Fundamentals of Aerospace Propulsion
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Lecture – 10

Now, we will discuss about the characteristics Mach number that we had earlier defined and expressed the special energy equation that we have derived.

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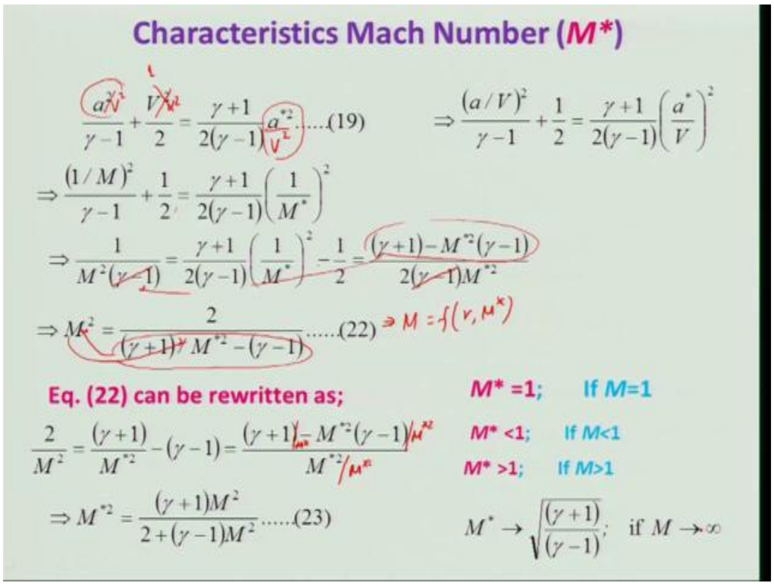

If, you look at, that is basically a square divided by gamma minus 1 plus V square divided by 2 is equal to gamma plus 1 divided by 2 gamma minus 1 a star square. What it indicates as I told earlier, that it indicates that, this what you call characteristics speed of sound remain constant along the stream line and we need to express this equation in terms of characteristics Mach number.

So, what we will do, we will just divide this explanation a 2 by V square and then naturally what will happen? This will become 1 and we will divide it by V square here. Both the l h s and r h s, I am doing, so what I will get here? I will get in this place 1 over Mach number square. Similarly, what I will be getting here, 1 over Mach star square. So, if I will do that I will get, I mean already I have done a by V square.

So, I will get 1 over n square plus half is equal to gamma plus 1 divided by 2 gamma minus 1, 1 over M star whole square right. Now, I need to express this in terms of like in terms of M star. So, for that let us what I will do, I will just take this over to the other side that is I can write down this term as 1 over M square into gamma minus 1 is equal to gamma plus 1, 2 gamma minus 1, 1 over M square minus 1 by 2.

So, I will just you know, subtract this thing, what I will get in the denominator 2 gamma minus 1 M star square and gamma plus 1, minus M star square gamma minus 1. What I am doing? I mean very simple, I am doing this thing. Then, I need to express this M, because M square in terms of M star, what I will do? I can just you know, cancel this term right. Can I not? I can cancel. So, then it will be and what I will do this portion, I will take to this side and then 2 M star square. I can take to this side and this term, I will take to this side.

So, what I will get? Then, N square is equal to 2 divided by gamma plus 1, M star square minus gamma minus 1. I can get very easily right? That means, what is saying that Mach number local max number can be related to the characteristics Mach number. Of course, you know what you can say that this Mach number, local Mach number is a function of characteristics Mach number and gamma right? That is it is saying, what it is saying? It is saying M is the function of gamma and M star, this expression says that for me is not it? Am I right? And we can express this just other way around that is characteristics Mach number in terms of local Mach number.

So, that is equation 2 can be rewritten as, I mean this is little algebra that is what I am doing, this is 2 by M square is equal to gamma plus 1 divided by M, M star square minus gamma minus 1. I have taken this term basically over here and this over there. Then, what is coming there like basically M star square in the denominator and then gamma plus 1 minus, M star square gamma minus 1. If, I divide by this you know both the numerator and denominator by M star square, M star square.

You know, I can get very easily that M star square is equal to gamma plus 1, M square 2 plus gamma minus 1 M square. And let us, look at how this characteristics Mach number. You know, is dependent on the local Mach number. If, local Mach number is equal to 1, what will happen to the characteristics Mach number? It will be also equal to 1. Is it? So, definitely yes because this will be 1 and this is 1 and gamma minus 1 plus 2 become gamma plus 1 that becomes 1. If, it is Mach number less than 1, local Mach number. What happens to the characteristics Mach number? It will be also less than 1 yes or no?

Similarly, if it is local Mach number great than the 1 that is your supersonic flow. Then, the characteristics Mach number also will be great than 1 for a particular of course gamma. We are considering, let us say gamma is equal to 1.4 and then looking at it, but if the Mach number tending towards infinity or it is a very big local Mach number, what will happen?

Then, It will be dependent on gamma plus 1 and gamma minus 1 root over. That means gamma plus 1 divided by gamma minus 1 root over. How I am getting this? It is in the limit when M is tending towards infinity. When you talk about limit. Then, how you will get this? You will have to use L'Hospitals rule. So, this I am leaving as an assignment to you and you will have to you know, do it and let me know, whether you are getting or not.

That means, what it is saying this simple expression says that it is almost you know, similar to the characteristics like characteristics Mach number is you know, similar to the local Mach number, but when the Mach number is tending to infinity it is not. It is some constant value. So, that is you know, this expression sometimes will be using this characteristic Mach number. Particularly, when we are dealing with, what you call normal shock and may be something Rayleigh flow kind of things, but I will be using Rayleigh flow give an assignment, how you can use it for a normal shock.

