

Fundamentals of Aerospace Propulsion
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Lecture - 33

Let us start this lecture 33 and as usual we will recall what we learnt in the last lecture, we basically looked at the why we will go for real cycle analysis, and how to handle the losses in case of ideal in the losses incurred in the engines, and incorporate in the ideal cycles such that it will be almost closer to the real cycle right, in an air craft engine, I mean only for the gas turbine engines. So, what are the those things we have looked at various components.

For example, we have defined the special recovery factor in case of a in air intake, then we have also looked at isentropic efficiency for air intake, and for compressor, turbine, but then we also looked at that it is not good enough to use isentropic efficiency, particularly for the compressor and turbine. And we will be using polytrophic, efficiency and know that we got a relationship how you can relate the polytrophic efficiency with the isentropic efficiency right.

But; however, in case of nozzle and air intake will be using the isentropic efficiency right, but that is not really playing very major role. Why because, the heat addition or the heat extraction is not there except during expansion and compression like that is the not extraction, rather it is being change in case of compressor and turbine you are getting some work done. In case of turbine and you are giving work to the fluid or the energy transfer takes place, when you give work to the compression.

So, I am doing all those things you know a parameters, and also a parameters which we will be considering in case of combustors or burner efficiency, and pressure ratio across the burner right. Today we will be looking at how to incorporate those you know efficiencies, and other pressure ratios into our calculation, and carry out a parametric cycle analysis for turbo jet engine, and we will derive this similar expression what we have done, in case of ideal turbo jet engine then we will take an example.

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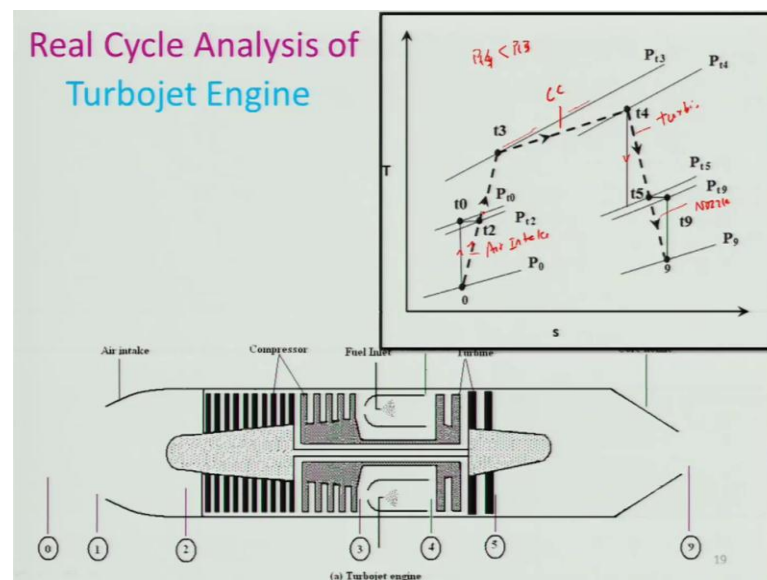
Lecture 33

Science should not be considered as a mere body of knowledge
but rather a way of thinking, and acting rationally in our own life.

D. P. Mishra

See, how we can you know use that for solving the problem or looking at you know various aspect and what it really does.

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So, we will as I told you real cycle analysis we will be looking at you know carrying out for a turbo jet engines, and this is as usual the turbo jet engine I am just repeating some of them. So, that it will be in ingrain your mind that it is having air intake; that means, all the process of compression in air intake is taking place the station 0 to 2. And as you

know very well that there is a compressor, and turbine compressor, and the turbine and the work done by the turbine is mean to run the compressor right.

And the thrust obtained by turbo jet engine is basically due to expansion in the nozzle, in this case I have used a what you call converging nozzle as shown. But; however, a converging and diverging nozzle can be used, and we will be restricting our discussion a particularly to the converging nozzle, and I will be mentioning about the converging diverging nozzle also. So, what are the processes in the T s diagram and if you look at 0 to 2 that is your what you call air intake right.

In case of your isentropic because, this is the solid line right what we have considered right, and keep in mind that all the time we will be using the dash line to show the real processes. Why because, this is non isentropic in nature and as it is non-isentropic, so you cannot show in a T s or a P v diagram, you must be knowing from the thermodynamic point of view. But; however, we have joined these lines right as a dash line to make it more prominent for that you can appreciate.

So, from station 2 to 3 is the compression which have shown here as what you call real process that is dash line, and keep in mind that this some process here I have not drawn. Because, the pressure you know if I could have gone by this it could have been higher than the P t 3, and this is the combustion chamber where if you look at the process is taking place where pressure P t 3 line to the P t 4 right keeping in mind that P t 4 is less than P t 3 right.

Because, this is the lower pressure and there is the losses in pressure, in ideal case the heat addition could have taken place in this line you know this line along with the P t 3 right. In real case it is the less than P t 4 is less than P t 3 is that clear; that means, there is a loss in the total pressure in case of combustion chamber, not only that there will be also the, you know in complete combustion in the combustor as the result the burner efficiency has to be included in the analysis.

And this is your basically combustion chamber and the gas is expanded in the turbine right this is turbine from station t 4 to the t 5 right. The ideal passes could have been taken place in this solid line, and the dash line indicates the real passes, and this is expansion in what you call nozzle that is from t 5 to the P 9. So, if you look at you might

be thinking why these two line are in the same, you know like t_5 and t_9 because, here we are assuming the flow to be adiabatic although it is isentropic, but adiabatic.

