 minus P naught right.

So, what we will do we will have to basically you know find out the flight what to call velocity, what it would be V_{naught} will be can be easily determine because, we know that temperature T_{naught} we know the mach number. So, V_{naught} will be γ you know this will be $\gamma a R T_{naught}$ into mach number, so all these values are given to you can get substitute this values. And you will get 254.6 meter per second right, and for isentropic flow we know that T_{t2} is equal to $T_{t_{naught}}$ right keep in mind this air intake we are not adding you know any work or nor some heat is going out.

So, therefore, the temperature remains constant and $T_{t_{naught}}$ in can be you know related to T_{naught} in terms of mach number, and which we know very well and to substitute the mach number γa will get this values. So, of course, the pressure we can get $P_{t_{naught}}$ by P_{naught} for an isentropic can be related to $T_{t_{naught}}$ by power T_{naught} power to the γa divided by γa minus 1 and it substitute those values you will get 1.6 it happens too this.

This is the what due to the RAM pressure basically right because, it is coming and then you are saying you will attempt the stagnation what it would be the pressure ratio right total pressure to the static pressure.

(Refer Slide Time: 07:34)



So, we let us just keep this in mind these are the turbo fan symmetric with the station number 0, 2 and of course, there is 13 for the fan, and 19 for the fan that the nozzle and the station 3 to 4 is combustion chamber of course 2 to 3 in the case of core engine is the

compression and 4 to 5 is turbine and 5 to 9 nozzle. But, actually nozzle will start from here that is 7, but we are saying the station 5 is same as the station condition; that means, property like total pressure, total temperature is remaining same, but in real situation it owned be, but in this even in the real cycle we are considering that.

So, if we look the passes is 0 to T t 2 it is in the air intake, and from 2 to 3 is your compression and 3 to 4 is your combustion chamber and 4 to 5 is turbine, both the high pressure and low pressure turbine, and 5 to 9 is a nozzle. And this is of course, for the fan engine right this is fan engine and this is 0 to 2 is your air intake, and 2 to 3 is your fan and 13 to 19 you know this point is 19 basically that is your fan nozzle.

So, you should keep this station number because, as you go along we will be you know solving and using this station number.

(Refer Slide Time: 09:13)

As the PRF in the air intake is 0.91, the total pressure at station (2) becomes

$$\Rightarrow P_{t2} = \frac{P_{t2}}{P_{t0}} \frac{P_{t0}}{P_0} P_0 = 0.91 \times 1.6 \times 26.5 = 38.6 \text{ kPa}$$

For Fan Stream

The pressure ratio across the fan π_F becomes

$$\pi_F = 2.0 = \frac{P_{t13}}{P_{t2}} \Rightarrow P_{t13} = 77.2 \text{ kPa}$$

Now, we can estimate temperature at station (13) as

$$\frac{T_{t19}}{T_{t2}} = \frac{T_{t13}}{T_{t2}} = \left(\frac{P_{t13}}{P_{t2}} \right)^{\frac{\gamma_a - 1}{\gamma_a \eta_{sp}}}$$

$$\Rightarrow T_{t13} = T_{t19} = 324.56 \text{ K}$$

The critical pressure ratio across the nozzle can be determined as

$$\frac{P_{t13}}{P_c} = \frac{1}{\left[1 - \frac{1}{\eta_s} \left(\frac{\gamma_a - 1}{\gamma_a + 1} \right) \right]^{\frac{\gamma_a}{\gamma_a - 1}}} = \frac{1}{\left[1 - \frac{1}{0.95} \left(\frac{1.4 - 1}{1.4 + 1} \right) \right]^{\frac{1.4}{1.4 - 1}}} = 1.964$$

So, as the PRF pressure equation factor in the air intake nozzle is 0.9, we can very easily find out total pressure ration station 2 that is P t 2 by P t naught. And this is nothing, but your PRF which is 0.91 and P t naught by P naught already we determined that is 1.6 and P naught of course, we know from the altitude that is 26.5 and when you substitute this values, and get that that will you will get P t 2 as 38.6 kilo pass, here what do I doing each point your solving sometimes you may find it is not required.

But, particularly real cycle it will be very essential, but in ideal cycle you can skip some of the calculations if you use it intelligently. But, in real cycle it will be little treating you know for fan stream the pressure ratio across the fan that is p_{t13}/p_{t2} becomes because, it is a straight forward P_{t13}/P_{t2} right, and that is given as 2 you can get directly P_{t13} because, this is known right P_{t2} is known already have evaluated. So, you can get directly P_{t13} , and we can estimate the temperature at the station 13 by you know assuming the T_{19} is same as the 13.

I mean of course, this is T_{t13}/T_{t2} will be again P_{t13}/P_{t2} , and this is what you call fan pressure ratio is given P_{t13}/P_{t2} that is nothing, but 2 polytropic efficiency we are not considering we will have to consider that further fan we need to consider the polytropic efficiency right p_F right we need to consider that. And when will substitute this values, you will get as you know T_{13} is equal to 324.56.

And critical pressure ratio across the nozzle can be determined as you know because, we need to find out whether it is the, where critical or not keep in mind we are assuming that the nozzle is a conversion nozzle. Because, generally in the turbo fan engine nobody really use in conversion diversion nozzle because, it not required right, and mostly it will be flying at not only the subsonic flow, but in case of even fighter aircraft people are using now a day's turbo fan engine, with very low by pass ratio right not high bypass ratio of 5 generally it is being use for the long range passenger aircraft.

Where which will be moving at all in the level flight of 0.8 my flight mach number 0.85 generally around 88.5. But, here we assuming the nozzle to be a conversion nozzle right, and that assumption are not mentioned, but whenever we are doing you need to it is not given in the problem I am a right. So, therefore, we need to assume that right, and critical pressure ratio across the nozzle can be determined P_{t13}/P_c , and of course, the nozzle efficiency is given and γ_z is given right 1.333.

