Fundamentals of Aerospace Propulsion

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Lecture - 29

Let us summarize what we have learnt in the ramjet engine analysis we have basically derived expression for various parameter, like specific thrust specific fuel consumption and other efficiency ratio.

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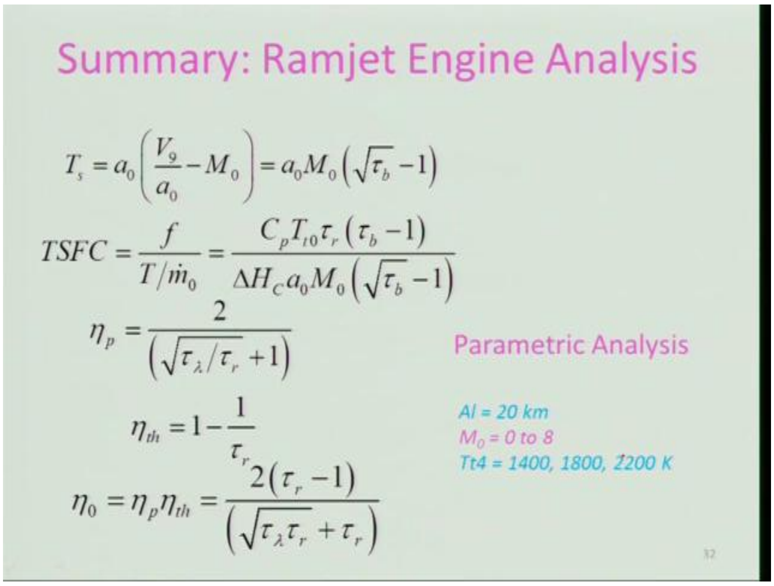

If you look at this thrust efficiency fuel consumption, we are look that it and derive this expression you can note from this expression that specific the thrust specification fuel consumption is dependent on the altitude. If you look at a naught and it is altitude which is tell us whether it is you know what temperature and flight Mach number. It will be dependent upon how much heat you have added the tau b that is the burner or for the combustor. So, we can find out expression for TSFC and these are parameters apart from this what you call tau b M naught a naught is also dependent on tau r.

It is again function of Mach number if you look at this and of course the kind of fuel you are using that is heat of what you call combustion and we have looked at propulsive efficiency. Here, we are dip using another parameter that is tau lambda and apart from the tau r, so thermal efficiency is really does not depend on the tau b. You know when I am saying tau lambda tau b is you know inside this propulsive efficiency. So, overall efficiency is nothing but you know multiplication of propulsive efficiency thermal efficiency what we will do with this equations. You can say look we have derived it, but our objective is to carry out the parametric analysis.

So, we can see which parameter is important how to optimize and how to go about under ideal conditions and that is why this is known as design analysis this can be use as a tool for designing. You can think and keep in mind that we do not have any geometry, we are just talking about. Therefore, this sometime known as Robber engine, you know like nothing is there, but thinking heat will be there and then looking at it. So, what we will do, we will look at parameters and we will take this you know data that is altitude. Let us say ramjet is flying at altitude 20 kilometer, generally it will be till higher altitude and Mach number.

We have varying 0 to 8, of course 0, I cannot say because I have just taken you know just to completeness and T t 4 that is the turbine, sorry that is the exit temperature of the combustors it is 3. We have taken 1,400, 1,800 and 2,200 Kelvin. We are basically using this equation may be in a computer program or something to do a lot of calculation one can do in hand, but generally you can write a small program.

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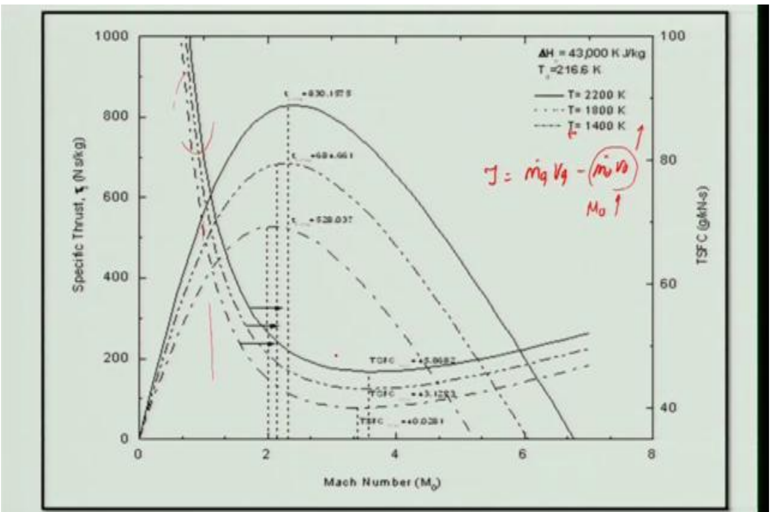

Variation of T_s and TSFC with M_0 for T_{t4} = 1400, 1800, 2200 K for ramjet engine

Then, generate data and plot it and see how it is varying and analyze these data plotting is not enough. You will have to derive some inference from the data as I always say all information you know cannot be converted into knowledge. Of course, information does not mean knowledge, knowledge has to obtain from the information all informations are not data.

So, you need to understand that so let us see that what we are doing here, we are basically plot plotting this specific thrust verses the Mach number and for p temperature like 1,400 1,800 and 2200 Kelvin. If you look at this curve is basically for the 1,400 Kelvin temperature T t 4 this is basically T t 4. If you look that this is T t 4, so what you can note from here that is specific thrust with the Mach number for a particular com exit temperature combustor exit temperature. It goes on increasing till it attains a particular maximum values at particular Mach number, then again it decrease question arises why it is happening.

Another interesting thing you can observe that at when Mach number is 0, the thrust or the specific thrust is 0. If I will increase this temperature to 1,800 Kelvin you will get the similar features almost like it is increasing and decreasing, but however its peak value of course it has increase from the lower temperature to higher temperature.

Also, the peak specific thrust occurs at a little higher Mach number and similarly for the when you go for the 2,200 Kelvin. You will have similar value only the little shift in the maximum specific thrust at a little higher Mach number only that is the difference, but it is similar if you look at that, the Mach number increases what happens you get the ram pressure in case of ramjet engine.

So, the pressure is higher and you will get the higher thrust, of course you are adding heat, but as soon as it will or when it attains a maximum value like that an optimum Mach number. Basically, then what happens why it decreases because suppose you are flying at you know let say 3 Mach number for a lower temperature. So, what will happen at a temperature at the exit of your air intake, it will much higher and then there is not much heat you can add to the fuel because the restriction on the temperature 1,400 Kelvin.

So, then you know like what will happen some of the thing really decreasing out decreases and it became whatever your energy you are giving and at the higher Mach number whether it is coming. So, it cannot be converted into thrust because the drag if you look at the inlet drag will be much higher in this case what we are doing. We are saying that exit thrust because what you call high velocity hot gas is coming out of the chamber. As a result you are getting a thrust, but here as I am increasing, for example like if I am increasing the flight Mach number, then your thrust will be decreases.

For example, if you look at m 9 v 9 minus M naught, V naught, so as I am increasing this Mach number, you know if I am increasing the Mach number what happen this become higher as compare to this because this is limited by air. How much heat, you can add and how much you know kind of things, therefore as a particular heat input you know this goes on increasing. So, that mean negative, therefore this heat decrease that make sense to you any doubt because this part is decreases because your inlet drag is a higher with the higher Mach for the same engine keep in mind same engine.

So, there is another interesting thing, you can look at it that is what you call the specific thrust it decreases like at a lower Mach number. It is quite high as it Mach number increases Mach number I mean then it reaches the small value. Then, after that you goes on increasing, but in a very low, so if you look, but in this region like where the Mach number is very low.

Although you are getting a specific thrust in this Mach number, but the TSFC is very high, but when what happens at the 0, 0. It will be infinite very high that means it has no meaning. You know like that way therefore, you know the problem of 0 static thrust at Mach number 0 is really difficult problem for the ramjet engine.