Similarly, you know in case of air intake that is t_1 is equal to t_2 right because, we are considering this is an adiabatic. But, in real situation is it possible the process would be adiabatic in case of either air intake or in nozzle, certainly no there will be some losses will be there, but we will not considering in real cycle. Because, it is difficult to estimate and those numbers are very small, you know particularly air intake it will be small, but in nozzle it will be higher because, the temperature is higher right.

And it is taking place in the residence time is small also right because, it will be nozzle it will be very high velocity. So, therefore, considering this fact we are not considered that, this is again little bit idealization we have using at this moment, so that is one point you should keep in mind. So, I am doing this now we will carry out, the similar analysis what we had done in case of ideal cycle; however, we will incorporate the what you call various parameters, which can take care of the losses in each component.

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Turbojet

The uninstalled thrust T for a turbojet engine is expressed as

$$\frac{T}{\dot{m}_0} = (\dot{m}_9 V_9 - \dot{m}_0 V_0) + \frac{A_9}{\dot{m}_0} (P_9 - P_0)$$

From Eq. (1), we can obtain an expression for specific thrust T_s as

$$T_s = a_0 \left(\frac{\dot{m}_9}{\dot{m}_0} \frac{V_9}{a_0} - M_0 \right) + \frac{A_9 P_9}{\dot{m}_0} \left(1 - \frac{P_0}{P_9} \right) \quad (1)$$

But, $\frac{\dot{m}_9 A_9 P_9}{\dot{m}_0} = \frac{\dot{m}_9}{\dot{m}_0} \frac{A_9 P_9}{\rho_9 V_9 A_9} = \frac{\dot{m}_9}{\dot{m}_0} \frac{P_9}{\rho_9 V_9} = \frac{\dot{m}_9}{\dot{m}_0} \frac{R_g T_9}{V_9} = \frac{\dot{m}_9}{\dot{m}_0} \frac{R_g T_9}{a_0^2} = \frac{\dot{m}_9}{\dot{m}_0} \frac{R_g (T_9/T_0)}{a_0^2} = \frac{\dot{m}_9}{\dot{m}_0} \frac{R_g (T_9/T_0)}{a_0^2} \frac{a_0^2}{\gamma_0 R_g T_0} = \frac{\dot{m}_9}{\dot{m}_0} \frac{R_g (T_9/T_0)}{\gamma_0 R_g T_0} = \frac{\dot{m}_9}{\dot{m}_0} \frac{T_9/T_0}{\gamma_0}$

However, we can write \dot{m}_9/\dot{m}_0 in terms of fuel-air ratio f by striking a mass balance across the turbojet engine as

$$\dot{m}_9 = \dot{m}_0 + \dot{m}_f \Rightarrow \frac{\dot{m}_9}{\dot{m}_0} = 1 + f \quad f = \frac{\dot{m}_f}{\dot{m}_0}$$

Similar to an ideal cycle analysis, we can express V_9/a_0 in terms of temperature ratio as

$$\frac{V_9}{a_0} = \sqrt{\frac{\gamma_9 R_g T_9}{\gamma_0 R_g T_0}} \cdot M_0$$

So, the uninstalled thrust for the turbo jet engine is expressed as we know that T is equals $\dot{m}_9 V_9 - \dot{m}_0 V_0$ this is due to the momentum. You know change of the flow and this is the pressure thrust $A_9 (P_9 - P_0)$, this is the area of the nozzle exit and $P_9 - P_0$ and we had neglected this term pressure thrust. That means, we are

saying it is fully expanded, but in real situation it can never be happened right except at the designed condition right.

So, therefore, we need to take care of that, and equation you know 1 we can express as a specific thrust if I will do what I will do I can divide it by m_0 here, similarly m_0 here I can divide it by m_0 here. So, if I divided that I will get an expression is basically I can take a m_0 out also m_0 divided by m_0 right for and then V_9 by a m_0 if you look at this term is usual like we have already derived relationship for this, and this is your flight Mach number minus m_0 that is your flight Mach number right.

And A_9 if I take P_9 out what I will get in the bracket is $1 - P_9$ divide by P_9 and m_0 I have added. So, if you look at we have already you know looked at either V_9 a m_0 terms we will be looking again little bit, just to see how this losses will be incorporated in that. And let us look at this portion you know which we have neglected in the case of your ideal cycle, we will consider this portion first see that what whether we can express in terms some certain known quantities particularly V_9 by a m_0 by m_0 or not something or m_0 is it possible.

We can convert that in terms of m_0 divided by m_0 and V_9 by a m_0 , it is possible. Let us see how we can, if we do that then it will be easier for to do a parametric analysis, keep in mind that when we will be solving problem you need not to do that. Here we are interested to club or the separate the parameters which you can vary right that is the beauty of this analysis, which is meant for you know to be used as a design tool.

So, let us look at A_9 by P_9 divided by m_0 I can write down m_0 right in the I can write down here m_0 divided by $\rho_9 V_9 A_9$ right. And that is nothing, but that, so I have done in the same, so; that means, m_0 by m_0 this is the term already we have got, which is similar to the momentum thrust expression, you know like and A_9 and P_9 divided by $\rho_9 V_9 A_9$ and using ideal gas law right I can express ρ_9 as $P_9 / R_9 T_9$.

