

Fundamentals of Aerospace Propulsion
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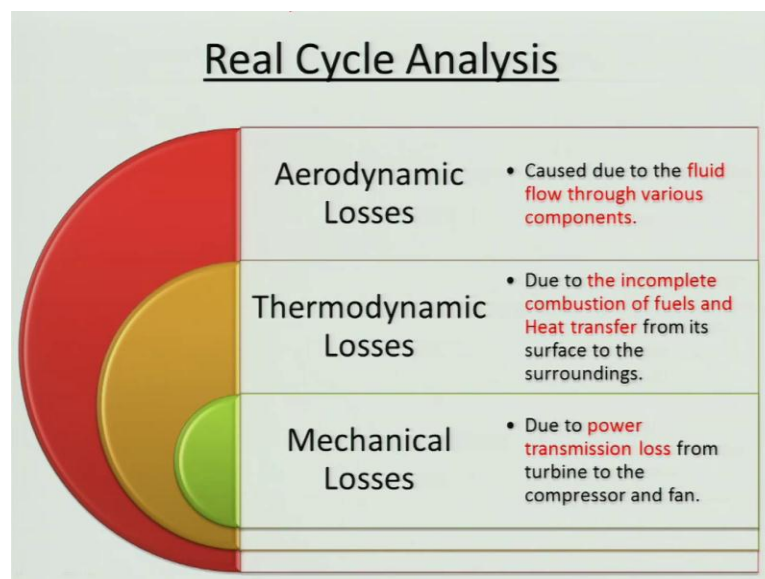
Lecture - 32

Let us start this lecture 32 and let us recall what we learnt in the last few lecture. We basically look that the ideal cycle analysis covering you know various engines namely ramjet engine and turbojet engine, turbofan engine, and turbojet with octagon and the turbo prop engine and we have taken several examples.

To illustrate how we can estimate the various performance parameters and we also had carried out the parametric analysis to understand which fact is placed important role in the you know performance parameter. Keep in mind that, this is all this things are based on the what you call without in a geometry, we are just playing around as a designer to look at what really want to have machine requirement.

Now, we will be discussing about how to use to carry out a real cycle, because all those things we did under several as assumptions. Now, will remove those assumptions and see how we can really carry out, analysis which will be more realistic still we are doing some kind of or will be doing some kind of approximation as well, but that is good enough that is rather better then the ideal one.

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So, what will be doing in this case will be looking that the losses incurred in the ideal in the what you call real and we have not consider any losses in the ideal cycle analysis, however we need to look at this. So if you look at losses all the losses kinds there are various kinds of losses. We can divide them into how many categories, what are those losses is for example, loss is occurring in a air intake it is occurring through the compressor or the turbo machinery, you know turbine that means; there will be some losses due to frictional losses there might be separation there might be mixing.

So, those are losses beside this there will be some transmission, you know and transmission of power from turbine to compression turbine to power pillar and there will be all the power given by the turbine cannot be transferred to the compression. And there will be another kind of losses, which will be basically due to the you transfer and we because the when the flow is taking place it is combustor, so it can.

So, broadly it can be divided into three categories, one is aero dynamic losses, other is thermo dynamic losses and the third one is the mechanical losses. Aero dynamic losses as I told it is basically due to the fluid flow through various components of the engine, which is quite complex in nature particularly, turbo machine and it will be a diverse special guide in compression air intake and the other places.

If you look at your combustion chamber it is quite complex, particularly the main combustor even after one the black bodies, there will be mixing there will be losses. So, those are dynamic losses and thermodynamic losses is due to incomplete combustion of fuels not that all the fuels you cannot really convert into the energy, is not possible very situation. Because, it will be operating over a wide range during its operation and beside this heat transfer is invertible

Although we have assumed there is no heat transfer, it is inevitable to the you know from the component to its surrounding and as I told you the power transmission you know has to be taken place from the turbine to the compressor to the power pillar and then there will be some losses.

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How does real cycle performance differ from that of an ideal cycle?

- ✓ Variation in gas properties throughout the engine owing to both temperature, pressure and composition.
- ✓ Non-isentropic diffusion process in the air intake and compressor.
- ✓ Non-isentropic expansion in the turbine and nozzle.
- ✓ Change in kinetic energy and potential energy between the inlet and outlet can be ignored in all components of gas turbine engine.
- ✓ Bleed air extractions from the air intake and compressor for turbine cooling.
- ✓ Incomplete combustion of fuel.
- ✓ Gas dynamics losses in the combustor due to heat addition.
- ✓ The flow in the exhaust nozzle may not be fully expanded to the ambient pressure. Rather, the flow in the exhaust nozzle can be either over- or under-expanded.

So, all this losses we need to now, consider and let us see how will be working with it and let ask a question how does the real cycle performance differ from that of an ideal cycle what are the those things. If you look at we have assumed in the your all the components you know having the all same properties like air, we say air standard ideal cycle. But, in real situation it is not the air alone from the combustor onwards it will be the several other combustion products, fuels and other things will become number 1. Number 2 temporarily changing from intake to the northerly and of course, big temperature will be occurring in the combustion.

So therefore, this C_p value will be changing, γ will be changing several kinds of things will be occurring, which we are not considered in our ideal cycle. So, as I told you variation in gas properties throughout the engine owing to both temperature, pressure and composition it is not the composition, which is changing, but also the pressure and temperature is changing.

So, and non-isentropic diffusion process in a air intake and compressor similarly non-isentropic expansion in the turbine and nozzle, but in real you know like ideal situation, we have all considered isentropic processes both for the compressor and expansion. So, changing kinetic energy and potential energy between the inlet and outlet can be ignored all the components of gas turbine engine after that will be considering because, we are

interested to look at total pressure and of course, potential energy very important particularly for case of the gas turbine engine.