And then you can find out it happens to be 1.92 we are seen in the tub site also, you know like these values if your efficiency is 0.95 you will get that, otherwise it will be little bit smaller values or higher values depending upon the efficiency of the nozzle. So, now, we need to find out what will be the pressure ratio you know $P_{t13}/P_{t_{naught}}$ right.

(Refer Slide Time: 13:24)

But, the actual pressure ratio across the fan nozzle with respect to ambient pressure is

$$\frac{P_{t13}}{P_e} = \frac{77.2}{26.5} = 2.91$$

Since $\frac{P_{t13}}{P_0} > \frac{P_{t13}}{P_c}$, the nozzle is considered to be choked. So, the static temperature at the nozzle exit T_{19} becomes

$$T_{19} = T_c = \left(\frac{2}{\gamma_a + 1} \right) T_{t5} = \left(\frac{2}{1.4 + 1} \right) \times 255.56 = 213 \text{ K}$$

Now, the static pressure at the nozzle exit P_9 becomes

$$P_{19} = P_c = \left(\frac{P_c}{P_{t13}} \right) P_{t13} = 40.2 \text{ kPa}$$

By using equation of state, the density of gas at the nozzle exit is estimated as

$$\rho_{19} = \frac{P_{19}}{RT_{19}} = \frac{40.2 \times 10^3}{287 \times 213} = 0.657 \text{ kg/m}^3$$

So, the actual pressure ratio across the fan nozzle with respect to ambient pressure will be P_{t13} by P_c or P_{naught} you can say this basically not you can say. So, that will be 2.91 that what is the meaning; that means, the pressure ratio across this will be higher, if it is higher than it is choked or un choked it is basically choked. So, if it choke; that means, it is having a critical velocity, what is that, that is equal to the speed of sound at the exit.

So, the static temperature nozzle exit can be determine very easily that is because, mach number is 1. So, from that condition you will get T_{19} is equal to T_c 2 divided by gamma a plus 1 T_{t5} keep in mind that, we are using here gamma a right, but there what I have use is basically could have been use the gamma a as well because, we are not adding any heat right. So, therefore, that correction has to be made, but and then you will get 213 Kelvin 2 divided by 1.4 plus 1 255.56 that is 213 Kelvin, you got my point there that.

So, the static pressure at the nozzle exit P_9 becomes P_{19} by P_c and they P_c by P_{t13} we have already evaluated. So, you will get the P_{19} by 40.2; that means, the P_{19} is not same as that of the your what to call ambient pressure, which happens to be 26.5 kilo Pascal, but now it is 40.2 kilo Pascal. So, the by using the equation of state the density of gas at the nozzle exit can be determine because, we use the equation of step, and then

you find out because, know P_{19} and P_{naught} and then substitute those values, and you will get these things right.

(Refer Slide Time: 15:40)

Hence, we can evaluate V_{19} as

$$V_{19} = C_{19} = \sqrt{\gamma R T_c} = \sqrt{1.4 \times 287 \times 213} = 292.54 \text{ m/s}$$

Let us evaluate $\frac{A_{19}}{\dot{m}_F}$ using the continuity equation as

$$\frac{A_{19}}{\dot{m}_F} = \frac{1}{\rho_{19} V_{19}} = 5.2 \times 10^{-3} \text{ m}^2/\text{kg}$$

The specific thrust with respect to total mass flow rate contributed by the fan can be evaluated as

$$T_{s,F} = \frac{\alpha}{\alpha + 1} \left[(\check{V}_{19} - \check{V}_0) + \left(\frac{A_{19}}{\dot{m}_F} \right) (\check{P}_{19} - \check{P}_0) \right]$$

$$= \frac{5}{5+1} \left[(292.54 - 254.6) + 5.2 \times 10^{-3} (40.2 - 26.5) \right]$$

$$= 31.67 \text{ Ns/kg}$$

And similarly we can find out V_{19} which is same as that the speed of the sound, and that is here again this will be $\gamma R T_c$ right you will put 1.4, 287, 213 and that is 292.54 because, you substitute this values get. So, by knowing this I can find out A_{19} by \dot{m}_F which is very easily from the continuity equation, you know we can get this and that is nothing, but $1/\rho_{19} V_{19}$ you will just substitute this values, you will get this 5.2×10^{-3} meter square second per k g.

And the specific thrust with respect to total mass flow rate contributed by the fan can be evaluated as because, we know that $T_{s,F}$ is $\alpha/(\alpha + 1)$ α is by pass ratio which is nothing, but your what α is equal to 5 in this case, we substitute value you know V_{19} we are evaluated V_{naught} have evaluated, this portion we also evaluated and this is with the respect to F right, we have are already evaluated this is P_{19} by P_{naught} .

So, if we look at P_{19} we know, and P_{naught} we know because, P_{19} is higher than the P_{naught} in this case. So, when is you substitute this value you will get the 31.67 neutron second per k g I think this will be kilo neutron second per k g.