So, I can you know cancel this P_9 and a what else I can cancel, I can cancel this A_9 right area of the nozzle exit right. So, I will get m_0 divided by m_0 and $R_9 T_9$ by V_9 what I will do I will multiply it you know, what you call in the numerator a

naught square that is the speed of sound at the what you call system and divide by gamma a R naught T naught. Because, I am just doing a naught square and this is nothing, but if you look at a naught square right can I not do that and there is nothing harm in doing am I right the same thing.

So, I will get here and I can write down this R 9 is nothing, but R g we are using R g means 1 value with respect to the higher temperature, we will be using. And similarly R a for the air what we will be using this is you keep in mind, this is the universe sorry gas constant right, which is the gas constant specific gas constant right. So, and T 9 by T naught I can write down, and V 9 by a naught, if you look at this term is similar to that of the momentum thrust, you know and gamma a, gamma a is we will be using 1.4.

So; that means, with this term you know this term we have expressed in terms of m naught 9 m naught divided by m naught 0 and V 9 a naught and all those things are known, and this is also another term which we will be using right, we can get expression for this. Then we will get then we can also evaluate this portion, where we need not to know the a 9 right I should know that because, here it is the what you call is a rubber engine what we are doing, we are not decided yet the what the dimension.

So, therefore, a 9 may not be available to you right, but; however, we can carry out a analysis you know in the preliminary analysis, we call it half designed analysis right. Because, we have not designed we are right later on we can do the undersign analysis off course that is not a part of your course, but it can be similarly it can be done right; however, we can write m naught 9 divided by m naught in terms of well ratio by sticking a mass balance across the turbo jet engine, which we have already done.

But, let us do that m naught 9 is equal to m dot naught plus m dot f, and if I divide it by this m naught here I will get m naught 9 divided by m naught is equal to 1 plus m dot f. Because, f is nothing, but f by m naught that we know well a ratio we have already known, so and we can write down V 9 by a naught as root over gamma g R g T 9 divided by gamma a R a T 9 into M 9 right. And earlier what we are doing, we are saying this gamma g gamma a is same, but know we won't consider again that you know the real effect is coming to picture.

Similarly R g need not to be same as R a right, then it is coming to picture of course, T nine by T naught this would be T naught right T 9 by T naught will be again we will

have to determine and M_9 will determine. So, that we can get this term, we will be following the same way I mean there is nothing difference except how we are now considering various effects of γ γ_a γ_r γ_c γ_b γ_t γ_n you know all those things. And as efficiency will be coming to pictures pressure losses will be coming to pictures right.

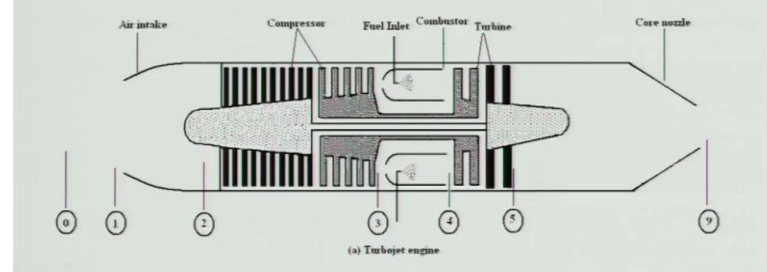
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By using the isentropic relation, we have

$$M_9 = \sqrt{\frac{2}{(\gamma_g - 1)} \left[\left(\frac{P_{t9}}{P_9} \right)^{\frac{\gamma_g - 1}{\gamma_g}} - 1 \right]}$$

$$\frac{P_{t9}}{P_9} = \left(\frac{P_{t9}}{P_{t5}} \right)^{\gamma_n} \left(\frac{P_{t5}}{P_{t4}} \right)^{\gamma_b} \left(\frac{P_{t4}}{P_{t3}} \right)^{\gamma_c} \left(\frac{P_{t3}}{P_{t2}} \right)^{\gamma_d} \left(\frac{P_{t2}}{P_{t1}} \right)^{\gamma_r} \left(\frac{P_{t1}}{P_9} \right)^{\gamma_t} = \pi_n \cdot \pi_b \cdot \pi_c \cdot \pi_d \cdot \pi_r \cdot \pi_t \cdot \frac{P_{t1}}{P_9}$$

From previous analysis, we also know that

$$\frac{T_9}{T_0} = \frac{T_{t9}/T_0}{(T_{t9}/T_9)} = \frac{T_{t9}/T_0}{(P_{t9}/P_9)^{\frac{\gamma_g - 1}{\gamma_g}}}$$


(a) Turbojet engine

So, by using isentropic relation we can get, you know in the similar manner M_9 is root of 2 by γ_g minus 1 P_{t9} by P_9 γ_g minus 1 divided by γ_g minus 1 . This similar from the you know like which we have already done from the isentropic relationship, now what we will do, we will write down this P_{t9} by P_9 right same as that of P_{t9} by P_{t5} into P_{t5} by P_{t4} P_{t4} into P_{t3} into P_{t3} by P_{t2} into P_{t2} by P_{t1} P_{t1} into P_{t1} by P_{t1} and P_{t1} by P_9 .