Because, all will be in the similar label, but when you use the centrifugal compression centrifugal turbine we need to worry about, but those contribution will be very less, but in case of steam turbine and other thing you might be knowing is potential energy, but here in this case we need not worry about it. So, there is another one which of course I would not be discussing, but you cannot that bleed air extractions from the air intake and compressor you know for turbine cooling.

Because, we need to cool the turbine like and it is basically the this air which is causing problem in case of your air intake and compressor, you will just transfer this problem due to what problem due to surging flow separation. And other things due to adverse. In case of, air intake and compressor those which are the problems now, you know you will be using for cooling the turbine it is win-win situation that is the kind engineers they have developed, you know how to overcome the problem at the same times solve another problem.

So, incomplete combustion of fuel gas dynamic losses in the combustor due to heat addition the flow in the exhaust nozzle may not be fully expanded to the ambient pressure. Rather it will be over expanded under expanded depending upon its you know operating if you look at gas dynamics. We have already seen that a rally flow you know that is basically, we have already talked about it how do we take care of it when ever heat addition it will be their there will be losses in the compression total pressure.

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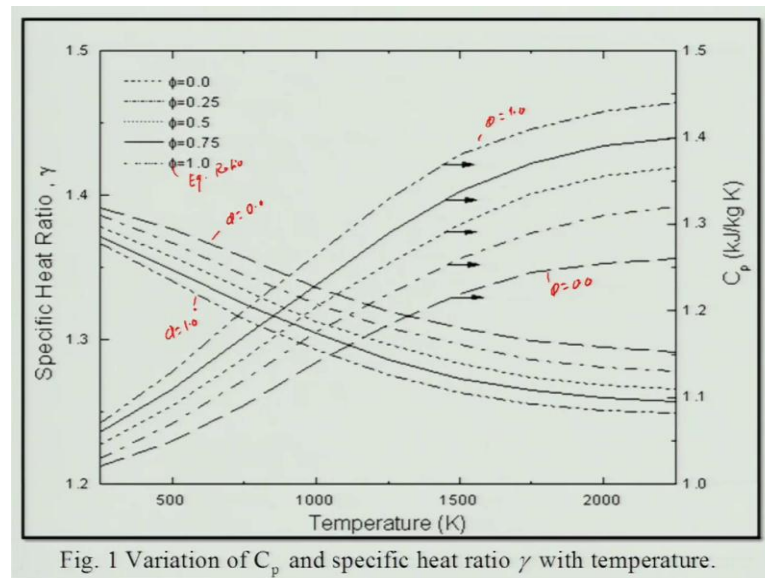


Fig. 1 Variation of C_p and specific heat ratio γ with temperature.

So, let us look at how this C_p will vary with the temperature because, across the engine it changes from temperature 300 Kelvin it can go to even 2000 Kelvin particularly, so if you look at the C_p values you know it will be dependent on ϕ . What is this ϕ ? ϕ is basically equivalence ratio which we have already defined.

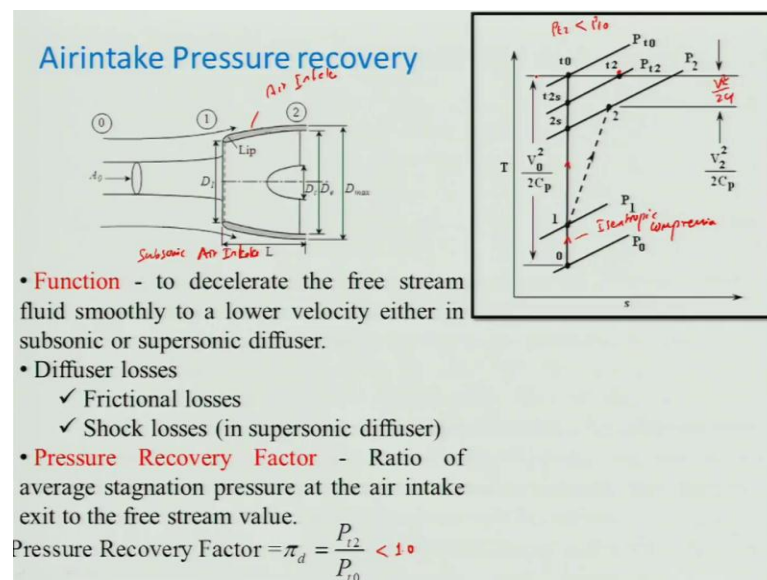
So, if it is a one γ will get a higher C_p values and keep in mind that the C_p have shown these are the C_p values C_p is increasing with respect to temperature that means, it is increasing and as you are having what we call one and this is ϕ is equal to one and as it decreases and it becomes a 0. You know this ϕ is equal to 0 small, but whereas, the specific ratio gamma that is $C_p / \gamma C_v$ like it is decreasing with respect to the temperature for all equivalence ratio and keep in mind that this is basically ϕ is equal to 0 and this is ϕ is equal to one and it is just opposite of that of the C_p .

So that means, way to take care of while analyzing the real cycles because each component there will be changing it, but however, our computer program one can do this thing, but for hand calculation will be not using this because it will be very cumbersome why our air intake that is a change in temperature. Now, I will have to calculate along with the air intake how it is a similarly in the compressor and combustion engine it will be very difficult job through a hand calculation, but in computer program I can have because C_p is a function of temperature.

We can get coefficients and it is also a function of spaces it is depended on spaces for methane for oxygen for some other then you will have to calculate average C_p . So, it is quite comparison therefore, will be using till compressor one gamma that is gamma 1.4 and for the combustor onwards will be using another gamma γ and similarly, C_p u will be using 1.005 kilo joule per k g till compressor exit of compressor and in let of combustor will be using another C_p that means, 2 C_p will be using that means, C_p a C_p γ that is for the simplicity again.

