Fundamentals of Aerospace Propulsion
Prof. D. P. Mishra
Department of Aerospace Engineering
Indian Institute of Technology, Kanpur

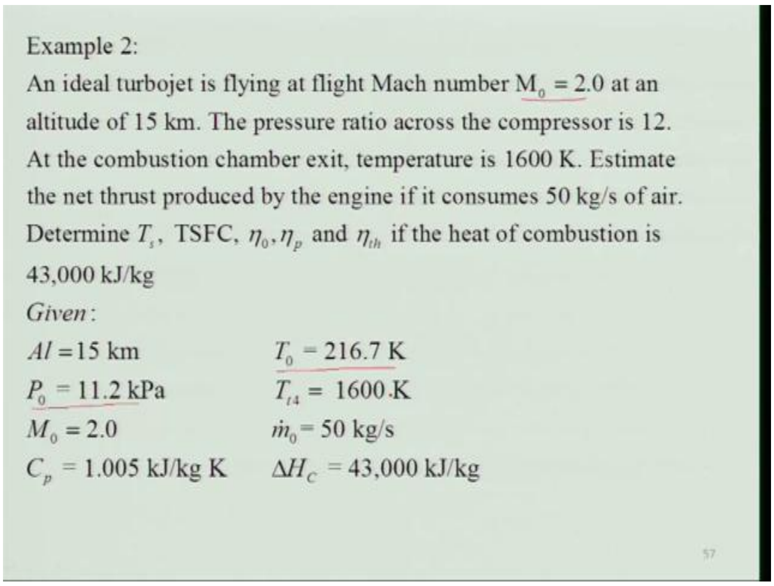
Lecture - 30

Let us start this lecture with a saying from Chanakya who says what qualities of energy, bravery, resentment, quickness and dexterity. These are the qualities of energy; you know it is about the human being nothing to do with your propulsion system. Let us recall that what we learnt in the last lecture, we basically discussed about the how to conduct or carry out a parameter cycle analysis for turbojet engines.

We have done some parametric variations and looked that how this transpacific consumption will be varying with pressure ratio for three flight Mach numbers. Then, we have seen that how it is varying and then also the TSFC thrust specific full consumption are you know varying with the com compression pressure ration right or the pressure ratio across the compression.

We found out that it is you know occurring at the minimum TSFC would not be occurring at the maximum specific thrust. Then, we look that propulsive specific what we will do now that we will now take of an example in the turbojet engine and look at how we can solve a problem particularly not of carrying out parametric analysis, but to understand what is happening.

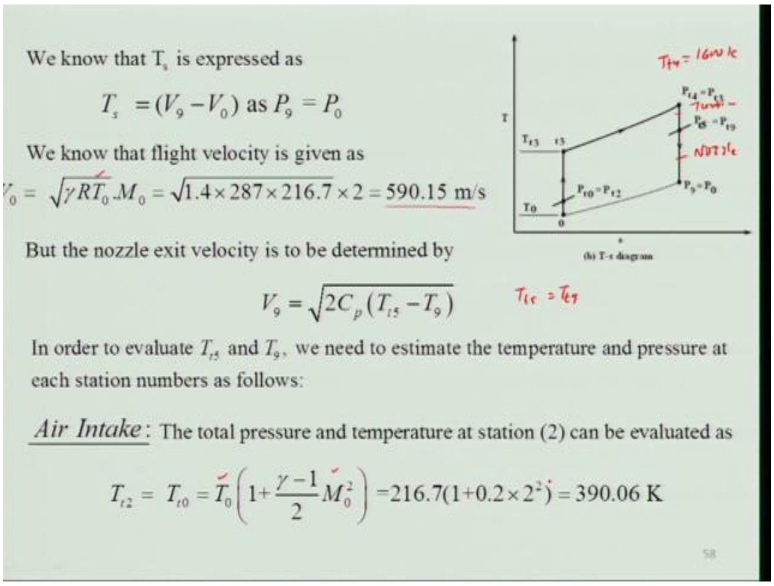
(Refer Slide Time: 02:11)



We will use the method two what I had discussed in the ramjet engine, now onwards whenever I will be taking an example, I will be using the method two that you will have to go in each point and then do that. So, then the ideal turbojet engine is flying at flight Mach number of 2 and an altitude of 15 kilometers. Pressure ratio across the compressor is 12 at the combustion chamber exit the temperature is 1600 Kelvin and we will have to determine net thrust produced by the engine. If it consumes 50 kg of air, 50 kg per second of air.

We need to determine specific thrust, I mean like then TSFC of propulsive efficiency thermal efficiency and overall efficiency and these are the if you look at values are given or data are given like altitude 15 kilometer. So, I can find out what will be the T naught from a table and a pressure P naught and Mach number of course is given. We can assume this C P because an ideal cycle I can take that C P as 1.005 kilojoules per kg Kelvin. T t 4 is given 1600 Kelvin and mass flow rate also mass flow rate air entering into the engine given as 50 kg per second.

(Refer Slide Time: 03:25)



So, what will be doing we will be basically you know looking at this processor in T s diagram and move from one point to another and evaluate all the properties whatever required and then we do that. Here, we will get the physical feeling, we will see what we are doing for example, and we are interested to find out specific thrust or thrust. So, we know that specific thrust is equal to V 9 by V naught for an ideal cycle, so we need to evaluate V 9. We need to evaluate V naught, V naught is very easy to evaluate because we know the T naught and by knowing the T naught, I can get the speed of sound a naught and Mach number is given.

So, by knowing the Mach number I can get what you call very easily the flight Mach number, sorry flight velocity that is V naught is equal to root over gamma r T naught, T naught is given to you and gamma you know 1.4, r is 287. That is the specific gas constant 287 for air and temperature we have already seen and you will get 590.15 meter per second whenever it substitutes the values, you get this which is quite you know higher.

If you look at the most of the turbojet engine is basically used for the high speed or high velocity of application like fighter aircrafts and other thing. Therefore, it can operate in the higher flight velocity, so the nozzle exit velocity is to be determining because we are interested the finding out T s and V 9 is the nozzle exit velocity.

So, we need to find out that, so if you need to find out that basically we can write down that V 9 is equal to root over 2 C P T t 5 minus T 9. So, you know really T 9 or is it same as that of the T naught T 9 is here, certainly it is not because it is not in the same place in the T s diagram. Of course, mechanically you can think of other one is it can never be because you adding heat and you are of course expanding in the nozzle, but still the temperature will be higher than the ambient temperature. So, we do not know T 9 and do you know this T t 5 why we have written T t 5 because T t 5 is equal to T t 9 in principle.

It could have been T t 9 by definition V 9 is equal to root over 2 C P T t 9 minus T 9, T t 9 means total temperature at the station 9, but we do not know T t 5, but of course why know this T t 4 T t 4 is what is given a 1600 Kelvin. I know this temperature T naught here, but I do not know what is this temperature T t 3, if I know this T t 3, then you know I can find out what will be the pressure here. Of course, from this, here I can find out how much expansion is taking place in the turbine because this is turbine expansion and this is your nozzle.

So, I do not know I can start from here and do that, but I need to know also the pressure, therefore I have to go in a systematic way. So, let us look at how I will do that, so in order to evaluate T t 5 and T 9, we need to estimate pressure at each station number because at each point I need to know the temperature and pressure. Sometimes, you can avoid particularly for an ideal cycle, but however in real cycle, you will have to go through the same systematic procedures, otherwise you cannot really handle it.

Of course, some of you used your intelligence to do that, the different thing but it is better to follow your systematic way. So, let us look at air intake, what is happening total pressure and temperature that is total temperature station two can be evaluated. It very easy, so we know that T t 2 is equal to T naught. That means in the air intake there is no heat transfer or it is adiabatic process, therefore T t 2 is equal to T naught which is nothing but T naught plus 1 plus gamma minus 1 divide by 2 m naught square. So, m naught is given to you gamma is not given and this is given.