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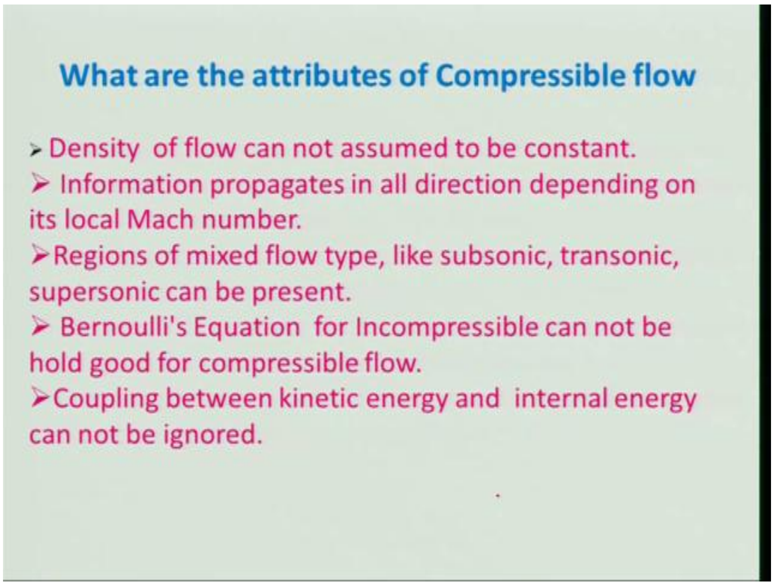

So, coming to that, let us now look at what are the attributes of compressible flow just to summarize the density of flow cannot be assumed to be constant. What we do generally for incompressible flow, Rayleigh says that, incompressible flow means density will not be changing, but as I told you that you cannot have that it is a myth, but for practical purposes we use it.

Information propagates in all direction depending on its local Mach number. We have already discussed and we will be using this concept to talk about the shock, shock formations and other things. So, regions of mixed flow like kind of things subsonic transonic, supersonic can be present particularly in transonic, both the subsonic and supersonic flow will be there.

We have seen and Bernoulli's equation, what we generally use for incompressible flow like P plus half rho V square without any contribution from the what you call this height or potential energy is remain constant that cannot be applied for compressible flow which we have already you know derived an equivalent for the compressible flow from the energy equation and the last, but not the least. That is the coupling between kinetic energy and internal energy cannot be ignored. What is the meaning of it? If you look at your energy equation we always say that h plus V square divided by 2 is constant that is of course, for al1 the stream line.

Here, if it is compressible flow. Then, what happens the kinetic energy? It will be converted into the Enthalpy. When you talk about Enthalpy, it consists of internal energy and p v, p v is your flow off. Therefore, this energy will be converted into kinetic energy, this will be converted into the internal energy and it is quiet. You know, this thing, so it cannot be really ignored, which we ignore. In case of incompressible flow because the change is very minimum. So, we have looked at the basics of the compressible flow right and today. We will start this lecture 10 with a statement from Richard P. Feynman ,who is my favorite.

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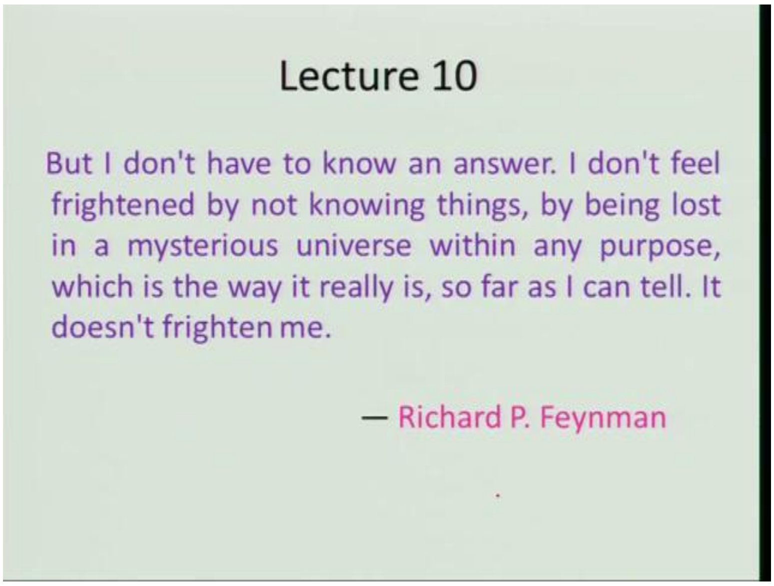

He says that, “but I do not have to know an answer for asking a question”. You know, last time I talked about question, “I do not feel frightened by not knowing the things, by being lost in the mysterious universe without any purpose, which is the way it really is, so far as I can tell. It does not frighten me”.

What he is saying? He is saying that he is not really frightened by for asking the question, because unless otherwise you ask a question you cannot do really any meaningful activities in the life. Not only in the science and technology, but also in the life. Therefore, it is very important to ask question and if you look at what is the performance statement he has given. Therefore, I would urge you people to look at what is the importance of life and how you can contribute and how you can do for the science and technology.

