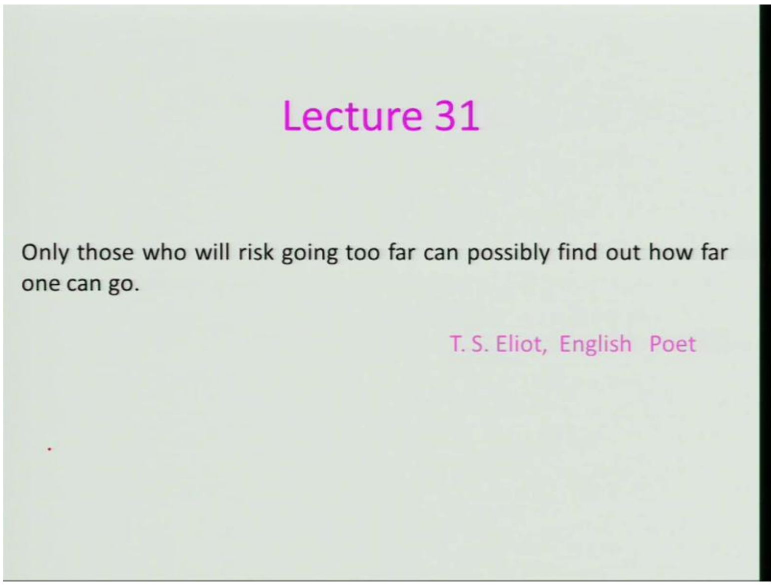
Fundamentals of Aerospace Propulsion
Prof. D.P Mishra
Department of Aerospace Engineering
Indian Institute of Technology, Kanpur

Lecture - 31

We will start this lecture with a thought process from T. S. Eliot, English poet.

(Refer Slide Time: 00:23)



Who says that, only those who will risk going too far can possibly find out, how far one can go. And let us recall as usual, what we had learned in the last lecture, we are basically looking at the ideal cycle analysis. We have already cover the turbo jet engines and the last lecture, we discuss about the turbo fan engines. We derive the relationship for the specific thrust, thrust specific fuel consumption and various efficiencies and then, we carried out a parametric analysis to look at, really how this performance parameter behave with the various parameters.

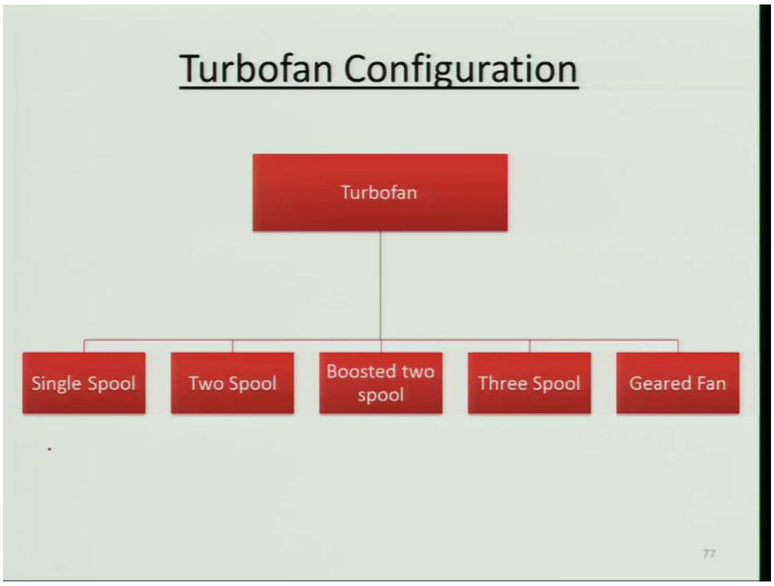
What are those parameter we looked at, those parameters are pressure ratio across the compressor, pressure ratio across the fan, by pass ratio and of course, the altitude and other things one can think of. So, if you look at it, you will find that, by pass ratio and fan pressure ratio plays a very important role for affecting the overall performance. In moderns days, you might be knowing that, turbo fan engines are being used for a large air craft and whenever you talk about large aircraft, we need to provide the higher thrust.

And on the other hand, we will have to reduce the fuel consumption or lower TFSC and we have looked at the conventional turbo fan engine, where there are two stream. One is fan stream and other is, what you call the course stream and overall pressure ratio is goes on increasing to enhance the thrust. But, whenever it increases then, what happens, it will be having a problem with the instability in the combustion chamber. Because, the pressure ratio across the compressor increases, as it is increases then, there will be problem of surging.

Generally of course, some of you might be think your pressure surging can occur in the low pressure, it is generally not lightly to occur. But, however of course, when you will study about this problem of surging in a compressor particularly then, you will appreciate that, it will be occurring at higher pressure. Whenever the pressure ratio occurs the compressor increases beyond certain limit then, it will subject to surging, if surging will occurs then, it is a problem.

Now, how to overcome that problem of surging, which will cause the stability in the whole engine is a very very important aspect. And also you want to make it more compact, otherwise your efficiency, your performance parameter will be affected by the effect of surging or by the unset of surging. So therefore, various configuration have been developed over the years, I will show you some of them.

(Refer Slide Time: 03:58)

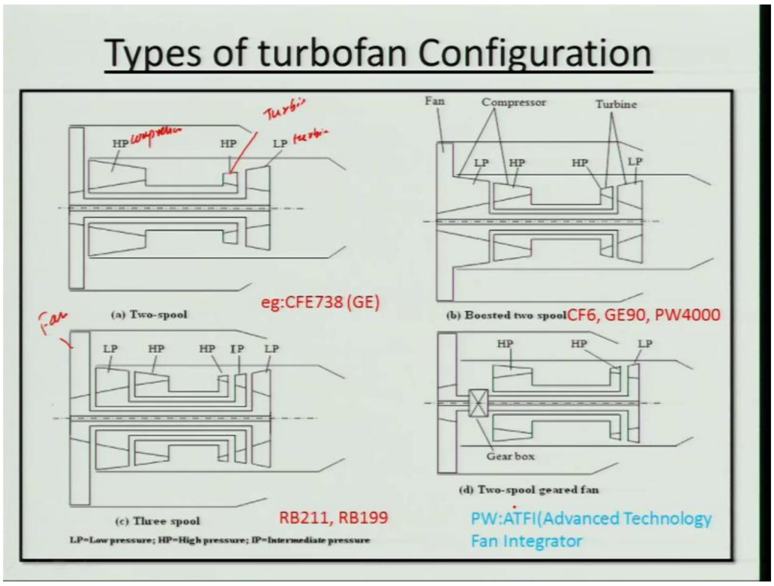


And the important one is, what we call, configuration is the single spool engine, single spool means what, is basically single shaft. The shaft which will be connected to both the low pressure compressor high pressure turbine and all this thing, low pressure compressor, high pressure compressor then, high pressure turbine and low pressure turbine. And two spool engine, where you are having two shaft, which will take care of surging and boosted two spool engines and 3 spool engines.

Of course, there is a another varieties, which have come up is the geared fan, there is the another configuration which have also come up, it is not from the problem, not from the prospective of avoiding the surging. But, however to compactness, mixed fan engines, what we have done the separate fan, that means mix flow engines like a fan flow and the core flow is separately be expanded in two different nozzle.

But, there is a turbo fan engine, where you can mixed together and then, expanded in nozzle such that, the noise level can be reduced. Of course, you can get the thrust and the those things I am not going to discuss.