(Refer Slide Time: 17:30)

For Core Stream
Compressor:

The total pressure at station (3) is given by

$$\frac{P_{t3}}{P_{t2}} = 20 \Rightarrow P_{t3} = \frac{P_{t3}}{P_{t2}} \times P_{t2} = 20 \times 38.61 \text{ kPa} = 772 \text{ kPa}$$

Using the polytropic efficiency of the compressor, let us now evaluate the total temperature ratio T_{t3}/T_{t2} as

$$\frac{T_{t3}}{T_{t2}} = \left(\frac{P_{t3}}{P_{t2}} \right)^{\frac{\gamma_a - 1}{\gamma_a \eta_{pc}}} = 20^{\frac{1.4 - 1}{1.4 \times 0.85}} = 2.74$$

$$\Rightarrow T_{t3} = \frac{T_{t3}}{T_{t2}} \times T_{t2} = 700.2 \text{ K}$$

By carrying out energy balance across the combustor, we can find out the expression for fuel air ratio as

$$f = \frac{C_{pg} T_{t4} - C_{pa} T_{t3}}{\eta_b \Delta H_c - C_{pg} T_{t4}} = \frac{1148 \times 1600 - 1005 \times 700.2}{0.94 \times 43000 \times 10^3 - 1148 \times 1600} = 0.029$$

So, the total pressure ratio across at the station 3 is given by P_{t3} by P_{t2} which is 20 right because, why we have taken this 10 into 2 that is the fan pressure ratio therefore, you can taken 20 right that you keep in mind. So, P_{t3} will become you know 20 into 38.61 the valid determined what will be P_{t2} right, so you can get this using the polytropic efficiency of the compressor let us now evaluate this T_{t3} by T_{t2} into P_{t3} by P_{t2} eta minus 1 eta p sorry gamma minus 1 divide by gamma a and eta p c.

So, when you substitute this value you will get 2.74 and then from that we know T_{t2} , so you can get T_{t3} as 700.02 Kelvin, by carrying out energy balance across the combustor we can find the expression fire fuel ratio. I mean we have already done this thing, we know all those values T_{t4} we know that is given T_{t3} already evaluated and ΔH_c is given eta b is given, so all this substitute and get that values f that is fuel air ratio.

(Refer Slide Time: 18:55)

As the pressure ratio across the combustor is 0.95, the pressure at the turbine inlet P_{t4} becomes

$$P_{t4} = \pi_b \times P_3 = 0.95 \times 772 = 733.4 \text{ kPa}$$

Turbine:

The Total temperature at the exit of the turbine is to be evaluated. We know that the function of a turbine is to supply the requisite power to the compressor. Hence, we have

$$\dot{W}_c + \dot{W}_f = \eta_m \dot{W}_t$$

From the above equation, we can estimate the turbine exit temperature as

$$T_{t5} = T_{t4} - \frac{C_{pa}}{C_{pg} \eta_m (1 + f)} \left[(T_{t3} - T_{t2}) + \alpha (T_{t13} - T_{t42}) \right]$$

$$= 1600 - \left(\frac{1005}{1148} \right) \frac{1}{0.95(1 + 0.029)} \left[(700.2 - 255.56) + 5(324.56 - 255.56) \right]$$

$$= 892.84 \text{ K}$$

So, as the pressure ratio across the combustor chamber is 0.95, so you can get basically P t there is pi b, pi b is 0.95 into P t 4 no P t 3 this will be P t 3. Now, P t 4 will be basically pi into P t 3 because, pi b what P t 4 by P t 3 station number, if you look at combustor chamber right it is 3 and this is your 4. So, pi b will be P t 4 by P t 3, so P t 3 is equal to pi b into P t 3 right 0.95 into 772 that will be little lower than the P t 3 that is 733.4 kilo Pascal.

So, we will look at turbine the total temperature the exit of the turbine is to be evaluated, we know that the function of a turbine is to supply the requisite power to the compression and the fan. So, we can write down that the compression work plus fan work will be supplied by the turbine, and there is an efficiency you know mechanical efficiency, which will be always lower than the one. So, if we write in terms of you know temperature and C p like enthalpy change, then you will arrive at this relationship T t 5 is equal to T t 4 minus C p a divided by C p g eta m 1 plus f then T t 3 minus T t 2 alpha T t 13 minus T t 2 right that is T t 2.

So, when you substitute this values you will get as 892.84 because, all this things we know I mean C p g eta m f already we evaluated T t 3 is evaluated T t 13 evaluated this is evaluated, this evaluated, so it just substitute and those value.

(Refer Slide Time: 21:05)

Then, the total pressure, P_{t5} , can be determined by using the polytropic relationship

$$\frac{P_{t5}}{P_{t4}} = \left(\frac{T_{t5}}{T_{t4}} \right)^{\frac{\gamma_g}{(\gamma_g - 1)\eta_p}} = \left(\frac{1196.1}{1600} \right)^{\frac{1.333}{(1.333-1)0.88}} = 0.069$$

$$\Rightarrow P_{t5} = \frac{P_{t5}}{P_{t4}} \times P_{t4} = 0.069 \times 733.4 = 50.6 \text{ kPa}$$

Nozzle:

The critical pressure ratio across the nozzle is determine as

$$\frac{P_{t5}}{P_c} = \frac{1}{\left[1 - \frac{1}{\eta_n} \left(\frac{\gamma_g - 1}{\gamma_g + 1} \right) \right]^{\frac{\gamma_g}{\gamma_g - 1}}} = \frac{1}{\left[1 - \frac{1}{0.95} \left(\frac{1.333 - 1}{1.333 + 1} \right) \right]^{\frac{1.333}{1.333 - 1}}} = 1.914$$

But, the actual pressure ratio across the nozzle with respect to the ambient pressure is

$$\frac{P_{t5}}{P_0} = \frac{50.6}{26.5} = 1.90$$

So, P_{t5} by P_{t4} we can evaluate using again isentropic for the what you call the turbine, and will substitute this values and get this number. So, by this we can find out P_{t5} as the 50.6 kilo Pascal, so now, for nozzle we need to determine the critical pressure ratio across the nozzle you know, and we do the similar thing we will find it is happens to be 1.914 and keep in mind here we will use gamma g because, it is hot.

So, therefore, we will use the gamma g and in case of a fan will be using gamma a that is 1.4, and the actual pressure ratio across the nozzle will be P_{t5} by P_{naught} which is 1.9. If you look at this ratio is less than that have critical pressure right of course, it is very little if we look at that way it will be almost critical right. So, therefore, one has to be little careful in because, the situation will change.