So, you can find out 390.06 when you substitute these values and you can see that it has increase from 216.7 Kelvin to 390 because the Mach number is 2 is higher. If Mach number is small, then this will be not really varying. For example, you know if it is Mach number 5 or 0.2, 0.3, then this you know static temperature T naught will be almost similar to the T naught, so that you should appreciate.

(Refer Slide Time: 08:49)

Let us now evaluate the pressure ratio P_{t2}/P_0 as

$$\frac{P_{t2}}{P_0}=\frac{P_{t0}}{P_0}=\left(\frac{T_{t0}}{T_0}\right)^{\frac{\gamma-1}{\gamma}}=\left(\frac{390.06}{261.7}\right)^{3.5}=7.82$$

$$\Rightarrow \qquad P_{t2}=\frac{P_{t2}}{P_0}\times P_0=87.6\text{ kPa}$$

<u>*Compressor*</u>:

The total pressure across the compressor is given as

$$\frac{P_{t3}}{P_{t2}}=12\Rightarrow P_{t3}=\frac{P_{t3}}{P_{t2}}\times P_{t2}=1051.2\text{ kPa}=P_{t4}$$

Let us now evaluate the total temperature ratio T_{t3}/T_{t2} as

$$\frac{T_{t3}}{T_{t2}}=\left(\frac{P_{t3}}{P_{t2}}\right)^{\frac{\gamma-1}{\gamma}}=12^{0.286}=2.035 \qquad \Rightarrow T_{t3}=\frac{T_{t3}}{T_{t2}}\times T_{t2}=794\text{ K}$$

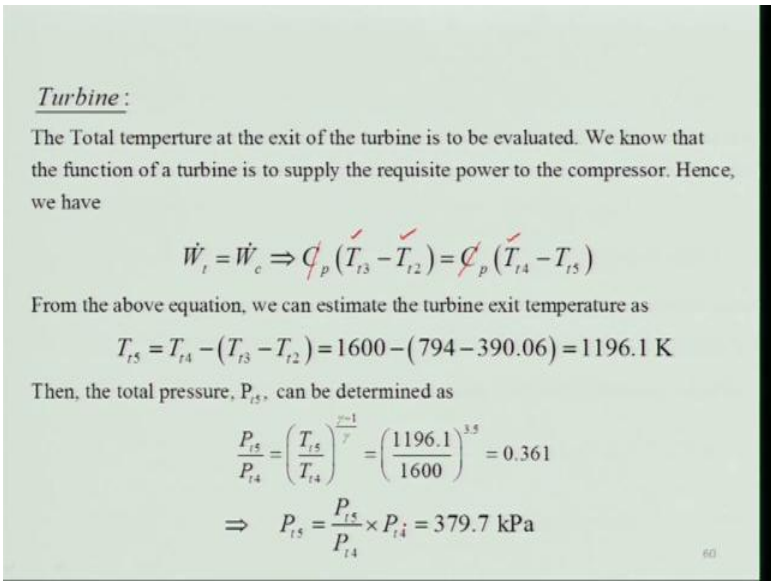
We need to now evaluate the pressure ratio P t 2 by P naught which is again we will be using isentropic process because at ideal cycle we are considering. So, P T naught is equal to P t 2, I mean like then I can T naught by T naught and if you look at these values, we can substitute these over here 3.5. When you substitute it, gamma is equal to 1.4, so I can get this ratio as 7.82 and I know the P naught.

So, I can get the what will be the P t 2 will be 87.6 kilo Pascal, when I substitute value, but sometimes you do not need to calculate this you can manage with the ratio. I have just done intentionally just to have a feel for it, now what is the pressure happening, but in your calculation you may not when you are doing particularly mechanical. You know in your exam and other thing when a time is a problem, so you need not to do that the compressor total pressure ratio across the compressor can be you know we know it is given P t 3 by P t 2 to 12. So, by knowing the P t 2, I can find out P t 4, so it is 12 time of that P t 2. So, it is just simple multiplication keep in mind P t 4, 3 is equal to P t 4 because in the combustion chamber there is no loss of pressure under ideal condition.

Therefore, it is in real situation, we will see that there will be losses, we will have to take care of that in real cycle analysis. So, let us evaluate total temperature ratio, so which is even you can use isentropic process and substitute these values, you will get this is the 2.035 and from that you can get T t 3 is basically you take this ratio and that is 794 Kelvin. So, this temperature will be inlet to the combustors because it is going from here, so if you look at all the combustion will be taking place with not in ambient air or the cold it is quite hot also 790 Kelvin.

Of course, it may not to be more than the self ignition temperature, but however it is quite close to that of the some of that. So, it is the very important thing we should appreciate that point and turbine. If you look at temperature, the exit turbine evaluated, we know that function of turbine is to supply the requisite power to the compressor.

(Refer Slide Time: 11:24)

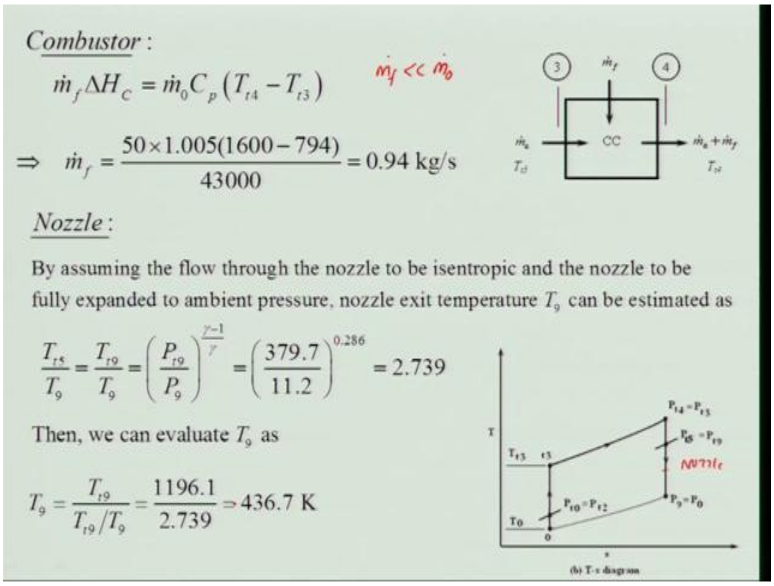


In turbo turbojet engine, the work harness by the turbine is mean to run the compressor; therefore you know we need to have that. So, we will say that W dot t is equal to W dot c, so if you look at this, you know this is basically on in terms of enthalpy by using energy equation for the steady flow passes and one dimensional. All those things T t is known T t is known and of course c P is same as that what we considering for ideal T t 4 is known.

So, I can get what is T t 5, now you must appreciate why I went to through those calculation and looking at compressor T t 3 P t 3 all those things because I need to find out T t 5 and T t 5 is nothing but T t 9. From the above equation we can estimate turbine exit temperature, just substitute values you will get 1196.1 Kelvin. That means from 1600 Kelvin to 1190 that much of expansion taking place as result the temperature change occurs due to expansion in turbine.

Now, rest of the thing has to be expanded in nozzle to get the thrust by the turbojet engine so that the total pressure P t 5 can be determine as again isentropic relationship because you know this. So, I will get you know when I substitute these values, the ratio is 0.361 and so P t 5 I can get I know this is P t 4, so I can get because P t 3 P t 4 is equal to P t 3.

