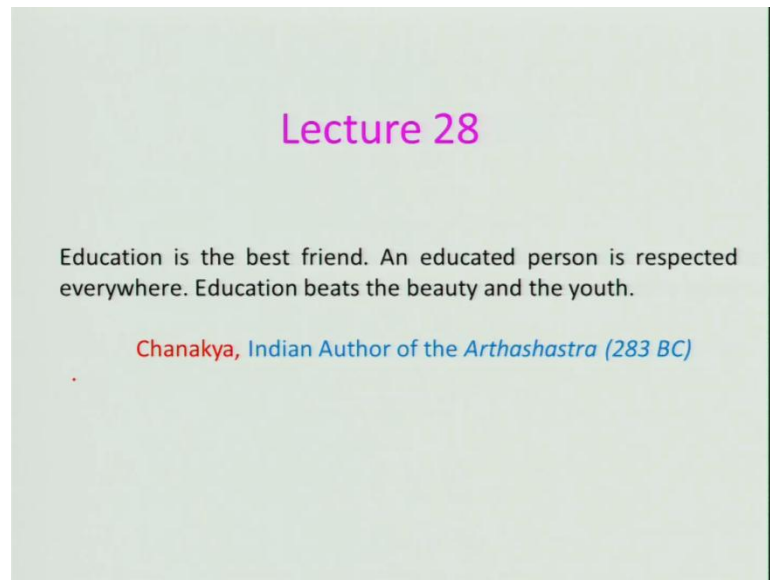


Fundamentals of Aerospace propulsion
Prof. D. P. Mishra
Department of Aerospace Engineering
Indian Institute of Technology, Kanpur

Lecture – 28

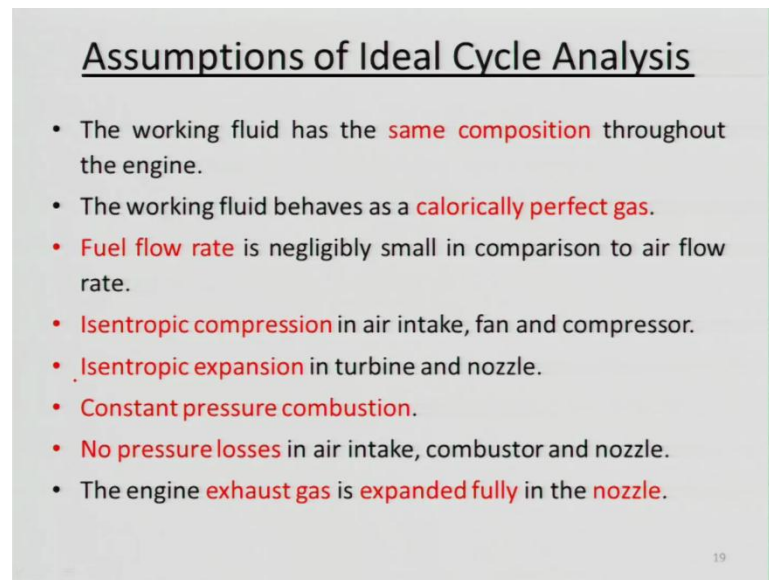
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Let us start this lecture from Chanakya one of the greatest person on education as I call him enhanced in India. And he has written a book known as Arthashastra, which you must be aware, and he says the education is the best friend and educated person is respected everywhere, education beats the beauty, and youth which was a statement what he has given. But if I look at modern education, it is not really subscribe with that therefore, there is a need to look at modern education and see that how it can help us in pursuing our life towards excellence.

So, let us look at, what we did in the last lecture, we are basically looking at how to carry out cycle analysis and various notations we have used. And we have defined certain terms like temperature ratio, pressure ratio for the various components. And what we will be doing to today, we will be basically looking at now the various assumption of ideal cycle analysis.

(Refer Slide Time: 01:36)



Assumptions of Ideal Cycle Analysis

- The working fluid has the **same composition** throughout the engine.
- The working fluid behaves as a **calorically perfect gas**.
- **Fuel flow rate** is negligibly small in comparison to air flow rate.
- **Isentropic compression** in air intake, fan and compressor.
- **Isentropic expansion** in turbine and nozzle.
- **Constant pressure combustion**.
- **No pressure losses** in air intake, combustor and nozzle.
- The engine **exhaust gas** is **expanded fully** in the **nozzle**.

19

Because to simplify whatever I will tell you now is basically to simplify the analysis which all of you know, because you have already done the course on thermodynamics and propagation in your earlier times. So, in the particular video, the working fluid has the same composition throughout the engine. When I say same composition generally we take is air. If you will look what I was assuming I will be talking about in thermodynamics known as air standard assumptions; that means, you know this we know that combustors in which you will be adding fuel it will be converted into various components various species, but, however, you are not considered just for the simply of sake.

An working fluid behaves as a calorically perfect gas. The perfect gas you know of course, it will be valid all the time because although the pressure will be high, but temperature is being higher you can do that. But here we were saying calorically perfect gas, because the properties would not be changed although the temperature will be changing as you go from various components of a gas turbine engine or a ramjet engine of course, ramjet engine we will be discussing today. So, but that we are assuming for the simplicity and fuel flow rate is negligibly small in comparison to air flow rate, because if you look at the ratio, air to fuel flow rate is around 30 kind of things of course, you can vary from 20 to 40 or even more right, but we are neglecting.