We are doing an approximation here just to make a problem tractable which can be solved in a hand.

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So, let us look at air intake pressure recovery and if you look at this an air intake and these are the lips in which the air will be entering into the air intake, if you look that is the diversion of the streamlined. What it indicates what it indicate there is divergences here you know it is divergences what does it indicates do you continuity balance mass balance you would do what will happen area is increasing if there increasing what will happen you keeping the density remaining the same.

So, velocity will be decreasing that means this is the hydration dissimulation or diffusion you can call and this is what you call external diffusion external compression you can say because, there will be increasing study and of course, however, it is going through the when is entering this is known as internal compression. Because, if you look at air

intake is a compression you know a air getting compressed because, this related the flow is being this related and there might be a situation.

Where it will be other where on will be accelerated in the external portions 0 to 1 will it really occurred when for example, under what condition standing or it is about to take a or it is climbing you know at that kind this will be a this different lines if you draw it will block. So, I am not going to discuss, but you please think about that so, if I look at the processes in t s what is happening this p_0 is your the precious static precious.

This steam static precious and that is a compression and this is compression 0 to 1 will do what isentropic compression am I when I say isentropic can I say are not because, there is no surface and also this compression is the heat losses will be very low. So, adiabatic and reversible because that is no losses in friction and solid surface is not there so, therefore, it will be like this isentropic compressible, but whereas, this process if you look at p_1 to p_0 or if you look at p_0 or p_t this point is equal to the t_2 , so this process is basically the isentropic compression, but however, as a solid surface.

Which is their it will be having a lower pressure over here p_2 that means, p_2 is less than p_0 agreeable decide it is not an isentropic this is not an isentropic process whereas, the t_2 s am saying this is isentropic s for isentropic am using whereas, this t not isentropic. That mean, this gas line you know is not an isentropic so, and the pressure p_2 is a static pressure that means, there will be you know change compression and am indicated I have indicated in the gas line keep in mind this velocity v_2 square by $2 C_p$ this portion v square by $2 C_p$ this portion.

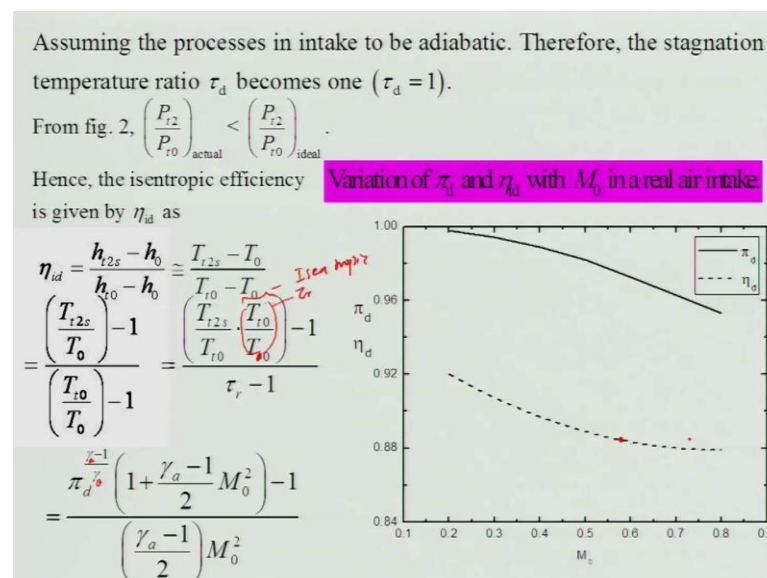
So, what will have to do now, how to handle that is because as I told it is basically air intake to ((Refer Time 15:05)) to a lower velocity either in subsonic or supersonic and this is basically a subsonic, there in the supersonic is there you need to take care of that which am not discussing, but it can be handle in the similar manner I will be talking little electron. So, what are the causes of losses frictional losses and shock losses in supersonic defuses then this is a rare both are interchangeable can be useful.

So, pressure recovery factor is basically we need to defined a pressure recovery factor that is basically the ratio of average stagnation pressure, at the air intake exists to the free steam value by average because in each point you know it may be changing. So,

therefore, I need to take average in various situations here we are considering into be plot to be one m s realistic situation they can never be one m s so, will take a average values.

So therefore, it will be p_{t2} divided by p_{t0} is nothing, but pressure recovery factor π_d always it will be less than one this will be less than one p_{t2} divided by p_{t0} p_{t0} is here if I convert all the portal and the velocity 0. You know $p_0 + \frac{1}{2} \rho V_0^2$ this is this portion total will be p_{t0} so, p_0 is much higher or rather is not much i it is higher than the p_{t0} so, therefore, it is less than one in idea l cycle we have assumed to be one π_d is equal to one, but it is not one is that clear in real situation.

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So, are assuming the processes in air intake to be adiabatic, but however, we are considering the processes to be adiabatic in the why because, the changing the temperature between the ambient and this thing will be a very low. So therefore, we are assuming again in the essence of because it can never, so the p_{t2} as I told you p_{t2} by p_{t0} actual will be less than p_{t2} by p_{t0} and p_{t2} by p_{t0} is basically one in case of ideal it will be less than one hence isotropic efficiency is given by we can define η_{id} is equal to $s_{t2s} - h_0$ that is the divided by $s_{t0} - h_0$ and we can assume again C_p is same during this passes.

Therefore, it is approximate, but it is not really true here C_p will be changing has in our real analysis we assume, that C_p will be remaining constant till the compressor exist then we can manage this with this is that clear. So, if you look at this is basically, what I

am saying this is a $\frac{t_2 s - p_0}{p_2 - p_0} \frac{t_0}{t_2 s - t_0}$ that is the isotropic efficiency where a is air intake where and air intake.