Earlier what we were saying that P_{t1} is same as P_9 right, we cannot afford to say at this moment it is not fully expanded right. I cannot say P_{t1} is equal to P_9 , in the case of real cycle it can be if it is fully expanded, if it is not it will be different it can be over expanded, it can be under expanded.

So, now, what are these terms if you look at P_{t9} by P_{t5} is nothing, but your this is π_n this portion is π_n nozzle P_{t5} by P_{t4} is nothing, but your π_b turbine and P_{t4} by P_{t3} is nothing, but your π_c burner P_{t3} by P_{t2} is nothing, but your π_d P_{t2} by P_{t1} is nothing, but your π_r and this is π_t right yes or no. You look at diagram which I have shown here, this portion P_{t5} by P_{t9} is your nozzle and 5 to 4 is your turbine right and

4 to 3 is your compressor engine right and 3 to 2 is your compressor and 2 to 0 is your diffuser or air intake right of course, $T_{naught 2}$ total to the P_0 static is nothing, but your p_{ir} .

So, and if you look at like you know these are the things which we need to consider, and we must know what is P_9 by or P_{naught} divided by P_9 value. P_{naught} will be given to you, but whether we know P_9 we really do not know that right that must be given or you will have to evaluate right. How we will evaluate, if it is the converging diverging nozzle it is quite difficult right.

Student: You can find.

You can find how isentropic relation we cannot apply.

Student: ((Refer Time: 20:07))

But, we are not using polytropic efficiency for your nozzle right, so therefore, and you do not know really what will be the because, there are various losses are there, and this P_9 will be decided by the various losses right. And P_{naught} of course, you can find out what is an ambient, so therefore, it is not that easy to find out that right, but; however, if it is a convergent nozzle you can do very easily.

So, from previous analysis we know that T_9 by T_{naught} is nothing, but $T_{t 9}$ divided by T_9 in the numerator denominator will be $T_{t 9}$ by T_9 $T_{t 9}$ by T_9 you get I will write down P_t by p_9 γg minus 1 divided by γg right. And $T_{t 9}$ we need to evaluate what it would be we will be using the similar way, you please keep this diagram with the station in mind, and we will be using the same way what we did for the pressure.

Basically we are expressing $T_{t 9}$ by T_9 in terms of temperature ratios across each components, this is for the total. But, now we are making it each components to carry out a cycle, what we call analysis right that is the reason.

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where $\tau_n = 1, \tau_t = 1, \tau_b = 1, \tau_c = 1, \tau_d = 1, \tau_r = 1$

$$\frac{T_{t9}}{T_0} = \left(\frac{T_{t9}}{T_{t5}}\right) \left(\frac{T_{t5}}{T_{t4}}\right) \left(\frac{T_{t4}}{T_{t3}}\right) \left(\frac{T_{t3}}{T_{t2}}\right) \left(\frac{T_{t2}}{T_{t1}}\right) \left(\frac{T_{t1}}{T_0}\right) = \tau_n \cdot \tau_t \cdot \tau_b \cdot \tau_c \cdot \tau_d \cdot \tau_r = \tau_t \cdot \tau_b \cdot \tau_c \cdot \tau_r \quad (2)$$

Now, let us strike an energy balance across the combustion chamber by invoking the First Law of Thermodynamics for steady flow process as

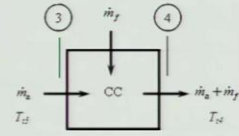
$$\dot{m}_0 C_{p0} T_{t3} + \eta_b \dot{m}_f \Delta H_c = \dot{m}_4 C_{p4} T_{t4} = (\dot{m}_0 + \dot{m}_f) C_{p4} T_{t4}$$

By algebraic manipulation, we can express Eq. (2) as

$$\frac{T_{t3}}{T_0} + \eta_b \left(\frac{\dot{m}_f}{\dot{m}_0}\right) \frac{\Delta H_c}{C_{p0} T_0} = \left(1 + \left(\frac{\dot{m}_f}{\dot{m}_0}\right)\right) \frac{C_{p4}}{C_{p0}} \frac{T_{t4}}{T_0}$$

$$f \frac{\eta_b \Delta H_c}{C_{p0} T_0} - \frac{C_{p4} T_{t4}}{C_{p0} T_0} = \left(\frac{C_{p4} T_{t4}}{C_{p0} T_0}\right) \left(\frac{T_{t3}}{T_{t4}}\right) - \left(\frac{T_{t3}}{T_0}\right) \Rightarrow f = \frac{\tau_\lambda - \tau_r \tau_c}{\frac{\eta_b \Delta H_c}{C_{p0} T_0} - \tau_\lambda}$$

where $\tau_\lambda = \frac{C_{p4} T_{t4}}{C_{p0} T_0}$



So, where T_{t9} by T_{t4} we can write down T_{t9} by T_{t5} into T_{t5} by T_{t4} T_{t4} by T_{t3} T_{t3} by T_{t2} into T_{t2} by T_{t1} T_{t1} by T_{t0} . If you look at what is this thing, this is your tau nozzle, this is your tau turbine, this is your tau b, this is your tau c and this is your tau d, and this is your tau r right. Now, in this case we know that this will be tau d will be 1 right total temperature remain constant in case of diffuse.