So, this third process we can start this lecture and as we do recall the things in the beginning, what we have looked at in the last lecture, about the basic concepts of the compressible flow starting from the stagnation point, the stagnation properties. Then you look at specialized energy equation, then you look at characteristics Mach number, how it can be caught in the energy equations or express the specialized energy equation in terms of the characteristics Mach number. We have related characteristics Mach number to the local Mach numbers and also now we will look at basically Quasi-One Dimensional Flow.

Why you are looking at Quasi-One Dimensional Flow, because we will be dealing with the analyzing. Basically, we will be analyzing the propulsive ducts where flow is likely to be 1 dimensional not all the cases. However, in case of a nozzle or in case of a diffuser we can happily consider the flow to be 1 dimensional of course, for the simplicity provided, there is no flow separations and other things are occurring.

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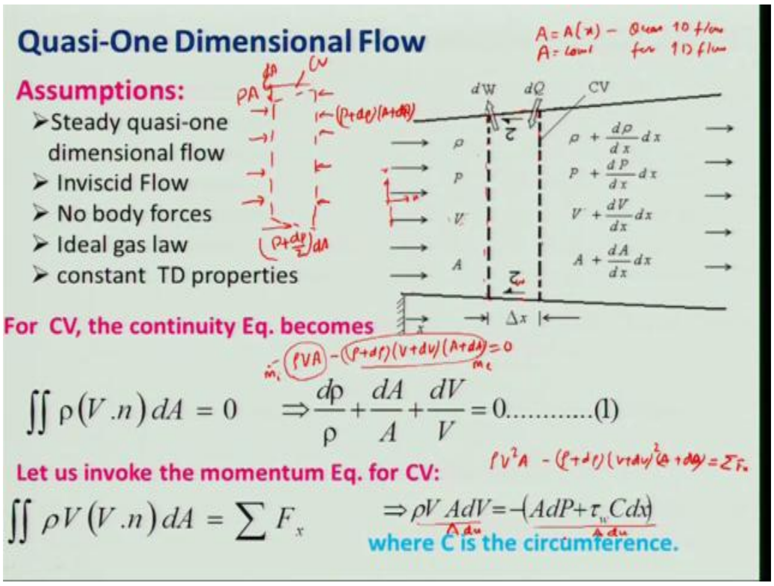

I have already explained you, what do you mean by Quasi-One Dimensional Flow? Let us consider, there is a duct, which is diverging in nature right and through which the fluid is coming with a uniform velocity and having the velocity V and of course, the pressure P. Here, and density is here and the cross sectional area is A, we can consider. It is the flow, is going and then of course we need to analyze this, what is happening here. If you look at whether we can take this flow in this duct as a Quasi-One Dimensional or not. If you look, the flow can be 3 dimensional in nature. For example, if I take x along the actual direction and r along the radial direction and theta is the in the theta direction.

So, r theta z coordinate system because in propulsive devices, we use that duct right duct with a circular cross section. Therefore, what will be the flow in this case even if you are having a uniform velocity profile in this region. Still, can I call it as a flow to be 1 dimensional and because flow is you know, if it is a circular cross section. I can consider the flow to be you know, symmetric along the theta direction right. Therefore, it will be 2 dimensional.

Then, there will be flow along the x direction, along the radial direction, but can we consider the flow to be one dimensional, provided the rate of change of this area is very small. That means, this rate of change of area what I have shown. Here, if it is small and then the change in the velocity along the x direction is more as compared to the change in the r direction. If you look at, this is my r direction and this is my x direction. So, in r direction will be very small, then we can call it as a Quasi-One Dimensional Flow. In this, let us say there will be work interaction, in case of your compressor and turbine.

There will be work interaction, but in case of your nozzle and air intake. There would not be any work interaction because it is having no rotary motions kind of thing. We just enhance the flow in a nozzle and decrease the flow in an air intake. Therefore, we are now talking about generalized one, there will be some heat interaction that means, you know, in depending up on kind of propulsive duct we are talking about.

So, we need to analyze generalize you know, equation will be deriving and we will make some assumptions. What are those assumptions? Which is, I have already told you, that is the flow will be steady and Quasi-One Dimensional Flow and then we are considering the flow to be inviscid, but however, I have kept here. You know, the shear stress on the wall just to make it more generalized because we will be using this equation later on and we will make some assumption of it inviscid flow as you go along.

There is no body forces and the gas can be considered to be an ideal and that means we need to use the perfect gas equation and constant thermo-dynamical properties. We will be using and when the properties remain constant that means, it is not changing when it is flow is taking place the properties. For example, C P, C V although it is a function of temperature, we are not considering to be function of temperature that means, C P will be remaining constant as you go along.

We will use some variation where you know, in propulsive ducts, we will see in the cycle analysis what changes. It can make and when you do that, is basically what you are doing you are using calorically perfect gas. So, now we need to analyze this, for that we will take this control volume, what I have shown. Here, either dash line right and apply the conservation of mass momentum and energy equation and then we need to in vocal.

So, the equation of state for calorically perfect gas. Then, we will solve that. Let us consider now, for this control volume continuity equation for a steady flow. They will be there, would not be any steady term, so that became rho V n d A, is equal to 0. In this case, if you want to look at, what it will be, if you just look at this point because this is the inlet to the control volume and the flow is going out of this control volume.

So, if I look at here it will be rho V A that is rho V A, rho V A minus rho plus d rho, in this case rho plus d rho at the exit of your control volume. Of course, this is canceling out, I can just show that it is the change in density. Therefore, I have taken right into V plus d V into what will be area in case of a plus d A, because this is what this is a Quasi-One Dimensional Flow, when I talk about Quasi-One Dimensional Flow area is a function of x. Here, right for Quasi 1 d flow, but for A 1 d flow what will be area? Is not a function of rather area is constant. It is like a pipe, therefore A plus d A comes into this is equal to 0.