(Refer Slide Time: 05:36)



What I am now concentrating on discussing about the engines, if you look at the single spool engines, basically it is the very simple one and it is being use in a like a fighter aircraft and other places. But, however some of the modern fighter air craft also uses two spool engines, but if you look at the sigma and other series, they are having let us discuss about two spool kind of engine. If you look at here, the high pressure turbine, this is basically a turbine, high pressure, HP stand for high pressure, it will be running the high pressure compressor and the low pressure turbine will be connected to the fan.

Because, the fan cannot operate at the higher rpm why, because the team particularly will be have a high speed and it may exceed the speed of the sound. If it will exceed then, the shock will formed and these are not designed for that. Therefore, the losses will be very very high, is all subsonic turbo machinery what being used as of now. In future, may be transonic or supersonic will be used I mean, may be used, may be some of you will be involving in designing that, which is the very very challenging job.

So, but this kind of engines is being used and particularly I have taken an example this is GE engine CFE738 and which is quite compact and is being used. But, it is having limitation, because the rpm will be, what you call deciding that, how much air will be entering into and also the number of stages what one can have particularly in turbine. And so, also in the compressor, if the rpm is the very very constant or may be very high, then, actual you cannot go for more number of stages.

Because, the pressure will be and you cannot go for a high pressure then, naturally you will have to go for a more number of stages then, compressor will be not operating properly. Particularly, last stages of the compressor will subjected to the surging due to the adverse pressure gradient, which will be... So, in order to overcome this problem, people thought about using the boosted two spool, because there is limitation of the pressure ratio, what one can achieved in a compressor in case of two spool engine.

But whereas, my requirement is higher pressure ratio without surging problem, so naturally I will have to go for boosted like kind of thing, where they have thought idea is that, why not have a compressor, which is attach to fan, which will be rotating. This is the shaft, the low pressure turbine is connected to that and you can get some kind of a pressure, more pressure ratio as compare to two spool engine. Of course, they are successful, but go back beyond certain limit, you cannot really avoid the surging problem.

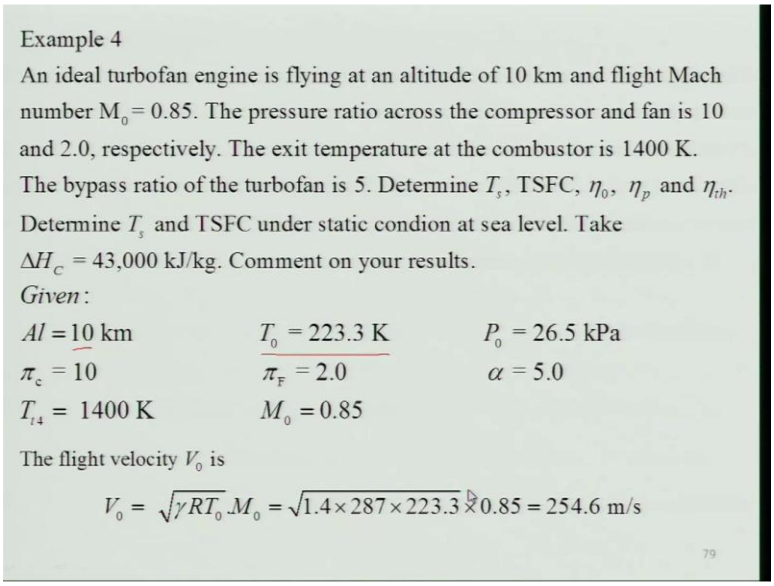
So therefore, there is a need to go for a 3 spool engines and there are several engines, people have designs quite popular one in few years back, but today the 3 spool engine is being used very rampantly like CF6GE9, GE is basically General Electric and CF also general, in earlier days they use to have, but today they are more systematized the nomenclature of the engine and PW is Pratt and Whitney 4000, which is being used in your big air craft kind of engines.

So, 3 spool engines, where you can have high pressure turbine is running, the high pressure compressor that is, intermediate pressure. Intermediate pressure turbine will be running, the basically low pressure compressor and the low pressure turbine will be running the fan, this is basically a fan. And here, it is you can have a more range and you can go for a higher pressure without really encountering the problem of surging.

So therefore, this is being used and of course, you can get varieties of engine like Rolls Royce like they use this thing 211 and 199 engine, there can be several engine, I have taken few of them example. But, there is a another engine, which is a quite, what we called being promoted by the pattern widely, two spool geared engines, instead of using a what you call to connect this low pressure turbine with a fan they are using gear box. But, keep in mind that, design of a gear box is not that easy, because the power level will be quite high.

And we will see, this kind of problem particularly in turboprop engine, but here the power from the low pressure turbine, which will be transport to fan will be relatively low as compared to the turboprop engines. So therefore, they are designing this gear box, which is quite little bit noise, like it makes a lot of mechanical stop. So, you can release, control this speed and you can manage to have a higher efficient fan over a whole range of operation. So therefore, they are already using this advanced fan and advanced technology fan integrator engines, but it is not being very much popular as of now, even today.

(Refer Slide Time: 11:50)

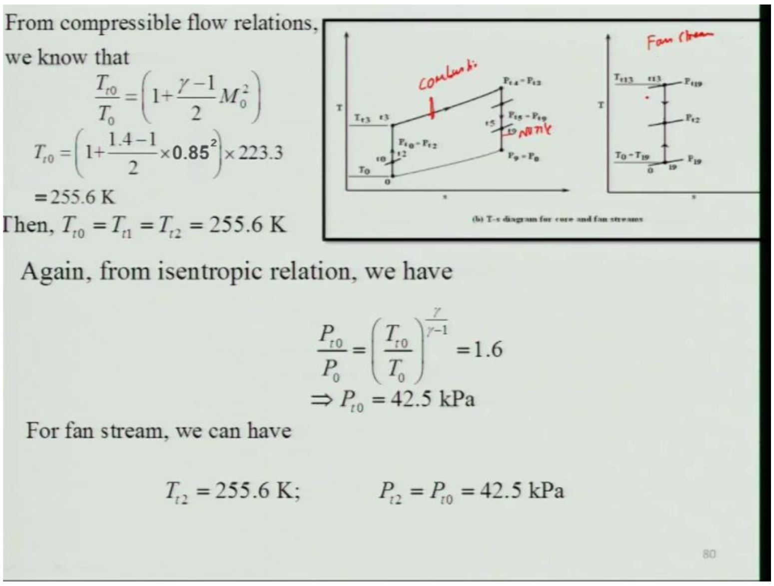


So now, what will we do, we will take of an example of, how to solve a problem like an ideal turbo fan engine is flying at an altitude of 10 kilometer, flight mach number of 0.85, the pressure ratio across the compressor and fan is 10 and 2 respectively. If you look at the total compression ratio in the core stream will be 20, 10 into the fan pressure ratio is the 20, keep it that in mind.

Bypass ratio of turbo fan is 5 in this example, will have to determine the specific thrust, thrust specific fuel consumption, propel fuel efficiency, thermal efficiency, overall efficiency. And we can determine the static thrust and TFSC under the static condition to the sea level, so these are the datas are given as I discuss. So, first we will have to find out, what is the flight velocity, we can get that very easily, because we know this altitude, from this altitude we will get this temperature.

And then, V naught is equal to root over gamma R T naught into multiplied by the flight mach number. When we substituted these values, you will get 254.6 meter per second and as an emphasizing that, whenever you want to solve a problem, it is better to solve in a systematic way, not going into the formulas. But, whenever you are going to do a parametric studies, it is advisable to use those formulas. So that, you can write a program and do it very easily, repetition jobs, you need not to waste your time. But, to appreciate the, how whatever these things, it is important to solve this problem.