And similarly tau n is 1, so if you look at then I can get write down all those things because, in this case this is 1 and this is 1. So, I will get tau t tau b tau c tau r right, and if you want to do further you know you can be written tau n and tau d I can improve this real cycle analysis. But, I must know how to do that how to put a values for that it should be known right, so that part portion of course, is not being used generally, but however it can be done in a very you know if one can think little bit critically and find out where the ((Refer Time: 23:03))

So, now, let us strike an energy balance across the combustion chamber by invoking the first law of thermodynamics for a steady flow practices right. So, if you look at all whatever we are doing is for steady flow processes, but in aircraft need not to be steady except you passenger aircraft in a noble fight it will be may be steady. But, still sometimes the turbulence will be coming into picture some unsteadiness will be there right. So, therefore, it cannot really be the steady, but you can assume it to be positive steady right.

So, now, combustion chamber having a station number 3 and 4 at the inlet and exit respectively. And we know this, you know air is entering, you are adding some heat fuel you know being burnt and then you can get, so you know product. So, if you look at $\dot{m} C_p T_3$ that is the amount of enthalpy entering into the $\dot{m} \Delta H_c$ and that is the heat released due to the combustion. But, all the fuel cannot be converted into the complete product, so therefore, there will be an efficiency this is known as burner efficiency.

We have already defined and it will be less than always 1 is equal to $\dot{m} C_p (T_4 - T_3)$ and \dot{m} is nothing, but \dot{m}_o plus \dot{m}_f and C_p is g. So, what I will do I can divide this thing you know by all the terms $\dot{m} C_p T_3$, and similarly $\dot{m} C_p T_4$ if I do that I will you know manipulate this simplification I think this is not equation I will get T_3 right, if I will this will cancel it out and then this will be nothing, but your $\eta_b \dot{m}_f \Delta H_c$ by $C_p T_3$ and 1 plus \dot{m}_f divided by $\dot{m} C_p T_4$.

And keep n mind this is nothing, but your f this is nothing, but your f what I will do I will get an expression for f basically by little manipulation I will take this f into this side and take this you know or I can take this term to that side anywhere, so that f will be 1 side. So, if I will do that I will get $f \eta_b \Delta H_c C_p T_3$ minus $C_p (T_4 - T_3)$ right and is equal to $C_p (T_4 - T_3)$ minus T_3 right, but how I am doing that because, I am basically dividing this by the what you call T_3 right if I divided by this T_3 here.

Similarly, T_4 here and T_3 here right I can do that. So, I am getting that and instead of T_3 I can write down T_2 you know because, we know T_2 is equal to T_3 right. So, what this term you could recognize you know can you recognize some terms like $C_p (T_4 - T_2)$ by $C_p T_3$ this term that is your τ right. So, similarly this is also problematic, so I can get and what is this one, τ_c this is your τ_r yes or no.

So, now, I can write down you know as I told you $\tau \lambda C_p (T_4 - T_2)$ divided by $C_p T_3$. So, I can get an expression f is equal to $\tau \lambda$ minus $\tau_r \tau_c$ divided by $\eta_b \Delta H_c C_p T_3$ minus $\tau \lambda$, so this expression you know

when you look at you are earlier you know it was not that complex right I am a right, it was simple now lot of terms have come out and that.

But, here you know eta b will be 1 you will find that it is same as that of the ideal cycle, so; that means, if you know the formula for the real cycle you can get very easily ideal cycle that is the cracks of the this analysis.

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Let us strike an energy balance between the compressor and turbine

$$\dot{W}_c = \dot{W}_t \Rightarrow \dot{m}_0 C_{pa} (T_{t3} - T_{t2}) = \eta_m \dot{m}_4 C_{pg} (T_{t4} - T_{t5})$$

where η_m is the mechanical efficiency. By algebraic manipulation, we get the expression for τ_t as

$$\tau_t = 1 - \frac{1}{\eta_m (1 + f)} \frac{\tau_r}{\tau_\lambda} (\tau_c - 1)$$

By knowing τ_t , we can find the total pressure ratio across the turbine π_t by

$$\pi_t = \tau_t^{\gamma_g / (\gamma_g - 1) \eta_{pt}}$$

where η_{pt} is the polytropic efficiency of the turbine. Similarly, the total pressure ratio across the compressor π_c can be determined by

$$\pi_c = \tau_c^{\gamma_a \eta_{pc} / (\gamma_a - 1)}$$

So, you will see as you go along let us strike a energy balance that is for you know that all the work, you know hardness by the turbine is being used by the compressor right that is for the turbo jet engine. So, then we can write down by using the first law thermodynamics that is $\dot{m} C_p (T_{t3} - T_{t2}) = \eta_m \dot{m} C_p (T_{t4} - T_{t5})$ what is this mechanical efficiency, basically this is the because, the work transferred from turbine to the compressor through a shaft.