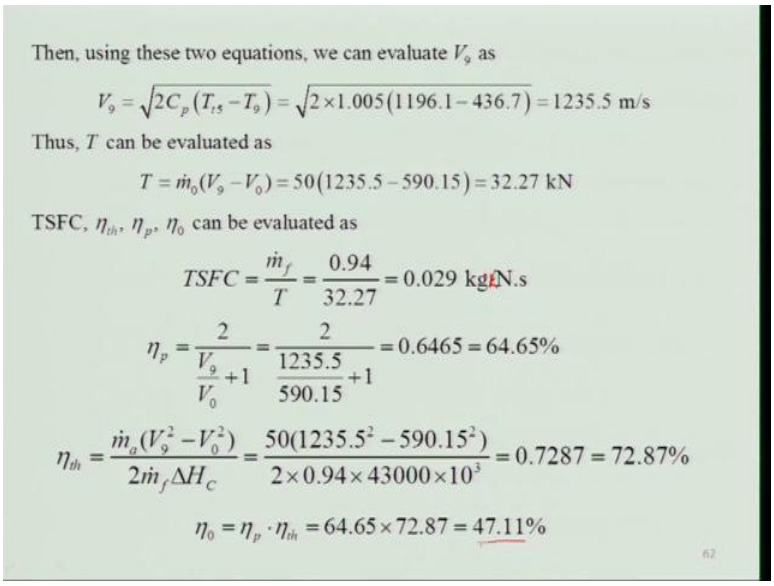
(Refer Slide Time: 13:29)



Therefore, we can easily get these values and combustion chamber, I can take this you know I can have this expression m dot f is equal to delta into multiplied by delta h c is equal to m naught c P T t 4 minus T t 3. Keep in mind that we are assuming here less than m dot, you know not that is the assumption we are doing in the right hand side, but in the left hand side we are keeping the m dot f because this is comparison what we will be doing. So, when you substitute these values m dot f, you know you will get this what you call values here, over here and this 0.94 kg per second.

So, if I know these values and you know it is important to find out TSFC, let us look at nozzle in the nozzle this is your nozzle expansion. So, to be isentropic and we need to find out exit temperature by you know using isentropic process. So, T t 5 is equal to T t 9 as I told earlier is nothing but you know T t 9 by T 9 can related to the pressure ratio and you can get this and you know already T 5. So, you can substitute these values, then you can get T 9 is 436.7 Kelvin which is higher than your what you call ambient temperature. Although then nozzle is fully expanded, so if you know this T 9 you know T t 5, you can evaluate the V 9, 2 C p T t 5 and T 9.

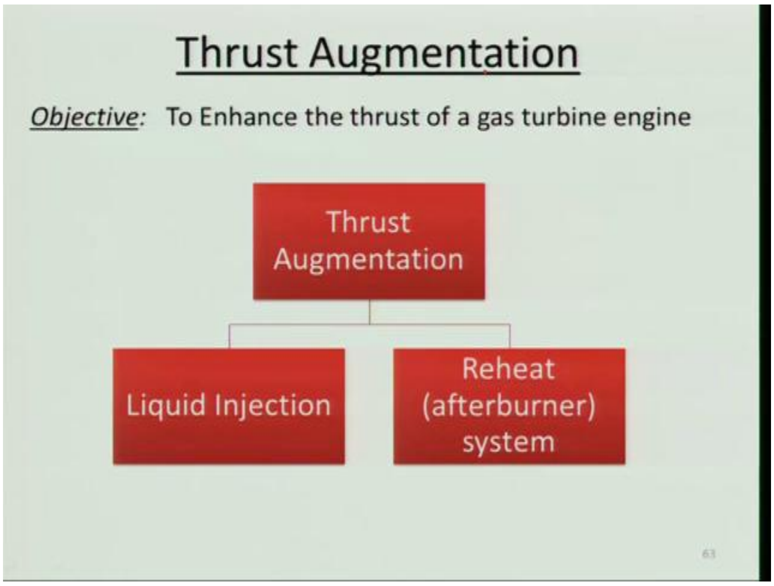
(Refer Slide Time: 15:09)



You will get this 1235.5 meter per second although your Mach number is 2, it is this exit velocity much higher than the flight velocity. So, T 9 can be thrust can be evaluated because by knowing m naught V 9 by V naught. So, you can get 32.27 kilo Newton, so you can substitute these values, similarly TSFC thrust specific can be evaluated by just substituting these values kind of thing. So, propulsive efficiencies and then thermal efficiency and overall efficiency, if you look at overall efficiency is I mean lower than that the total propulsive and thermal efficiency. So, if you look at you get and you have a feel what is happening and kind of things, so there is a small here, there will be kilo Newton kg per kilo Newton second that value.

Now, we will move to that how to enhance the thrust because whenever flight fighter aircraft you are having or even takeoff, you know you need to have higher thrust than the level flight at that place. So, you need to augmentation, so what are the ways and means of enhancing the thrust how I can enhance temporarily. Of course when I am designing, I can enhance by increasing what you call what you call more kind of velocity or the nozzle or you know I can change or I can reduce for the same turbine inlet temperature kind of thing, I can reduce the compressor work.

(Refer Slide Time: 16:16)



That is one way out to doing where reduce more energy can be for when I you know it can be utilize for an expanding in the nozzle get the higher thrust. That i can do, but I want it temporarily, how i will do that after burner is one way and another ways that we can inject you know like inject the liquid in the compressors such that the work can be reduce for the same pressure ratio. That means work input for the compressor for the same pressure ratio can be reduced, because suppose I put some alcohol or I put some water, a combustor particularly the high pressure combustors where temperature is high.

So, what will happen that will be vaporized and then some heat will be can be observed and temperature will be reduced. As a result this is known as inter cooling kind of things, you know which you might be study in your thermodynamic.

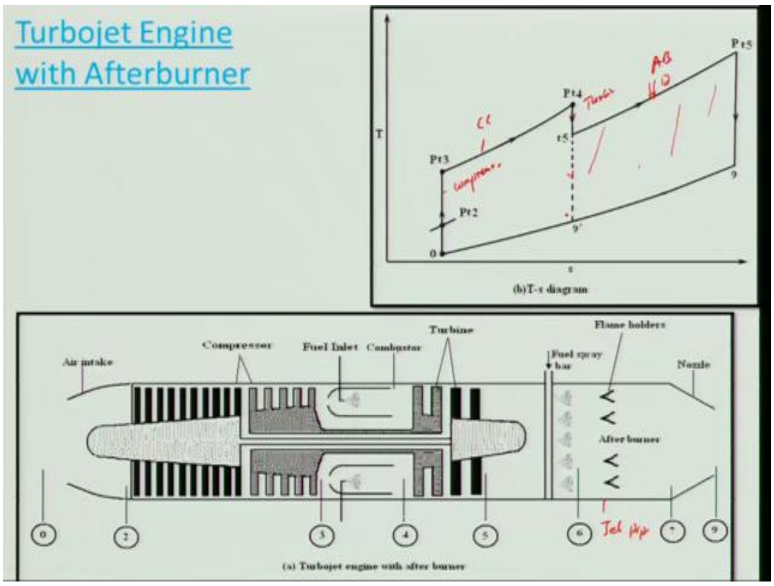
So, we will be doing temporarily inter cooling, so we will get what we you call the decrease in the work input to the compressor because we are decreasing the work input. As a result, what will happen, there will be what you call increases in the pressure ratio for the same work input because the turbine is already connected, turbine will be giving the same work. Then, you will increase in the pressure ratio for the particular engine for a fixed engine, then when it is even then rapidly go up and more mass flow rate will also come also into because for the same fixed cross sectional area.

So, there will be increasing thrust augmentation that is one way, but however it is not being used because to carry those and other things, it requires a more amount of thing there will be also choking of the compressor. Surging might occur, there is lot of other problem associated, therefore it is not being used because the critical turbo machinery in case of a turbojet or the any gas turbine engine in the compressor. Therefore, to play around that it will be difficult right and therefore, that is not being used.

However, it is a you know need not to increase the weight of the engine, but whereas the after burner which is being used because there will be lot of amount of oxygen which will be available even after it is combustor fuel being burned. So, you will add little amount of fuel or more amount of fuel it the exit of the turbine and enhance the temperature to greater than what is limited by the turbine material.