If I simplify this thing you know, neglecting the higher order terms right and simplifying further, I can get an expression which says that d rho by rho plus d A by A plus d V by V is equal to 0. So, that means if you want to see that, what it really says? If I say that means it says it will be. What is the meaning of this? If you look at this thing it is basically mass flow rate that means, mass flow rate entering here and mass flow rate exiting out of the control volume.

This is your mass flow rate, if you look at mass exit and mass dot in this, is equal to 0. I mean we have already talked about d M by d t is equal to M dot. You know, e minus M dot half, you can say that. So, you can get this expression and let us invoke the momentum equation for a steady Quasi-One Dimensional Flow. That is rho V, V dot M into d A, is equal to summation of forces, like x.

So, if you look at what are those forces will be 1, will be pressure forces other one. We are considering is, this is a friction. You know, without considering the boundary layer, If it is anything, it is a little other way around. So, let us look at what are these things in this portion? What it would be? It will be rho V square a minus rho plus d rho V plus d V whole square A plus d V, d A summation of f x. Can I not get this left hand side of this equation? I am doing, so and you can see them.

We will have to put substitute these things, that is as I told you, pressure and when you talk about this pressure how will do that can I take this control volume. For example, if I take this control volume here, what will be pressure acting over in this surface? What will be pressure P and what is the area P A will be? P A that means the force acting on this surface will be P A. What is the force acting over here? P plus d P into A plus d A right keep in mind that this thing is in equilibrium.

Therefore, it will be opposite, but is there any force which will be acting on this surface? Certainly yes, that force we can take as a P plus d P by 2 into what area it will be. Certainly no, this will be d A what is d A , if you look at it is this area is d A. So, if you do that and what is the force acting? Here, tau w tau w into c. If I take this circumference into delta x is this. So, that is the force is acting which is opposite to this x. Therefore, the pressure force will be negative, so also the frictional force.

So, when you do this thing, you will get an expression, it will simplify higher order term. If you neglect you will get rho V A, d V is equal to minus A d P plus tau w c d x, as I told you that c is basically circumference of this control volume. Of course, which circumference you will take? You will take the average right circumference will be changing here.

It will be changing there right, so we take that and what I will do, I will divide this equation by A and d x similarly. If I divide this equation A by d x, what I will get? See this A will cancel, it out this became rho V d V by d x is equal to A, A cancel it out d P by d x minus d P by d x minus tau w c by A.

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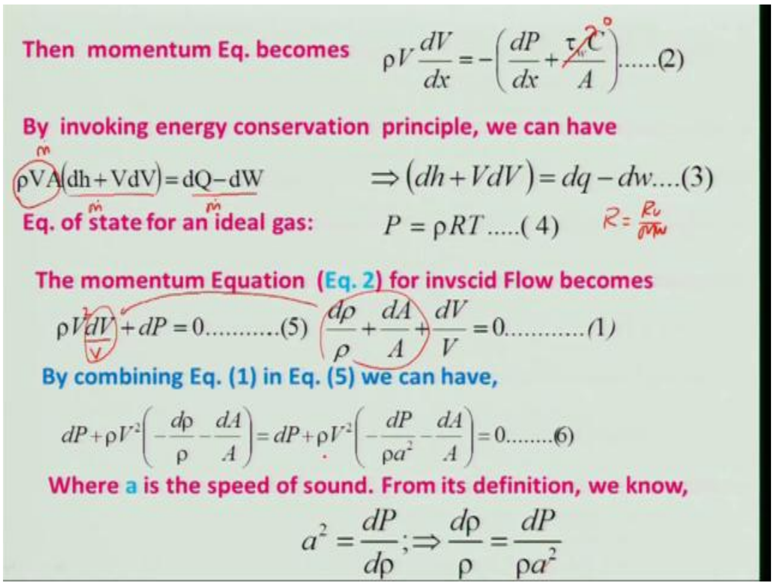

So, if I will do that momentum equation becomes rho V d V by d x is equal to minus d P by d x plus tau w c divided by A. Keep in mind that in the whole, use bracket minus because it is in opposite direction, the frictional force always oppose the opposite direction to the flow. Now, we will invoke the energy conservation principle and in the similar way we can derive it rho V A d h plus V d V is equal to d Q minus d W. What is this term? Here, this is your mass flow rate which is remaining constant although it is a diverging duct in this example, but mass remains constant because it is a steady flow. Am I right?

So, I can divide this both the left hand side and the right hand side by this 1. So, that this will cancel it out. What I will get? Then I will get d h plus V d V is equal to d Q minus d W. Basically, is heat per unit mass, the heat interaction per unit mass, flow rate or mass. This is the work interaction unit mass right, so now we will have been work this equation of state for an ideal gas which we already know that P is equal to rho r, t keep in mind that this r is specific gas constant that means this is equal to r u by molecular weight.

Universal gas constant divided by molecular gas is a specific gas constant, that means this specific gas constant will be dependent on the kind of fluid, you are handling. If it is air it will be 287 some number, I am just telling, but it will be different for different gases, the momentum equation for a inviscid flow what it becomes, this will be 0. So, when it is 0 that is rho V d V plus d P is equal to 0, because you know, it is A d x d x. So, rho V d V is equal to, if I take to this side it become is equal to 0 and combining this equation 5 with the 1 what is that equation 1.