So, if I look at this is basically $\frac{t_2 s}{t_0}$ I can write down again divide this by t_0 and I will get $\frac{t_2 s}{t_0} \div t_0$ that is a little bit problem. Here, write this is not so, these I can write down this is basically not $\frac{t_2 s}{t_0}$ basically $\frac{0}{t_2 s} \div t_0$ in to $\frac{t_0}{t_0}$ by t_0 and we know that $\frac{t_0}{t_0}$ by divide with t_0 is nothing, but one plus γa minus one divided by $2 M_0^2$ is not it isotropic flow because this portion is isotropic.

So therefore, I can write down this is nothing, but $\frac{t_2 s}{t_0}$ this is nothing, but $\frac{\pi_d \gamma a - 1}{\gamma a}$ keep in mind that here it is γa this is γa and enough course if I put this basically this is nothing, but you were τ_r this portion. So, if I what I am doing it is basically π_d is a you know function of flight mug number and also it can be related to the π_d if I know this π_d add values because π_d mug number is very low it is towards going to the one π_d one.

Because, there is no flow it will be one towards one and it will be decreasing with the mug number and if I know this values I can get what will be the η_d which is lower than this. So, you can get a relationship between the isotropic efficiency Air intake with respect to the what to call the pressure ratio.

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The overall pressure ratio of the air intake can be expressed as

$$\pi_d = \pi_{d,max} \cdot \pi_r$$

where $\pi_{d,max}$ = certain portion of ram total pressure due to wall friction
 π_r = certain portion of total pressure due to ram recovery

For subsonic and supersonic flow, we can use an expression for ram recovery π_r as:

$$\pi_r = \begin{cases} 1 & M_0 \leq 1 \\ 1 - 0.075(M_0^2 - 1) & 1 < M_0 < 5 \\ \frac{800}{M_0^4 + 935} & 5 < M_0 \end{cases} \quad \text{Supersonic flow}$$

The isentropic efficiency is very handy for use in cycle analysis. However, It is **not easy to measure total temperature** used in defining isentropic efficiency in comparison to total pressure.

So, the overall pressure ratio of the air intake can be expressed as π_d because we need know this π_d π_d will be maximum into π_r and this is pressure is due to ram wave ramming effective know because particularly supersonic flow the ramming effect will be very higher and this π_r in the subsonic flow as the low subsonic flow will be almost equal to one will see that. So, the π_d maximum certain portion of ram total pressure due to wall frictional because, wall friction will be there.

So therefore, this value will be having certain things which is due to wall friction whereas π certain portion of total pressure due to ram recovery and now, to get this π_r is not that easy what we will be using in empirical relations which of course, was been derived long time back in maybe 60s that till we are using π_r is equal to one in m net be no less than equal to one very low. Subsonic or the subsonic flow it whereas, the m 0.1 to 0.5 will have to use this expression and when m 0 you know is beyond π we will have 2 uses expressions.

Particularly, supersonic very high supersonic flow we have to use and this case is this is basically supersonic. So, isotropic is very handy in you know handy to use in cycle analysis, but however, it is not easy to measure total temperature not the static temperature that easily however, one can measure the total pressure very easily so, therefore, in experiment people use this total pressure and in cycle analysis we always use the isotropic efficiency.

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Hence, the ram efficiency η_{rd} is defined as

$$\eta_{rd} = \frac{P_{t2} - P_0}{P_{t0} - P_0}$$

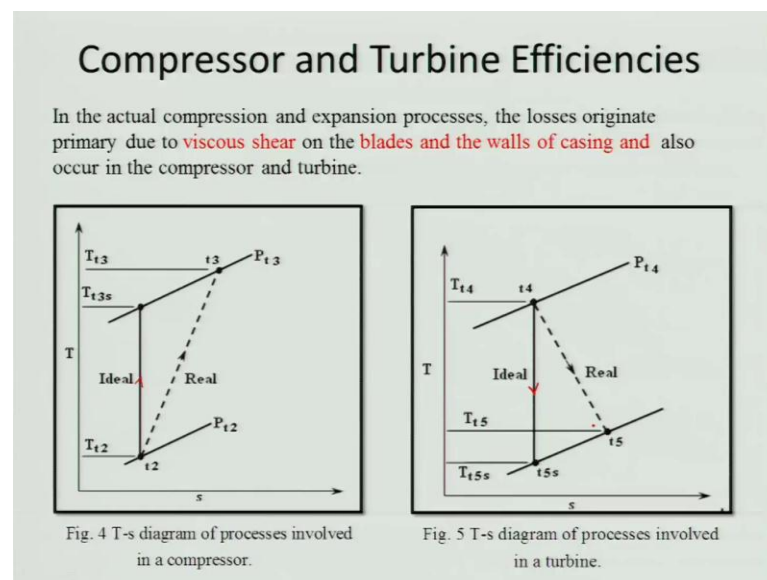
Note:

1. η_{rd} is found to be almost same as η_{id} .
2. It is easier to evaluate experimentally, but it offers no specific advantages over η_{id} .
3. Also, the location of the engine in an aircraft (in the wing, pod or fuselage) dictates the air intake efficiency.

So, that you should, but of course, there is a way of doing it and ram efficiency is defined as $\frac{p_{t2} - p_0}{p_{t0} - p_0}$ keep in mind that this is not η this is η_r . Of course, one can convert into the η and these η_r is found to be almost same as η , but it is not I mean this is approximately same.