(Refer Slide Time: 13:36)



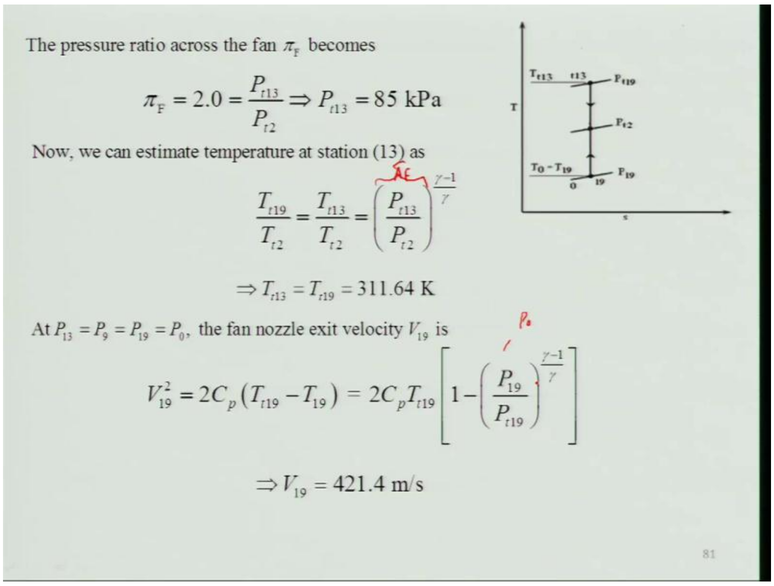
So, what are the happening, I have already talk about that thing in the T s diagram, so this is basically compression, 0 to t 2 is your fan and t 2 to t 3 is your compressor and t 3 to t 4 is basically combustion, this is the combustion and expansion is 4 to 9. In this case, it is a turbine, low pressure turbine, high pressure turbine and then, the nozzle, this is a nozzle portion. And similarly, in the fan stream, this is your fan stream like it is going from P naught to the P t 2 and then, P t 2 by P t 3, 13 and of course, again expanded because ideal, so it will be coming back to the P 19, which is same as the P naught.

So, from compressible flow relation or isentropic flow relation, we can get T t naught divided by T naught is equal to 1 plus gamma minus 1 divided by 2 M naught square. We can substitute this values, we will get the total temperature T t naught is 255.6. Keep in mind that, this is not very different than 223, because the mach number is being very small. And we know that, T t naught is equal to T t 1 is equal to T t 2, because adiabatic process.

So, again from isentropic, you can get the pressure ratio, because what we are doing, we are going on each point and then, get getting this thing. Sometimes, you may avoid doing this systematic manner, I know like whenever you will be in examine, take a little shortcut root, but you may commit an error in the process, one has to be careful the P t naught is equal to 42.5 kilo Pascal. So, for the fan stream, we get actually T t 2 is equal to 256.6 Kelvin and P t 2 is same as the P t naught 42.5.

That means, we are looking at fan streams and by the same way, we can look at the compression, in the compressor how much change in the temperature is occurring.

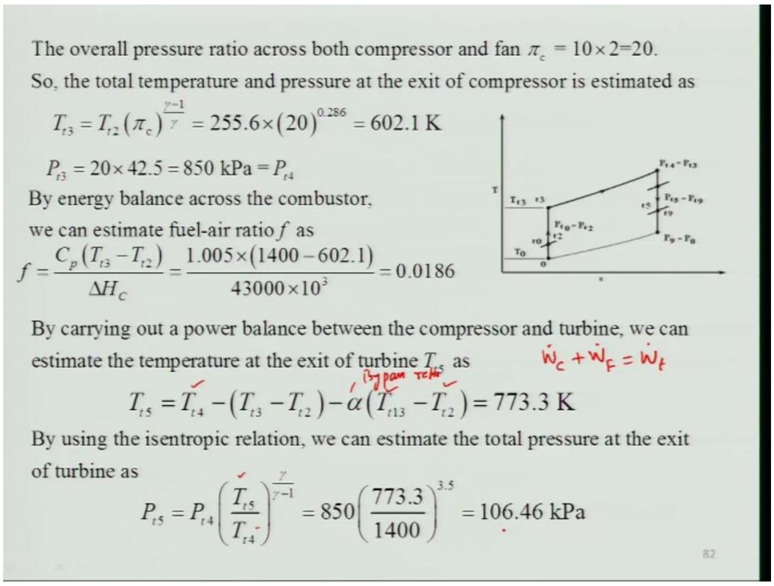
(Refer Slide Time: 16:03)



So, if you look at the pressure ratio occur, the fan pi F becomes P t 3 by P t 2, which is given, 2 is given, so you can get P t 3 is 85 kilo Pascal. And we can estimate the temperature by using the isentropic relationship and which is basically it is given like this is nothing but, your pi F, this portion. So therefore, you just substitute this value and get that and T t 2, so you can get 311.64 Kelvin, so there is a very very less change in the temperature. And as we know that, P 19 is equal to P 9 is equal to P 19 is equal to P 9, that is for an ideal cycle, for real cycle it need not to be right.

So, V 19 square we can get 2 C p T t 19 minus T 9, we can take it out and we can express in terms of pressure ratio. So, this already we know, P 9 we know, is nothing but, your P naught and P 19 we know, so we can put substitute these values and when you will do, you will get V 19 is 421.4 meter per second. So, if you look at, there is a increase in the velocity due to the work addition to the flow by the fan.

(Refer Slide Time: 17:44)



So, what are pressure ratio across the both compressor of fan or due to compression in the core engine, is pi c is 20 as I told. So, you can get T t 3 is equal to T t 2 pi c gamma minus 1 divided by gamma, this for isentropic relationship, you can get the 602.1. And similarly, you can get P t 3, because you know the pressure ratio, so you know the P t 2, so you can get that very easily. By energy balance across the combustor, we can estimate fuel air ratio or f, which is we have done for the turbo jet engine is similar to that, just you substitute the value, because it is given here 1400 Kelvin, T t 3.

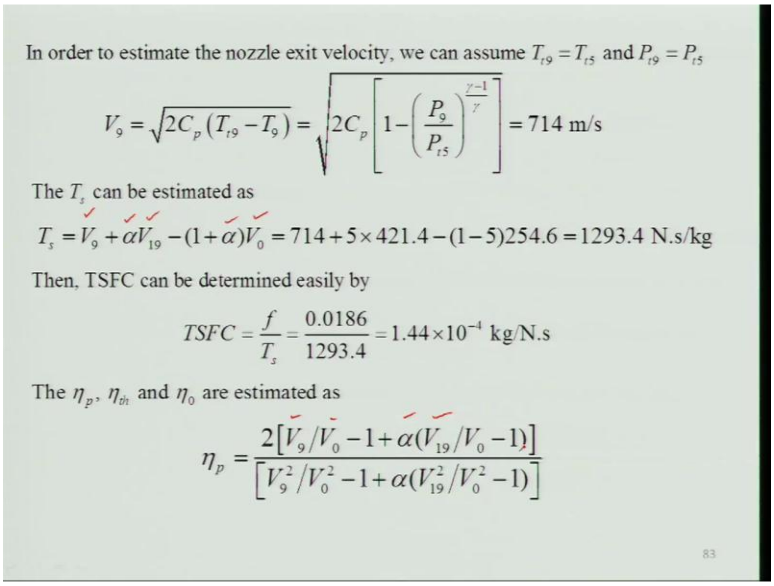
So, T t 2 you know, you can get these values, by carrying out power balance between the compressor and turbine and also the fan, we can estimate temperature. For example, we know that, W c plus W fan is equal to W turbine, if I take this and use that first blocks of dynamics for this then, I can get this an expression and we as using the same C p, but keep in mind the real cycle will be using different C p. So, in this case, this C p is cancel it out, you will get T t 5 is equal to T t 4 minus T 3 minus T 2 minus alpha T 13 minus T 2.