Whenever we are comparing we will neglect, but in other places we would not be neglecting. For example, if I am talking about TSFC - thrust specific fuel consumption or thermal efficiency can I neglect say that $\dot{m} \cdot f$ will be zero, I cannot manage to do that right. So, that way you should understand. Comparison whenever we are comparing we will have to neglect and isentropic compression in air intake, fan and compression. So, compression process occurs in all this three components. So, we are saying isentropic, but in real situation it would not be. Isentropic means reversible and adiabatic, how can that possible? It is not possible of course, one can say that if the fluid is flowing through the compressor and air intake very fast, maybe heat lost to the ambient will be very low, you can neglect it, but it cannot be reversible, friction will be there without that you cannot really do much right particularly in compression.

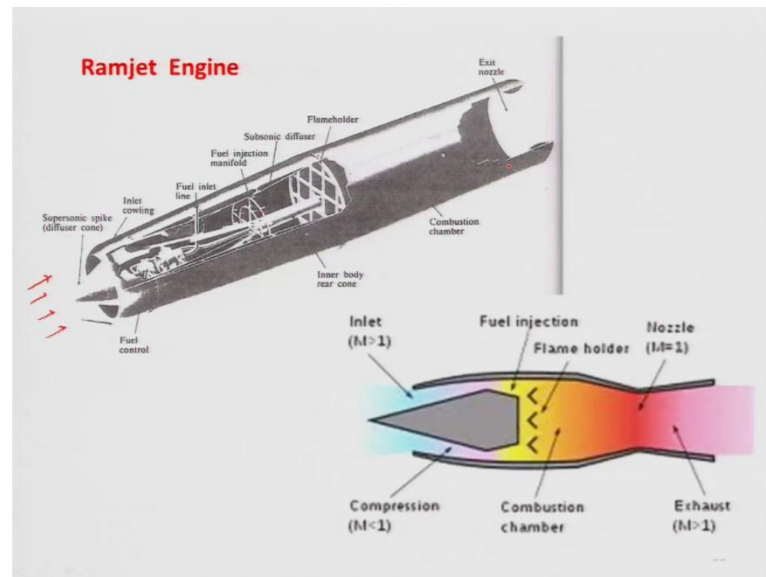
So, therefore, you cannot have, but we are doing for simplicity an isentropic expansion in turbine and nozzle right and constant pressure combustion we are assuming right because this is the deflagration and unlike it is not a combustion chamber at that close volume where combustion taking place. So, here the pressure is remaining constant. One example I always give as if the combustion is occurring in a balloon, but if it a closed surface, can I say pressure is remaining constant during combustion, it will be increasing like in your IC engine internal combustion it is modeled as a constant volume process. We have seen that adiabatic temperature under constant volume and constant pressure will be different.

So, similarly this is again we are making and it is not that bad, but; however, we are also assuming in these the total pressure loss is during the combustion is negligibly small, which is not the case. No pressure losses in air intake and combustor nozzle as I told just now not only combustor, but air intake and nozzle there is no pressure losses. It is not possible in real situation, but we are doing, we are assuming that, but engine exhaust is expanded fully in the nozzle right; that means, it is what you call the thrust is contributed due to the momentum and reduced.

Of course, that is ideal situation because particularly in rocket engine other than which we not considering, but there you cannot have air may be in some ways in it will be expanded fully another ways in it would not. So, in this assumption now we will be you know moving into ramjet engine keep in mind these assumptions will be valid for ramjet and then turbo fan engine turbo jet engine turboprop engine what we will be discussing

under I just said. And what we are doing we are basically looking at parametric cycle analysis; that means, we will be varying those parameters and see how it is affecting the various performance parameter that is the objective of this cycle analysis unlike your other analysis whatever the that it can be used as a design too.

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So, let us look at ramjet engine. If you look at ramjet engine have as shown as schematic diagram. This having portion ((Refer Time: 06:45)), in this case the what you called a will be encoding and form the supersonic ((Refer Time: 06:53)), because ramjet engines operates some whether, supersonic speed kind of the and it will be entering, and then thus inneractive, that means compression of they are will be taking place. And then it will be burned in the combustor ((Refer Time: 07:10)) there. And once this high temperature, high pressure, gas ((Refer Time: 07:14)), it will be expended nothing but a CD [FL].