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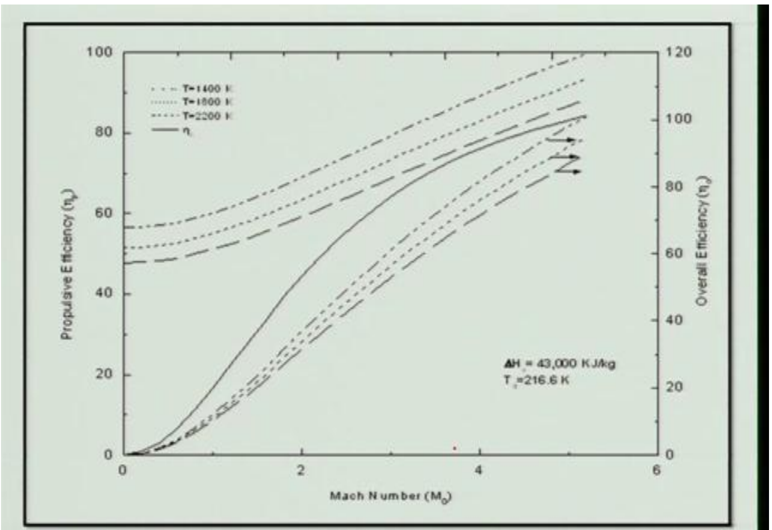

Variation of η_p and η_{th}, respectively, with M_0 for ramjet engine

So, you can also note that TSFC goes on decreasing as basically you are decreasing the temperature. In other words when I am increasing the exit temperature from the combustors that TSFC goes on which is obvious because as you are want at higher temperature you need to consume more fuel.

So, if you look at the propulsive efficiencies these are basically what you call is increases with the flight mac number for the same temperature, overall thermal efficiency does not depend on the Mach number on the increases in temperature. It does depend on the Mach number because it goes on increasing with the flight Mach number and overall efficiency multiplication of 2. You can see these are the overall efficiency which is increases with the flight Mach number. So, I mean this is the thing what we have learnt in this and we can also carry about several kind of parametric variation try to understand what is happening with this case.

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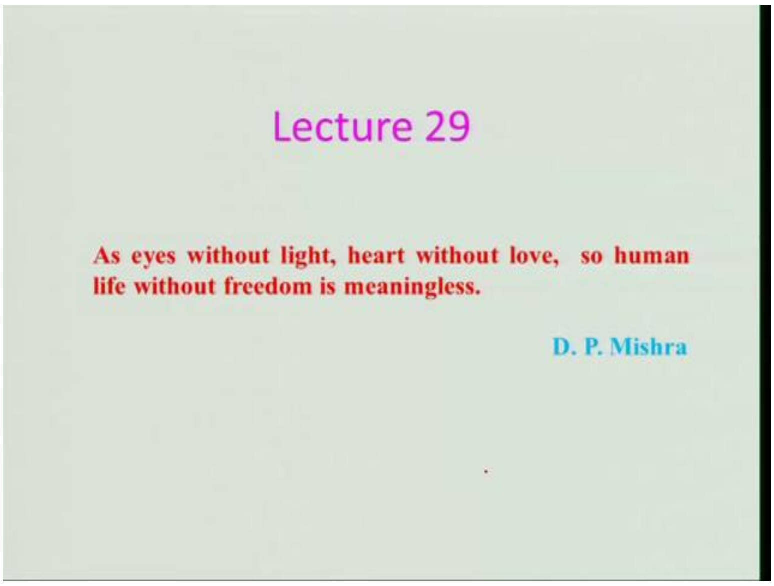

So, let us start this lecture with the thought process as eyes without light heart without love. So, human life without freedom is meaningless, so let us look at what we learnt in the last lecture we have basically looked at ramjet engine and carried out the analysis. We have also looked at parametric studies and how these thrust or the specific thrust will be varying with the flight Mach number for a range of the temperature. Temperature means combustion exit temperature combustor exit temperature and we have looked at propulsive efficiency, overall efficiency, thermal efficiency, now what we will do, we will be looking at basically an example.

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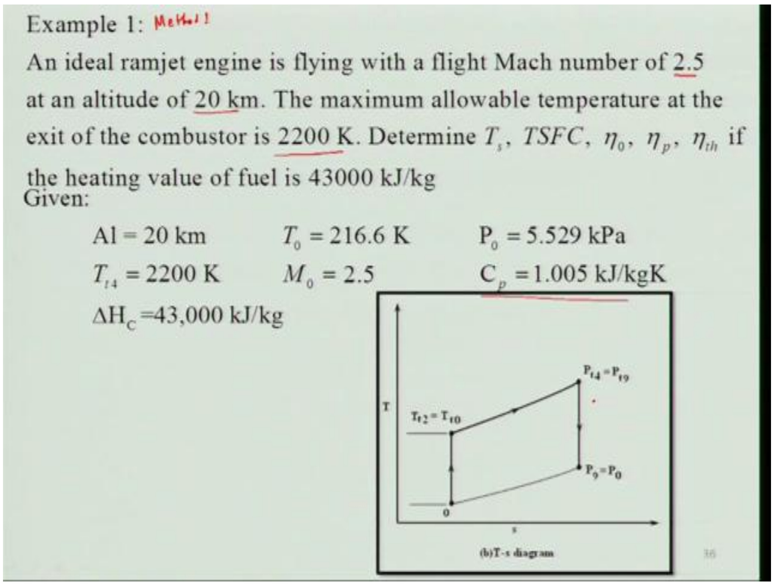

I would take the same example and used another method you can say this is the methods one and I am doing that. Now, this method, so what is that an ideal ramjet engine flying with a flight Mach number 2.5 altitude of 20 kilometer, maximum allowable temperature is 2,200 Kelvin. We will have to determine specific thrust plus fuel consumption and all other efficiencies and these are the given like altitude is given and temperature t 4 all those things i have already talked about and C p.

We are assuming 1.005 kilojoules per kg Kelvin, so this is the process what to be we have looked at T s diagram, you know like 0 to 1 is 0. Basically, compression or RAM compression and this is the heat addition combustion from t 2 the station 3 to the 4 and this is your expansion nozzle. So, what we will do we will basically know these values we will look at it kind of things and then do the input method one.

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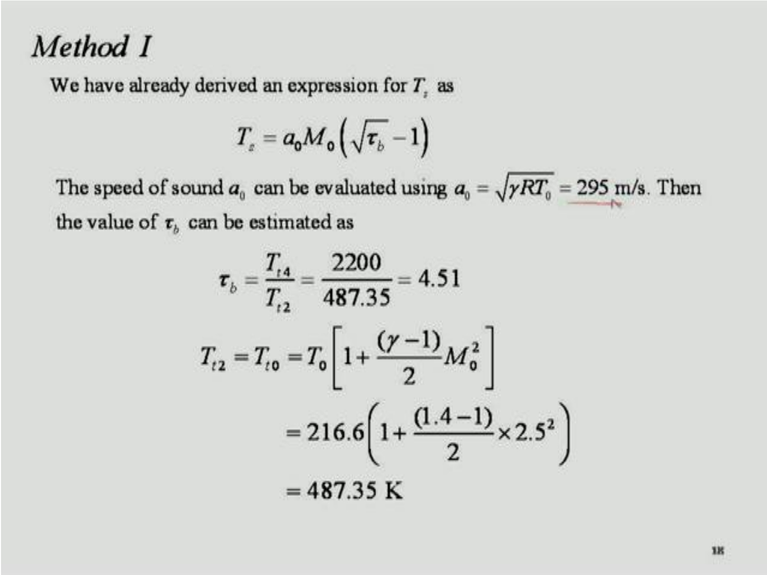

As I told you have already drive those expression for the specific thrust, so what we will do we will just we know these values this is 2.5, do you know that and we will find out a naught. Of course we know altitude we know the temperature and knowing that we can find out a naught is equal to root over gamma r t. So, that is a naught is equal to root over gamma R T naught and we get this, so we do not know that and if you look at this specific thrust. Then, we need to find out tau v tau v is what that is by definition it is T t 4 by T t 2 we know T t 4 that is 2,200 Kelvin which is given and we need to find out T t 2.