As a result, you will get the thrust, but there will be a problem so when you add heat it may chock your exhaust nozzle that sometimes it will be thermal chocking. Sometimes, it will be aerodynamic chocking; therefore it is important to have a variable nozzle whenever you are using an afterburner, so I mean that is very important to keep that in mind.

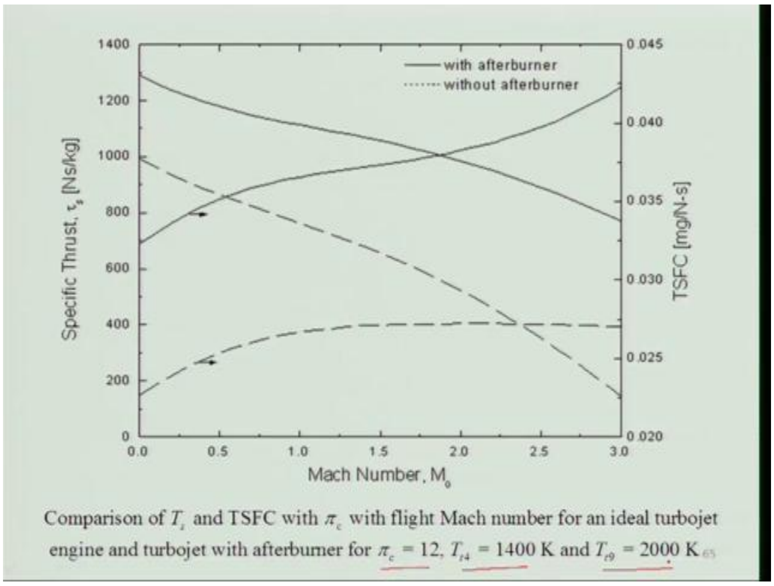
(Refer Slide Time: 20:18)



So, let us look at a simple turbojet engine with an afterburner, if you look at the fuel spray by the air, it is having fuel being sprayed and it is you know gutters are there or the flame holders various kinds of flame holders, which will help you in stabilizing flame. This is basically jet pipe right jet pipe which is there and then it will be expanded in nozzle. So, if you look at the processors what we are looking at it, of course 0 to 2 and then 3, it is basically air intake and then this is compressor and this is your combustion chamber and this is your turbine.

Then, you will have to you know, then add heat this is your afterburner, add heat and then you will take the temperature much higher than the what is can be obtain at P t 4, let say 1600 Kelvin, but you can go to 2000 Kelvin here because there is no rotatory parts. So, therefore, you can get a higher thrust and so this is a additional thrust what you will be getting you know these are the additional thrust what you will be getting, but that is temporarily and sometime and if you are not using afterburner. So, you will get you can expanded over here and get the thrust P 9.

(Refer Slide Time: 21:54)

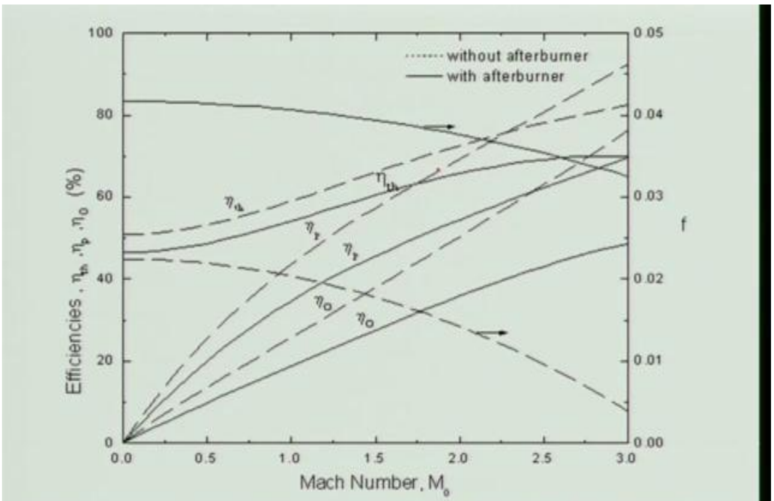


So, if you look at all these things are similar analyze analysis, so I will be not looking at it and only I will be looking what really happening carrying out a parametric analysis and what we are doing. We are taking the pi c as 12 and T t for as 1400 Kelvin and T t 9 as 2000 Kelvin T t 9 means afterburner at the exit of afterburner you know you can have it that temperature that will be detected by the materials. Also, how much you can add heat and if you do this and you do Mach number variation Because you are fixing this pi c and vary you will find that this solid line is basically with afterburner and dash line without afterburner.

That means you will see that specific thrust you know is varying decreases with flight Mach number, but whenever it is the afterburner heat being added and there is a increase in specific, but trend is similar. Of course, there is a little more differences in case of high of flight Mach number and if you look at TSFC without afterburner is a low which is expected because less amount of fuel will be consume, but at when the afterburner is being used. You see that the TSFC having similar trend, but except in these region there is there is increase in the TSFC that means thrust specific fuel consumption fuel consumption will be more.

So, therefore it is expected to be more because you are adding more and of course here it is divers because flight Mach number. You know that more amount of heat has to be given to get that kind of you know these things particular thrust, so you need to up higher increase in the TSFC.

(Refer Slide Time: 23:48)



Comparison of η_p, η_{th}, η_0 and f with π_c with flight Mach number for an ideal turbojet engine and turbojet with afterburner for π_c = 12, T_{t4} = 1400 K and T_{t9} = 2000 K

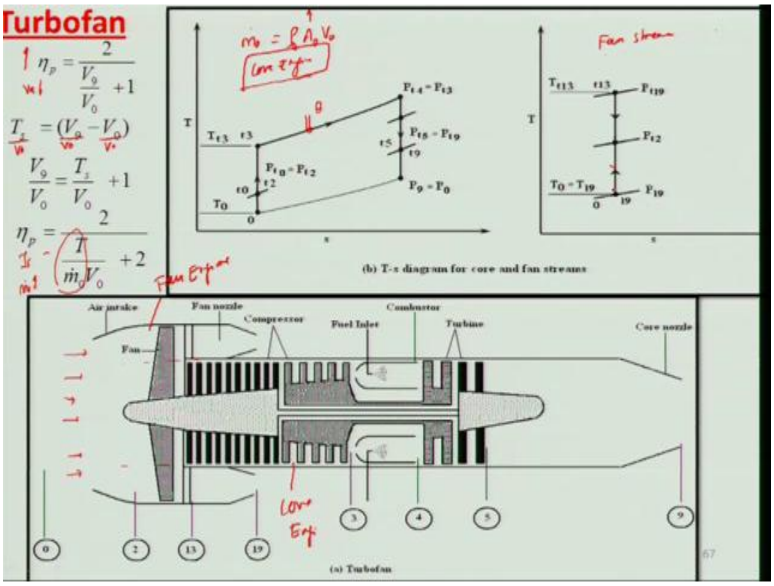
If you look at what really we are gaining in the propulsive efficiency and propulsive efficiency if you look at it is decreases because the exit Mach number exit velocity from the nozzle will be increasing. So, the heat will be decreasing and overall efficiency of course the thermal efficiency will be what you call increases. When you add this, you know when you add without any afterburner that is higher values than that of that with afterburner.

So, overall efficiency decreases, I mean with the what you call any Mach number whenever it take, so what we can learn from here that means you can get the Mach higher thrust, but paying a penalty of lower thermal efficiency and higher TSFC. So, here it is nothing you know you can really get, but however if you want to have a you know emergency particularly you know like when we are going taking a person who is you know emergency, you know hospital will have use ambulance at a higher speed.

Similarly, whenever you are winning a war or something important, then you will have to pay penalty, of course one should not worry about that only war for that technology has to be used it has to used for peaceful thing. So, that is very important, but however what we are saying that there is lot of scope or with the afterburner, there is a way of augmentation within the existing system under emergency conditions.