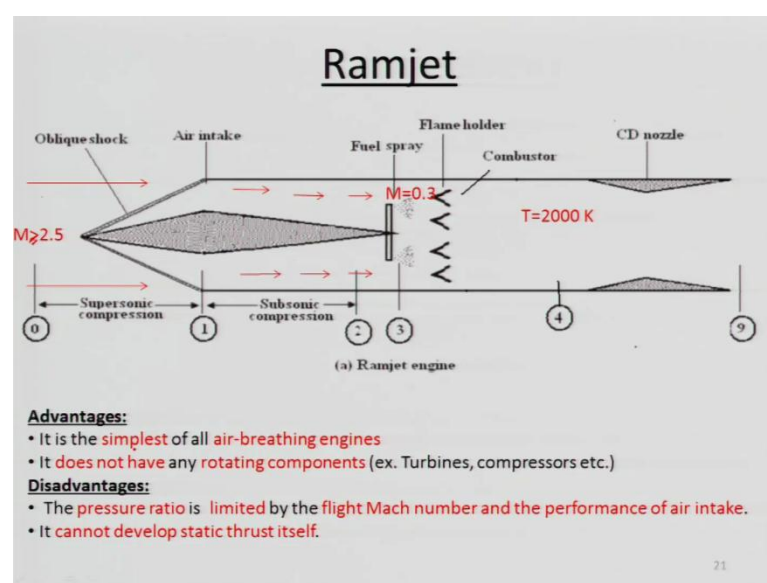
Let us look at actual cut away view of a ramjet why the name ramjet is there because in this case the ram pressure is been utilized. For example, when the what you call airs coming at a supersonic speed, and it will be having a supersonic spy and will act as a diffuser and what you call this is known as supersonic compression right. That means, the air will be decelerated from supersonic speed to the sonic in the what you call in the intake supersonic intake and then from sonic to the subsonic, you know it will be taking place. Beside this you need to have you know fuel because the fuel has to be burned right

and for that you need a pond. And then inlet will be there it is injected for the atomizers you know then we fuel manu forth air fuel will be being stored generally, you know like in your boing aircraft where fuel is being stored, it is stored in the wing right kind of thing.

So, therefore, in the fuel injections that atomizers it will be there and then it will be mixed with incoming air and it should be burned good, and then when you burn the flame has to be you know stabilize with the help of flame holders pretty simple stabilization mechanism is being used. And when this hot gas is being produce at a high pressure it is what you call expanded in nozzle.

If you look at this is the conversion diversion which is very very small know in this case it is been shown, but then what about this thing because there in the mixing and combustion will be taking place it will be uniform. So, that you know and combustion will be complete because it is at a little higher speed. And keep in mind that here the subsonic of combustion occurs not supersonic combustion. So, now, if you look at it is a quite simpler system as compared to the gas turbine engine right, why because it is not having compression, if compression is not there; that means, it will be not having any turbine. So, and there is no and then the complexities you know it is very very simple thing.

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Let us look at a schematic form a ramjet which will be used in and keep in mind that zero we are saying is a station number and zero to one is your supersonic compression. And basically it is the air decelerated from supersonic speed the sonic condition and this from one to two is a subsonic compression generally people design for to achieve point three mach number mach number of point three at the station two at the end of the or the exit of the air intake. And the total zero to two is nothing but your air intake right combined the combination of both supersonic and subsonic and then the efforts there is no compression although we are keeping the same station number three two one three is almost same although it is not shown in this figure.

And then three to four is your combustion where the fuel will be spread at wings and then you know stabilize with the help of flame holders which have basically gutters is being used by the flame stabilizers and expanded in nozzle that is four to nine right. So, why we do not need a compression here because of fact that the air will be entering at a very high speed. For example, if it is entering Mach number around 2.5 greater than or equal 2.5 or you can say it is equal to 2.5. And we will have to now compress it or decelerate it you know because we are compressing by decelerating the flow at the end of the air is changed to point three we could do a simple calculations of isentropic flow, you will find the pressure ratio of something around seventeen eighteen you can get.

So, then if I am getting a pressure ratio of eighteen or seventeen you know it is good enough I need not to really go for a compression which is very faulty and costly at the risk. So, after the turbine again, but problem with this kind of thing is that it cannot start from the beginning right unless it achieve certain flight velocities particularly supersonic kind of thing that compression will be very very low right or it would not be zero it will be zero at the standing condition.

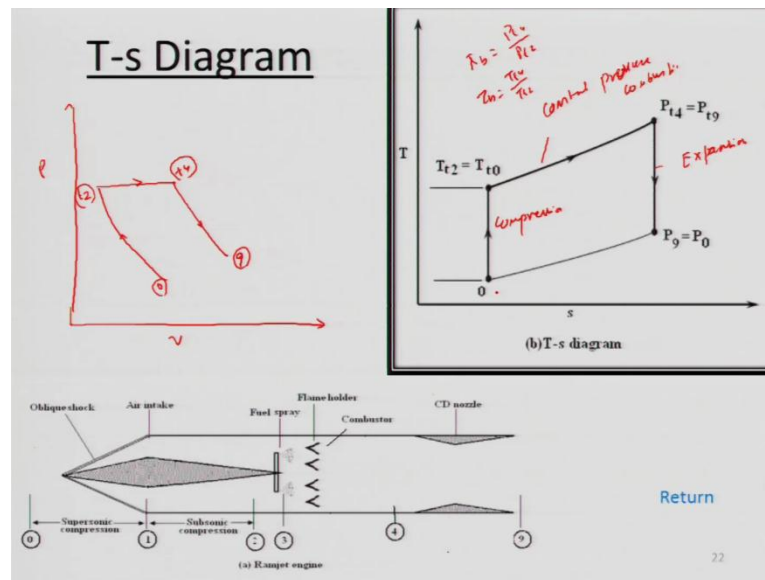
So, from the ground it cannot be really fly. So, that is the limitation of this. So, if we look at there is a another important aspect you can have you know add more amount of fuel such that it can go to the temperature of 2000 Kelvin because there is no rotary elements in it. So, there is no material problem although material problem about the combustor liner will be there, but that is being stationary. So, you can manage to cool it design properly because with the now days good exerting materials have come up you can cool little more beyond the two thousand Kelvin.