(Refer Slide Time: 22:24)

Since $\frac{P_{t5}}{P_0} < \frac{P_{t5}}{P_c}$, the nozzle is not choked. Then $P_9 = P_0$.

Hence, we can evaluate V_9 as

$$V_9 = \sqrt{2\eta_n C_p T_{t5} \left[1 - \left(\frac{P_9}{P_{t5}} \right)^{\frac{\gamma_g - 1}{\gamma_g}} \right]}$$

$$= \sqrt{2 \times 0.95 \times 1148 \times 892.84 \times \left[1 - \left(\frac{26.5}{50.6} \right)^{\frac{0.333}{1.333}} \right]} = 539 \text{ m/s}$$

The specific thrust for the core stream can be estimated as

$$T_{s,c} = \frac{1}{1 + \alpha} [(1 + f)V_9 - V_0]$$

$$= \frac{1}{1 + 5} [(1 + 0.029)539 - 254.6] = 50 \text{ Ns/kg}$$

The total specific thrust for the turboengine can be estimated as

$$T_s = T_{s,c} + T_{s,f} = 50 + 31.67 = 81.67 \text{ Ns/kg}$$

But, in this case we consider that nozzle is not choked because, $P_{t5} > P_{naught}$ by P_{less} than P_{t5} by P_c and then if it is not choked P_9 is equal to P_{naught} ; that means, nozzle is full expended. So, we can evaluate these things and this $2\eta_n C_p T_{t5} \left[1 - \left(\frac{P_9}{P_{t5}} \right)^{\frac{\gamma_g - 1}{\gamma_g}} \right]$ for the nozzle right. So, by we know this value T_{t5} we know this P_9 P_{t5} C_p this will be C_p η_n all those things.

So, if you look at I am using $\eta_n C_p$ 1148 into 892.84 then you will get this 539 meter per second, the specific thrust for the core stream we can evaluate because, there is no contribution of thrust due to the pressure right. So, is only due to the momentum, and you will get that specific thrust core engine is 50 neutron per second I think my mach it will be kilo neutrons because, it is looks to me.

So, you will have to cross to check, the total specific thrust for the turbo engine can be estimated as you know T_s plus this thing that is you know you just add and you get this number 81.67 kilo neutron per second per kg it looks to me like that.

(Refer Slide Time: 24:00)

Then, TSFC can be determined easily by

$$TSFC = \frac{f}{T_s} = \frac{0.029}{81.67} = 355.1 \text{ mg/N.s}$$

The η_p , η_{th} are estimated as

$$\eta_p = \frac{2M_0 \left[(1+f)(V_9/a_0) + \alpha(V_{19}/a_0) - (1+\alpha)M_0 \right]}{\left[(1+f)(V_9/a_0)^2 + \alpha(V_{19}/a_0)^2 - (1+\alpha)M_0^2 \right]}$$

$$= \frac{2 \times 0.85 \left[(1.029)(1.794) + 5(0.976) - (6)0.85 \right]}{\left[(1.029)(1.794)^2 + 5(0.976)^2 - (6)0.85^2 \right]} \times 100$$

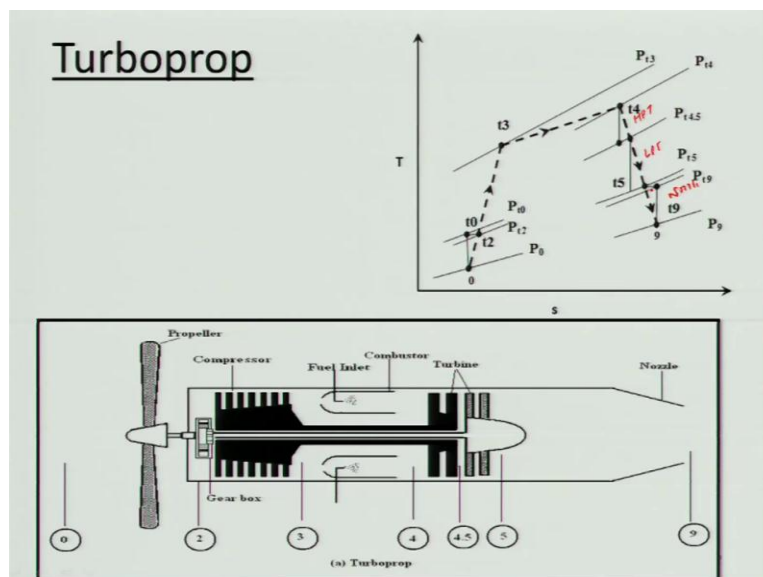
$$= 73.92\%$$

$$\eta_{th} = \frac{a_0^2 \left[(1+f)(V_9/a_0)^2 + \alpha(V_{19}/a_0)^2 - (1+\alpha)M_0^2 \right]}{2f\Delta H_C}$$

$$= 13.45\%$$

So, TSFC can be evaluated as TSFC f divided by T s 0.029 divided by 81.67 you will get 355.1 milligram per neutron per second. So, we can evaluate the purposively efficiency write and substitute this values, and you will get the same things what you have done keep in mind that this purposively efficiency 73.92 percent which is happens to be quite bit high of course, the thermal efficiency being reduce, so the smaller values.

(Refer Slide Time: 24:38)



So, with this you know we will stop over and we will discuss this may be you having doubt little later on, and now we will move into the turboprop engine. So, if you look at

turbo prop engines which is having you know we are already discuss in the ideal cycle, it is having propel having a compressor, which is you know. And then combustor chamber that is the high pressure turbine, and low pressure turbine is connected to the propeller gear box right.