So, this alpha comes into picture, because of mass being separate, this is nothing but, your bypass ratio. So, when you substitute, because alpha is known, T 13 is known, T t 2 is this is also known, this is known, so T 4 is known that is, your 1400. So, you can get T t 5 if you look at, in the turbine it is expanded from 1400 to 773.3 Kelvin in temperature. By using this isentropic relation, we can estimate total pressure at the exit of turbine, the

P t 5 is equal to P t 4, we know this T t 5, we know T t 4, substitute these values, you will get the pressure at this point 106.46 kilo Pascal.

So, which is the very systematic way it is done then, nothing really to understand, because it quite easy.

(Refer Slide Time: 20:29)



In order to estimate the nozzle exit velocity, we will have to assume of course, T t 9 that is equal to T t 5, is not we are assuming, it is because it is adiabatic process and under then ideal condition, there is no losses. So, P t 9 is equal to P t 5, so when you substitute this value V 9 is equal to root of over 2 C p T t 9 minus T 9. And we know this pressure ratio, which can be express 2 C p 1 minus P 9 P t 5 power to the this gamma ratios, gamma minus 1 divided gamma.

And when you substitute these values, we will get V 9 is 714 meter per second and T s we can estimate very easily, because we know V 9 and alpha is known, V 19 already we have found out and this V naught is known, this is known, so we substitute, we will get the specific thrust, what it would be, that is basically 1293.4 Newton second per kg. So, TSFC can be estimated, because we have already estimated f divided by specific thrust, so you will get 1.44 10 power to minus 4 kg per Newton second.

So, we will estimate this propel fuel efficiency, thermal efficiency and overall efficiency, if you look at, this we have already derive, we will just use that or we can like use from your own thing that is, the thrust power, total thrust into the velocity, this portion divided by that, you can use that. So, V 9 is known, V naught is known, alpha is known, V 19 is known, so you can estimate these and substitute those values and get that.

(Refer Slide Time: 22:24)

$$\eta_p = \frac{2\left[\dfrac{714}{254.6} - 1 + 5\left(\dfrac{421.4}{254.6} - 1\right)\right]}{\left[\left(\dfrac{714}{254.6}\right)^2 - 1 + 5\left(\left(\dfrac{421.4}{254.6}\right)^2 - 1\right)\right]} \times 100 = 65.32\%$$

$$\eta_{th} = \frac{V_0^2\left[V_9^2/V_0^2 - 1 + \alpha(V_{19}^2/V_0^2 - 1)\right]}{2 f \Delta H_C}$$

$$= \frac{254.6^2\left[\left(\dfrac{714}{254.6}\right)^2 - 1 + 5\left(\left(\dfrac{421.4}{254.6}\right)^2 - 1\right)\right]}{2 \times 0.0186 \times 43000 \times 10^3} = 63.06\%$$

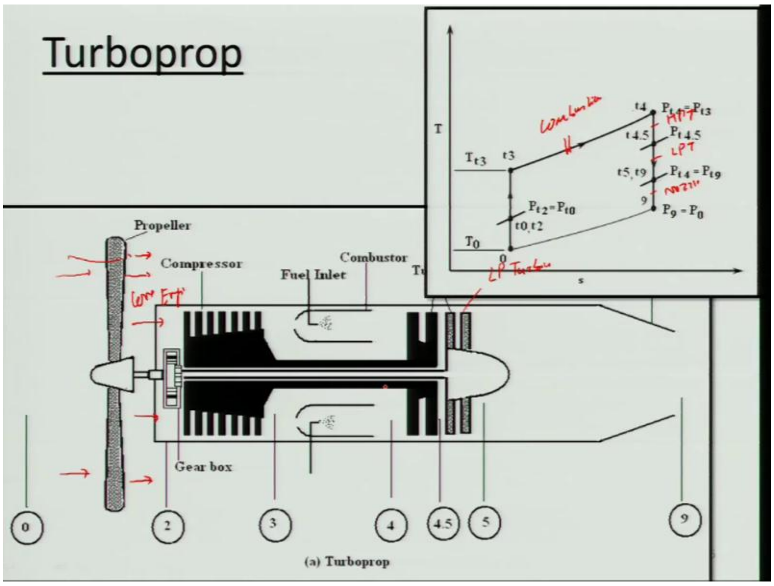
$$\eta_0 = \eta_p \cdot \eta_{th} = 41.19\%$$

84

What I would suggest that, although I have use here derive the relationship for propel thermal efficiency, but you need not, you go to the basic definition and then, do that, so that you need not to memorized it. So, you will get that propel efficiency 65.32 and similarly, when you substitute these values of thermal efficiencies for all velocities like V naught, V 9 and V 19 and alpha. Alpha is the bypass ratio and the heat of combustion, so you will get 63.03 percentage and the overall efficiency when we just multiplied both, you will get it is 41.19.

If you look at the overall efficiency, if you compare of course, the same conditions you will find that, turbo fan engine is having higher overall efficiency than the turbo jet or the engine kind of the thing. So, you must appreciate that point and particularly, I will be giving some problem in the assignment, where you could get the chance to fill for it by solving yourself.

(Refer Slide Time: 23:38)



So now, we will moving to the turboprop engine and which is basically being used whenever we want to have a higher thrust level kind of things and of course, it will be what you will call whenever we need have also the TFSC kind of thing. So, the typical turboprop engine is being used, being shown here the schematic diagram that is, it is having a propeller, instead of a fan it is a popular, which is unducted, there is no duct. And keep in mind that, the length of the blade in a propeller is much longer as compare to the length of the blade in a turbo fan engine or the fan.

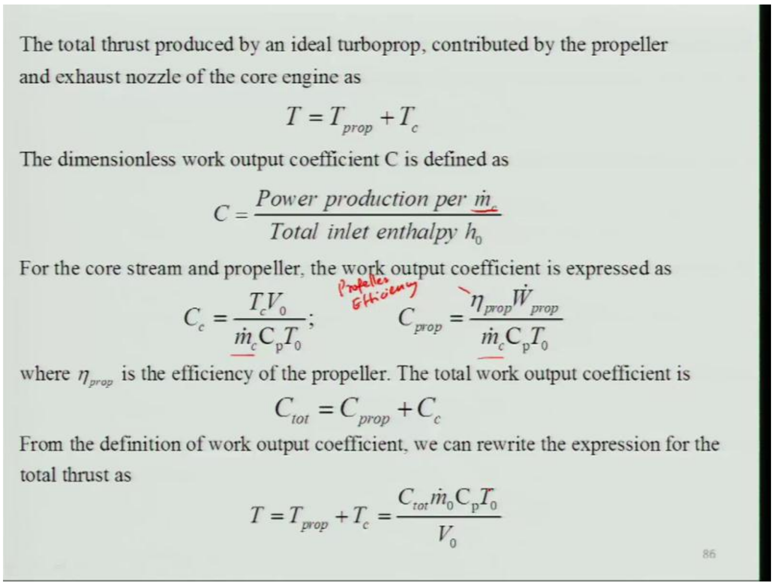
There is a two difference, one is it is the unducted fan you can say, but not only that, but it also length of the blade is higher. So therefore, it has to be operated at a very low rpm why, the same reason that, if the length is higher then, at the tip, the velocity or the local velocity there fluid will be much higher, omega into r. So, if you look at, that is the very higher values and therefore, it has to be rotated at a lower rpm and therefore, the gear box is being used to reduce this rpm.