So, how will find out T t 2, T t 2 is equal to t T naught, we know T naught into 1 plus gamma minus 1 divide by 2 M naught square. We will multi substitute these values, this is given from the altitude 216.6 Kelvin when you substitute values, you will get 487.35 Kelvin and then you substitute you get these values. Once, we get these values, we know basically what about the parameter in specific thrust that is a naught is known M naught is known tau b is known, so you can substitute those values and get the specific thrust.

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Then, T_s can be easily determined as

$$T_s = a_0 M_0\left(\sqrt{\tau_b}-1\right) = 295\times 2.5\left(\sqrt{4.51}-1\right) = \mathbf{828.71\ N.s/kg}$$

The expression for TSFC can be used to evaluate its value as

$$TSFC = \frac{C_p T_0 \tau_r \left(\tau_b - 1\right)}{\Delta H_C a_0 M_0\left(\sqrt{\tau_b}-1\right)} \qquad \frac{T_{t0}}{T_0} = \tau_r = 1 + \frac{\gamma-1}{2} M_0^2$$

Substituting the respective values in the above equation, we get

$$TSFC = \frac{1.005\times 216.6\times 2.25\times\left(4.51-1\right)}{43000\times 295\times 2.5\left(\sqrt{4.51}-1\right)} = 4.82\times 10^{-5}\ \text{kg/N.s}$$

$$= 48.2\ \text{mg/N.s}$$

Now, η_p can be determined as

$$\eta_p = \frac{2}{\left(\sqrt{\tau_b}+1\right)} = \frac{2}{\left(\sqrt{4.51}+1\right)} = 0.6403 \overset{\times 100}{=} 64.03\%$$

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So, if you look I am just substituting these values and then I am getting 828.71 Newton per second per kg because heat is per unit mass of air flow rate specific thrust is be thrust divided by the mass flow rate of air which is entering into the inlet of the engine. So, the expression for the TSFC can be used to evaluate its values, we know this by definition C p T naught and tau b is already we evaluated delta s c known a naught is known M naught is known.

You just substitute these values and you get, so do you know this tau r tau r is a very simple thing what we will have do tau r is equal to 1 plus gamma minus 1 divided by 2 m r square. So, I know these values I know gamma, so I can find out what it is and tau r is basically what you call t T naught by d naught or p t 2 by d naught what about rather the basic definition will be T naught.

So, if I substitute these values, I will get 48.2 milligram per Newton second, I have put it milligram so that it will be easier. Otherwise, you can put it in SI unit kg per Newton second and propulsive efficiency can be determine very easily and 60. You know if you substitute tau v in this place, you will get 64.03. Keep in mind that I need to multiply by 100, so in percentage we always feel comfortable, therefore we put that.

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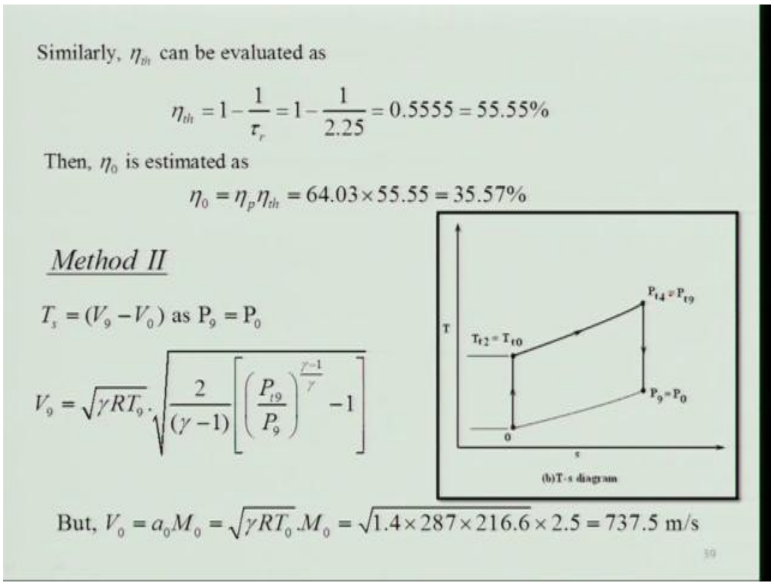

So, similarly thermal efficiency we know this tau r 2.25 and you can get to 55 percentage and when you multiplied these things, you will get the thermal efficiency and overall efficiency 35.57 percentage even under ideal condition which is the quite low value. You know if you compare to your other engines, now what we will do when we are doing these things, you do not have a feel about let us say you know what is the exit Mach number or what is the exit velocity and what is really happening. So, there is another way, you know of that you need to remember those you know formulas particularly in examination.

So, what will be doing we will be looking at direct you know this one and look at each point like stations points and see what we can do. Sometimes, we may used this you know methodology to some extend to make this, but this will give a physical feel when you are looking at it, but those things whatever we have derived those expressions are mean for computer program. So, it is more mechanical in nature, so I would ask to use the method two particularly while you are solving and try to understand the problem, but whenever you want to mechanize it. You want to do a several repeated calculations, then you can adopt the method one and which is essentially design for a what you call parametric studies.

Nowadays, computer is available you can use that very easily without really thinking much. So, what we will do we will basically looking at a specific thrust is equal to v 9 by minus V naught and of course we are assuming p 9 what is the v 9 v 9 is root over gamma r t 2 divided by gamma minus 1 p t 9 by p 9 power to the gamma minus 1 gamma divided by gamma minus 1. So, that means I need to determine this t 9, I need to determine this p 9, do I know really p 9 p 9 I known it is nothing but your p naught, but do you know p 9. You know the all things you know the p 9, similarly we will have to find out p t 9 and p naught we need to find out.

That is a very simple one which is same as that of previous method that is a naught M naught and you just substitute these values. You will get 737.5 meter per second. At least you are having a feel what is V naught, but in that other method you do not know really you substitute the values of M naught. If you do not what will be the velocity with which you know ramjet is flying, so that gives a feel and as I told you earlier what will have to do. We will have to basically find out from here and go to this point, find out various properties pressure, temperature whatever it required.

Similarly, I will have to go this point to t 4 and then find out what will be the temperature T t 4 if it is not given if it is generally given or sometimes you need to find out you know that you do not know really and you will have to cross check whether it is that. Then, of course this will you need to know or you will have to find out this example or ideal situation p 9 is equal to p naught.

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Let us now evaluate the pressure ratio P_{t9}/P_9 as

$$\frac{P_{t9}}{P_9} = \frac{P_{t9}}{P_{t4}} \cdot \frac{P_{t4}}{P_{t2}} \cdot \frac{P_{t2}}{P_{t0}} \cdot \frac{P_{t0}}{P_0} \cdot \frac{P_0}{P_9} = \cancel{\pi_n}^{1.0} \cdot \cancel{\pi_b}^{1.0} \cdot \cancel{\pi_d}^{1.0} \cdot \pi_r \quad (\text{as } P_9 = P_0)$$

But, for an ideal engine $\pi_n = \pi_b = \pi_d = 1$. Therefore, the above equation will become

$$\frac{P_{t9}}{P_9} = \pi_r = \left[1 + \frac{(\gamma - 1)}{2} M_0^2\right]^{\frac{\gamma}{\gamma - 1}} = \left[1 + \frac{0.4}{2} \times 2.5^2\right]^{3.5} = 17.09$$

Now, we can evaluate the value of M_9 as

$$M_9 = \sqrt{\frac{2}{(1.4 - 1)}\left[(17.09)^{0.286} - 1\right]} = 2.5$$

It is known that the Mach number at the nozzle exit is same as that of the flight Mach number.