We have that, is nothing but your continuity equation which states that d rho by rho plus d A by A plus d V by V is equal to 0. What we will do now we will basically substitute over here, that means if I say this is d V by V , what I will do? I will just divide here V and it will become V square, because I am putting V here and then I can do that in place of d V by V. What I will do? I will put minus d rho by rho and minus d A by A.

In this place, in this place, I will be using these terms keeping in mind that with a minus sign when I will do that, what I will get? I will get d P plus rho V square minus d rho by rho minus d A by A and we know that we can relate this d rho with the d P, how from the definition of speed of sound that is nothing but a square is equal to d P by rho d rho and so d rho by rho becomes d P s square into rho. So, I will substitute over here these values, so if you look at it, if I take this out here what I will say rho can be cancel it out and V square a square. If you look at what it becomes? This is nothing but your Mach number square. So, then this term becomes, if you look at d P. If I take it out it will be 1 minus M square.

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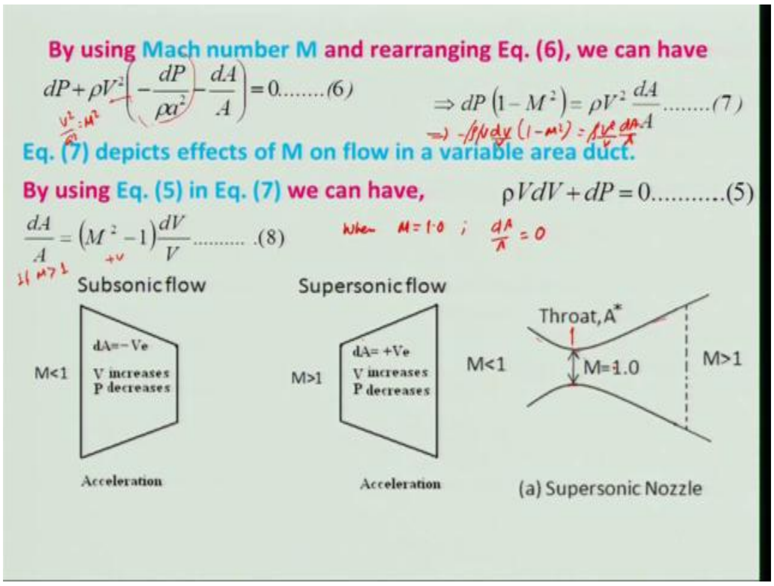

We will do that, so I have already talked about this equation. So, as I told you earlier that if I take this term, this term you know, outside here and rho will cancel it out and it becomes basically V square by a square. This is nothing but your V square by a square is nothing but your M square, so d P, if I take it out it become 1 minus M square is equal to rho V square d A by A. That means, this expression is relating not only the pressure difference and Mach number, but also the change in area. So, let us you know what, You can simplify this further.

As I told you earlier that equation 7 depicts the effect of Mach number on the flow in a variable area duct, because if you look at the area is changing and Mach number is there. Of course, for a certain pressure gradient. How it will be related? It is being depicted in equation 7 by using equation 5 in equation 7. What is that equation 5? It is nothing but your rho V d V plus d P is equal to 0. So, we will substitute here in case of d P, what I will do, I will put minus rho V d V right in place of d P, what I will do ? I can put it is a minus rho V d V. I mean if I do it here like minus rho V d V 1 minus M square is equal to rho V square d A by A.

So, rho cancel it out rho and V cancel it out V. So, if I divide it by this V here, so what it comes it comes d V by V 1 minus M square is equal to d A by A. In other words, I can write down as d A by A is equal to M square minus1 d V by V, so yes fine. So, we will arrive at d A by A, is equal to M square minus 1 d V by V , I mean this is the relationship and if you look at this relationship, what is saying it is relating the change in area with the change in the velocity if M is less than 1.

What will happen, if M is less than 1 and that means, if I want to accelerate the flow, I will have to use a decrease in area, what you can see here when there is a decrease in area what happens, the velocity increases as I told and also pressure decreases now. If the M is greater than 1 what happens like this term become higher that means it becomes positive, this become positive if M is greater than 1 and if it is positive that means area has to what you call increase for increasing the velocity right that means V 2 should be greater than V 1 or V exit area should be greater than inlet area. Therefore, I will get an acceleration, of course.

Here, again the pressure decreases and velocity increases, but if I combine this together what I will get, I will get a duct which will be basically make the flow to be supersonic. How it is? Because, if you look at M when M is equal to 0, when M is sorry, M is equal to 1, what it would be? It will be d A by A is equal to 0, right? That means there would not be change in area. It can be of course, maximum or it can be minimum, but generally maximum would not give you physical results. Whereas, the minimum 1 will give you the figure here, that means the Mach number will be 1 here.

In the throat, where if you look at this area is the minimum as it is converging and again diverging converging and it is diverging. Therefore, they are the throat, this is known as throat right and where Mach number will be equal to 1. So, let us consider the other way around that is basically we want to decelerate the flow. If you want to decelerate the flow then what will have to basically decelerating flow means V 2 should be less than V 1.