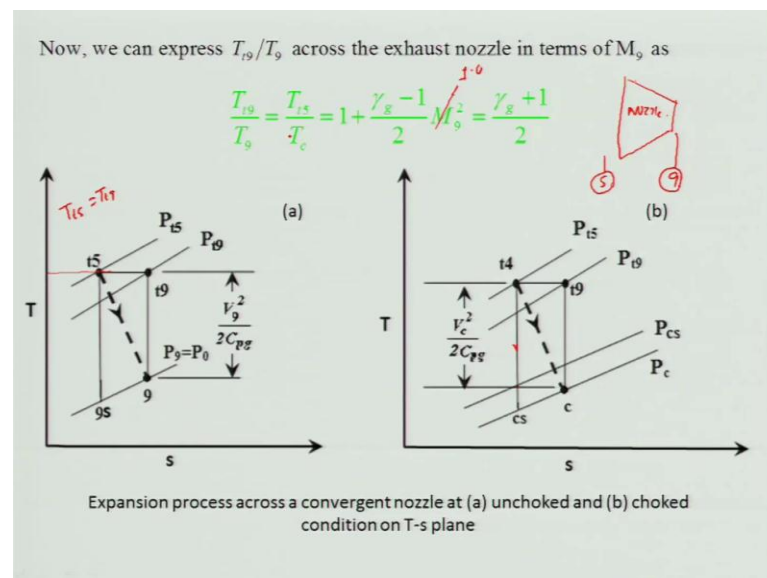
So, all the work cannot be transferred it will be you know less than the what has been transferred can be reached the compressor. So, that is because of sap and friction and other things you know like therefore, it will be always less than 1, so we are using this again. So, by algebraic manipulation we can get you know τ_t is equal to $1 - \frac{1}{\eta_m (1 + f)} \frac{\tau_r}{\tau_\lambda} (\tau_c - 1)$. If you look at for I mean like this small algebra I am just skipping right you can do yourself.

And if you look at this is same as that of ideal cycle, under what condition when f is what will be 0 right and η_m will be 1; that means, these term will become 1 for ideal cycle, then τ_t is same as that you can look at your node you will get that it is same. So, now, you know we need to find out τ_t you know like we have already now we need to find out π_t by knowing the τ_t .

Because, τ_t is known to us if I know τ_c right τ_c how we will get, I should know the pressure ratio across the compressor, if I know the pressure ratio at the compressor I can get τ_c or not, how we are using polytropic efficiency which we have derived in the last lecture right. And if I know this τ_r by the flat Mac number, τ_λ I know, so I know all those things I can get that τ_t I am getting that we can get the π_t because, that is the very important right π_t will be τ_t power to the γ_g divided by γ_g minus 1 η_{pt} this is polytropic efficiency of turbine right of turbine.

So, similarly total pressure ratio across the compressor can be determined you know right if I know the τ_c or τ_c can be determined if I know the π_c by using the polytropic efficiency of the compressor and the γ off course is known to you will be using γ is 1.4 for γ .

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So, now, we can express this because, we have now, come to the turbine now we need to look at the nozzle right nozzle part means, we can express this T_{t9} by t_9 , you know across nozzle in terms of Mac number right, you need to know that right. And how we

will do that, that is the question when the losses there and if you look at for that we can basically look at whether is the conversion nozzle, whether it is choked or unchoked right.

For example, if I consider the what are the processes happening under a unchoked conditions. So, this is my condition is a $P_t 5$ right $P_t 5$ is the my inlet for the nozzle right I can consider and whereas, the $P_t 9$ is the lower than that, what is that total pressure corresponding to the nozzle exit that this p_9 right, this is the static pressure p_9 right.

And here we are assuming you know it is unchoked in a converging nozzle, if you look at we are considering a convergent nozzle only this you can say as a 5 this is 9 right or you can consider this as a 9, but there is a difference in the pressure right this is your nozzle. Now, if you look at in this case $T_t 5$ is equal to $T_t 9$ because, it is a adiabatic process therefore, I put it here you know $T_t 5$ is as the $T_t 9$, but if it is unchoked right then expansion will be there you need not worry about.

Because, it is $P_t 9$ is equal to P_{naught} in the convergent nozzle, you know like there is no over expansion under expansion kind of thing right this all this over expansion under expansion will come when it is convergent divergent nozzle. So, therefore, it is very easy to look at it and you can you know, but if it is a choked one what will happen that if the process is taking place here it is to be chock, this is a choked condition in a non isentropic flow condition.

But, isentropic it will be see as here which will be the same pressure if I go for, but in actual situation it will be see as isentropic critical pressure like that is the V_t square by $2 c g$ right. Now, the main problem will come how we can determine whether the it is choked or non-choked; that means, we need to determine that by considering the pressure ratio across the nozzle is less than equal to the chock pressure ratio or not right, if it is less than then it is unchoked, if it is greater than or equal to then it is choked.

But, all those things we cannot consider here because, it is not isentropic, so the losses will be there, now if losses are there we need to consider the nozzle efficiency how we will do that. So, that we will see now, so and that is the question and once you know that then you will get the basically this is a choked conditions, and then that will be corresponding to the choked temperature right or the this thing condition to the T_c right

if I know T_c I can get V_9 is equal to that root over gamma $R g t c$ right then, I will get V_9 , otherwise V_9 will be different depending upon unchoked condition.

So that is the part we will look at and we choked means what as I told you T_{t9} divided by T_{t5} is equal to T_{t5} by T_c , if I say this is choked condition T_9 is same as the T_c we are saying, then it will Mac number is equal to 1. So, this is gamma g divided by 2 right, now from knowing this you know gamma g , so you know this T_{t5} , so you can get very easily T_c or T_9 . Because, T_{t5} will be known here from whatever you have done the calculation we will be knowing.

But, we won't be knowing whether this you know choked or not, suppose if not choked and you have taking this condition then you will be in trouble.