And most of the thrust is being obtain of the propeller, and small portion is being obtained by expending gas in the nozzle, generally conversion nozzle is been used. And the passes will be similar to the turbo jet engine only difference with be here that in this portion that is the expansion in the high pressure turbine, this is high pressure turbine. And low pressure turbine will be much higher as compare to the nozzle, nozzle is very, very small that you should keep in mind right.

And rest of the things is similar to the turbo jet engine or the core engine of the what you call turbo fan engine.

(Refer Slide Time: 25:58)

Turboprop

The specific thrust of the core engine is given by

$$T_{sc} = \frac{T_c}{\dot{m}_c} = a_0 \left[(1+f) \frac{V_9}{a_0} - M_0 + (1+f) \frac{R_g(T_9/T_0)(P_9 - P_0)}{R_a V_9/a_0 P_9 \gamma_a} \right] \quad (1)$$

From the definition of work output coefficient for the core stream C_c , an expression for C_c can be derived using Eq. (1)

$$C_c = \frac{T_c V_0}{\dot{m}_c C_p T_0} = (\gamma_a - 1) M_0 \left[(1+f) \frac{V_9}{a_0} - M_0 + (1+f) \frac{R_g T_9/T_0 (P_9 - P_0)}{R_a V_9/a_0 P_9 \gamma_a} \right]$$

The expression for V_9/a_0 is obtained in a similar manner as was done for turbojet engine

$$\frac{V_9}{a_0} = \sqrt{\frac{2 \tau_\lambda \tau_{tH} \tau_{tL}}{\gamma_g - 1}} \left[1 - \left(\frac{P_9}{P_{19}} \right)^{\frac{\gamma_g - 1}{\gamma_g}} \right]$$

So, what will do we will basically look at the similar expressions I mean which we have already done this for the core, write engines we are not considering the popular part. And which we recognize this are the similar terms I am like you know V_9 by a naught and flight mach number, and then T_9 by T naught we have which we already seen and V_9 by a naught you know.

And these are the terms which we are very familiar we let me not spend time on this, from the definition of work output coefficient for the core stream one can really get the $T_c V_{naught}$ divided by the $\dot{m} c_p$ and T_{naught} . So, if you look at you can get an expression like this you know we can derive it very easily, we are done similar thing in the case of your ideal cycle, except this term, this term is due to the pressure thrust which is extra.

Because, here nozzle is not you know fully expanded right therefore, this term is extra which will be similar to that what we done earlier for the case of turbo jet engine or turbo fan engine. So, the expression for V_9 by a naught will be similar, but only terms what you will get here, we are separated as τ_{tH} and τ_{tL} instead of that we generally use that τ_t right, we never really separate this high pressure and low pressure turbine temperature ratio, this is only difference rest of the things are fine, and this you keep in mind this will be $E_{gen} - 1$.

(Refer Slide Time: 27:50)

By carrying out an energy balance for the main combustor in the core engine, we can derive

$$f = \frac{\tau_\lambda - \tau_c \tau_e}{\frac{\eta_b \Delta H_c}{C_{pa} T_0} - \tau_\lambda}$$

Now, we must carry out an energy balance between the high pressure turbine and compressor to get $\dot{m}_a = \dot{m}_c$

$$\dot{W}_c = \eta_{mH} \dot{W}_{tH}; \Rightarrow \dot{m}_a C_{pa} (T_{t3} - T_{t2}) = \eta_{mH} (\dot{m}_a + \dot{m}_f) C_{pg} (T_{t4} - T_{t4.5})$$

$$\tau_{tH} = 1 - \frac{1}{\eta_{mH} (1 + f)} \frac{\tau_r}{\tau_\lambda} (\tau_c - 1)$$

where η_{mH} is the mechanical efficiency of the high pressure shaft.

Power consumed by the propeller = Power supplied by low pressure turbine

$$\dot{W}_{prop} = \eta_{gb} \eta_{mL} \dot{m}_{4.5} C_{pg} \left(\frac{T_{t4.5}}{\tau_{t4.5}} - T_{t4.5} \right) \Rightarrow \dot{W}_{prop} = \eta_{gb} \eta_{mL} \dot{m}_{4.5} C_{pa} T_0 \tau_{tH} \tau_\lambda (1 - \tau_{tL}) \dots (2)$$

where η_{mL} = Mechanical efficiency of the low pressure shaft
 η_{gb} = Efficiency of the gear box

Handwritten notes on the slide:
 $\lambda = \frac{C_{pg} T_{t4}}{C_{pa} T_0} \left(\frac{\tau_r}{\tau_\lambda} \right)$
 $= \frac{C_{pg}}{C_{pa}} \left(\frac{T_{t4}}{T_0} \right) \left(\frac{\tau_r}{\tau_\lambda} \right)$

So, the carrying out the energy balance for the main combustor in the core engine, we will get the similar expression as have shown here, like this is same for both turbo fan, turbo trap, turbo you know jet engines. So, we need not to really do anything, but this is the very important part we must look at it because, the energy balance between high pressure turbine and compressor. And also will be looking at the energy balance between the low pressure turbine and propeller.

So, if you look at \dot{W}_c is nothing, but $\eta_m H$ into $\dot{W}_t H$ right, and you can write down $\dot{m} C_p a$ and this $\dot{m} a$ we also write this as $\dot{m} c$ keep in mind right, $\dot{m} a$ this a is equal to $\dot{m} c$ what we use. So, you should not get confuse and multiplied by $T_{t3} - T_{t2} \eta_m h \dot{m} f$ plus $\dot{m} a$ plus $\dot{m} f$ into $C_p g T_{t4}$ is minus $T_{t4.5}$ keep in mind, this is very important this one, this for the high pressure turbine.