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So, you know, but in when you go for real cycle you would not be knowing these things, you need to find out it p 9 would not be known some cases, so let us evaluates this p t 9 by p 9.

If I am use this method or you can go by some other method also step by step, so these are the same thing I have done. So, if you look at these pi d will be what pi d will be 1 pi n will be 1 and what about pi b can I say it is as 1 and pi b is also will be one because there is no pressure loss in the total pressure loss in the burner. So, this is also be 1, so that turns out to be pi r and pi r you can find very easily so that you know this pressure ratio and p t 9. You can find out I mean you need not to you can really substitute these values and find out M 9 is this much 2.5 and interestingly this M 9 is happens to same as that of the what you call flight Mach number.

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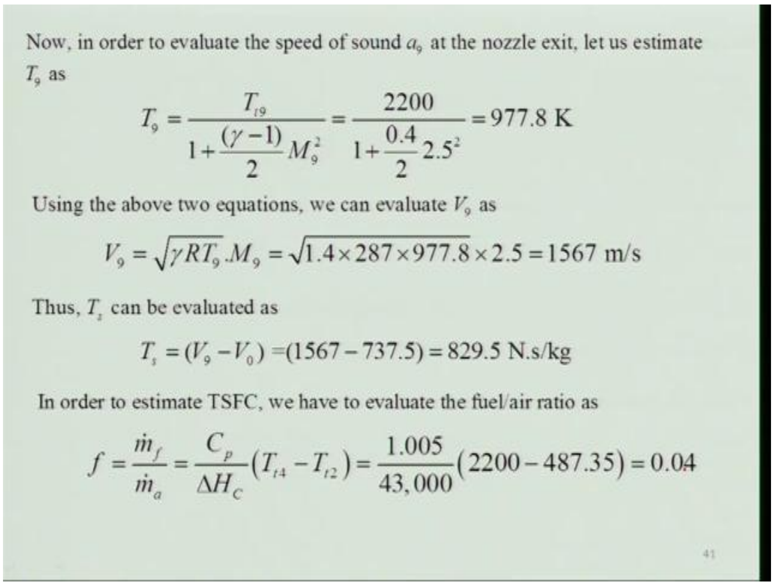

That will be valid only for an ideal place, now in order to evaluate the speed of you know sound a 9, I need to find out n a T 9, T 9 you can find out V 9 by this what you call T 9 kind of things and you will get that. It happens to be 977.8 Kelvin, so using above two equations, we can evaluate V 9, I know this V 9 and I know M naught I can find out. So, this V 9 happens to be 1,567 meter per second and which is not same as that of the flight velocity which is the 700 or not same.

So, when you substitute those values you know you will get 829.5 Newton second per k g and you will see that it is almost same as that of the earlier method. So, in order to estimate TSFC we can evaluate all those things C p delta H C T t 4 times T t 2 this is we are not using tau v tau r, but you can put that it is happens to be 0.004.

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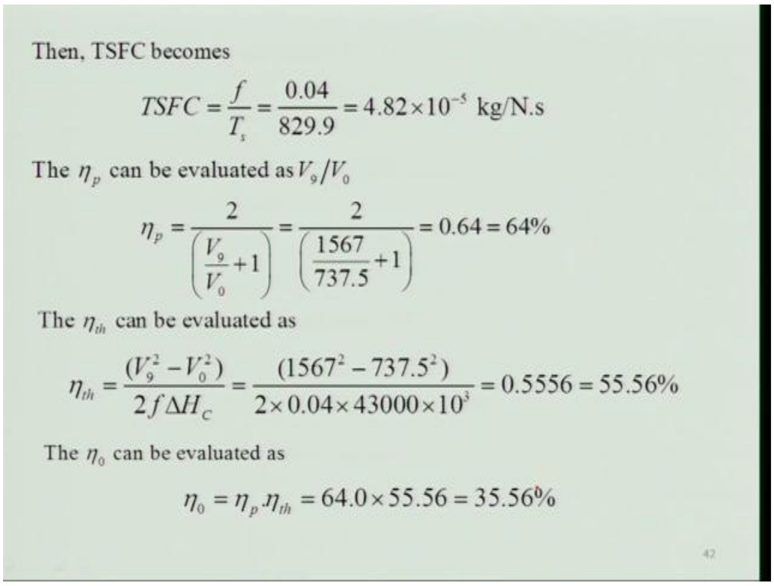

Keep in mind that this is what you call ratio, so using this t we can find out TSFC is equal to f by that t s that will be 4.82 into 10 to power minus 5 k g per Newton second and which is same as that. So, propulsive efficiency can be evaluate as V 9 by V naught and we know V 9, we know the V naught. So, you can get a propulsive efficiency very directly and thermal efficiency is same as that when you substitute V 9, but here you can see that what difference you know in that velocity is and how much change in the kinetic energy is occurring. You can have a feel for it where you cannot get there it is just a number and what you cannot.

So, the overall efficiency will be 35.6, so what I am suggesting you that you if you want to have a feel you will go for this method 2 and I would are you people to use in your exam and other places the method to analysis. Otherwise, station particularly in some assignment I will be giving in some question where you need to carryout parameters. Therefore, method one has to be adopted for those problems rest of the problems the method two is to be adopted is that clear to everybody so method two gives a physical feels therefore, it is very essential you should do that to have a feel for the what is happening what you want to examine.

So, now we will get into the turbojet engine, we know that in ramjet engine there is always two problem to get the you know overcome the problem of 0 specific thrust or the specific. As a result we cannot really make it air burn from what you call plane itself, so you need have to angular units to overcome that one can think of using the compression and when you talk about the compression. You know because it has to give some ram pressure to make it fly to make it good itself.

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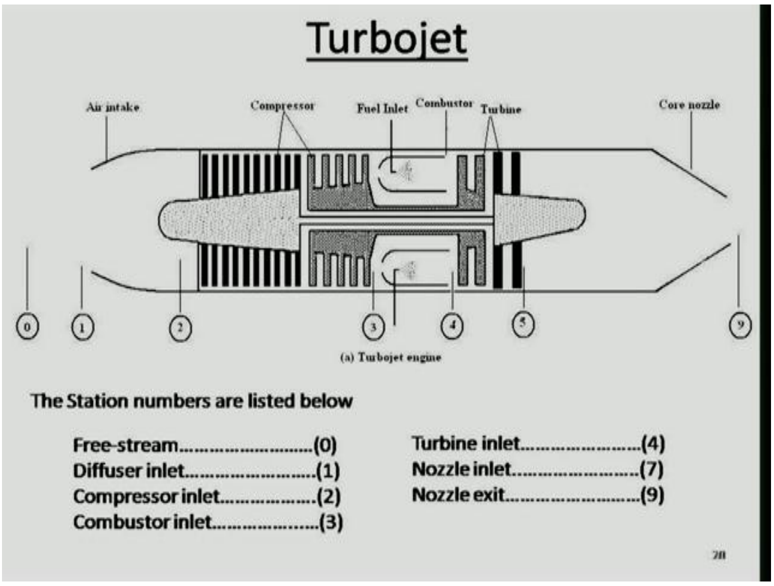

So, for that reason we need to have adds a turbine, so a turbo jet engine if you look at it is nothing but similar to the ramjet engine it a turbo machinery if you look at there is a compression and to run the compression we need a turbine. Keep in mind that the work obtain by the turbine is just meant to the nozzle compressor that is the turbo jet engine and remain all the thrust is being produced by expanding the gas in the nozzle. That means in case of turbo jet engine, the thrust is being produced by expansion of gas in nozzle rather nozzle is a component which gives you the thrust not the turbine or the compressor.