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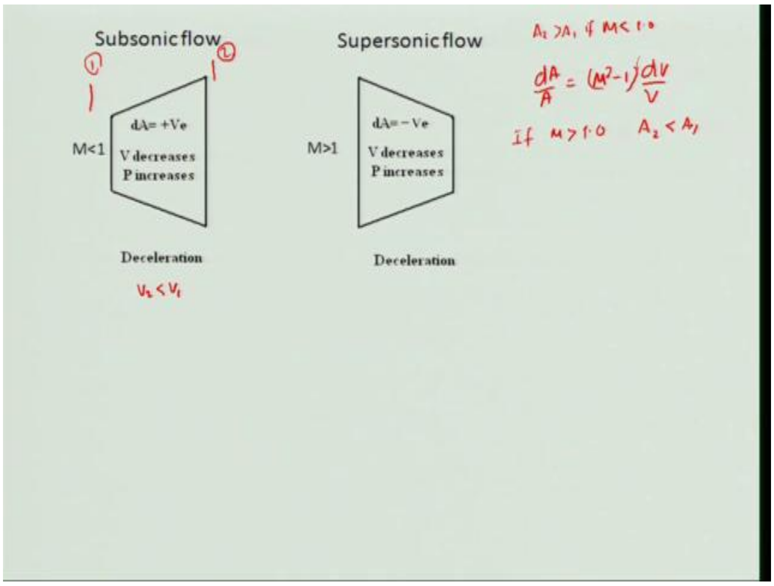

For example, if I take this is, my 1 and this is my 2 that means my flow should be for deceleration of the flow V 2 should be less than V 1. That means, what will be that d V will be negative, so what we have looked at the expression is we have looked at right that means this, if this is negative right, this term is negative. We want to have, what we will have to do. If M is less than 1 that means this is become negative.

So, negative, negative become positive, so that means area should be greater, that means A 2 should be greater than A 1. If M is less than 1 that is what is being shown here. That means if it is divergent duct the velocity will decrease and pressure, of course increases. Similarly, if M is greater than 1 what we will get, we want to decrease this because this term is negative and this is positive. Therefore, area should be less than A 1 right so then we can decelerate.

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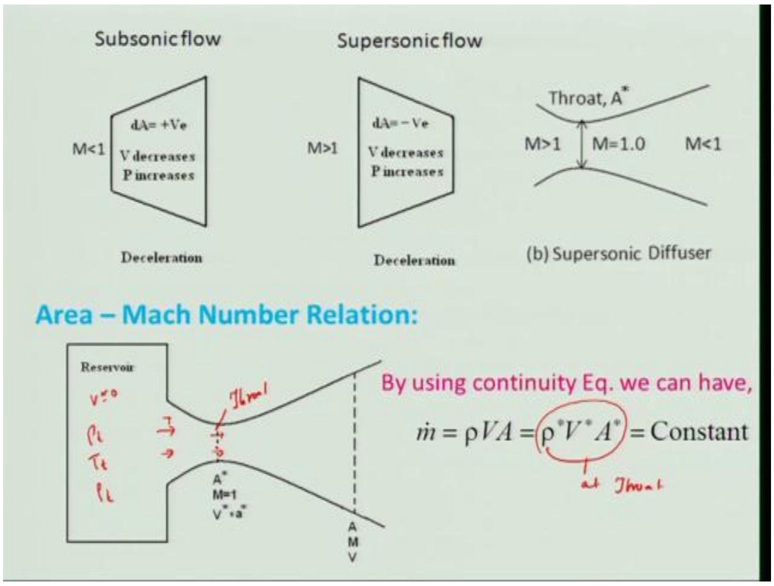

So, if you look at, this is basically the supersonic, diffuser where the flow is subsonic at the entrance and it goes out decreases, because the flow is supersonic and then it decreases further after passing through the throat. This is being used in your air intake kind of thing, which we will be discussing little later on, when you are discussing about cycle analysis.

Now, we look at area Mach number relationship let us consider a convergent divergent duct, which I have shown is attached to a reservoir, reservoir means, what that means here velocity will be almost equal to 0. That means it is a very larger area and if velocity is almost 0. Then, what will be the properties? Properties will be total or the stagnation properties like P t, T t and rho t kind of things right total temperature.

This is our throat area where the flow is likely to be sonic condition. Therefore, I am using star here and of course, any area in it can be Mach number. It will be different depending on what really for it is designed. So, also the velocity what we will do we will just take the mass flow rate because here we are using a steady flow. Therefore, mass will be remaining constant in each cross section. So, I can take M dot is equal to rho V A is equal to rho star V star A star. What is the meaning of this? This is basically adds the throat that means whatever mass flow is coming over here. It will be same as that of at this point is it agreeable.

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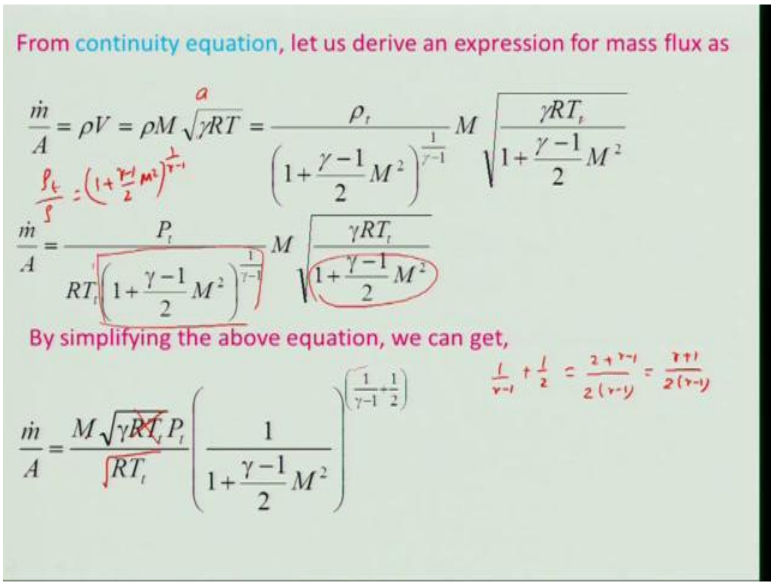

So, from the continuity equation, we can derive an expression for mass flux, mass flux means what mass flux means mass flow rate per unit area, it is nothing but A rho V and what is this V, V we can express velocity right V is the velocity of the fluid at particular cross section in the duct convergent divergent duct.