So, when you do that and where $\eta_m H$ is the mechanical efficiency of when you simplify it and express in terms of temperature ratio, you will get as τ_{tH} is equal to $1 - \frac{1}{\eta_m H^{1+f}}$ or divided by τ_r by τ_λ multiple by $\tau_c - 1$ right. So, if you look at this is the same as that off the what you are getting for the turbo jet engine or the core engine of the turbo fan is nothing, much different, only thing in place of these we are using the τ_H here there it will be τ_t in this case turbo prop engine τ_{tH} .

And we know that power consumed by propeller is equal to power supplied by low pressure turbine, with this we can write down this power produce consumed by the propeller is equal to η_{gb} this is the gear box efficiency η_{mL} that is the mechanical efficiency of the low, what you call pressure turbine shaft right and $\dot{m}_{4.5} C_p g T_{t4.5} - T_{t4}$.

And when you do little manipulation and you will get basically you know this expression, how we get like I can divided by this is as $T_{t4.5}$ and this is $T_{t4.5}$ and that is 1 right. And if you look this is nothing, but your τ_{tL} right, and when I divide this I need to multiply by here 4.5 yes or no right, so and then these expression $T_{t4.5}$ and $C_p g I$ can express in terms of $\tau_\lambda C_p a T_{naught}$ and τ_{tH} I am just leaving it because, if you look at the definition what is λ , λ is equal to $C_p g T_{t4}$ divided by $C_p a T_{naught}$ yes or no right.

And in place of a T_{t4} let me just do little bit and then leave it T_{t4} by $T_{t4.5} T_{t4.5}$ divided by T_{naught} . And this is what is this one this is nothing, but your τ_{tH} is it τ_{tH} or $1/\tau_{tH}$ that is $1/\tau_{tH}$ yes or no by definition $T_{t4.5}$ divided by T_{t4} is τ_{tH} right. So, therefore, it will be $1/\tau_{tH}$ and you will do that you will get this values I will leave it here for you to look at this.

(Refer Slide Time: 32:23)

By using Eq. (2), we can get an expression for C_{prop} as

$$\dot{W}_{prop} = \eta_{gb} \eta_{mL} \dot{m}_{4.5} C_{pa} T_0 \tau_{tH} \tau_{\lambda} (1 - \tau_{tL}) \dots (2)$$

$$C_{prop} = \frac{\eta_{prop} \dot{W}_{prop}}{\dot{m}_c C_{pa} T_0} = \eta_{prop} \eta_{gb} \eta_{mL} \underbrace{\frac{\dot{m}_{4.5}}{\dot{m}_c} = (1+f)}_{(1+f)} \tau_{tH} \tau_{\lambda} (1 - \tau_{tL})$$

Now, $C_{tot} = C_{prop} + C_c$. Hence, total specific thrust and workout coefficient for the turboprop becomes

$$T_s = \frac{T}{\dot{m}_c} = \frac{C_{tot} C_{pa} T_0}{V_0}$$

The expression for specific power W_s from the turboprop engine is given by

$$\dot{W}_s = \frac{\dot{W}_{tot}}{\dot{m}_c} = C_{tot} C_{pa} T_0 \quad \quad \frac{T_{tot}}{\dot{m}_c} = \frac{C_{tot} C_{pa} T_0}{V_0}$$

Now, let us derive the expressions for TSFC and PSFC as

$$TSFC = \frac{f V_0}{C_{tot} C_{pa} T_0}$$

So, now, we you know look at this co efficient of the propeller, so, that is C propeller, if you look at eta propeller, W dot propeller and is divided by m dot C p a T naught. And we have already looked at that it eta W dot propeller will be eta g b and eta m L m dot 4.5 C p a T naught tau t H tau lambda 1 minus tau t L that we have already looked at, if we divided you will get basically what you have doing, you are basically multiplying you know this substituting the value here right.

When we have substitute then C p T a will cancel out here m dot 4.5 by m dot c is nothing, but 1 plus f, so that is why it is coming over here right and the rest of the things as usual. So, if you look at C propeller we can express in terms of these, so hence the total you know total the co efficient of the power will be equal to C propeller C c and then total specific thrust work out you know co efferent, we can basically T s is equal to T divided by m dot c and this is C total C p a and T naught divided by V naught.

So, expression of specific a power will be W dot the total divided by m dot c keep in mind that all the time in case of prop engine, we are divided by the mass flow rate to the core engine. So, that is nothing, but C total C p a and T naught, so let us derive an expression for TSFC and PSFC, so TSFC will be f into V naught divided by C total C p a and T naught this is same what we are done for the ideal engine ideal cycle analysis.

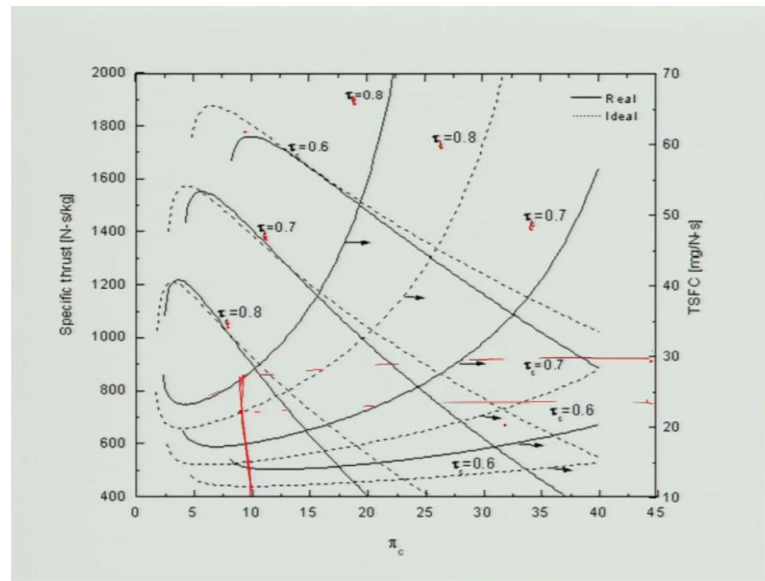
(Refer Slide Time: 34:32)