So, this is a very important concept you should keep in mind unlike the other you know particular turboprop engine. So, we will be using this station number like 0 to 2 is your air intake and 2 to 3 is your compressor 3 to 4 is combustion chamber or combustor and 4 to 5 is your turbine that is expansion and 5 to 8. Of course, 5 to 6 and the 7 you can say this will be 7, this is known as jet pipe, but however 7 to 9 is your nozzle. So, I have let us discuss about these things, now we will be what you call looking at how the processes you know in a T s diagram.

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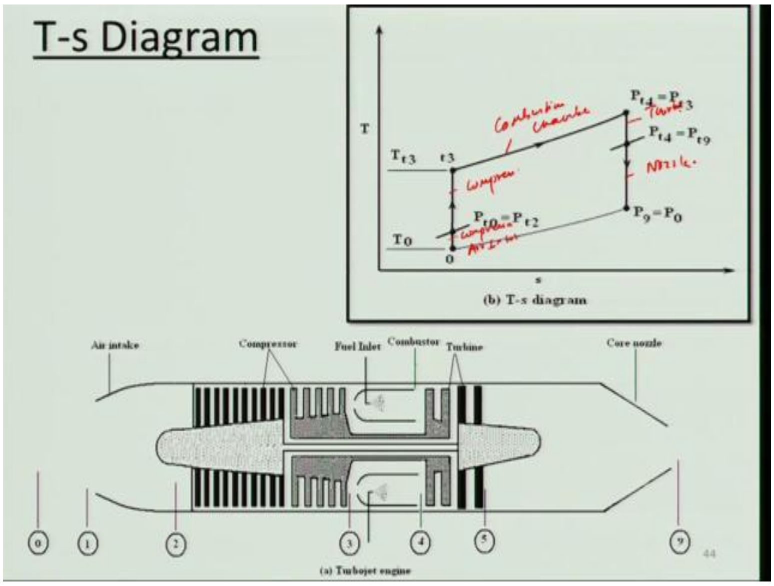

So, if you look at this process is basically what you call compressions compression intake air intake and this is your compression and this is your combustion chamber constant pressure heat addition. This portion is your expansion in nozzle, sorry expansion in turbine and this is in your nozzle grate expansion. It is similar to what that ramjet only think you are having a com contribution from compression and a turbine that is all, but it is similar. If you look at PV diagram, it will be similar to that only the division will be there in the compression and expansion.

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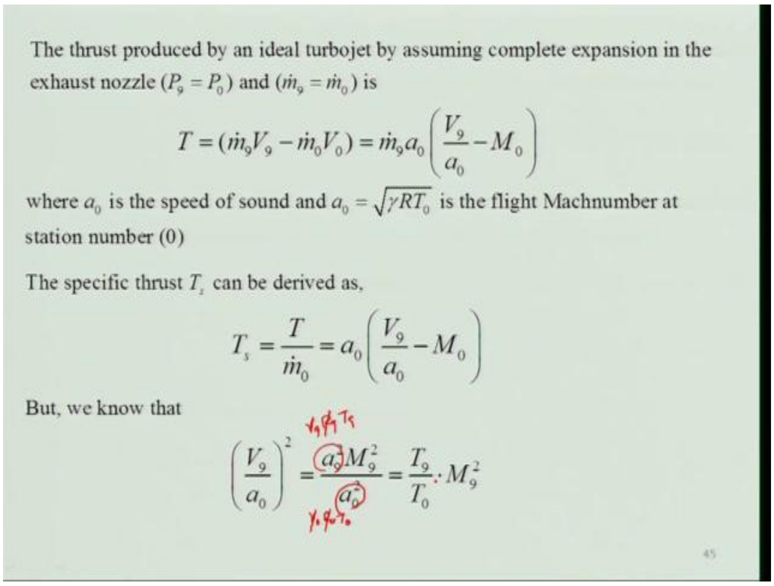

So, now what we will do we will do the similar you know methodology to derive the expression for the thrust produce when ideal turbojet engine will be same that what we have derived M naught 9 v minus M naught V naught. If I take this a naught out, you know and M naught 9 out, I will get V 9 by a naught minus M naught M naught is a flight Mach number. So, a naught is speed of sound and we have already know these things and the specific thrust can be you know defined as I told you earlier the thrust divided mass flow rate of air is nothing but a naught V 9 by a naught minus M naught.

This same thing what we have done, so we know that V 9 by a not square is nothing but a 9 square M 9 square and a naught 9 we know that it is gamma 9 or 9 T 9 and a naught square, you know that has gamma naught or naught T naught. So, this is cancel it out gamma is same as that, so we will land in getting T 9 by T naught into M 9 square, so what we will be doing, we will be doing the similar thrust of relating the various you know pressure and temperature parameters in each component. It is a just extension what we are done for the ramjet engine, so p T 9 by p you know 9, we know that it can be express for isentropic flow in terms of Mach number and exit Mach number.

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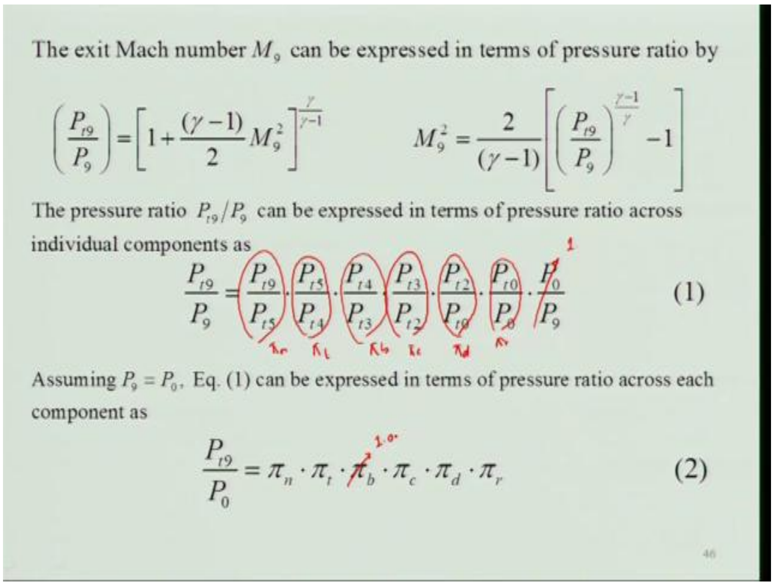

If I just rewrite back, that it will be M 9 square is equal to 2 by gamma minus 1 p T 9 by p 9 gamma power to the gamma minus 1 divided by gamma minus 1. So, we will be looking at this p T 9 by p 9 and look at a various pressure ratios, so you look at a similar way and I have just written down it here and if you look at this is a fully expanded. So, this will be 1 and p t by p naught what it would be this will be your pi r and p t by p t, p t 2 by p T naught. This is nothing but your pi d and this is p t 3 by p t 2 is pi c and p t 4 by p t 3 is nothing but pi b and p t 5 by p t 4 is your pi turbine and this is your nozzle you can say.

We know that this what you call and we can express in this similar in terms of pressure ratios and under ideal condition, what we can say we can say that pi b is equal to 1. What happens to your turbine pi t can I say 1 pi c i can say one no and pi d can I say 1, total pressure ratio what happens in the pi d no compressor no turbine. You are saying because we have you know getting sometimes doing the work on it and extracting the work in the turbine, so what about pi d. What we did in the ramjet engine total pressure what is happening ram pressure will be what total pressure will be remaining same or it will be different this is isentropic process.

So, what about nozzle this also one because we are just converting you know kinetic energy into the where into the static pressure or a dynamic air to the static air in case of the air intake and this is other way around then nozzle is just opposite. The static way you know like static pressure we are converting into dynamic, but the total pressure is remaining constant. Therefore, you will have to be you know understand this concept, so pi n pi b and pi d is equal to 1.