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we can invoke the definition of η_{in} to derive an expression for temperature ratio T_{cs}/T_{t5} for isentropic process in terms of non-isentropic temperature T_c/T_{t5} as (Since, the expansion process is non-isentropic)

$$\frac{T_{cs}}{T_{t5}} = 1 - \frac{1}{\eta_{in}} \left[1 - \frac{T_c}{T_{t5}} \right] \quad (3)$$

By using Eq. (3) and isentropic relation, we can derive an expression for P_c/P_{t5} as,

$$\frac{P_c}{P_{t5}} = \left(\frac{T_{cs}}{T_{t5}} \right)^{\frac{\gamma_g}{\gamma_g - 1}} = \left[1 - \frac{1}{\eta_{in}} \left(1 - \frac{T_c}{T_{t5}} \right) \right]^{\frac{\gamma_g}{\gamma_g - 1}}$$

By substituting the expression for T_c/T_{t5} in terms of η_{in} , the P_{t5}/P_c can be expressed as

$$\frac{P_{t5}}{P_c} = \frac{1}{\left[1 - \frac{1}{\eta_{in}} \frac{\gamma_g - 1}{\gamma_g + 1} \right]^{\frac{\gamma_g}{\gamma_g - 1}}} \quad (4)$$

So, for that we invoke the definition of the efficiency right to derive, efficiency means isentropic efficiency for the nozzle to derive an expression for temperature ratio T_{cs} by T_{t5} for isentropic process, in terms of non isentropic temperature T_c by T_{t5} as since expansion process this is non isentropic. For example, these T is the isentropic T_{ts} by T_{t5} is equal to you can write down $1 - \eta_{in} (1 - T_c/T_{t5})$, it is basically coming from the definition of what, definition of η_{in} right, it is coming from the definition of η_{in} isentropic efficiency.

So, by equation by using you can say is relation we can derive a P_c by T_{t5} is nothing, but T_{c5} divided by T_{t5} this is an isentropic process. So, we can very easily do that, and γ_g divided by $\gamma_g - 1$ and what we will do, we will put this expression 3 here and get that right. That means, we if I know this right I know this T_c and T_{t5} right and I can find out what will be P_c by P_{t5} right. So, then we can express that P_{t5} by P_c right in terms of you know this expression will come over here.

So, I will get minus 1 over η_i in $\gamma_g - 1$ $\gamma_g + 1$ right, so we can get this expression, if we know this right then because, η_i is known and γ is known. So, you can get this expression and once you know this then you can find out whether it is choked and not choked.

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From Eq. (4), we find that pressure drop across the nozzle increases due to frictional losses in the nozzle. With the help of Eq. (4) and other equations, we can determine the specific thrust as

$$T_s = a_0 \left[(1+f) \frac{V_9}{a_0} - M_0 + (1+f) \frac{R_g T_9 / T_0}{R_a V_9 / V_0} \frac{\left(1 - \frac{P_0}{P_9}\right)}{\gamma_a} \right]$$

It must be noted that equation for specific thrust is a bit complicated as compared to the ideal turbojet cycle analysis. The next step is to find out TSFC from the following

$$TSFC = \frac{f}{T_s}$$

For CD nozzle P_0 must be known.

And from equation 4, we can we find that pressure drop across the nozzle, increases due to the frictional losses in nozzle right this thing, and with the help of equation 4 and other equation we can determine specific thrust. So, if you look at we are basically finding out all this expression V_9 by a_0 M_0 is known $1 + f$ we are derived expression T_9 by T_0 we have already found out how to go about it, and you know we need to find out this or in the case of convergent diverging nozzle, you know for C D nozzle right generally P_0 by P_9 must be known right.

If you know this then you can derive if it is choked one you can do that, so it must be noted that a equation plus specific complicated, as compared to ideal turbo jet engine. Next step of course, is to find out that TSFC I am following that f by T s.

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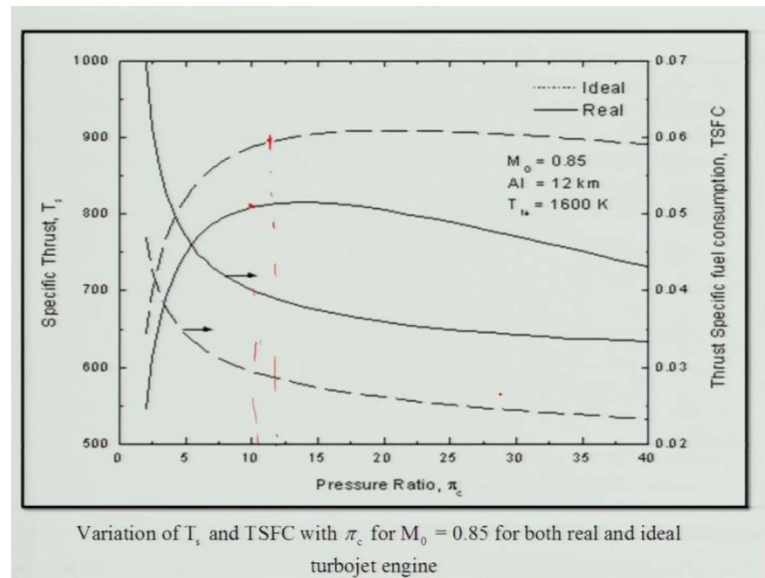
$$\text{Propulsion efficiency, } \eta_p = \frac{2T_s V_0}{a_0^2 \left[(1+f)(V_9/a_0)^2 - M_0^2 \right]}$$