But earlier days it was people were stuck into 1415 hundred Kelvin kind of thing even in ramjet because, but when the designer you know they have used their mind and up with the ideas how to you know develop how to operate the ramjet engine with the even what you call existing material. So, that is the what we call innovativeness of the designer. So, therefore, we always work with constant and whenever we work with constant we get a lot of ideas, so whichever whenever you face problem you learn to see that I will have to do something, so that I will use my mind, so that I will get problem to solve right. So, that is very important and it is the simplest of air breathing engine because it is devoid of gas turbines and rotating elements like turbines and compressors.

Of course, as I told the pressure ratios limited by the flight mach number and performance of air intake that we will see as you go alone. You cannot develop the starting thrust itself, but questionnaires you it cannot develop. I cannot really use this unless otherwise I am having a another aid to do that, for example, I can take you know carry a missiles of you know based on their ramjet with a rocket or an a aircraft. You know supersonic air fighter, then I launch, but if want to know no I am not happy with that I want to use the ramjet from the ground itself.

What I will have to do what are the ideas can anybody tell me pulsejet itself I can use why should we use ramjet right rocket is already being used sold to aircraft, but I would want to do something. So, what I will have to do. So, what I am thinking I will leave this course into you right and think about it if you come up some ideas without looking at existing one then you know it will be interesting to discuss otherwise some ideas I can give I do not want give at this moment later on we will see.

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So, let us look at how what are the process that are involved as you told that zero to two is basically the compression in the t s diagram I can write down zero to two that is the compression right isentropic process isentropic compression. But if I look at in a p v diagram how it will be compression process zero to two into it will be right and then T t two from this station to what you call four it is the compression that is constant pressure combustion right.

This is compression right. So, if I look at in p v diagram is will be very simple that is a constant pressure pressure is not remaining constant and we give four to p nine it will be expansion expansion where expansion in another that is isentropic expansion. So, my it will be T t four and this station is basically lined. So, and I am just saying it is basically four station we can say because total we are talking about totally keep in mind. So, whenever I am talking about ratio and combustion chamber what I will be saying it will be p_4/p_2 will be p_9/p_0 right total I am considering what if I say it is τ_4/τ_2 bit will be similar if you code by p t two right.

, but whereas, when I am coming over here it is a starting condition that the exit of the nozzle at the inlet it is also starting condition keep this in mind very important point right. So, we will be using those things and see that whether we can you know use how we can use this thing forever and is that clear? Any doubt in this, because it is a very important things and we will be using similar tau you know processes describing in the

process diagram like t s p v I can have h s I can have some other thing as well right I can have t v right. So, you should think about this how I can you know describe the process in various ways because that will help you to enrich your mind.

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Thrust equation for ramjet engine:

$$T = (\dot{m}_9 V_9 - \dot{m}_0 V_0) + A_9 (P_9 - P_0) \quad (1)$$

Assuming complete expansion in the exhaust nozzle $P_9 = P_0$ and $\dot{m}_9 = \dot{m}_0$, then Eq. (1) becomes,

$$T = \dot{m}_0 \left(\frac{V_9}{a_0} - M_0 \right) a_0 = \dot{m}_0 a_0 \left(\frac{V_9}{a_0} - M_0 \right) \quad (2)$$

where a_0 is the speed of sound station at (0) ($a_0 = \sqrt{\gamma_0 R T_0}$)

Now, we can write,

$$\left(\frac{V_9}{a_0} \right)^2 = \frac{a_0^2 M_9^2}{a_0^2} = \frac{\gamma_9 R_9 T_9 M_9^2}{\gamma_0 R_0 T_0} \quad (3)$$

However, $\gamma_9 = \gamma_0 = \gamma$ and $R_9 = R_0 = R$ for a calorically perfect gas.

So, the thrust equation for ramjet engine is basically is thrust is equal to $\dot{m}_9 V_9 - \dot{m}_0 V_0$ plus $A_9 (P_9 - P_0)$ and keep in mind that what we are assuming in ideal cycle it is an fully expanded nozzle. So, therefore, P_9 is equal to P_0 is equal to P_0 . So, therefore, it is thrust is equal $\dot{m}_9 V_9 - \dot{m}_0 V_0$ only contributed from the momentum what we are trying to do we are trying to now instead of this equations in terms of various ratios. So, that we can vary and see what we are doing that is you should keep in mind if it is the inverse sort of algebra.

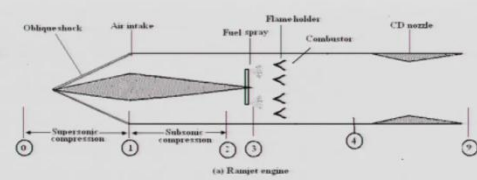
So, and as I told it is $P_9 = P_0$ and then $\dot{m}_9 = \dot{m}_0$ is equal to \dot{m}_0 like that we are assuming; that means, whatever the mass is exiting it is whatever the equal to mass is entering. That means, we are neglecting the mass of fuel being added which is not a you know right thing, but we are doing for simplicity right. What I am saying maybe I can write on here $\dot{m}_9 = \dot{m}_0 + \dot{m}_f$ this we saying it is small as compared to \dot{m}_0 that is why it is neglected right. So, is equal to \dot{m}_0 approximately it is not too.