It can be anywhere, this is generally we are talking about and this velocity can be expressed in terms of Mach number because by definition we know, Mach number is equal to V divided by A. So, this is M into A and A is nothing but your root over gamma R T. So, in place of this we need to relate to the reservoir properties. What are the reservoir properties? Those are total temperature pressure density like that.

So, what I will do? We have already derived expression for the temperature ratio that means rho t divided by rho is equal to 1 plus gamma minus 1 divided by 2 M square, whole power to the 1 minus gamma minus , we have already derived. You are not getting ,I think, let me write down. Here, rho t by rho is equal to 1 plus gamma minus 1 M square, power to the 1 minus gamma minus 1.

So, in place of rho what I will do I will write down the rho t by 1 plus gamma minus 1 divided by 2 M square power to the 1 minus gamma minus 1 and M remains same here. In place of T, T you know this is the t here I will express this t that is the static temperature right with respect to the total temperature corresponding to the reservoir in this example. That is gamma R T divided by 1 plus gamma minus 1 divided by 2 M square.

So, what we will do now ? I will instead of rho t you know, I can use equation of state for a perfect gas and then express in terms of pressure. So, instead of that I can write down P t by rho t in place of rho t, I will be writing r P t divided by R T and of course, same thing remains. We would need to simplify, if you look at, I can simplify very easily you know which may, I can do here or I can do later on.

So, if you look at this term and this term is similar in nature right, so we can clap this together because this says 1 plus gamma minus 1 divided by M square to the power 1 by 2. Here, it is 1 by gamma minus 1 the same term, so we will be trying to club it. So, if you look at it, I can write down 1 divided by 1 plus gamma minus 1, 2 M square power to the 1, minus gamma minus 1 plus 1 plus half. What will be, if I take this term gamma 1 plus 1 by 2 what it would be? It will be 2 gamma minus 1, 2 plus gamma minus 1 is equal to gamma plus 1 into 2 gamma minus 1. That means in place of this term I can write down this and keep in mind that this, if I cancel it out this become root. That means this will go away.

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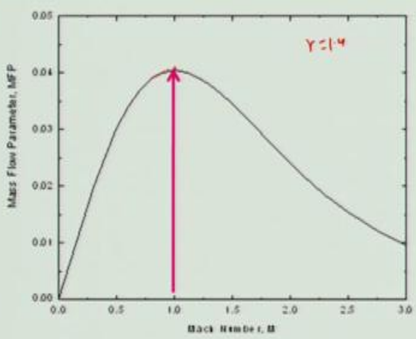
By rearranging the above equation, we can have,

$$MFP = \frac{\dot{m}\sqrt{T_t}}{AP_t} = \frac{\sqrt{\gamma}}{\sqrt{R}} M \left(\frac{1}{1+\frac{\gamma-1}{2}M^2} \right)^{\frac{\gamma+1}{2(\gamma-1)}} = f(M,\gamma) \ldots\ldots(9)$$

Note, $\frac{\dot{m}\sqrt{T_t}}{AP_t}$ = MFP



So, what you saying, this mass flux ratio will be function of P T, function of Mach number. Then, gamma which we will little bit simplify in the next and see that. So, by rearranging this, I can get this M dot root over t, A P T what I am calling it that mass flux parameter M F P which is not seen here. So, is equal to root over gamma root over R because T, I have taken M 1 over 1 plus gamma minus 1 divided 2 M square power to the gamma plus 1 divided by 2 into gamma minus 1. What it says that this mass flux parameter is the function of Mach number and gamma that means it is the function of flow velocity right Mach number.

We use it for convenience and gamma, is the c p by c v that means it is dependent on the kind of fluid being used. So, as I told you, I am saying this term as the mass flow parameter. So, this if I plot this expression you know for gamma is equal to 1.4. What I will see? I will see this mass flow rate with Mach number it increases. You know, it increases and reach a certain value then it decreases. That means, if I am increasing this mass flow rate than it will be on increasing and then decreases and these peak values occurs at Mach number is equal to 1.

This peak values occurs as A, of course this you can call it as a subsonic flow it will be on increasing mass flow parameter and decreases. What is the implication of this? We will be discussing it little later on what is the implication, of course, it is increasing and decreasing. So, what you can just immediately see that what is happening to this mass flow rate, it will be you know, dependent on what dependent on pressure total temperatures you know kind of thing.