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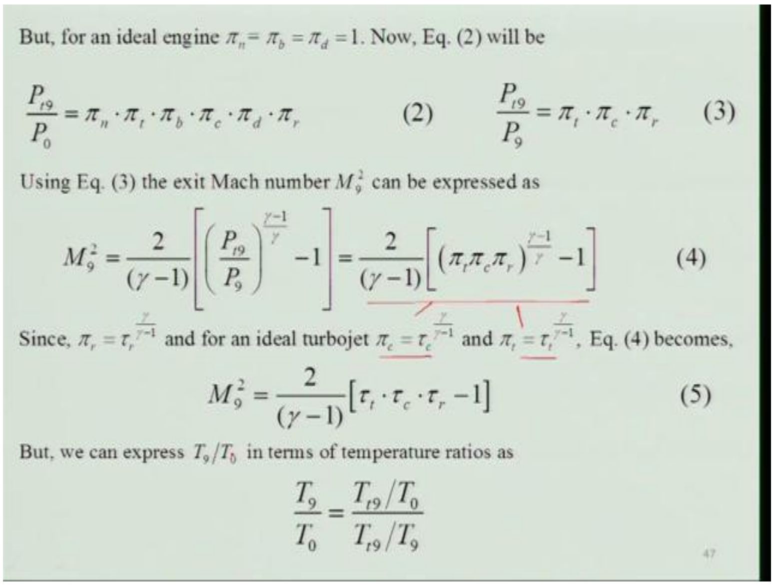

So, this equation becomes you know like p t by p 9 pi t pi c pi r in case of ramjet engine pi t is equal to 1 pi c because there is no compression, no turbine. So, you can directly get this expression that means if I know this equation, you know expressions I will get ramjet turbojet just making some parameter one. You know that why that is a beauty of this analysis, I need not to do anything if I say pi d pi c is equal to 1. That means it is ramjet that is the beauty of this relationship where computer you can do very easily. I can put a condition and do that ramjet same equation you know so that you should must appreciate of this relationship and using equation 3 exit Mach number, I can get because I have already derived that is nothing but same as that and except this.

You know pi t an d pi c being coming to as compare to that ramjet engine, so if you look at this similar to that what we have done for the ramjet only two terms are come for the turbojet pi d and pi c.

So, pi r you know is we can relate to the tau r gamma power to the gamma minus 1 and similarly, for pi c and pi t these are isentropic relationship, pi means it is the pressure ratio tau means it is the temperature ratio. So, we know that pressure ratio can be related to the temperature ratio with the help of this gamma, you know of the index. So, M 9 you know square, we can write down here itself if I put these values over in this place, you know and also for pi r i can get instead of pi i can get in terms of tau that is tau t tau c tau r.

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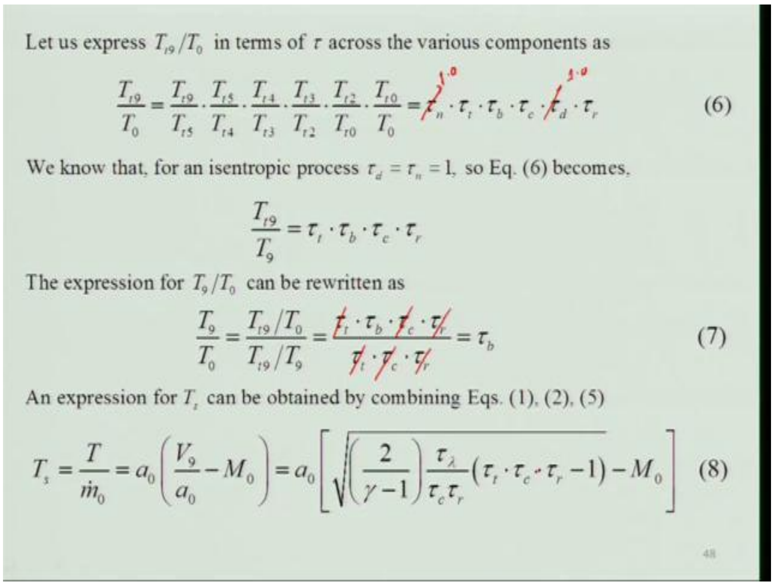

So, we will do the similar way of getting this T 9 by T naught you can write down t T 9 by T naught if your t T 9 by T 9 and we will go on doing that all those things if you look at it will you will be get this tau n tau c tau t tau b tau c tau d and tau r. If you look at tau d is what is it one because the adiabatic process in there air intake there is no heat. Therefore, that will be one, similarly in the nozzle is it you are adding some heat, you cannot you are not doing anything. Therefore, it will be 1, so what about tau c and tau t can I make it i 1 and tau b is certainly no because I am adding some amount of heat can i make it.

I cannot make the tau t and tau c as 1 that means the total temperature across the compressor and across the turbine are changing. So, it cannot be one because in one case compress you are giving you know amount of work and in that the turbine you are extracting the work from the fuel it. So, there will be changing total temperature otherwise if you will get other thing. Therefore, you cannot really make it one, so t T 9 by T naught it will be tau t tau b tau c tau r and again you can say this here you can appreciate this point the tau c and tau t will be one in case of ramjet this same thing I am repeating. So, that it will entering into your mind so the expression T 9 by T naught can be really if look at is a interesting.

If I put this t T 9 by T naught all those things this is can and also t T 9 by T 9 you will see that it is can be cancel it out. This, can be cancel it out and the tau c, you can cancel it out it happens to be tau b and what we have seen in case of your ramjet engine yes or no. We are doing the same thing, but make it little complex, but this is cancel it out which is obvious because the turbo in the turbojet engine. The, work you know harness by the turbine is being utilized by the compressor therefore, it must be T 9 by T naught will be tau b it will be how much heat added into the combustors that is the thing we were saying and it is true also Ramjet.

So, the T s you can just substitute these values V 9 by V naught and you will get a expression which looks to be little frightening, but however it is quite simple. I am not expecting that you should remember this expression, but however you must know how to go about how to derive it that is expected.

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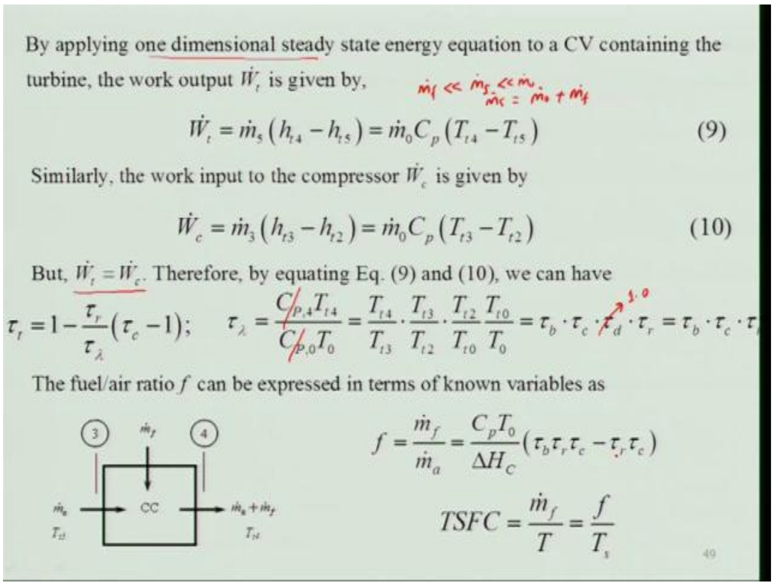

So, which will be quite simple to derive this expressions and that is very important; we need to now relate this turbine work into the compressor work, because we will have to get the relationship between the compressor and turbine. So, we will assume the one dimensional steady state energetic way, you know these things. We apply this to a energy equation control volume and the turbine work will be m dot 5 is equal to s t 5 minus s t 5, because the in the turbine. You keep in mind that here we are assuming, you know like the mass is continuity is maintain berceuse one dimensional flow we are talking about M naught 5 is equal to M naught 4.