So, it is easier to evaluate experimentally, but it offers no specific advantages over η particularly. So, and location of the engine in the aircraft for example, in the wing pod or fuselage dictates the air intake efficiency for example, aircraft you know you were a intake will be in the diffusion, itself and there will be a sometimes we will use therefore, efficiency will be different than.

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So now, we will be looking at out to handle this losses in compressor and turbine efficiencies in actual compression and expansion process losses. Basically or caused, due to viscous shear on the blades and the walls of casings and also the mixing you know where two blades will be there missing at the downstate in the ((Refer Time 24:08)) regions or all those things because of, that the losses will be and how to take care of that is basically we need to look at a passes in $t-s$ diagonal.

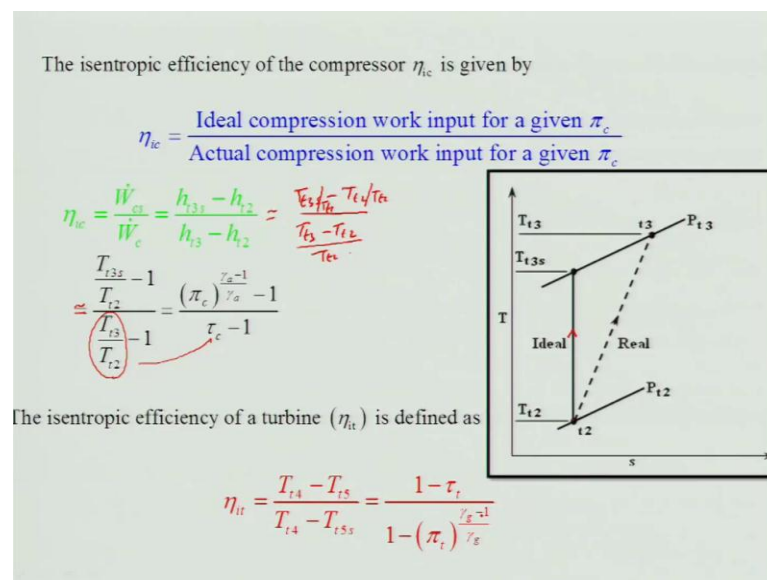
If you look at this is your station t_2 and the pressure corresponding p_{t2} and temperature T_{t2} and if you look at this is the ideal or isotropic process from t_2 to t_{3s} p_{t3s} per isotropic and whereas, the real one which is shown in a dash line will be from P_2

p_{t3} and of course, temperature t_{2t} to t_{p3} what it indicates that means, for the same pressure ratio in a compressor or getting compress are getting then you need to provide more amount of work.

In case of real situation than that of the ideal that is the meaning because, lot of things will be lost and which you need to provide to get the same pressure, that is very important point you should keep in mind and similarly, will let us look at for you know the expansion passes in a turbine that is station from t_{42} to t_5 . If you look at this is the isotropic process the vertical striped line and the dashed line is for your real, what does it indicates what is it indicating it indicates that is the more amount of work will be reduce foreign ideal case for the same expansion ratio.

In other words less amount of work will be producing real situation as, compared to the ideal one for the same pressure ratio of expansion compression it is other way around that means, you need to give more amount of work in the situation to produce the same pressure ratio. So, we will be looking at how to handle is that and we can define very easily with the using the similar what we did for air intake.

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The isotropic efficiency of the compressor can be given as ideal compression work input for a given p_i divided by actual compressor work input for a given p_i . So, and what is that meaning is that like what is the ideal work is basically, $h_{t3s} - h_{t2}$ this portion divided by the amount of work for ideal will be $h_{t3s} - h_{t2}$ this portion divided by the amount of

So, you know approximately is that so, I am then I will do the I can write down here t^3 minus t^2 divided by if I divided by this t one t^2 and here also t^2 . So, this is t^2 i will get three s divided by t^2 minus one and t^3 divided by t^2 minus one if you look at this is nothing, but you your tau and this is for isotropic process because, we are talking about here the isotropic.

So, question arises we need to whether use this isotropic efficiency all we will go for something because, isotropic efficiency will be you know changing in a compression because, if you look at in modern day the compressor pressure ratio of forty people are contemplating to viewers. That means, there are several number of pages and whether these isotropic efficiency can assume to be a constant for such a high pressure ratio and you will be surprised to know that it is not that to views the constant isotropic efficiency across the all stages of a modern compression particularly more number of stages.

Polytropic Efficiency

The diagram is a Temperature-Entropy (T-s) plot for a compressor. The vertical axis is Temperature (T) and the horizontal axis is Entropy (s). A dashed vertical line represents the isentropic compression path from state 2 to state 3s. A solid curve represents the actual compression path from state 2 to state 3. The area under the isentropic path is labeled P_t (ideal work). The area under the actual path is labeled $P_t + dP_t$ (actual work). The difference between these areas is the heat input dQ_t . The temperatures at states 2, 3s, and 3 are labeled T_{t2} , T_{t3s} , and T_{t3} respectively. The pressure at state 3 is labeled $P_{t3} = P_{t4}$. The pressure at state 2 is labeled P_{t2} . The entropy at state 2 is labeled s . The diagram illustrates that the actual work input is greater than the ideal work input due to the heat input dQ_t during the compression process.

Polytropic efficiency of a compressor : Ratio of ideal to actual work for an infinitesimal step during the compression process.

Fig. 6 T-s diagram (polytropic process) for a compressor

So therefore, we need to define poly-tropic processes, poly-tropic efficiency is basically what you call a ratio of ideal to actual work for an finite decimal step during the compression processes. That means, if I am going let us say you know from prey shift p t 2 p t three and then there will be number of then the number of stages which will be there and what I am saying this is a you know in your very in finite decimal let us say this is p t and this is p t plus d p t when d p t is tending towards 0 are very small in in finite decimal is small d p t.