So, one can use it, but that depends upon how we can use our mind to use for particular peaceful purposes. Now, we have looked at turbojet engines, but however if you see the propulsive efficiency is being lower and TSFC is being higher when you compared to the turbofan engine. In other words, is there any way we can enhance these what you call TSFC or the propulsive efficiency or the overall efficiency, is there any way we can do that so one way of doing is that you reduce the V 9.

(Refer Slide Time: 23:36)



Let us look at the definition of propulsive efficiency for the particular flight Mach number or a particular flight velocity V naught, how can I enhance this propulsive efficiency given under ideal condition when I am when I am decreasing the V 9. That means if I decrease this exit velocity from nozzle, you know then only I can what you call enhance that means when V 9 will decrease the propulsive efficiency will increase.

However, what will happen to the thrust, so thrust if I do that if I can multi you know divided this by V naught and if I divide by V naught, I will get V 9 by V naught is equal to specific thrust. This is T s specific thrust divide by V naught plus 1 if I substitute these values in this propulsive equal relation for propulsive efficiency, I will get 2 divided by t by m naught that is nothing but your T s if you look at this is nothing but a specific thrust and V naught.

So, I want to enhance my propulsive efficiency for a particular flight velocity the flight velocity. So, how I can do that and particular thrust eleven, what is the way out that means I will have to increase this m naught what is this m naught m naught is rho naught, you can say a naught and V naught V naught is specific. So, rho naught is also fixed particular altitude if I take only one way is that I can do by increasing the cross sectional area of the engine. So, when I do that, then I can get the higher propulsive efficiency that means when I am actually increasing this m naught, what is happening.

This become a smaller quantity if this become a smaller quantity it is the propulsive efficiency will be increasing for a fixed thrust and fixed flight velocity that you keep in mind make sense to you does it make this sense to you. So, for that I will have to enhance this a naught because I cannot you know for a particular altitude when I am operating I cannot change the rho naught. I cannot change the flight velocity because that is fixed I am talking about that way.

So, therefore there is one way of doing and when you do that you will get a turbofan engine that means what we are doing we are for the same flight velocity using a fan and which will having a more cross sectional area. It will be you know setting a into that and we are having two is this thing, one is this is known as core engine and this is known as fan engine. That means you keep in mind that there is two part, one is core engine core engine will from here you know 0 to you can think of like this core engine air is coming and entering. We will be using the same station number that means you know zero o two and 3, 4, 5, 9 for a core engine.

For the fan engine, this is 0 to 39 degree, keep in mind that you will get a thrust in case of fan engine when it is expanded in nozzle that is nozzle that means there is 2 nozzles, here one is your fan nozzle other is your core nozzle right and if you look at the processes what is happening.

It will be you know this is for what you call core engine right and 0 to 2 is your air intake and 2 to 3 which is nothing but your pressure ratio across the both fan and the compressors, please keep this in mind. If i given in you know if it is compressor pressure ratio, it is for this one and where fan ratio as to be multiplied. That means that is the compression which is taking place you know from what you call 2 that station 2 to 3 because it is including fan.

Now, that is a usual way from station 3 to 4 the heat being added here, this is what you call combustion chamber then expansion in what you call 4 to 5 and what you call 4 to 5 is basically you know in the turbine. Then, in the nozzle core nozzle and this is for the fan stream and what is happening you are going from here 0 to what you call 2 that is your air intake. Then, this 2 to 13 that is your fan and then again you are coming back you know expanding it here till P 19.

Keep in mind that for a ideal cycle if it is going vertically and again these things in T s diagram and what will happen to the P V, I will leave in your imagination and think about it in the P V diagram. So, what we will have to do, we will need to find out basically you know expression for thrust. That means thrust will be having two components or the thrust will be contributed by both by the core engine and the fan engine or the stream whatever you call.

(Refer Slide Time: 29:48)

The exit Mach number M_9 can be expressed in terms of pressure ratio by

$$\left(\frac{P_{t9}}{P_9}\right) = \left[1 + \frac{(\gamma - 1)}{2} M_9^2\right]^{\frac{\gamma}{\gamma - 1}} \qquad M_9^2 = \frac{2}{(\gamma - 1)}\left[\left(\frac{P_{t9}}{P_9}\right)^{\frac{\gamma - 1}{\gamma}} - 1\right]$$

The pressure ratio P_{t9}/P_9 can be expressed in terms of pressure ratio across individual components as

$$\frac{P_{t9}}{P_9} = \underbrace{\frac{P_{t9}}{P_{t5}}}_{\pi_n} \cdot \underbrace{\frac{P_{t5}}{P_{t4}}}_{\pi_t} \cdot \underbrace{\frac{P_{t4}}{P_{t3}}}_{\pi_b} \cdot \underbrace{\frac{P_{t3}}{P_{t2}}}_{\pi_c} \cdot \underbrace{\frac{P_{t2}}{P_{t0}}}_{\pi_d} \cdot \underbrace{\frac{P_{t0}}{P_0}}_{\pi_r} \cdot \cancelto{1}{\frac{P_0}{P_9}} \qquad (1)$$

Assuming $P_9 = P_0$, Eq. (1) can be expressed in terms of pressure ratio across each component as

$$\frac{P_{t9}}{P_0} = \pi_n \cdot \pi_t \cdot \cancelto{1.0}{\pi_b} \cdot \pi_c \cdot \pi_d \cdot \pi_r \qquad (2)$$

If I just rewrite back, that it will be M 9 square is equal to 2 by gamma minus 1 p T 9 by p 9 gamma power to the gamma minus 1 divided by gamma minus 1. So, we will be looking at this p T 9 by p 9 and look at a various pressure ratios, so you look at a similar way and I have just written down it here and if you look at this is a fully expanded. So, this will be 1 and p t by p naught what it would be this will be your pi r and p t by p t, p t 2 by p T naught. This is nothing but your pi d and this is p t 3 by p t 2 is pi c and p t 4 by p t 3 is nothing but pi b and p t 5 by p t 4 is your pi turbine and this is your nozzle you can say.

We know that this what you call and we can express in this similar in terms of pressure ratios and under ideal condition, what we can say we can say that pi b is equal to 1. What happens to your turbine pi t can I say 1 pi c i can say one no and pi d can I say 1, total pressure ratio what happens in the pi d no compressor no turbine. You are saying because we have you know getting sometimes doing the work on it and extracting the work in the turbine, so what about pi d. What we did in the ramjet engine total pressure what is happening ram pressure will be what total pressure will be remaining same or it will be different this is isentropic process.

So, what about nozzle this also one because we are just converting you know kinetic energy into the where into the static pressure or a dynamic air to the static air in case of the air intake and this is other way around then nozzle is just opposite. The static way you know like static pressure we are converting into dynamic, but the total pressure is remaining constant. Therefore, you will have to be you know understand this concept, so pi n pi b and pi d is equal to 1.