You are not adding anything expect you know, so same fuel it is going therefore, which is nothing but that M naught not keeping in mind that here I am saying it is very very less than M naught 5 or is very less than M naught. Therefore, I am saying this is M naught is equal to M naught 5 basically M naught 5 is equal to M naught plus M naught f, but I am assuming this.

Therefore, M naught f C p T t 4 minus T 2 5, similarly I can have for the compressor, which is same as that M naught C p T t 3 minus T t 2 and when we quit this work done by the turbine is same that as work you know taken by the compressor. Then, we will get an expression you know tau t is equal to 1 minus tau r by tau lambda in bracket tau c minus 1. Keep in mind that we can express in terms of tau lambda because that is a thing how much heat you know being added or the total enthalpy at the exit of the combustor divided at the amount of enthalpy entering into the engine.

So ,if you look at it is this, you know it can be rewrite this because this is cancel it out T t 4 by t t 3 into t 3 by T t 2 into t 2 by t T naught and t T naught by T naught. So, you can write down in terms of all these tau b tau c this is compressor and tau d is for air intake and tau r. You know that tau d is equal to 1 and because of it is adiabatic process we are saying no heat is going out, but in real situation it cannot be, but in ideal situation it is ok. So, the fuel air ratio can be express in terms of known variables as you know we will have to consider these combustors, which we have done a similar thing in case of ramjet engine it will same.

So, we can write down m dot f divide by m dot a nothing but C p T naught delta h c which you will see that tau b tau r tau c minus tau r tau c. You see some step I have omitted here, but you can do very easily and in case you will find some problem. Let me know and TSFC you can get very easily that is f divided specific thrust and you will put this values. You know now we are having several terminologies are coming like several ratio temperature ratio of pressure ratio. Keep in mind that we can you know change, this tau c in terms of pressure ratio compressors. Similarly, tau p we can you know some places wherever it required we can change into the pressure ratio across the turbine I use in the isentropic relationship.

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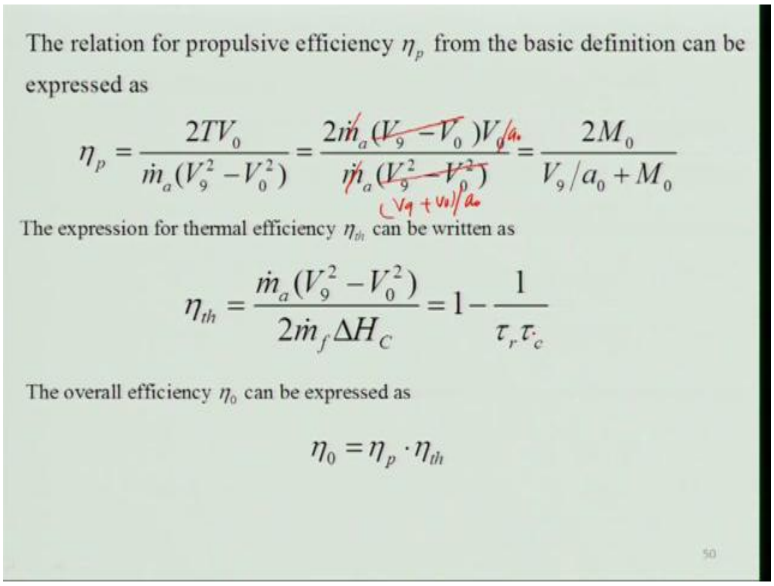

So, the relationship for propulsive efficiency we can write down 2 T V, basically the thrust power divide by kinetic energy change in the engine that is m dot V 9 square minus V naught square divide by 2. If I just you know write in in place of thrust in this expression m dot into V 9 minus V naught, you will see that I cancel it out and this will cancel it out and I will get V 9 plus v naught. If I divided by a naught square here, similarly a naught square. Here, I will get two Mach number divide by V 9 by V naught plus M naught. So, this is an easier way of saying that because I know expression V 9 by a naught I can do very easily.

Also, you can see that how these Mach number is really affecting your propulsive efficiency and thermal efficiency you can get you know do all those thing algebra. You will get 1 minus divide by tau or tau c keep in mind that here tau c is coming to the picture in case of the turbojet. Earlier, it was only 1 minus 1 over tau r in case of Ramjet tau c is equal to 1 in case of Ramjet. So, propulsive efficiency overall efficiency will be nothing but propulsive efficiency multiplied by thermal efficiency will give nozzle that means by this we have derived all the expressions. We are armed with all the expression to the carry out parametric studies, and just to summarize those what we have derive.

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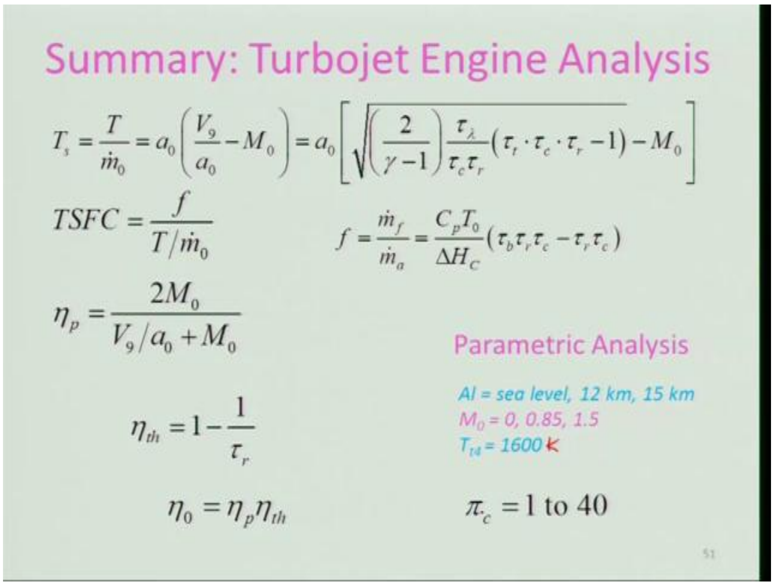

That is the specific thrust which we have done and these are the expression which in terms of various tau b tau c tau r. We are using also tau lambda and in place of tau lambda may be you know this term tau lambda divide by tau c tau r is nothing but tau b some places you can use tau b as well. So, what is helping this specific thrust expression is basically separating each parameters. It is arming with us or helping with us say that we can do parametric analysis that you must appreciate. Otherwise, I need not to go for this you know unless I am not interested in parametric studies and which is essential when I am trying to understand you know how it is the performance is affected by the various parametric efficiency.

That is f c and we know that f is you can be express in terms of tau b tau r tau c minus tau r tau c multiplied by C p T naught divide by delta H c. and these are expressions for propulsive efficiency thermal efficiency which we have discussed just now and what we will do now we will take three cases. One is the Mach number of je w that is the sea level conditions. Another we will take a long range you know vehicle or other aircraft that we use passenger rate at 0.85 which can operate at 12 kilometers altitude. Then, the fighter aircraft which is corresponding to 15 kilometers altitude value and we are keeping this the turbine, you know like inlet temperature or combustor exit temperature is 1600 Kelvin.

So, and what we will do now what are the variables we will be using we will be basically using this, you know we can vary this pressure ratio across the compression unit. You can say that why not vary the pressure ratio across the turbine. You can do as well both are you know can be related basically with the work you know because a work what about you are getting from the turbine is being utilized by the compressors, so generally compressor is very important one.

Therefore, we use the compressor as parameter that you will be varying from 1 to 40 and see that how we can choose a compressors. How we can choose a, you know pressure ratio, because the compressor size and you know cost will be dependent on the pressure ratio you are want to have for each, so that is a very important one.