And this of course, the shaft if you look at, it can be very clearly seen, it is the two spool kind of engine, where the low pressure turbine, this is your low pressure turbine. Low pressure turbine is being connected through a gear box to the fan, of course the high pressure turbine is being connected to the compressor and this is your combustion chamber and station number we are using in the similar one. Now, let us look at the process what is happening, it is 0 to 2 is your compression and there will be some compression due to the propeller.

And keep in mind, this is your core engine, air is also flow, this is your core engine and this is the propeller, propeller air will be coming over here and going out and there is a compression in the, what you call in the compressor. And there is a heat addition, constant pressure heat addition, combustion and there is a high pressure turbine from 4 to the 4.5. There is the expansion in the turbine high pressure and expansion in the low pressure turbine that is, 4.524 and this is your nozzle, low pressure turbine and this is high pressure turbine, HPT and LPT I am telling.

So, we need to carry out this analysis, keep in mind that, we do not know how much air is passing through the propeller, because it is unducted. So, how to handle, can we go by that, there is a one concept which is very important and another one is that, the amount of the power given to the propeller is not being converted into the kinetic energy, because always there will be slip. So therefore, we will be using efficiency known as propeller efficiency in this analysis and we are assuming the gear box efficiency is 1, which is not the case, that we will be we are using the ideal cases, the gear box efficiency is equal to 1.

(Refer Slide Time: 28:04)



So, the total thrust produced by an ideal turboprop engine can be contributed by what, two thing, one is your core engine through core it is going and another is due to the propeller. So, if you look at, which is the major portion in a turboprop engine, is this is the propeller will give you more or the core engine will give you more. Generally, propeller will give you the more thrust as compared to the provided by the core engine, because expansion in the nozzle will be very very small, although it is not being reflected in a T s diagram, please keep that in mind.

The dimensionless work output coefficient we can define as, C is the power produced per unit mass of flow rate through the core engine, this is very important one. You must understand this, because why you are using the amount of mass passing through the core engine. Because, we do not know really, how much it is going through that, because openly it is going and how much it is contribute in the thrust. So therefore, we use this work output coefficient, instead of now we will be operating both thrust.

And the work output coefficient trying to connect it, convert I am like connect both the thing, where all the other analysis we are using thrust, we are not bother about, what is the work output kind of things, because we are interested in thrust there and there is no problem of handling that. So, there is a little difference in analysis, please keep that in mind, so therefore, for the co-stream and propeller, work output coefficient can be express as C c, this is your thrust produced by the core engine that is, tau T c.

And then, V naught is your flight velocity divided by m dot C p T naught means, this is with the respect to total energy, which is coming, m dot, C p and T naught, C p, T naught is enthalpy into mass, enthalpy this is per kg and then, this will be 2. So, similarly probe, the work coefficient for the propeller will be eta propeller that is, the propeller efficiency, this is basically propeller efficiency and the work input to the propeller divided by m dot c C p T naught.

keep it in mind that, we are not using total mass, we are using all the time m dot c, so it is having implication when we interpret the data, because that is that why I am emphasizing this point. The total work out could coefficient will be C total is the equal to C propeller, work out put coefficient plus core engine C c. And from the definition of the work output coefficient, we can rewrite expression for total thrust, what is the definition, we have already seen, that is the total thrust into the flight velocity. So, total thrust will be basically, the propeller thrust plus the core thrust is equal to C total into m naught total C p and T naught divided by V naught, that will give me the total.

(Refer Slide Time: 32:06)

The expression for specific thrust developed by the core flow would be

$$T_s = \frac{T}{\dot{m}_0} = a_0\left(\frac{V_9}{a_0} - M_0\right) = a_0\left[\sqrt{\left(\frac{2}{\gamma-1}\right)\frac{\tau_\lambda}{\tau_c \tau_r}\left(\tau_t \cdot \tau_c \cdot \tau_r - 1\right)} - M_0\right]$$

Now, the work output coefficient for the core engine C_c can be expressed as

$$C_c = \frac{T_c V_0}{\dot{m}_0 C_p T_0} = \frac{V_0 a_0}{C_p T_0}\left(\frac{V_9}{a_0} - M_0\right) = (\gamma - 1) M_0 \left(\frac{V_9}{a_0} - M_0\right)$$

The specific power (SP), an important parameter for turboprop, is given by

$$SP = \frac{\dot{W}_{tot}}{\dot{m}_0 h_0} = C_{tot} C_p T_0 \text{ where } C_{tot} = C_{prop} + C_c$$

In order to find an expression for C_{prop}, let us strike a power balance across high pressure turbine and propeller as

$\dot{m}_c$

$$\dot{W}_{prop} = \dot{m}_{4.5} C_p \left(T_{t4.5} - T_{t5}\right)$$

87

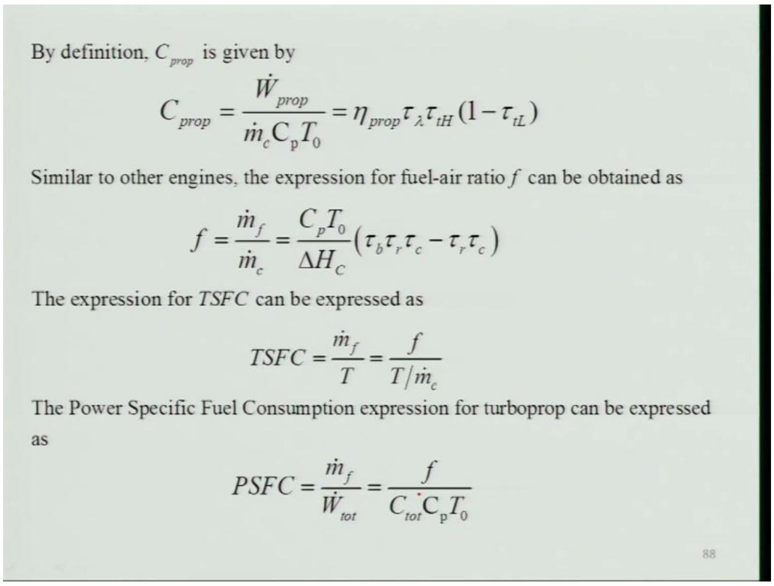
So, expression for the thrust developed by the core engine, flow rate can be expressed in this way by using the similar analysis or the same analysis rather, for the turbo jet engine. So, T s is equal to T by m naught is nothing but, a naught V 9 by a naught minus M naught and then, in place of this V 9 by a naught minus M naught, you can write down this figure expression, which we have already derived for the turbo jet engines. And keep it mind that, all this parameters are there like tau lambda, tau c, tau r and tau t.

So, these are basically temperature ratios across the components like your turbine, compressor, air intake and lambda of course is little different thing, which I had defined earlier. So, the work output coefficient for the core engine we can say that, T c by V naught divided by m naught C p T naught. And we can express this in terms of like V 9 and a naught, because we have already we can put this thing values here and over here and get that.

And you will find that, little analysis if you do, you will find out this gamma minus 1 M naught the bracket V 9 by a naught minus M naught, you will get these values. So, and if you look at, what you are using, we are basically expressing C p in terms of gamma. So, the specific power is an important parameter for turbo jet engine, which can be derived like SP that is, the specific power, is W dot total divided by m naught h naught, which is nothing but, C total C p and T naught and where, T total is equal to C propeller plus C c.

Because, if you look at this, you will have to go to the definition of the C total and then, you will get that, this is nothing but, C total C p T naught. So, in order to find an expression for C p, let us strike a power balance across the high pressure turbine and propeller. So, if you look at, this is a propeller work input is equal to m dot 4.5, this is nothing but, your m dot c, because this is the core engine, which is going, whatever the air is going through that core C p and T t 4.5 minus T t 5.