(Refer Slide Time: 33:14)

But, for an ideal engine $\pi_n = \pi_b = \pi_d = 1$. Now, Eq. (2) will be

$$\frac{P_{t9}}{P_0} = \pi_n \cdot \pi_t \cdot \pi_b \cdot \pi_c \cdot \pi_d \cdot \pi_r \qquad (2) \qquad\qquad \frac{P_{t9}}{P_9} = \pi_t \cdot \pi_c \cdot \pi_r \qquad (3)$$

Using Eq. (3) the exit Mach number M_9^2 can be expressed as

$$M_9^2 = \frac{2}{(\gamma-1)}\left[\left(\frac{P_{t9}}{P_9}\right)^{\frac{\gamma-1}{\gamma}} - 1\right] = \frac{2}{(\gamma-1)}\left[\left(\pi_t \pi_c \pi_r\right)^{\frac{\gamma-1}{\gamma}} - 1\right] \qquad (4)$$

Since, $\pi_r = \tau_r^{\frac{\gamma}{\gamma-1}}$ and for an ideal turbojet $\pi_c = \tau_c^{\frac{\gamma}{\gamma-1}}$ and $\pi_t = \tau_t^{\frac{\gamma}{\gamma-1}}$, Eq. (4) becomes,

$$M_9^2 = \frac{2}{(\gamma-1)}\left[\tau_t \cdot \tau_c \cdot \tau_r - 1\right] \qquad (5)$$

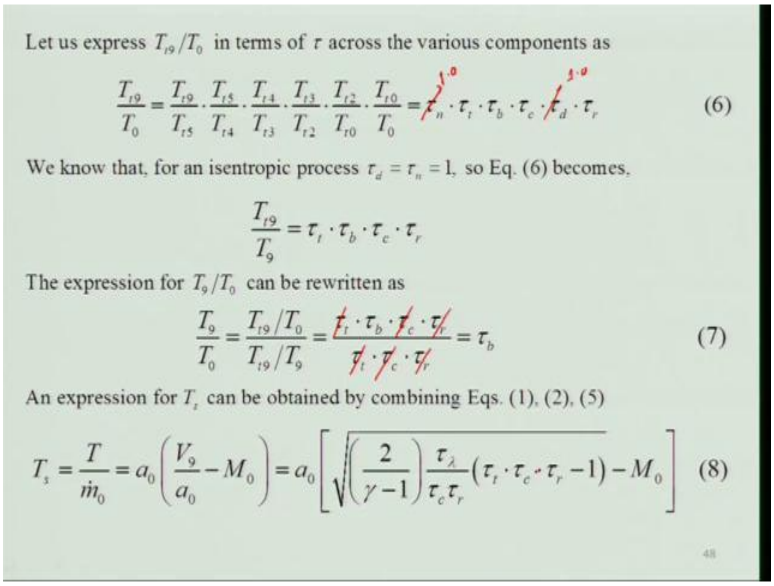
But, we can express T_9/T_0 in terms of temperature ratios as

$$\frac{T_9}{T_0} = \frac{T_{t9}/T_0}{T_{t9}/T_9}$$

So, this equation becomes you know like p t by p 9 pi t pi c pi r in case of ramjet engine pi t is equal to 1 pi c because there is no compression, no turbine. So, you can directly get this expression that means if I know this equation, you know expressions I will get ramjet turbojet just making some parameter one. You know that why that is a beauty of this analysis, I need not to do anything if I say pi d pi c is equal to 1. That means it is ramjet that is the beauty of this relationship where computer you can do very easily. I can put a condition and do that ramjet same equation you know so that you should must appreciate of this relationship and using equation 3 exit Mach number, I can get because I have already derived that is nothing but same as that and except this.

You know pi t an d pi c being coming to as compare to that ramjet engine, so if you look at this similar to that what we have done for the ramjet only two terms are come for the turbojet pi d and pi c. So, pi r you know is we can relate to the tau r gamma power to the gamma minus 1 and similarly, for pi c and pi t these are isentropic relationship, pi means it is the pressure ratio tau means it is the temperature ratio. So, we know that pressure ratio can be related to the temperature ratio with the help of this gamma, you know of the index. So, M 9 you know square, we can write down here itself if I put these values over in this place, you know and also for pi r i can get instead of pi i can get in terms of tau that is tau t tau c tau r.

(Refer Slide Time: 35:38)



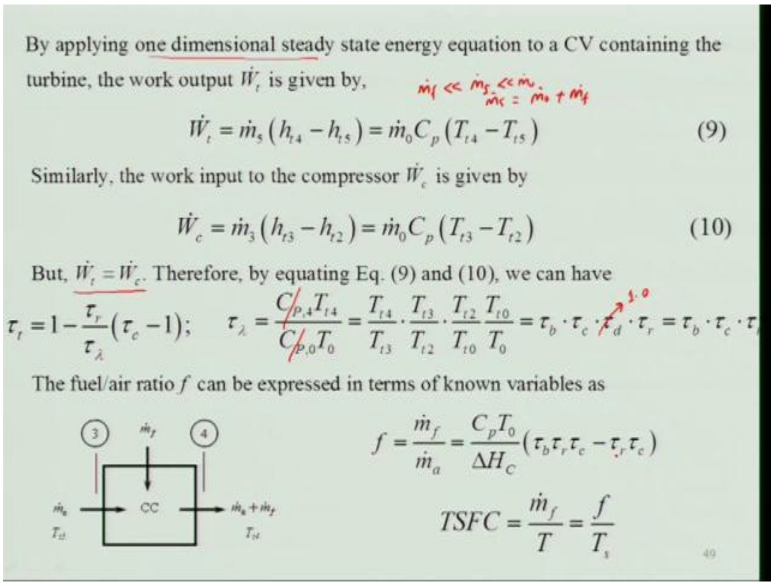
So, we will do the similar way of getting this T 9 by T naught you can write down t T 9 by T naught if your t T 9 by T 9 and we will go on doing that all those things if you look at it will you will be get this tau n tau c tau t tau b tau c tau d and tau r. If you look at tau d is what is it one because the adiabatic process in there air intake there is no heat. Therefore, that will be one, similarly in the nozzle is it you are adding some heat, you cannot you are not doing anything. Therefore, it will be 1, so what about tau c and tau t can I make it i 1 and tau b is certainly no because I am adding some amount of heat can i make it.

I cannot make the tau t and tau c as 1 that means the total temperature across the compressor and across the turbine are changing. So, it cannot be one because in one case compress you are giving you know amount of work and in that the turbine you are extracting the work from the fuel it. So, there will be change in total temperature otherwise if you will get other thing. Therefore, you cannot really make it one, so t T 9 by T naught it will be tau t tau b tau c tau r and again you can say this here you can appreciate this point the tau c and tau t will be one in case of ramjet this same thing I am repeating. So, that it will enter into your mind so the expression T 9 by T naught can be really if look at is a interesting.

If I put this t T 9 by T naught all those things this is can and also t T 9 by T 9 you will see that it is can be cancel it out. This, can be cancel it out and the tau c, you can cancel it out it happens to be tau b and what we have seen in case of your ramjet engine yes or no. We are doing the same thing, but make it little complex, but this is cancel it out which is obvious because the turbo in the turbojet engine. The, work you know harness by the turbine is being utilized by the compressor therefore, it must be T 9 by T naught will be tau b it will be how much heat added into the combustors that is the thing we were saying and it is true also Ramjet.

So, the T s you can just substitute these values V 9 by V naught and you will get a expression which looks to be little frightening, but however it is quite simple. I am not expecting that you should remember this expression, but however you must know how to go about how to derive it that is expected.

(Refer Slide Time: 38:49)



So, which will be quite simple to derive these expressions and that is very important; we need to now relate this turbine work into the compressor work, because we will have to get the relationship between the compressor and turbine. So, we will assume the one dimensional steady state energetic way, you know these things. We apply this to a energy equation control volume and the turbine work will be m dot 5 is equal to s t 5 minus s t 5, because the in the turbine.