Then we need to look at and then will define this isotropic efficiency and connect to you know what to call this poly-tropic efficiency to isotropic efficiency across the entire compression consisting of number of stages.

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Polytropic efficiency of a compressor - ratio of ideal to actual work for an infinitesimal step during the compression process.

$$\eta_{pc} = \frac{\text{Ideal compression work input for an infinitesimal } dP_t}{\text{Actual compression work input for an infinitesimal } dP_t}$$

Consider an incremental pressure rise P_t to $P_t + dP_t$ as shown in the T-s diagram (Fig. 6). Let the actual temperature rise accompanying the increase in pressure be denoted as dT_t and the ideal temperature as dT_{ts} . Then, η_{pc} becomes

$$\eta_{pc} = \frac{dT_{ts}}{dT_t} = \frac{dT_{ts}/T_t}{dT_t/T_t} \quad (1)$$

For an infinitesimal isentropic pressure change, the temperature and pressure ratio can be related as

$$1 + \frac{dT_{ts}}{T_t} = \left(\frac{dP_t}{P_t} \right)^{\frac{\gamma_a - 1}{\gamma_a}} \quad \frac{T_t + dT_{ts}}{T_t} = \left(\frac{P_t + dP_t}{P_t} \right)^{\frac{\gamma_a - 1}{\gamma_a}} \quad (2)$$

So, let us do that is let us poly-tropic efficiency that is ideal compression work input for an in finite decimal spatial change that is d p t d p t is very small divided by actual compression work input for an in finite decimal special like changes keep in mind we are doing, so compression and you will be I mean I know looking at also for the turbine. So, considering incremental in a pressure rise p t plus d p t as shown in t s diagram you can see that that is basically if you look at is efficiency will be you know p t d t s divided by d t will be nothing, but your poly-tropic efficiency.

Because, this is the isotropic this is for your isotropic and this is for your ((Refer Time 31:39)). So therefore, eta p C will be d t s divided by d t I can write down d t s divided by

$\frac{dT_t}{T_t}$ divided by $\frac{dT_{ts}}{T_t}$ I can write down there is very easily and for in finite decimal isotropic pressure change in the temperature and pressure ratio can be related for example, I can write down this as $\frac{dT_{ts}}{T_t}$ I can write down $\frac{dT_{ts}}{T_t}$ by $\frac{dT_t}{T_t}$ is equal to $\frac{dP_t}{P_t}$ by $\gamma_a - 1$ because this is isotropic.

So, I can write down the $\frac{dT_{ts}}{T_t}$ so, I can write on one plus this one plus this will be one plus that and then I can write down these and what I do now with this I will be using the binomial expansion because this is the real small quantities.

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$$\frac{T_t + dT_{ts}}{T_t} = \left(\frac{P_t + dP_t}{P_t} \right)^{\frac{\gamma_a - 1}{\gamma_a}} \Rightarrow \frac{dT_{ts}}{T_t} = \left[\left(\frac{P_t + dP_t}{P_t} \right)^{\frac{\gamma_a - 1}{\gamma_a}} - 1 \right] = \frac{\gamma_a - 1}{\gamma_a} \frac{dP_t}{P_t} \quad (3)$$

Expanding $\left(1 + \frac{dP_t}{P_t} \right)^{\frac{\gamma_a - 1}{\gamma_a}}$ by Binomial series and neglecting higher order terms, we have

$$\left(1 + \frac{dP_t}{P_t} \right)^{\frac{\gamma_a - 1}{\gamma_a}} = 1 + \frac{\gamma_a - 1}{\gamma_a} \frac{dP_t}{P_t} \quad (4)$$

Using Eqs. (1) and (4), we can rewrite the expression for η_{pc} as

$$\eta_{pc} = \frac{dT_{ts}}{dT_t} = \frac{dT_{ts}/T_t}{dT_t/T_t} \quad \eta_{pc} = \left(\frac{\gamma_a - 1}{\gamma_a} \right) \frac{dP_t/P_t}{dT_t/T_t} \Rightarrow \frac{dT_t}{T_t} = \left(\frac{\gamma_a - 1}{\gamma_a} \right) \frac{1}{\eta_{pc}} \frac{dP_t}{P_t}$$

By integrating the above equation between stations (2) and (3), we can arrive at an overall compressor pressure ratio across the compressor. Thus, we have

$$\frac{T_{t3}}{T_{t2}} = \left(\frac{P_{t3}}{P_{t2}} \right)^{\frac{\gamma_a - 1}{\gamma_a \eta_{pc}}} = \pi_c^{\frac{\gamma_a - 1}{\gamma_a \eta_{pc}}}$$

You know I can rewrite this as $\frac{dT_{ts}}{dT_t}$ by $\frac{dT_t}{T_t}$ plus $\frac{dT_{ts}}{dT_t}$ divided by $\frac{dT_t}{T_t}$ power to the $\gamma_a - 1$ and if you look at these expansion I can you know I can expand in a binomial in the series and neglecting the higher terms I will get one plus $\frac{dP_t}{P_t}$ divided by $\frac{dT_t}{T_t}$ is nothing, but one plus $\gamma_a - 1$ divided by γ_a $\frac{dP_t}{P_t}$. So, if I substitute these values here what I will get because in place of these you know I am substituting here what I will get I will get $\gamma_a - 1$ γ_a $\frac{dP_t}{P_t}$ by $\frac{dT_t}{T_t}$ isn't one cancel it out that will be.