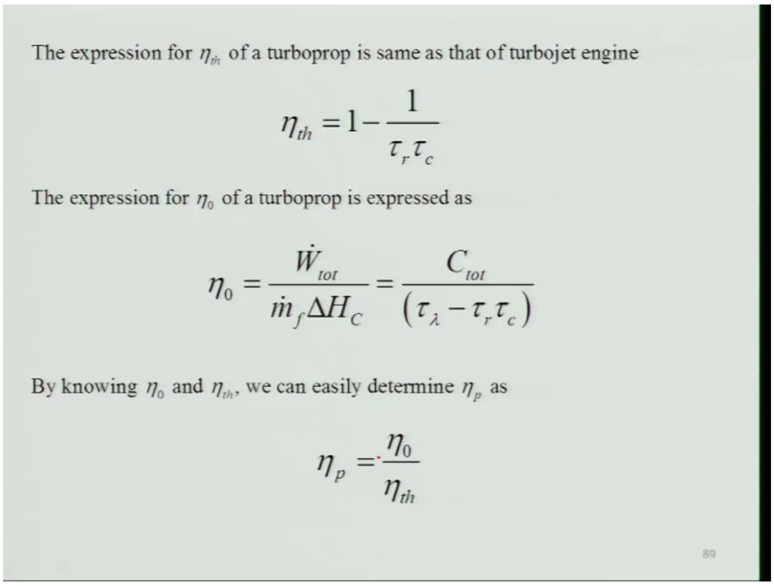
(Refer Slide Time: 35:01)



So, by definition we know that, C propeller will be basically W dot propeller into m dot C p T naught and there W dot propeller we have already seen that, it is nothing but, you are in terms of propeller efficiency and temperature. And if you just rewrite, you will get get T lambda tau t H, this is basically tau t for the high pressure turbine and tau t L is for low pressure turbine. Similar to other engines, the expression for fuel air ratio, you can get the similar expressions, which we have already derived for the turbo fan and turbo jet engines even.

So, and TSFC you can get in the similar way, m dot f divided by T is equal to f into this T divided by m dot c. Keep in mind, what we are doing, we are basically looking at even the mass flow rate through the core engine. The power specific fuel consumption expression, because it is a turboprop engine, therefore we are interested in the TSFC or the power specific fuel consumption. So, that is, m dot f divided by W dot total, we know this f and C total, C p and T naught, if you look at, you will substitute these values, you will get that.

(Refer Slide Time: 36:31)

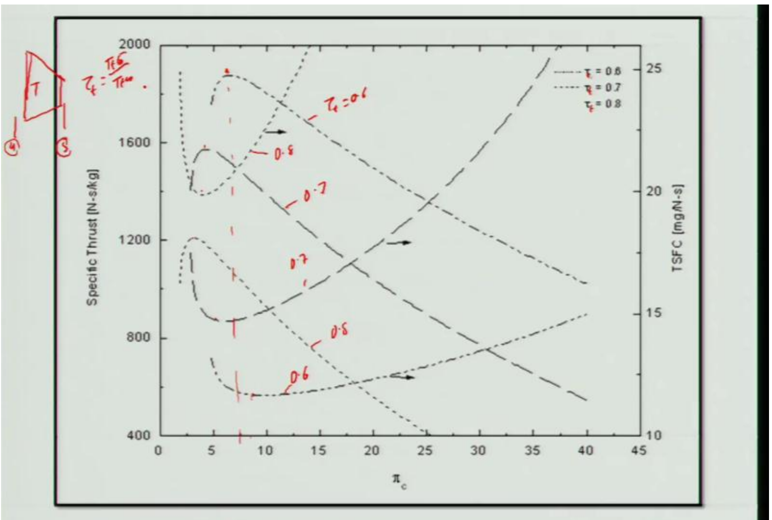


And the expression for the thermal efficiency will be similar to that the turbine jet engine and overall efficiency, it is more important that, you first really find out the overall efficiency in case of turboprop engine. But, in case of turbo jet and turbo fan engine, you find out first the propulsive efficiency, before you really find out overall efficiency. But, it is other way around, because we need to find out this and then, you will get, because we do not know really, how much it is going.

So, and this is the expression the similar way, I am in like a same coefficient, but this is the why definition m W dot total divided by m dot f delta H. It is how much work being put into a engine, how much work being produced by the engine and then, how much fuel being burnt. So, by knowing this overall efficiency and thermal efficiency, we can get the propulsive efficiency, this is keep in mind, this is not propeller efficiency, this is propulsive efficiency, which is due to the motion of the vehicle or the engine.

So, that you should keep it mind, so now, what we will do, we will carry out a parametric analysis, as we had done earlier.

(Refer Slide Time: 37:59)



Variation of T_s and TSFC with π_c for turbine temperature ratio τ_t=0.6, 0.7 and 0.8 for an ideal turboprop engine

And then, we will see what we are getting and here, what we are doing, we are basically taking the three tau t values, tau t is equal to 0.6, 0.7 and 0.8. And we are varying the pi c from 0 to 40 you can say, kind of things and then, we are plotting it, specific thrust versus the pi c for a certain altitude and certain flight mach number kind of things. You will find that, this when tau is 0.6, what is the meaning of that and when tau increases 0.8 what is the meaning of that tau that means, temperature ratio across the turbine tau t, this is tau t that is, your temperature ratio across the turbine.

Lower means, it is what?

Student: Less work extraction.

It is more work extraction, because the temperature at the exit of the turbine divided by the inlet. So, if it is a smaller value that means, it is being expanded well as compared to the bigger value, are you getting my point. For example, this is your basically 4 and let say this is your 5, this is your turbine that means, this tau is t is equal to T t 5 divided by T t four. That means, pipe is smaller that means, more expansion as compared to when the tau t is larger.

That means, smaller means more expansion, larger means less expansion, so keep in that in mind that, if you look at, this line is basically this is your tau t 0.6 and you will get that specific thrust increases and then, it decreases with the pi c. And as you go to the 0.7 and this your 0.8 and which is obvious that, it will be decreasing and it is also if you look at, peak value also decrease. That means, peak value is basically increasing point with the, what you called larger expansion that means, tau t beings smaller.

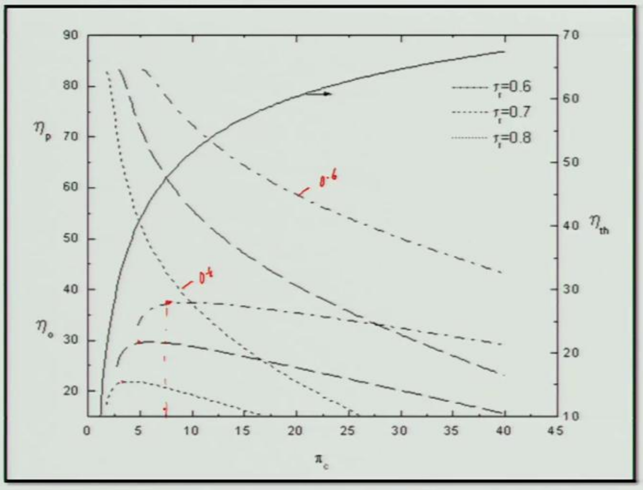
And it is occurring at a higher pressure ratio if you look at, because if the expansion is more, you need to provide the more compression ratio, pressure ratio. And of course, if you see this, this is your 0.8 and this is your 0.7 and this is nothing but, your TSFC, Thrust Specific Fuel Consumption and this your 0.6 and you will see that, minimum values are also minimum decreasing and so, also the overall like TSFC decreases.