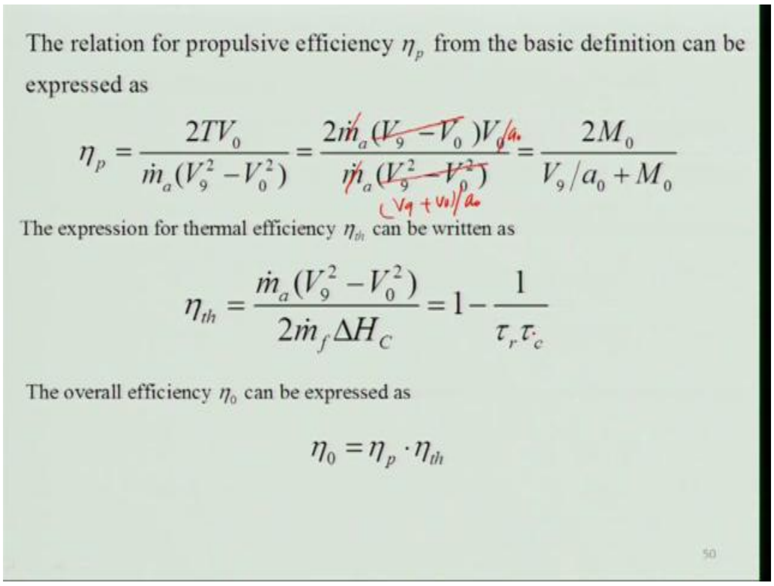
You keep in mind that here we are assuming, you know like the mass is continuity is maintain berceuse one dimensional flow we are talking about M naught 5 is equal to M naught 4. You are not adding anything expect you know, so same fuel it is going therefore, which is nothing but that M naught not keeping in mind that here I am saying it is very, very less than M naught 5 or is very less than M naught. Therefore, I am saying this is M naught is equal to M naught 5 basically M naught 5 is equal to M naught plus M naught f, but I am assuming this.

Therefore, M naught f C p T t 4 minus T 2 5, similarly I can have for the compressor, which is same as that M naught C p T t 3 minus T t 2 and when we quit this work done by the turbine is same that as work you know taken by the compressor. Then, we will get an expression you know tau t is equal to 1 minus tau r by tau lambda in bracket tau c minus 1. Keep in mind that we can express in terms of tau lambda because that is a thing how much heat you know being added or the total enthalpy at the exit of the combustor divided at the amount of enthalpy entering into the engine.

So ,if you look at it is this, you know it can be rewrite this because this is cancel it out T t 4 by t t 3 into t 3 by T t 2 into t 2 by t T naught and t T naught by T naught. So, you can write down in terms of all these tau b tau c this is compressor and tau d is for air intake and tau r. You know that tau d is equal to 1 and because of it is adiabatic process we are saying no heat is going out, but in real situation it cannot be, but in ideal situation it is ok. So, the fuel air ratio can be express in terms of known variables as you know we will have to consider these combustors, which we have done a similar thing in case of ramjet engine it will same.

So, we can write down m dot f divide by m dot a nothing but C p T naught delta h c which you will see that tau b tau r tau c minus tau r tau c. You see some step I have omitted here, but you can do very easily and in case you will find some problem. Let me know and TSFC you can get very easily that is f divided specific thrust and you will put this values. You know now we are having several terminologies are coming like several ratio temperature ratio of pressure ratio. Keep in mind that we can you know change, this tau c in terms of pressure ratio compressors. Similarly, tau p we can you know some places wherever it required we can change into the pressure ratio across the turbine I use in the isentropic relationship.

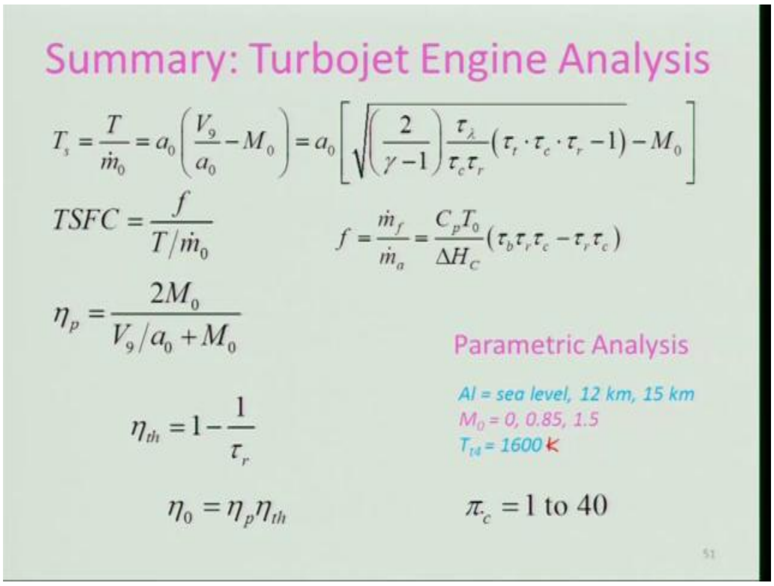
(Refer Slide Time: 43:06)



So, the relationship for propulsive efficiency we can write down 2 T V, basically the thrust power divide by kinetic energy change in the engine that is m dot V 9 square minus V naught square divide by 2. If I just you know write in in place of thrust in this expression m dot into V 9 minus V naught, you will see that I cancel it out and this will cancel it out and I will get V 9 plus v naught. If I divided by a naught square here, similarly a naught square. Here, I will get two Mach number divide by V 9 by V naught plus M naught. So, this is an easier way of saying that because I know expression V 9 by a naught I can do very easily.

Also, you can see that how these Mach number is really affecting your propulsive efficiency and thermal efficiency you can get you know do all those thing algebra. You will get 1 minus divide by tau or tau c keep in mind that here tau c is coming to the picture in case of the turbojet. Earlier, it was only 1 minus 1 over tau r in case of Ramjet tau c is equal to 1 in case of Ramjet. So, propulsive efficiency overall efficiency will be nothing but propulsive efficiency multiplied by thermal efficiency will give nozzle that means by this we have derived all the expressions. We are armed with all the expression to the carry out parametric studies, and just to summarize what we have derived.

(Refer Slide Time: 44:57)



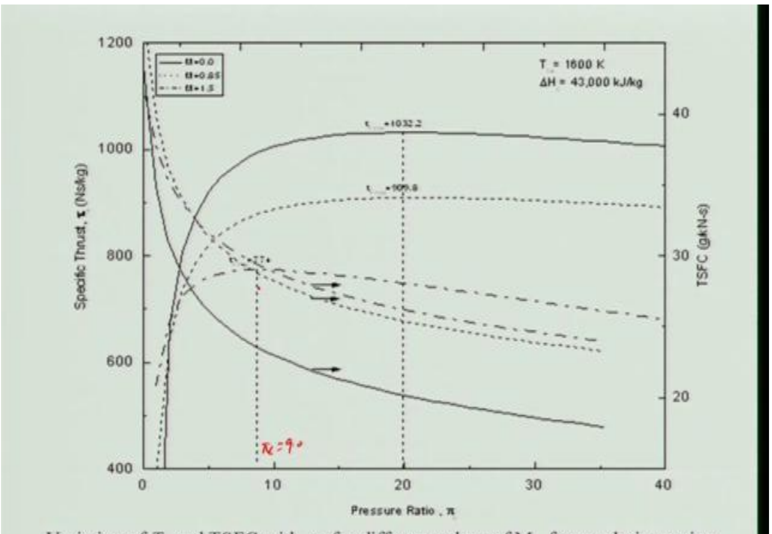
That is the specific thrust which we have done and these are the expression which in terms of various tau b tau c tau r. We are using also tau lambda and in place of tau lambda may be you know this term tau lambda divide by tau c tau r is nothing but tau b some places you can use tau b as well. So, what is helping this specific thrust expression is basically separating each parameter. It is arming with us or helping with us say that we can do parametric analysis that you must appreciate. Otherwise, I need not to go for this you know unless I am not interested in parametric studies and which is essential when I am trying to understand you know how it is the performance is affected by the various parametric efficiency.

That is f c and we know that f is you can be express in terms of tau b tau r tau c minus tau r tau c multiplied by C p T naught divide by delta H c. and these are expressions for propulsive efficiency thermal efficiency which we have discussed just now and what we will do now we will take three cases. One is the Mach number of je w that is the sea level conditions. Another we will take a long range you know vehicle or other aircraft that we use passenger rate at 0.85 which can operate at 12 kilometers altitude. Then, the fighter aircraft which is corresponding to 15 kilometers altitude value and we are keeping this the turbine, you know like inlet temperature or combustor exit temperature is 1600 Kelvin.