$$\begin{aligned}
 PSFC &= \frac{\dot{m}_f}{\dot{W}_s} = \frac{f}{C_{tot} C_{pa} T_0} \\
 \text{Thermal Efficiency, } \eta_{th} &= \frac{a_0^2 \left[(1+f)(V_9/a_0)^2 - M_0^2 \right]}{2f \Delta H_c} \\
 \text{Overall Efficiency, } \eta_0 &= \frac{\dot{W}_{tot}}{\dot{m}_f \Delta H_c} = \frac{C_{tot} C_{pc} T_0}{f \Delta H_c} \\
 \text{Propulsive Efficiency, } \eta_p &= \frac{\eta_0}{\eta_{th}}
 \end{aligned}$$

So, PSFC will be similar thing only difference a \dot{m}_f divided by the power produce by the engine total right. So, that will be $C_{tot} C_{pa} T_0$ and T_{naught} , you know f divided by $C_{tot} C_{pa} T_{naught}$, so thermal efficiency will be as usual, and keep in mind that this thermal efficiency we have evaluated real cycle without considering the pressure effect right. So, that you should keep in mind, and the overall efficiency will be basically \dot{W}_{tot} divided by $\dot{m}_f \Delta H_c$.

And keep in mind that propulsive efficiency you will get by determining the overall efficiency, and thermal efficiency in case of turbo prop engine right. But, in case of turbo jet and turbo prop we get directly, so that is the only different you should keep in mind, so far efficiency is concern rest of the efficiency is similar. Now, we will carry out basically parametric analysis and see what we are getting.

(Refer Slide Time: 35:53)



What we are doing here, we are taking this τ_t you know as various values like your 0.6, 0.7 and 0.83. And if you look at this curves like you know is corresponding to the specific thrust, which is plotted to the left hand side, and here of course, the pressure ratio across compression. You can see that the solid line is corresponding to the real cycle, and that is line corresponding to the ideal cycle. And in this case specific thrust goes on increase with the fan pressure ratio have a pick values, and then it goes on decrease right.

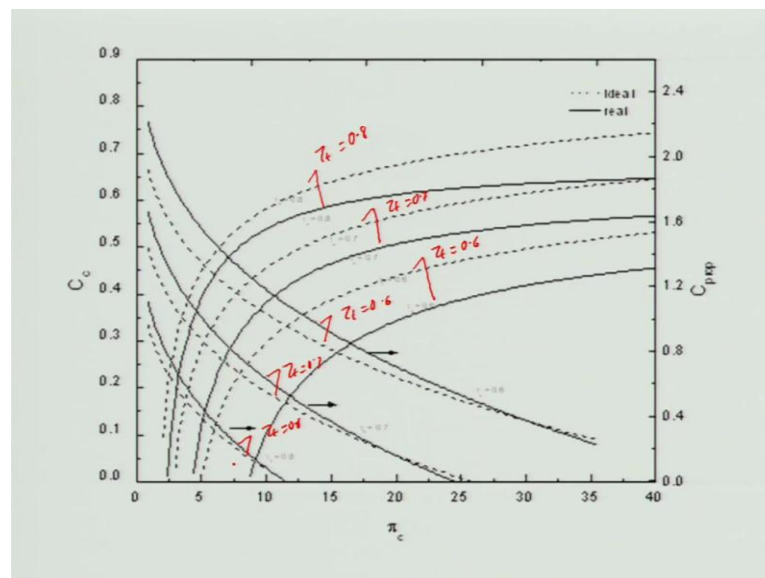
And in the case of ideal cycle it is a having similar nature, but only thing is that in the case of you know real cycle you are getting the lower values most of the pressure ratio arrange across the compression of course, in some place it is going up above the ideal cycle. But, and as you go on increasing this temperature ratio across the you know turbine right, it is the turbine this is the turbine. So, you will find the specific thrust decrease, what is the meaning of that because, the temperature ratio across the turbine is increasing means what, it is other way around.

That means, expansion in the turbine is low right because, this is $1 - 0.8$ it will be 0.2 percent, what it will be expansion will be there. That means, temperature ratio you know will be other way around because, it is define in the opposite downstream, divided by the upstream of a components right that always you will be find. So, therefore, it is a smaller value means, expansion is low, higher value means expansion is move right.

So, the specific thrust will be decreasing and of course, if you look at TSFC which is just opposite that is 0.8 values, you know the τ_t at 0.8 the thrust specific consumption it decrease and attempt a minimum values, and then again it increases for the real cycle. And similar thing you will get in case of an ideal only different between this real case and ideal case is that, that real case will have a higher TSFC for a particular values of pressure.

For example, if I take then you know you will get values of much higher, and if I put a this values you know this will be much lower. So, therefore, you will get higher that thrust specific consumption due to losses, which is I mean obvious case and as of course, the τ_t goes on decrease you will get a lower TSFC which is usual I mean expected.