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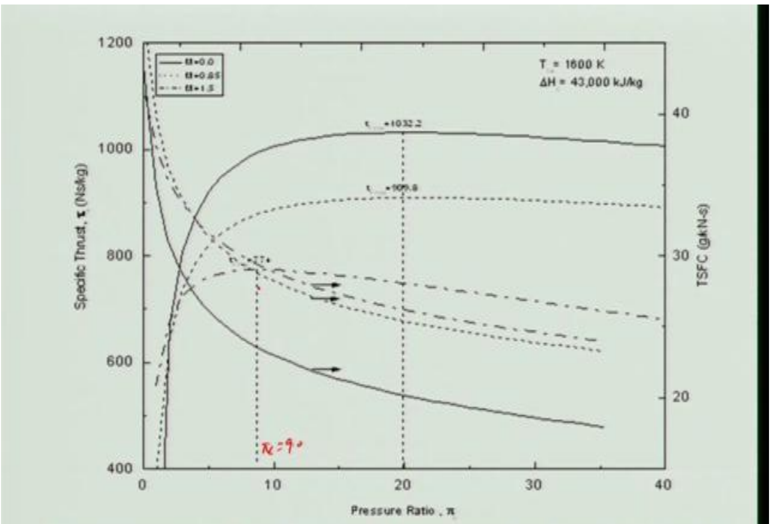

Variation of T_s and TSFC with π_c for different values of M_0 for a turbojet engine at TIT of 1600 K

So, what we are looking at this specific thrust being plotted on the y axis and the pressure ratio plotted on the x axis for a turbojet engine which is having turbine inlet temperature of 1600 Kelvin. We are also varying this Mach number as I told you three cases we have taken representative, case one is sea level conditions or you can say static conditions other is 0.8. Mach number flight, Mach number and other is 1.5 or you can note here that the when this flight Mach number is 0 this solid line the specific thrust increases with the increase in pressure ratio across the compressor. Then, it reaches the peak values here and then after that it decreases fully.

So, what it indicate that means there is a optimum value for the what you call pressure ratio across the compressor for which you will get the maximum specific thrust and the however if you look at TSFC which I have shown here. It is a very high values at the low the pressure ratio of one, you know because we cannot have zero pressure ratio across the compressor. So, one is you know that that means no compressor factors at that value it is going towards infinity very large. It has no meaning and you cannot have any meaning turbojet engine having the pressure ratio of 1, but however it decreases.

You know it decreases continuously as you increases in pressure ratio, you will see that this is having not a minimum values. Unlike in a ramjet engine, there is a minimum values and of course, you will have to choose that, but there is another interesting thing you can observe. When I will go for the flight what you call Mach number of 0.8. You will see that this is having, you know it is also having similar features goes on increasing and then of course, it is having a certain values of phone t and looking where you get the maximum specific thrust, but if you go for the Mach number of 1.5, you will have similar features of the curve.

It is decreasing you know after reaching a value of certain maximum value at a pressure ratio of 9, 5 t v is equal to 9, you are having and then it decreases little at a higher rate as compared to the both the sea level conditions and other static condition and the flight Mach number. What it indicates, it indicates that when the flight or the engine is moving at a higher speed, it can used that pressure and you need not to go for a high pressure ratio compression because the ram pressure can utilize two kind of for increasing its pressure.

So, therefore if you look at the fighter aircraft, we will having this smaller compressor because you would need a low pressure altitude, whereas you go for a long range passenger aircraft, you need to have to go for a twenty kind of engines. At the same diameter, what thrust you need to because at a static conditions with static process is also important. So, you will have to play around and see that what really you need and what is the level blight flight. So, these are the things you can get by just doing you know specific parametric analysis that is the beauty of this method. You can learnt a lot by just playing around and see what is happening, why it is happening and what are the reason whether you can have any scope to improve it further.

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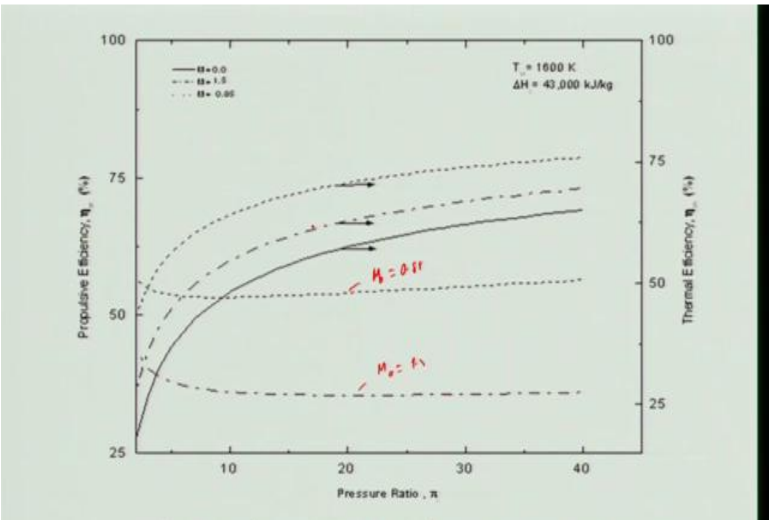

Variation of η_p and η_{th} with π_c for a turbojet engine

So, let us look at a propulsive efficiency and the thermal efficiency if you look at these are the curves which is you know basically the propulsive efficiency. These two curves and it is a static condition there will be no meaning of having propulsive efficiency even Mach number type number is equal to 0. So, what you call this one is basically Mach number of 0.85, it is having higher propulsive efficiency as compare to 1.5.

It is having similar feature features in the when it is pressure ratio increases, it is very higher over here and decreases may be at 7 pressure it will minimum which is not very obvious in this diagram. When you look at number, it indicate and thermal efficiency, it is goes on increasing, you know from the lower pressure ratio to higher pressure ratio and you will get a lower thermal efficiency. In this case it indicates you know zero flight Mach number and when you increases this 0.85, you know you will get thermal higher efficiency when it is the 1.5 kind of thing, then it decreases because of you know what your energy utilization wants.

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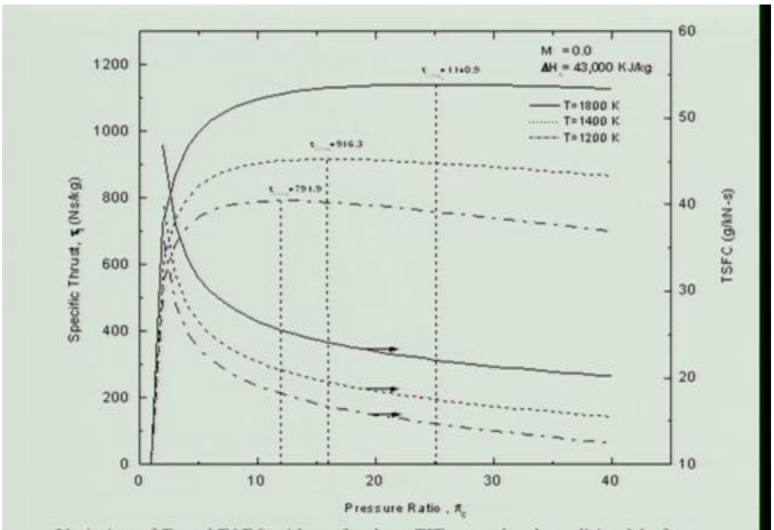

Variation of T_s and TSFC with π_c for three TIT at sea level condition M_0 for a turbojet engine

So, what we will do now, I will just look at you know what happens the effect of the temperature ratios and which is having similar views you can see that this is having highest temperature you know like 1,800 Kelvin. So, if you look at you will have a higher pressure ratio, you need to get to get a higher specific thrust and as a temperature decreases, you will get a lower specific thrust. You know like a maximum you know specific thrust at a lower pressure ratios, because you know you need to have a higher what you call the temperature to be achieved. Therefore, you need to give you know more amount of pressure and you can get a higher kind of value and after that you peak reaches.

So, the higher temperature when you want to get and you want to get also higher specific thrust. So, naturally you will pay penalty for having a higher TSFC and all having similar view you know a value what you call trend like as you goes on decreasing the turbine inlet temperature, you will go on decreasing the TSFC. So, you will have to make a these things where you want to go and with this I will stop over in the next class, we will take an example to see how we can solve this problem.