So, and what we will do now what are the variables we will be using we will be basically using this, you know we can vary this pressure ratio across the compression unit. You can say that why not vary the pressure ratio across the turbine. You can do as well both are you know can be related basically with the work you know because a work what about you are getting from the turbine is being utilized by the compressors, so generally compressor is very important one.

Therefore, we use the compressor as parameter that you will be varying from 1 to 40 and see that how we can choose a compressors. How we can choose a, you know pressure ratio, because the compressor size and you know cost will be dependent on the pressure ratio you are want to have for each, so that is a very important one.

(Refer Slide Time: 47:55)



Variation of T_s and TSFC with π_c for different values of M_0 for a turbojet engine at TIT of 1600 K

So, what we are looking at this specific thrust being plotted on the y axis and the pressure ratio plotted on the x axis for a turbojet engine which is having turbine inlet temperature of 1600 Kelvin. We are also varying this Mach number as I told you three cases we have taken representative, case one is sea level conditions or you can say static conditions other is 0.8. Mach number flight, Mach number and other is 1.5 or you can note here that the when this flight Mach number is 0 this solid line the specific thrust increases with the increase in pressure ratio across the compressor. Then, it reaches the peak values here and then after that it decreases fully.

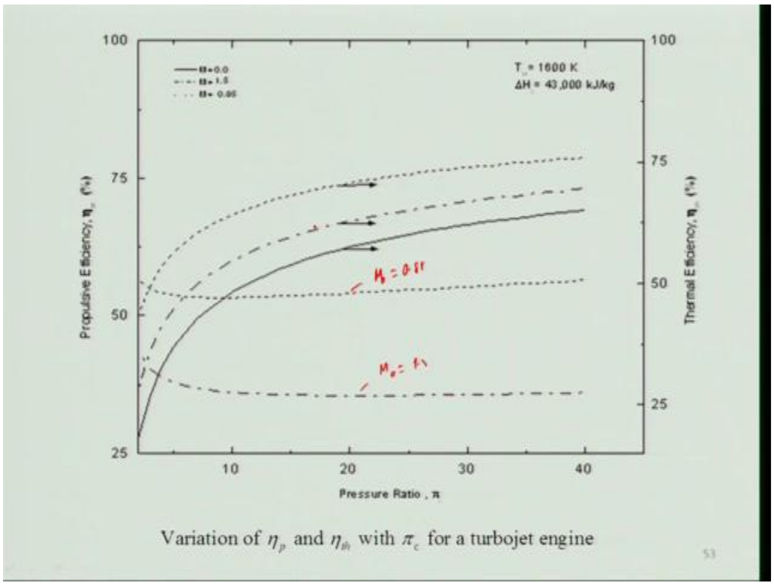
So, what it indicate that means there is a optimum value for the what you call pressure ratio across the compressor for which you will get the maximum specific thrust and the however if you look at TSFC which I have shown here. It is a very high values at the low the pressure ratio of one, you know because we cannot have zero pressure ratio across the compressor. So, one is you know that that means no compressor factors at that value it is going towards infinity very large. It has no meaning and you cannot have any meaning turbojet engine having the pressure ratio of 1, but however it decreases.

You know it decreases continuously as you increases in pressure ratio, you will see that this is having not a minimum values. Unlike in a ramjet engine, there is a minimum values and of course, you will have to choose that, but there is another interesting thing you can observe. When I will go for the flight what you call Mach number of 0.8. You will see that this is having, you know it is also having similar features goes on increasing and then of course, it is having a certain values of phone t and looking where you get the maximum specific thrust, but if you go for the Mach number of 1.5, you will have similar features of the curve.

It is decreasing you know after reaching a value of certain maximum value at a pressure ratio of 9, 5 t v is equal to 9, you are having and then it decreases little at a higher rate as compared to the both the sea level conditions and other static condition and the flight Mach number. What it indicates, it indicates that when the flight or the engine is moving at a higher speed, it can used that pressure and you need not to go for a high pressure ratio compression because the ram pressure can utilize two kind of for increasing its pressure.

So, therefore if you look at the fighter aircraft, we will have this smaller compressor because you would need a low pressure altitude, whereas you go for a long range passenger aircraft, you need to have to go for a twenty kind of engines. At the same diameter, what thrust you need to because at a static conditions with static process is also important. So, you will have to play around and see that what really you need and what is the level blight flight. So, these are the things you can get by just doing you know specific parametric analysis that is the beauty of this method. You can learnt a lot by just playing around and see what is happening, why it is happening and what are the reason whether you can have any scope to improve it further.

(Refer Slide Time: 52:44)

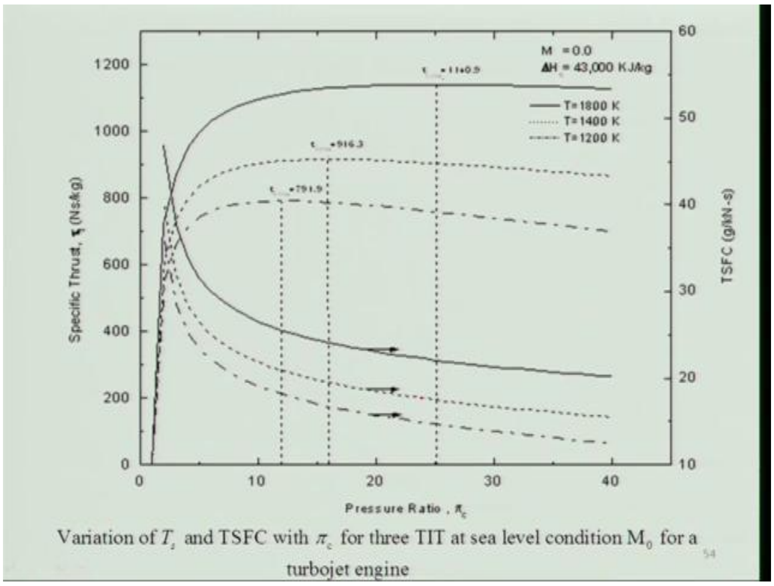


Variation of η_p and η_{th} with π_c for a turbojet engine

So, let us look at a propulsive efficiency and the thermal efficiency if you look at these are the curves which is you know basically the propulsive efficiency. These two curves and it is a static condition there will be no meaning of having propulsive efficiency even Mach number type number is equal to 0. So, what you call this one is basically Mach number of 0.85, it is having higher propulsive efficiency as compare to 1.5.

It is having similar feature features in the when it is pressure ratio increases, it is very higher over here and decreases may be at 7 pressures, it will minimum which is not very obvious in this diagram. When you look at number, it indicate and thermal efficiency, it is goes on increasing, you know from the lower pressure ratio to higher pressure ratio and you will get a lower thermal efficiency. In this case it indicates you know zero flight Mach number and when you increase this 0.85, you know you will get thermal higher efficiency when it is the 1.5 kind of thing, then it decreases because of you know what your energy utilization wants.

(Refer Slide Time: 54:08)



Variation of T_s and TSFC with π_c for three TIT at sea level condition M_0 for a turbojet engine

So, what we will do now, I will just look at you know what happens the effect of the temperature ratios and which is having similar views you can see that this is having highest temperature you know like 1,800 Kelvin. So, if you look at you will have a higher pressure ratio, you need to get to get a higher specific thrust and as a temperature decreases, you will get a lower specific thrust. You know like a maximum you know specific thrust at lower pressure ratios, because you know you need to have a higher what you call the temperature to be achieved. Therefore, you need to give you know more amount of pressure and you can get a higher kind of value and after that you peak reaches.

So, the higher temperature when you want to get and you want to get also higher specific thrust. So, naturally you will pay penalty for having a higher TSFC and all having similar view you know a value what you call trend like as you goes on decreasing the turbine inlet temperature, you will go on decreasing the TSFC. So, you will have to make a these things where you want to go and with this I will stop over in the next class, we will take an example to see how we can solve this problem.