So, using this equation because this equation 4 and equation one this is basically equation one if you look at so, what I will do I will get $\frac{dT_{ts}}{dT_t}$ by divided by $\frac{dT_t}{T_t}$ is nothing, but $\gamma_a - 1$ divided by γ_a $\frac{dP_t}{P_t}$ divided by $\frac{dT_t}{T_t}$ and $\frac{dP_t}{P_t}$ by $\frac{dT_t}{T_t}$. So, if I just a you know work it out I will get $\frac{dT_t}{T_t}$ by $\frac{dT_t}{T_t}$ is equal to $\gamma_a - 1$ divided by γ_a one over η_{pc} it η_{pc} plus is basically qualitative instances $\frac{dP_t}{P_t}$ by now if I you to

get you know this expression I can get is equal to basically $d t_3$ by $d t_2$ that is more the entire range.

Because, I have taken a very small thing and then I am integrating over the station t_3 to t_2 I will get p_3/p_2 power to the $\gamma - 1$ divided by γ a η_c plus c is equal to if you look at this is nothing, but your π_c . So, I can write down $\pi_c^{\gamma - 1}$ divided by γ a η_c plus one so, that means, this power you'll be get to the π_c by this using this one I can really go back and relate this poly-tropic efficiency into isotropic efficiency.

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Then, the overall isentropic efficiency of a multistage compressor can be related to η_{pc} as

$$\eta_{ic} = \frac{\frac{T_{t3}}{T_{t2}} - 1}{\frac{T_{t3}}{T_{t2}} - 1} = \frac{\pi_c^{\frac{\gamma-1}{\gamma \eta_c}} - 1}{\pi_c^{\frac{\gamma-1}{\gamma \eta_{pc}}} - 1}$$

Note: The differences between η_{ic} and η_{pc} increase with the number of stages and with increase in π_c because increase in temperature caused by friction in one stage causes more compressor work in the subsequent stages. This process is also known as *preheating effect*.

Similarly, we can derive an expression relating η_{it} and η_{pt} for a turbine as

$$\eta_{it} = \frac{1 - \pi_t^{(\gamma_g - 1)\eta_{pt}/\gamma_g}}{1 - \pi_t^{(\gamma_g - 1)/\gamma_g}}$$

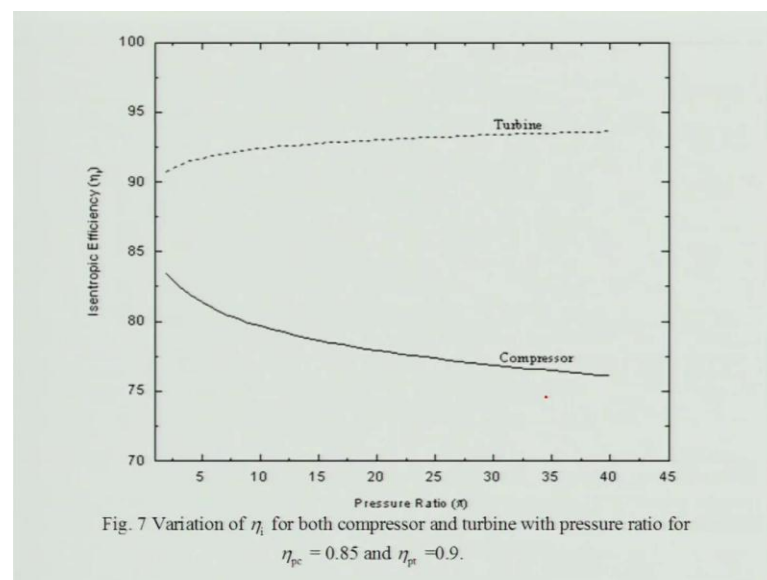
So, if you look at I will put this isotropic efficiency by definition it is the s divided by the $d t$ is minus one and this is nothing, but $\pi_c^{\gamma - 1}$ divided by γ . A just for isotropic process because, this is the isotropic process isotropic and whereas, t_3 is the by t_2 π is nothing, but your you know similar to isotropic, but there is a compression poly-tropic efficiency and it minus one.

So now, I can get this isotropic efficiency be related to the poly-tropic efficiency for a certain range of pressure you will be doing that in the following. The similar, ways you know you can also did I were the turbine, but keep in mind that the difference between these η_{ic} isotropic efficiency and poly-tropic efficiency compression increases with the number of stages and also with increase in π equation ratio of compression because, why it will.

So, and as the pressure ratio increases there will be increasing temperature caused by, the friction and this increasing temperatures will be because, more compression work in the subsequent stages and this process you know as polytropic compression that is the reason. Why inter cooling these the use to reduce the work input to the compression at the same pressure ratio that is why told you know like you know will be using some kind of a alcohol or a water injection in a to enhance the of the compression.

So, because the free heating effect as to be non-defined we know very well I keep the temperature is higher. So, to get the higher at the same pressure ratio you need to provide the more work did I the expression for η_i in terms of η_p and pressure ratio across the parallel.

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So, if I take this you know two values I am taking one is η_{pc} that is the poly-tropic efficiency of compression, point 8 pi and poly-tropic efficiency for turbine point 9 and varying the pressure ratio. You can see that this isotropic efficiency goes on increase in case of, a turbine and in the case of compression it is just opposite in the picture because, of free heating effect there it will be pulling just opposite to the end .

So, now, we need to look at the combustor efficiency and pressure loss keep in mind that you will be use in those whatever for analysis, we have carried out to take care of the losses in components various formula airming taking compression in turbine. It will be using near circumstances and compression combustor efficiency and pressure losses.

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Combustor Efficiency and Pressure Loss

The burning processes of fuel in a real gas turbine engine can be modeled by considering two distinct due to:

- **Inability of combustor to convert entire chemical energy of fuel**, which arises due to imperfect combustion of fuel whose energy can remain as partially combusted carbon in terms of CO, unburnt hydrocarbon (HC) or soot particles.
- **Stagnation pressure loss** across the entire combustor.