So, what I will do I can write down this expression and take this M_9 and a_9 then I am getting v_9 by a_9 right what I will do I will basically find out this a_9 right here. So, this is nothing, but your M right and what we will do if you can look at M is the speed of sound right is it not? This is the speed of sound where M , they are the free stream free stage and M is the free stream Mach number free stream mach number. And this A is nothing but $\sqrt{\gamma R T}$, and you can in some place you can see this is γ correspond into the γ which will be used in not in ideal cycle, but in the real cycle.

So, I can write down v_9 by a_9 whole square is equal to by definition a_9^2 square M_9^2 divide a_9^2 and this a_9^2 is nothing, but $\gamma R T_9$ and we are assuming this γ equal to the γ and T_9 is same as that T . So, what we will getting is right is basically T_9 by T M_9^2 that is right. So, if we look at I am getting here v_9 by M equals whole square is equal to T_9 by T M_9^2 I will be writing in the same thing in the next line.

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Hence Eq. (3) becomes

$$\left(\frac{V_9}{a_0}\right)^2 = \frac{T_9}{T_0} M_9^2 \dots (4)$$


The exit Mach number M_9 can be expressed as:

$$M_9^2 = \frac{2}{(\gamma-1)} \left[\left(\frac{P_{t9}}{P_9} \right)^{\frac{\gamma-1}{\gamma}} - 1 \right] \quad (5)$$

However, the pressure ratio at station (9), P_{t9}/P_9 can be expressed in terms of pressure ratio across each component as:

$$\frac{P_{t9}}{P_9} = \underbrace{\frac{P_{t9}}{P_{t4}}}_{\pi_n} \underbrace{\frac{P_{t4}}{P_{t2}}}_{\pi_a} \underbrace{\frac{P_{t2}}{P_{t0}}}_{\pi_d} \underbrace{\frac{P_{t0}}{P_0}}_{\pi_r} \cdot \frac{P_0}{P_9} = \pi_n \cdot \pi_a \cdot \pi_d \cdot \pi_r \quad (\text{as } P_9 = P_0) \quad (6)$$

24

So, is that clear. So, and the equation three becomes v_9 by a_9 whole square is T_9 by T M_9^2 . So, if you look at a_9^2 I can express for isentropic process right for the because expansion is taking place in a nozzle can I not

write down what will be the m_9 in terms of pressure ratios, just using isentropic relationship right. So, I can write down that m_9^2 is nothing, but 2 divided by $\gamma - 1$ p_{t9} by p_9 power to the $\gamma - 1$ divided by $\gamma - 1$ this is from the isentropic relationship for the p_{t9} by p_9 , right.

Nothing I will just read right. So, what we will do now we learn to basically look at p_{t9} by p_9 how it is you know how it can express in terms of various pressure pictures. So, p_{t9} by p_9 can be expressed in terms of pressure ratio across each component what I will do I can write down p_{t9} by p_9 is equal to p_{t9} by p_{t4} into p_{t4} by p_{t2} into p_{t2} by p_{t9} into p_{t9} by p_{t9} into p_{t9} by p_{t9} .

And this is equal to one why because p_9 is equal to p_9 fully expanded to for your... So, if you look at what is this one it is nothing, but your π_n the pressure ratio is a nozzle total pressure ratio is a nozzle and this is nothing, but your π_b p_{t4} by p_{t2} this is the combustors right and this is nothing, but your π_d and this is nothing, but your π_r

If we admire that this is p_{t9} by p_{t9} ; that means, total right to the static right. So, it is saying it is π_r by definition of course, you know because I have defined that thing you know next course. So, now what we will now we are saying this pressure; that means, the mach number exact mach number will be dependent on this pressure ratios.

You have getting one, but know it is you know two π_n right yes or no and the same thing we will doing for turbojet turbofan turboprop also right because we are now looking at each component who they will be affecting the exit velocity as exit velocity will be affecting the thrust right for a particular of course, the flight matter.

So, let us and if you look at what is this π_n by by your assumption π_n will be one or not total pressure ratio across the nozzle is is one because there is no loss. So, π_n will be one and what about this π_b that is also one burner π_d isentropic compression therefore, it will be one can I say π_r is equal to one can I say I cannot because mach number you know it is not total is is basically except all these thing pressure ratio this is called the pressure ratio which is electro static

So, this is nothing, but your what you call one minus gamma minus one divide by two p t by p nine right no sorry m nine square right that much. So, this is assumption we have made you look at your assumptions we have looked at the total pressure ratio across the combustor will be zero total pressure loss if total pressure loss is zero then p t nine divide by p t two will be one no pressure lost right, but; however, in real situation it would not ideal situation it is this.

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But for an Ideal engine, $\pi_n = \pi_b = \pi_d = 1$. Then, Eq. (6) gives

$$\frac{P_{t9}}{P_9} = \pi_r$$

Now, the expression for M_9 can be expressed as

$$M_9^2 = \frac{2}{(\gamma-1)} \left[\left(\frac{P_{t9}}{P_9} \right)^{\frac{\gamma-1}{\gamma}} - 1 \right] = \frac{2}{(\gamma-1)} \left[(\pi_r)^{\frac{\gamma-1}{\gamma}} - 1 \right] = \frac{2}{(\gamma-1)} \left[\tau_r - 1 \right]$$

Since, $(\pi_r)^{\frac{\gamma-1}{\gamma}} = \tau_r = 1 + \frac{(\gamma-1)}{2} M_0^2$, Eq. (7) becomes,

$$M_9^2 = \frac{2}{(\gamma-1)} [\tau_r - 1]; \quad M_9^2 = \frac{2}{(\gamma-1)} \left[1 + \frac{(\gamma-1)}{2} M_0^2 - 1 \right] = M_0^2$$

Note: The exit Mach number for an ideal ramjet engine is equal to flight Mach number.