That means, if I go for more expansion then, I am having the lower TSFC values and also it is coinciding if you look at minimum values here, it is almost closer to that, in comparison to the turbo jet and other engine. So, that is a new part of it that means, TSFC and the minimum TSFC and the peak specific thrust are almost closer you can say that, you can operate value and which is better as compared to.

And keep it mind that, here the specific thrust is much higher as compared to the turbo jet engine, if you look at the numbers here something 1800 kind of thing for tau t 0.6, is it something or is it wrong. What is that, it is right, because you have divided, this is the specific thrust with respect to the core mass flow rate that means, mass flow rate is very very small. But, in case of turboprop engines, if you look at, if I take total mass flow arte then, naturally it will be smaller.

The specific thrust always smaller for turboprop engine as compared to the turbo jet engine, but here it is an artifact, because the assumption at the way we have analyses, that you keep in mind, this is a very important point you may not I mean, like observe that thing.

(Refer Slide Time: 43:22)

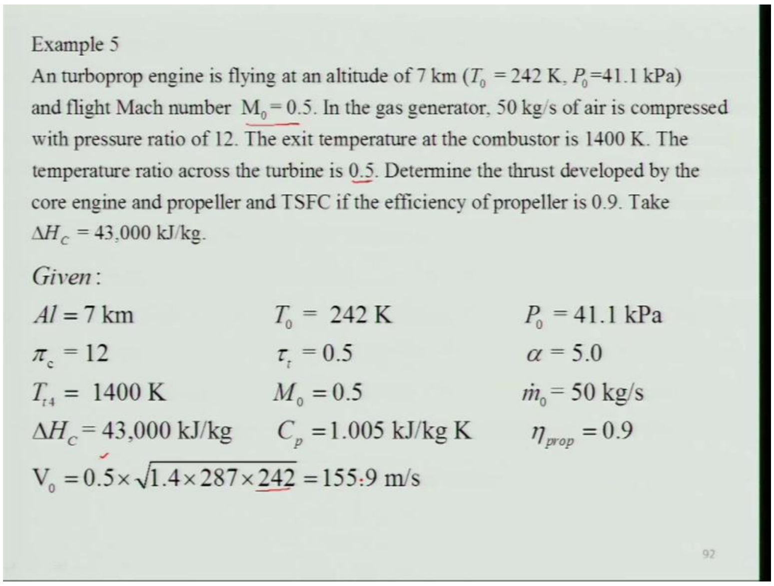


Variation of (a) η_p with π_c and (a) η_0 with π_c for an ideal turboprop engine

So, the propulsive efficiency and overall efficiency I have told, this is basically the thermal efficiency this line, which increases pi c, because it is the same as that of your turbo engine. But however, what you call this is 0.6 that is, your propulsive efficiency and this is 0.8 and whereas, the overall efficiency if you look at, it is increases and then, it decreases, it is having some peak values here.

And similarly, as you goes on increasing this tau, it gets reduced that means, the more expansion in the turbine, the better is the overall efficiency, that is one point. The other point is that, it is peak values, at which the peak overall efficiency occurs at a higher pressure ratio across the compressor. That means, my compressor will be more bulky, because number of stages will be higher so then, you need to look at this aspects while designing a turboprop engines. Of course, several analysis one can carry out, but I will not consider those things.

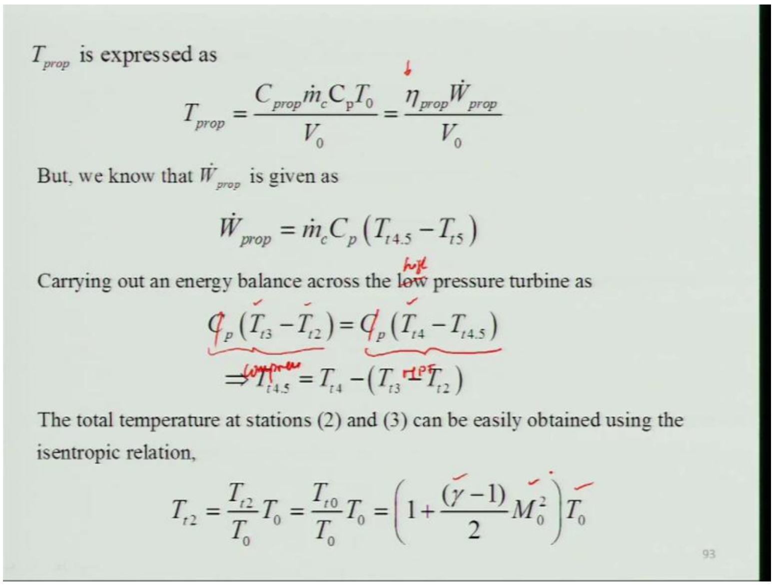
(Refer Slide Time: 44:42)



And let us look an example, a turboprop engine is flying at an altitude of 7 kilometer that is, P naught 242 Kelvin and P naught 41 and flight mach number of 0.5. In the gas generator, 50 kg of air is compressed with the pressure ratio of 12, the exit temperature at the combustor is 1400 Kelvin, temperature ratio across the turbine is 0.5 we are using. And determine the thrust developed by core engine and propeller and TFSC, if the propeller efficiency is 0.9.

And as usual, we can find out the flight velocity I mean, because mach number is given and we know this temperature T naught is the... And we know this specific gas constant, we substitute, which happens to be very very low 155.9 meter per second.

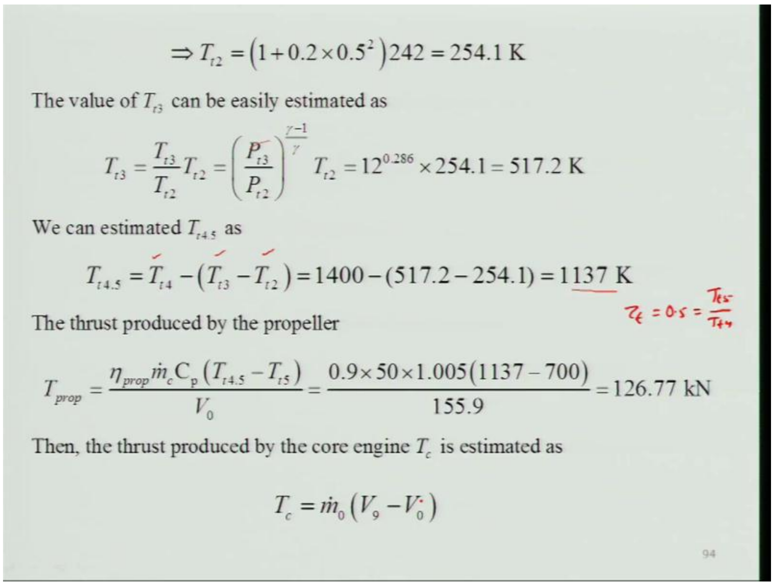
(Refer Slide Time: 45:42)



And a propeller thrust by the, propeller thrust can be express C propeller m dot c C p T naught by V naught and it is given the propeller efficiency W dot propeller we need to calculate and V naught already we have calculated. So, and we know that, m W dot propeller is nothing but, m dot c C p T t 4.5 minus T t 5. And when we carry out this C p and then, energy balance across the low pressure turbine and propeller, so we will carry out energy balance across the low pressure turbine and the compressor, this is not low pressure, this is basically high pressure turbine across the compressor.