Combustor efficiency is defined as the ratio of actual change in enthalpy due to combustion and total energy due to calorific value of the fuel.

$$\eta_b = \frac{(\dot{m}_a + \dot{m}_f)h_{t4} - \dot{m}_a h_{t3}}{\dot{m}_f \Delta H_c}$$

$\pi_b < 1.0$
 $\pi_b = \frac{P_{t4}}{P_{t3}}$

where $\dot{m}_f$ = mass flow rate of fuel
 $\dot{m}_a$ = mass flow rate of air
 ΔH_c = heating value of the fuel (for ATF, $\Delta H_c = 43,000 \text{ kJ/kg}$)

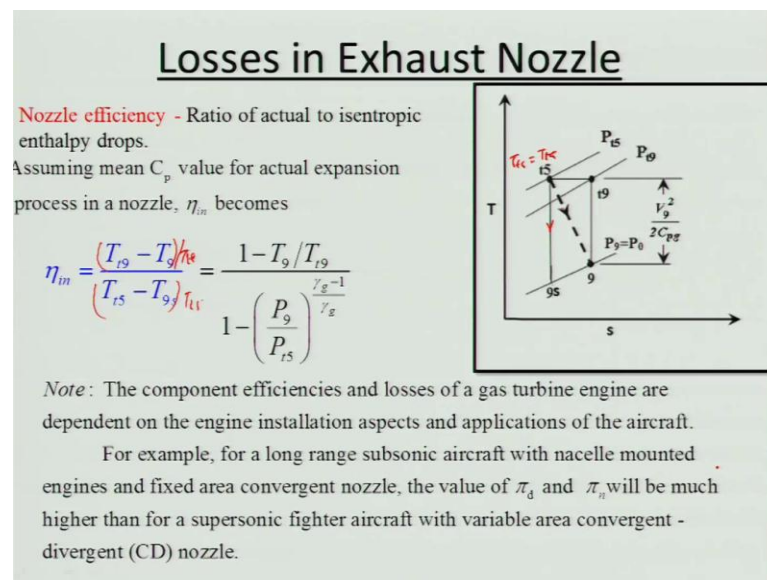
If you look at the this efficiency of what you call combustor will be effected will be lower than one. Because, it is not possible to convert all the chemical energy of a fuel due to the imperfection of imperfect combustion of fuel and air and there might be carbon monoxide and then hydrocarbon soot particles and other things.

Beside this there will be stagnation pressure losses across the entire combustor because, of heat additions and also there will be mixing the flow ease by complex. So, therefore, it will be losses that means, you know we need to take case of that that means, will be taking care of combustion efficiency what will be designing that is as a ratio of actual change in enthalpy due to combustion and total energy release, due to the calorie the value of the ideal you know you cannot get all the calorie value you know you convert all the calorific value you know to be converted into the thermal energy.

So, if I considered this combustion chamber yes within compression three and combustion 4 and certain amount of, mass is here its entering at temperature p three and leaving temperatures t t 4. So, i can get a tie efficiency nothing, but $m \dot{a} + m \dot{f}$ s p q 4 minus $m \dot{a}$ s p divided by $m \dot{f} \Delta h_c$ Δh_c is the heat of combustion or calorific value generally, for a t f be considered what is the thousand energy per k g keep in mind that there is π_b will be less than one always and what is this π_b π_b is a quality.

Basically, p_{t2} divided by p_{t1} and in ideal cycle we have consider p_{t2} as one, but here it will be 0.9 p_{t2} as flow rate possible. Due to go on designing 2 being to the are take this pressure ratio to be closer to the close to 1 means there will more happy I mean approach there will not achieve till there, but it is a you know good, I mean their progress well to.

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So, losses and exist nozzle a nozzle efficiency ratio of actual to the isotropic drops that these, will be looking a nozzle efficiency and I again your assuming means C_p value per actual expansion process in a nozzle. Than the if you look at the processes in the p l diagram is basically some stations 5 to the 9 8 there is several other things as they because, if you look at p t 5 is the exit of your wearing we will be there then it will be p t 7 and sorry if the is there will be 5 to 7 is there will be 7 8 9.

So, it is a we can say either it is p 7 or 5 both the same, but I am considering p t 5 than here actual for expansion will be for p t 5 59 whereas, the isentropic expansion will be p t 5 to t9 s 39s, but the pressure remains p 9 to p 0. But your consideration, but when I will talk about the what you call turbo jet engine then will be talking about choking commission whether it is choked.

I am not let is defined this nozzle efficiency, but is and keep in mind that this is the same this is p t 5 is equal to p t minus. Because, idea biotic we are considering therefore, p p 9 and minus p p 9 divided by t t 5 minus p, p plus and if I divide these by the you know p 9 and divided by t t 5 is basically same so, you can get these things and this is a ratio you

know and you will be getting all those values and you can get exposed this τ_9 as by τ_5 in terms of pressure ratio p_9 by τ_5 hour to be done.

The component efficiencies and losses of a gas turbine engine or dependent on the engine installation aspects and applications of the aircraft. Because, the different installation of the engine will be having different efficiency for example, in high temperatures are reinstalling the engine in the are just into the fuel line part of the whereas, engine other place is like a it will be hanging from the wing naturally it looks in some cases you may put in the on the backside of the fused less.

So therefore, but call it will be there having different inefficiency is and factors example for, a long range subsonic characters is nicely mounted engines and extra area of conversion nozzle value of p_d and p_n is much higher than the forest supersonic fighter aircraft with variable area convergent divergent c_d nozzle always. So, the fighter aircraft use a variable area convergent nozzle now people d_c will convergent variable area convergent so, with these I will stop out we will in the next class discuss about legal turbo jet engines carry out a recycling analysis of turbo.