(Refer Slide Time: 39:50)



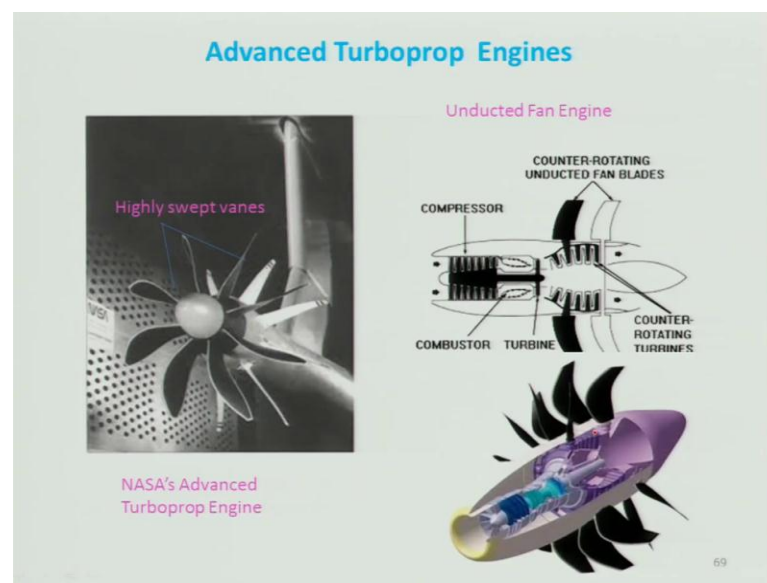
And if you look at the C_c I am in like a core engines, like you will find that C_c is basically decreasing right, as it is with the pie pressure ratio it sorry that C_c it goes on increasing and reaches a certain values, at the higher pressure ratio compression. And the real case as I told it is a solid line is having a lower value, as compare to the ideal one which is a dashed line for a particular condition. And of course, as it is decrease that this one basically τ_t is equal to 0.8.

And similarly this is τ_t is equal to 0.7, and this will τ_t 0.6 right, and which is expected because, the losses will be more. So, therefore, you will expect to have way work out put C_c efficiently will be lower in case of a real cycle right, for a particular

pressure ratio, particular turbine you know temperature ratio. And of course, the C propeller it is decreases with increasing pressure ratio across the compression, and this is having a higher value as compare to ideal cycle right.

Because, the fan you know which will be taking the more amount of work, and then it will be higher than that. So, of course, this is for τ_t is equal to 0.7, and this is for the τ_t is equal to 0.8. So, it is if you look at other way around you will get a fan pressure ratio which will be higher.

(Refer Slide Time: 41:51)



So, what we look at is you know advance turbo prop engines because, of fact that you know this propeller cannot really operate at a very high speed that is the one limitation or in other around if you look at the turbo prop engine cannot really fly at a high mach number. For a particular r p m of the engine you know when you fly then what will happen, it will be having a very much higher blade.

So, then the blade team will attempt a local velocity having speed more than the speed of sound. Then the shaft will be formed and then you will have several other problems or losses will be very much higher, and inner to overcome that thing what people are you know thinking about that why not reduce the length of the blade, and use the more number of blade. Because, the number of blades less then you know the work transfer to the momentum transfer to the air for getting thrust will be reduce right.

But, if it is other way if you increase then you are getting higher you know efficiency of transferring the momentum, so that you will get higher thrust. So, that was the idea which will people try to do that, and then they felt that they will have to use very high lift weft blades right. When you do that then as a result, so that you can fly at a high mach number right that is one advantages, and also you can have a higher thrust right because, the your static thrust should be much higher in case of turbo prop engine, so that is the advantages.

So, they come up with a ideas of a you know more number of blade they experiment at with 8 number of blade, generally you know 3, 4 peoples are used 8 number of blade which have much higher. But, this are highly swept vanes right, and this high swept vanes can be operated at a higher RPM as well because, when you reduce this speed then the gear box you know will have very big. Because, the suppose your engine is operated at 50,000 RPM you will have to bring 2000 RPM right it will big here ratio of 25 is to 1.

So, the gear box will be much higher because, the power level of the engine will be very high, in this case not like your auto mobile engine some few kilo it will be order of mega watts right. So, therefore, when you reduce the RPM to lower than then you will be reducing the gear box with as usual so, but the problem with this kind of you know wings it creates a lot of noise right. And of course, and it will be also give oxide kind of twisting right because, the pre factor as symmetric thrust will be generated when popular rotate it will create a symmetric thrust.

So, here it will be most in a stranger, so inner to overcome that thing they put you know way of course, this is static kind of thing one can thing, but there where another concept which was being used, you know known as un ducted fan engines right. But, if you want to if you look at these are the wings you can make it contra rotating; that means, if it is rotating in a clock wise, the other will be anti clock wise right. Then what you will get out of it, you will overcome the problem of the asymmetry thrust, and it will be more efficiency.

Because, the air is moving to the blade and it is going and it is the west, so you would not get the thrust because, it will be having a swirl component. So, what will give in the excel components will give with the thrust, so therefore, in these case you will avoid doing that. That means, at the if there is a two you know what to call stages of blade, at

two rows of blade, then you know like it is rotating other opposite direction, so then you can get a higher thrust.

So, people have got you know around 6 to 16 percent higher in the thrust right using this, but; however, it is not being use in practical situation. People are saying that the gear box is the problem because, you will have to transfer the what to call, the power from the low pressure turbine to the propeller with the help of a gear box right, which is propeller generally in the front right as you see it is in the front.

But, then why not you attach this you know propeller to the turbine itself right instead of you know using gear box, you can directly do that and rotate at little higher RPM right which is same as that has the low pressure turboprop. So, that was the idea which is people use that is a unducted fan engines right, if you look at they directly with the turbine it is attach the blades right, there are and which are counter rotating right it will be one in the clock wise direction, they may be lets first one and second will be the counter clock wise direction.

So, they could menace to get a higher thrust and overcome the problem of the gear box right, weight will be reduce the efficiency, but it is having a problem, problem is that noise, noise is very high. And there is another problem of leakage right, the leakage of this you know gas hot gas is another biggest problem, so therefore, of course, the g has the you know I have develop this engine quite some time back and they use very high list when as well you can see.

But, it is not being use in practice, but; however, may be in future it will be use, and we will be you know looking at the rocket engines in the next class onwards and I will stop over.