$$\text{Thermal efficiency, } \eta_{th} = \frac{a_0^2 \left[(1+f)(V_9/a_0)^2 - M_0^2 \right]}{2f \Delta H_c}$$

$$\text{Overall efficiency, } \eta_0 = \eta_p \cdot \eta_{th}$$

And is similar here we can find out propulsive efficiency T s V naught and you know all this things we know it is similar to that ideal cycle only thing that 1 plus f is coming to picture right rest of the things are similar. So, you can get the overall efficiency for this expression. Now, what I am thinking now I will taking an example I will be little fast in doing that, but in the some point where we will be looking at critical pressure I will spend more time.

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But, before that let us look at what is happening in the by carrying out is specific thrust right. So, if you look at this we have taken some condition until 12 kilometer M naught 0.85 and T_{t4} 1600 Kelvin right, and this is basically ideal we are using dash line as a ideal 1, and the solid line as d l and we are doing a variation we are looking at various specific thrust pressure ratio, and on the right hand side we are looking at what you call thrust specific fuel consumption right.

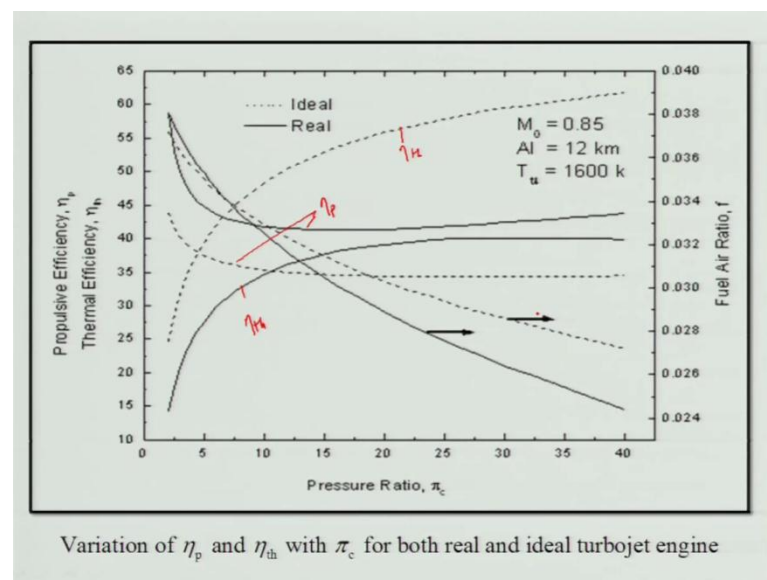
And if you look at this specific thrust you know increases, and reach a peak value you know then it decreases right. And if you look at for this real one the peak values will be somewhere occurring here this is your peak value or the higher and whereas, for ideal one you know it is having similar train. But, peak values will be somewhere, you know at different place kind of things or may be a like ear kind of which is not same as that of this different pressure ratio it is occurring.

And keep in mind that the real cycle at the real one the specific thrust is much lower as compared to the ideal specific. Because, the dash line is a ideal right and because, why there is the losses will be there, and as the result the mass flow rate you know what you need you have the same kind of thrust right will be higher. So, that specific thrust is lower because, the lot of resistance we will be getting right, and combustion efficiency wont be a you know proper it will be less than one and pressure losses will be there.

And similarly if you look at, this the real cycle that TSFC decreases with the increase in the pressure ratio across the compression. And for a ideal one it is having similar strength, but; however, the real you will be having higher values TSFC because, if you look at TSFC in this case soon it is much higher as compared to the ideal cycle right. And which is obvious because, the as I told that combustion you know efficiency will be lower, and you need to burn more fuel to have the same kind of thrust right.

So, therefore, you need to I mean like pay panel, I am like not pay penalty rather it is the reality is that ideal cycle will give you more TSFC as compared to the real cycle. So, if we incorporate more you know losses what we have neglected, then you may get in high TSFC and lower specific thrust.

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So, let us look at the propulsive efficiencies, and thermal efficiencies, if you look at with respect to the pressure ratio compressor for same flight Mac number, and same conditions of altitude 12 kilometer and $T_t = 1600$ Kelvin right. If you look at here this is your dash line is basically ideal one that is your thermal efficiency, this is your thermal efficiency. And which is much higher in case of ideal, but in real cases this is much lower values and whereas, the propulsive efficiencies in case of real one you know it is having higher value as compared to the ideal one right why it is, so.

Because, if you look at the case of the velocity exit velocity in case of you are real cycle will be smaller as compared to the ideal one. So, therefore, you will have higher

propulsive efficiency, and if you look at fuel consumptions you know you will be in case of real one it is having fuel a ratio will be lower. Because, you know in this pressure range, but whereas, in case of a lower pressure ratio it will be having the higher values and of course, it decreases with the pressure ratio across the compressor.

So, and this fuel air ratio because, what happened you need to have a may be large air amount of air which you need to be inducting into that to overcome the resistance. So, therefore, may be the fuel air ratio is been higher alright being lower in case of a real cycle analysis as compared to the ideal one. So, with this stopover and we will take an example in the next lecture.