So, what we will do we are getting pi n pi b pi d is equal to. So, p t nine by p nine is equal to pi r right and now expression m nine can be expressed as you know m nine is equal two divide by gamma minus one p t nine by p nine gamma minus one gamma minus one. So, what you say this is nothing, but your pi r if we look at pi r power to the gamma minus one is nothing, but you tau r yes or no because I can relate this isentropic temperature ratio to pressure ratio isentropic pressure ratio right. We we know the expression. So, therefore, but where else the pi r gamma tau r is nothing, but one plus gamma minus one divide by two m naught square if I will substitute over here what I will get I will get gamma minus one two m naught square minus one. So, this will cancel it out right. And multiply by two gamma minus one right.

So, what I will get I will get this also will be cancelling out. So, two two cancel it out is equal to m naught square what it saying it saying it is a very interesting statement it is saying the exit mach number for an ideal ramjet engine is equal to the flight mach

number. That means what really is happening; that means, whatever the velocity of air which will be entering into the ramjet will be same as the exit velocity of the engine certainly no it is the mach number which is same under ideal condition real situation it would not be right. So, that is the interesting part is here, but you should not assume that exit nozzle exit velocity v_9 is equal to the v_∞ or the flight velocity certainly no.

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$$\left(\frac{V_9}{a_0}\right)^2 = \frac{T_9}{T_0} M_9^2 \dots (4)$$

Now, the temperature ratios is given by $\frac{T_9}{T_0} = \frac{T_{t9}/T_0}{T_{t9}/T_9}$

We can express T_{t9}/T_0 in terms of τ across the various components as

$$\frac{T_{t9}}{T_0} = \frac{T_{t9}}{T_{t4}} \cdot \frac{T_{t4}}{T_{t2}} \cdot \frac{T_{t2}}{T_{t0}} = \tau_n \cdot \tau_b \cdot \tau_d \cdot \tau_r \quad (8)$$

(a) Ramjet engine

So, now we will look at how we can you know look at this T_{t9}/T_∞ and so T_{t9}/T_∞ temperature ratio. I can write down as T_{t9}/T_∞ divided by T_∞ , can I not write like this absolutely no problem, because it is just you V writing that. Now, we will be looking at how we can say this T_{t9}/T_∞ and keep in mind this T_{t9}/T_∞ is related to what pressure ratio or temperature ratio, all those thing can be related. And we will now look at T_{t9}/T_∞ in terms of temperature ratio, I can write down T_{t9}/T_∞ by T_{t9}/T_{t4} is equal to T_{t9}/T_{t4} divided by T_{t4}/T_{t2} into T_{t2}/T_{t0} into T_{t0}/T_∞ correct.

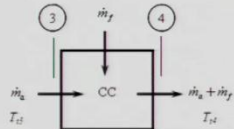
So, if you look at this is temperature ratio what is happening right this is nothing, but your this portion is nothing, but your τ_n this is nothing, but you τ_b this is nothing, but your T_{t2}/T_∞ τ_r no τ_d and this is of course, the τ_r right, but if I look at this can I say that τ_n is equal to one total because we are not adding any heat now you are taking any doubt of it adiabatic condition isentropic expansion right. So, therefore, I can say this is one.

Can I say that tau b is equal to one can I say I cannot why I am adding this right. So, therefore, the total temperature ratio after all the burner or the combustor cannot be worked what about tau d I can say it is one not constant one ratio is one right and so therefore, I will and tau r can I say it is equal to one totally no right because it is the flight dependent on the flight mach number right. It can never be one it will be more than one always unless otherwise flight mach number is zero then it will be one right. So, therefore, I can say that t nine by t naught is nothing, but tau b dot tau; that means, multiplication basically in this case I have used. So, t nine by t naught is nothing, but T t nine divided by t naught right and T t nine by t nine if you look that is equal to tau r we have already seen that earlier.

So, it will cancel it out you will get tau b; that means, t nine by t naught is nothing but tau b and of course, this I have already told you earlier that how it is T t nine by t nine is nothing, but you pi r gamma minus one that is nothing, but you tau right we have already derived this thing. So, we need not to repeat it.