Because, this is your compressor, this is your basically high pressure turbine, so I know this T t 4 is given and I need to find out this T t 3 and pressure ratio is given. So, I can find out the similar way T t towards to found out and of course, the C p C p will cancel it out, so I can get T t 4.5 is equal to T t 4 minus T t 2. So, we need to determine basically T t 3 and T t 2, T t 2 is basically T t 2 divided T naught into T naught and this already we know. So, we know these values, we know these values, we know gamma, we can determine what will be T t 2.

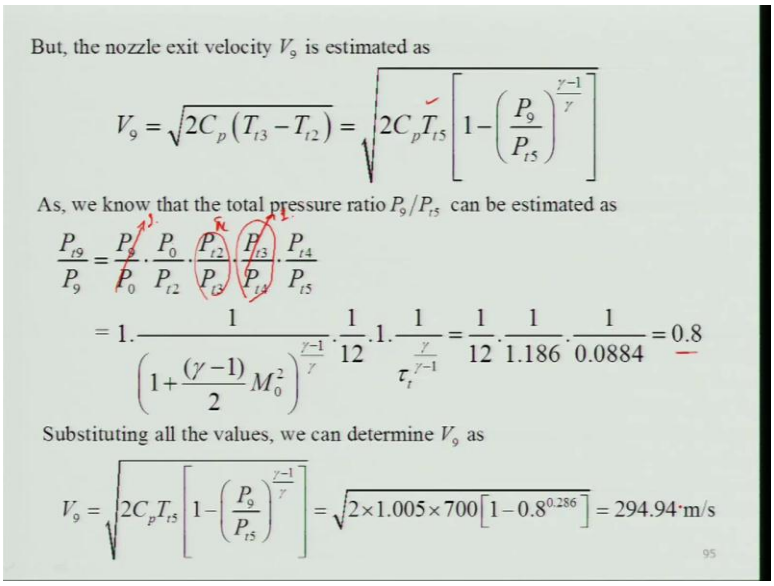
(Refer Slide Time: 47:46)



Similar way, what we have done and values of T t 3 can be easily estimated, because we know the pressure ratio across the compressor. So, we can find out that values very easily, because we know this is 12 and gamma we know 1.4 will be using and we can get this values. So, by knowing this value of T t 3 and T t 2 and T t 4, so we can get T t 4.5 and the thrust propeller we can get as T like eta propeller into m dot c, m dot c is given to you like how much air is passing.

That is, 50 kg per second, which is important parameter, C p is known and T t 4.5 minus T t 5 and V naught you can get this how much. So, because how will you get this T t 5, tau is given, tau t is given, so tau t is what, is it 0.5, so that is nothing but, your T t 5 divided by T t 4. So, from that, I can get because, the T t 4 is your 1400, 1400 and 0.5 T t 5 is equal to 700, so that is given, so you can find out very easily. So, the thrust produced by core engine will be T c is equal to M naught V 9 minus V naught.

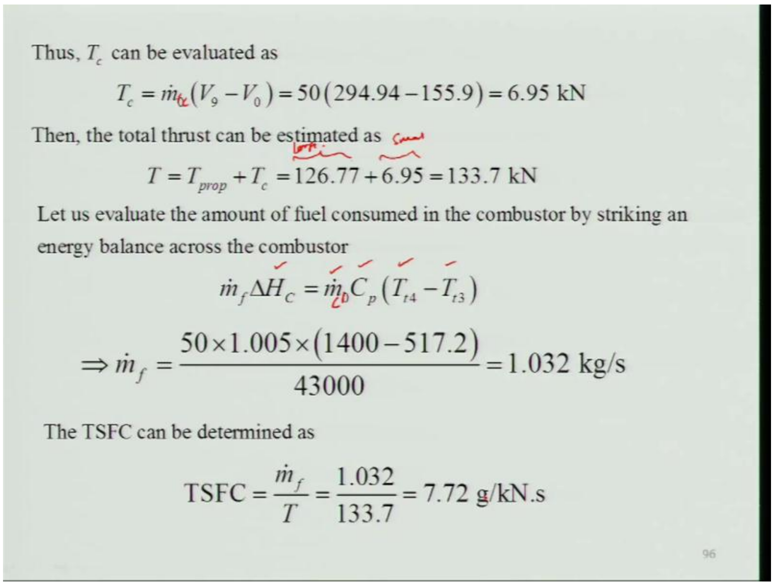
(Refer Slide Time: 49:29)



So, nozzle exit velocity you can get by this expression and we know this T t 5, we know C p and it is nozzle is fully expanded. So, we need to know the pressure ratio across the nozzle that is, P t 5 by P 9, so this you can do that and you can substitute and get that, it is happens to be 0.8. If you look at, P t 9 by P 9 is equal to P 9 by P naught, which is nothing but, your 1 P naught by P t 2, P t 2 by P t 3, this is nothing but, your pi c it is given P t 3 by P t 4, this is nothing but your 1 and P t 4 by P t 5 and tau t is given.

So, you can substitute those thing and get their value, tau t can be express in terms of your pressure ratio or the pressure ratio across the turbine can be express in terms of tau t. So, substituting all values, we will get V 9 is this values 294.94 meter per second.

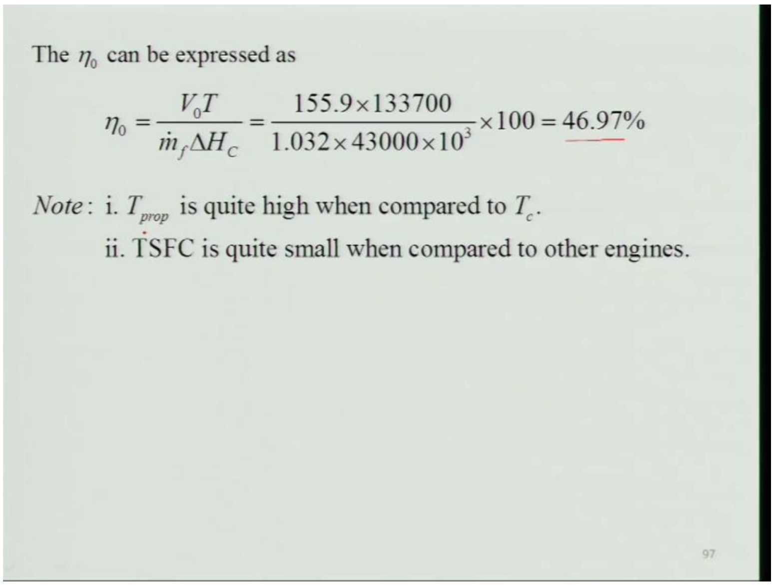
(Refer Slide Time: 50:52)



And after knowing this thing, we can get basically the T c, this will be m c and V 9 minus V naught. And we know this values, we can get T c and by knowing T c, we can get what will be the total thrust, because the thrust equal to propulsive thrust or propeller thrust plus the core engine thrust will give you the total thrust. You can see that, the propeller thrust is much higher as compared to the code thrust, it is a very very small, this is quite small, this is large.

So, it is evaluate the amount of well consumption by striking this valance, this will be m dot c basically and if you look at, you substitute this values, because this is known, ((Refer Time: 51:54)) this is known and this is known and this is given and this also given, so you can substitute and get the values. So, TSFC you can get, m dot f divided by T, so you can get those values.

(Refer Slide Time: 52:09)



So then, we need to find out the overall efficiency, because you know this T, the total this is basically total thrust into V naught m dot f delta H c, you can get very easily, substitute this values and get that is, 46.97. And you can find out the kind of thermal efficiency very easily and you can note that, I have already discuss about the propeller thrust is quite high when compared to T c, TSFC is quite small when compared to other engines. So, with this, we will stop over here and in the next lecture, we will be looking at real cycle analysis.