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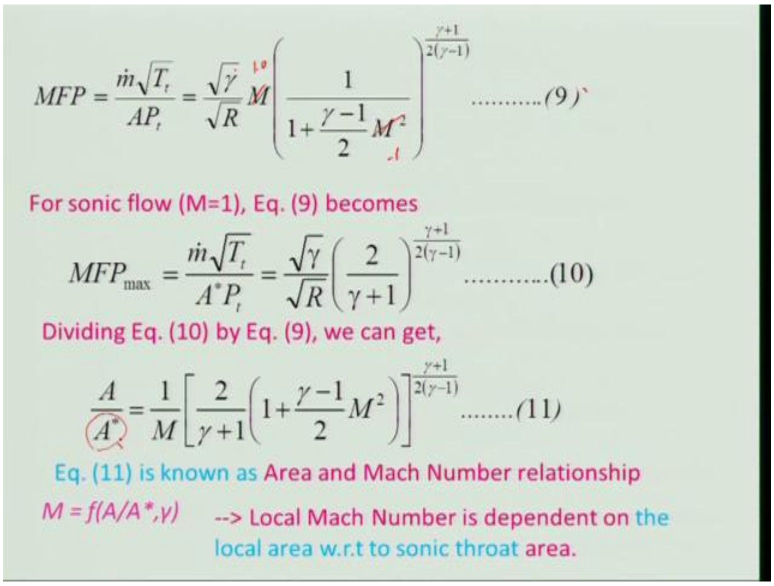

So, we will be looking at little later on, so just to talk about it like that for sonic flow M is equal to 1, what will get? Equation 9 becomes M F P max that is the maximum flow rate which is corresponding to M equal to 1 Mach number, it becomes like what I will do? I will put M is equal to 1 in this equation, M is equal to 1, that means this become 1 term right. If it is 1 what it becomes? This will be 2 plus gamma minus 1, that becomes gamma plus 2, so and 2 will go to the numerator. So, that becomes 2 divided by gamma plus 1, M here is equal to 1.

So, from equation 9, I can get this expression very easily, that means if 1 equation this is general, this equation 9 is for general. This is for the only sonic flow where Mach number is equal to 1. If I divide this expression, divide this equation 1 by equation 9, what I will get? I will get an expression A by A star ratio is equal to 1 over M 2 divided by gamma plus 1 into 1 plus gamma minus 1 divided by 2 M square.

Of course, the whole bracket here to the gamma plus 1 divided by 2 gamma. How I am getting that because if you look at this equation 10 is divided by equation 9. That means M f maximum divided by M F P. I am doing, so if I do this thing this mass flow rate will be canceling out. Fine, This is A by and root T, will be canceling out P t, will be canceling out what you will get? A by A star in the left hand side and in the right hand side what you will get? You will get 2 by gamma plus 1. This term and this gamma root over gamma and R will be canceling out with this equation 9.

So, and then this term will becoming just you know, what you call the denominator and as it is 1 over this becomes, 1 plus gamma minus 1 divided by 2 M square. So, what it indicates, this area ratio is a function of what is a function of Mach number and gamma and this is known as area Mach number relationships. What it indicates, so the local Mach number is dependent on the local area with respect to sonic throat area that means a star. If you look at this is your sonic throat area right. So, what we will do we will look at this variation that expression, what I have talked about here and let us see how it is varying with respect to the Mach number and when we are talking about it.

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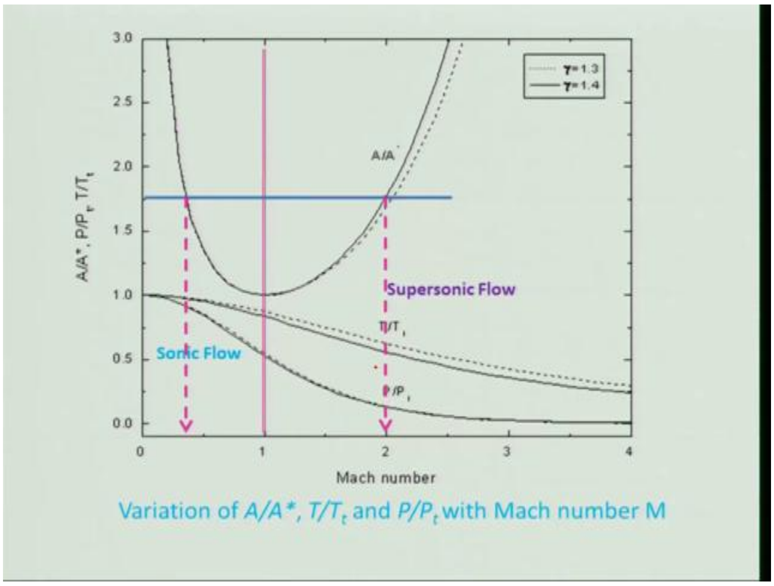

We will be looking at T by T t also and P by P t because the P is the local pressure of the static pressure and P t is the reservoir pressure or the total pressure. The curves will be looking like this, A by A star and Mach number are plotted here. If you look at this curve source, for the solid line source for the gamma 1.4 and gamma 1.3 for the dashed line and if you look at this Mach number is equal to 1. That means this side is your sonic, what you call subsonic flow and the right hand side is your supersonic flow, but if I take a certain area ratio let us say I am taking may be 1.75.

Here, is what it indicates, it cuts this area ratio to 1 that means I can get this. You know, 2 Mach number that means for the same area ratio, I will get 1 Mach number here which is may be around 0.4 kind of thing. The other 1 is around 2, what it says, I think I did not asked this question, if you go back and look at that equation, it will be having 2 solutions for the same area ratio.

Physically you can feel and the other thing is that, if you look at your this temperature ratio and pressure ratio is changing and for different. You know, specific ratio is constant that is gamma, it is different. With this I will stop over and we will continue in the next class. How we can use these equations and expressions for talking about nozzles and other things.