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Assuming the flow in the combustion chamber to be one dimensional, steady flow, we can write,



$$\dot{m}_3 h_{t3} + \dot{m}_f \Delta H_c = \dot{m}_4 h_{t4} \quad (9)$$

From the continuity equation, we know that

$$\dot{m}_4 = \dot{m}_f + \dot{m}_3 \quad (10)$$

Since, $\dot{m}_f \ll \dot{m}_3$, therefore, $\dot{m}_4 = \dot{m}_3 = \dot{m}_a$ and also, for ideal gas,

$$C_{p3} = C_{p4} = C_p, \text{ Eq. (9) becomes, } \dot{m}_f \Delta H_c = \dot{m}_a C_p T_{t2} \left(\frac{T_{t4}}{T_{t2}} - 1 \right)$$

Handwritten notes:
 $\dot{m}_a C_p T_{t3} + \dot{m}_f \Delta H_c = \dot{m}_a C_p T_{t4}$
 $T_{t3} = T_{t2} \Rightarrow \dot{m}_f \Delta H_c = \dot{m}_a C_p (T_{t2} - T_{t4})$

So, now we will have to look at the combustors and keep in mind that we are assuming the in the this combustion chamber the flow would be one dimensional, but in real situation it can never be and we are assuming that certain amount of air is entering and fuel being added and it is going out with certain mass flow rate of air and fuel. So, we can carry you know like considering the flow to be one dimensional and steady

flow that always we are assuming in this analysis which I think I did not mentioned that is a steady flow.

And even in real cycle we will be assuming the flow to be steady that you keep in mind. So, therefore, it is the $\dot{m}_2 h_{t3} - \dot{m}_f \Delta h_c$ is equal to $\dot{m}_4 h_{t4}$. And what we are assuming here that continuity equation you know that \dot{m}_4 is equal to \dot{m}_f plus \dot{m}_3 and we are saying \dot{m}_f in this equation particularly less than \dot{m}_3 very very small. So, therefore, we are saying that is \dot{m}_4 is equal to \dot{m}_3 is equal to \dot{m}_a how much air is entering.

And we are assuming calorie perfect gas. So, that c_{p3} is equal to c_{p4} and c_{p3} right. So, if I look at this if I can write down this equation I can write \dot{m}_f equation I am writing $c_{p3} T_{t3} + \dot{m}_f \Delta h_c$ that is the heat of combustion \dot{m}_a and $c_{p4} T_{t4}$ right.

If I could write this expression for the simplify what I will get I will get $\dot{m}_f \Delta h_c$ is nothing, but $\dot{m}_a c_{p4} T_{t2}$ because T_{t3} is equal to T_{t2} right. So, in this place I can write down you know T_{t2} . So, I can take this out $\dot{m}_a c_{p4} T_{t2}$ and T_{t4} by divide by $T_{t2} - 1$. So, what I can write down here basically $\dot{m}_f \Delta h_c$ is equal to $\dot{m}_a c_{p4}$ right and $T_{t2} - T_{t4}$ and I from that I can get this like if I take T_{t4} out no no sorry there is some problem $T_{t4} - T_{t2}$ ok.

(Refer Slide Time: 35:48)

Let us define $f = \text{fuel / air ratio} = \dot{m}_f / \dot{m}_a$, then Eq. (10) becomes,

$$f = \frac{\dot{m}_f}{\dot{m}_a} = \frac{C_p T_{t2}}{\Delta H_c} \left(\frac{T_{t4}}{T_{t2}} - 1 \right) = \frac{C_p T_{t0}}{\Delta H_c} (\tau_b - 1)$$

where $\tau_b = \frac{T_{t4}}{T_{t2}}$.

For an ideal ramjet,

$$T_{t0} = T_{t2} = T_0 \left(\frac{T_{t0}}{T_0} \right) = T_0 \tau_r$$

then,

$$f = \frac{\dot{m}_f}{\dot{m}_a} = \frac{C_p T_0 \tau_r}{\Delta H_c} (\tau_b - 1) \quad (11)$$

However, we shall write

$$\tau_\lambda = \frac{\text{Total enthalpy at burner exit}}{\text{Ambient enthalpy}} = \frac{C_{p,4} T_{t4}}{C_{p,0} T_0} = \frac{T_{t4}}{T_{t2}} \cdot \frac{T_{t2}}{T_0} = \tau_b \cdot \tau_r$$

16

So, if you take this out you will get $T_t 4$ by $T_t 2$. So, let define f is equal to \dot{m}_f by \dot{m}_a and then from that expression I will get \dot{m}_f divide by \dot{m}_a is equal to $C_p T_t 2$ divide by Δh_c is $T_t 4$ divide by $T_t 2$ and this nothing, but your τ_b by definition right because we are assuming $T_t 2$ is equal to T_t , there is no you know compression here. So, therefore, we are and keep in mind that this $T_t 2$ will be same as T_t naught right because T_t naught is equal to $T_t 2$ and no heat being added as shown in the what you call $t-s$ diagram. So, therefore, I can write down T_t naught is equal to T_t naught divided by t naught and this by definition is nothing, but $\tau_r t$ naught by τ_r

Then we can write down as f is equal to \dot{m}_f by \dot{m}_a is equal to $c_p T_t$ naught τ_r $\Delta h_c \tau_b$ minus one. So, now, we shall write down basically the one definition which we will by using I did not defined let me define that the τ_λ this is total enthalpy at the burner exit that is you can say it is a there is a mistake here. So, four and t four and this is not required, so $T_t 4$ by $T_t 2$ and $T_t 2$ by this. So, will be τ_b into τ_r right the total enthalpy at the burner exit right will be $c_p 4 T_t 4$ divide by this thing c_p naught by T_t naught and that I can say that $c_p c_p$ this will cancel it out. So, therefore, $T_t 4$ by $T_t 2$ is equal to $T_t 2$ by t naught is equal to τ_b by τ_r ; that means, what you are saying τ_λ is basically $\tau_b \tau_r$ right, and this will used in you know some equations.

(Refer Slide Time: 38:08)

Then, Eq. (11) can be written as

$$f = \frac{\dot{m}_f}{\dot{m}_a} = \frac{C_p T_{t0}}{\Delta H_c} (\tau_b \tau_r - \tau_r) = \frac{C_p T_{t0}}{\Delta H_c} (\tau_\lambda - \tau_r)$$

Now, we can write

$$\left(\frac{V_9}{a_0} \right)^2 = \left(\frac{T_9}{T_0} \right) M_9^2 = \tau_b M_9^2 = \tau_b M_0^2$$

As stated earlier $\tau_b = T_9/T_0$ and $M_9 = M_0$. Then, the expression for thrust will be

$$\frac{T}{\dot{m}_0} = \dot{m}_0 a_0 \left(\left(\frac{V_9}{a_0} \right) - M_0 \right) = \frac{\dot{m}_0}{\dot{m}_0} a_0 M_0 (\sqrt{\tau_b} - 1) \quad (12)$$

Rewriting Eq. (12) to get a relation for specific thrust,

$$T_s = a_0 \left(\frac{V_9}{a_0} - M_0 \right) = a_0 M_0 (\sqrt{\tau_b} - 1)$$

So, coming to that like I can write down you know this f is equal to $\dot{m}_f c_p t_{naught} \Delta h_c \tau_b \tau_r$ you can write down λ right and minus τ_r . So, v_9 a naught square if we look at what we have done we have find out t_9 by $t_{naught} m_9$ square and this nothing, but your τ_b and m_9 is nothing, but your m_{naught} square. So, that became $\tau_b m_{naught}$ whole square.

So, then our expression for the thrust if we look at it will be in this place what we will be using is basically root under $\tau_b m_{naught}$. So, if I take this m_{naught} out it will be $m_{naught} a_{naught}$ square m_{naught} root over τ_b minus one this is very very simple expression what it says to you it says that thrust will be dependent on the amount of mass flow rate and that is into the engine.

It will be dependent on also the flight mach number and it will be also dependent on τ_b ; that means, how much energy you have been added τ_b will be representing temperature ratio across the burner which will be detected by how much you know fuel you have been burned. So, that total temperature will be increased right.

So, this is the simple expression we are getting and we will be varying this parameters and see how it is you know dependent on what and what are the conclusion that is why we call it as a parametric cell rewriting equation get a specific you know get a specific thrust because if I divided by \dot{m}_a here \dot{m}_a or m_{naught} .

(Refer Slide Time: 40:32)

The expression for TSFC is given by

$$TSFC = \frac{f}{T/\dot{m}_0} = \frac{C_p T_{t0} \tau_r (\tau_b - 1)}{\Delta H_c a_0 M_0 (\sqrt{\tau_b} - 1)}$$

The propulsion efficiency is defined as

$$\eta_p = \frac{\text{Thrust Power}}{\text{Rate of kinetic energy of the incoming fluid}}$$

$$\Rightarrow \eta_p = \frac{2TV_0}{\dot{m}_a (V_9^2 - V_0^2)} = \frac{2}{(\sqrt{\tau_b} + 1)} = \frac{2}{(\sqrt{\tau_\lambda/\tau_r} + 1)}$$

So, then I will get specific thrust a naught into v nine by a naught minus m naught that you can write down a naught m naught root under tau b minus 1. Basically just same thing only this is the specific thrust right and that is the thrust. So, now, we will do we will look at TSFC, just you know put these values here and then specific thrust you will get there as several terms are coming into pictures. And of course, it is a function of similar topics same general coming into picture. And propulsion efficiency will be you know you can express in terms of tau b right in place of tau b you can talk about tau lambda by tau r because tau b is related to tau lambda or tau lambda is related tau b and tau r. We know all that tau lambda is equal to tau b tau r.

(Refer Slide Time: 41:13)

The thermal efficiency is defined as

$$\eta_{th} = \frac{\text{Rate of kinetic energy of the incoming fluid}}{\text{Total thermal energy}}$$

$$\Rightarrow \eta_{th} = \frac{\dot{m}_a(V_9^2 - V_0^2)}{2\dot{m}_f\Delta H_c} = 1 - \frac{1}{\tau_r}$$

The overall efficiency η_o , which is the product of η_p and η_{th} , is

$$\eta_o = \eta_p \eta_{th} = \frac{2(\tau_r - 1)}{(\sqrt{\tau_\lambda \tau_r} + \tau_r)}$$

31

And if we look at thermal efficiency it will be similar you can note that the thermal efficiency is equal to one minus tau r; that means, it is dependent on the only on the flight mach number it is not dependent on tau b and other thing like how much heat been added is not really distinct that is a very interesting result. And overall efficiency of course, is a multiplication of that which is dependent both on the tau r and tau b, because tau lambda when I am saying this expression is basically tau b right because tau lambda is tau b by tau r tau tau lambda is equal to tau b by tau r. So, it will be you know coming. So, what I say we are armed with all this you know expressions right what we will have to do we will have to varies these parameters and which will be govern by in a altitude right, it will governed by flight mach number it will be governed by the temperature you

know at the exit of the combustors right. So, we will be doing that may be in the next class right because you may be having class after this, fine.