Fundamentals of Aerospace Propulsion
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Lecture – 26

Let us start this lecture 26 with thought passes manipulative materialism makes a man maniac. And we usually now recall what we have done in the last 25 lectures, if you look at 5 or 3 to 4 lectures, I talked about various aspects of propulsive devices, then we spent around may be 22 lectures around that on fundamentals of the propulsion. That means, what are the tools are required to analyze the propulsive device. What are those one is compressible flow then we talked about thermodynamics, then we talked about chemical kinetics and then we talked about combustion. Now we have almost completed of course, I cannot say we have completed, but we look at it. So, that it will be helpful for analyzing the propulsive devices as the course is tell us to look at fundamentals of propulsion therefore, I am gave an emphasize for the fundamental.

Now, we will see how we can use those fundamentals principle for analyzing a propulsive device both air breathing and non-air breathing. Non-air breathing means rocket kinds of engine right. So, what we will do we will do now look at how to you know analyze propulsive device particularly thrust and the other performance parameters.

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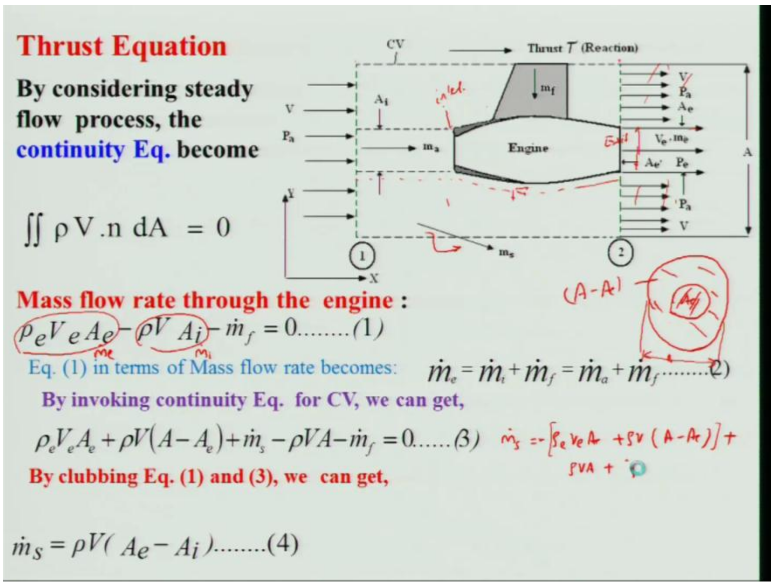

So, let us derive the expression for air, you know thrust pertaining to air breathing engine. And here I have considered a pot mounted engine, I mean we are not worried about what its contain, what are the components we know that air breathing engine will be various component like air intact, compressor, combustion, chamber, turbine and nozzle, we are not worried about it. We are saying that it is basically propulsive device or a engine or a propulsive duct and which is mounted in a pot. That means you know there is various ways the engine can be mounted to an aircraft. You might have seen engines are hanging from the wing and it is you know mounted near by the fuse less, and in fighter aircraft, it will be, engine where it will be located? Where?

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Professor: Close to fuse less, but on the front side. So, the air intact will go. So, there are various ways of mounting the engines, but here we are looking at general. What we look at basically a pot mounted engines. And it is hanging right from the wing, and keep in mind that the fuel flow rate will be entering through that engines I have shown here and through this is the inlet. This is the inlet of these things inlet, and this is your exit of the engine that some air will be entering and it is going out. And of course, you will add some fuel combustion will be taking place and then you add basically energy you will be giving energy, so that you can get the thrust.

Now when air is entering and what is happening here, it is going at a very high velocity. Suppose, it is entering with the you know V and then it is leaving with the velocity of V e which is higher why it is so, because you are added energy into it. And you are trying to accelerate the exit velocity, and keep in mind that we have done some approximations here is it that, it will be coming through that way that whatever vectors have shown. Of course, it will be true right provided that we assumed that is one-dimensional flow. In real situation, there will be boundary layer, there will be some swilling, you know because from turbine it is coming. So, there all those things we are not considering. Please keep this in mind.

And now, if I want to find out this thrust equation I need to invoke, the mass conservation equation, momentum conservation equations for that I need to placed the control volume, where to control volume, where to place this control volume is important. Of course, here it is I have shown here, but why I will put like this. For example, why my control volume will be just exactly at the exits of the engine, why not it is far away? Why my this control surface bottom bottom and the top control surface is away from engine, why not nearby, adjust it here itself, and similarly why my this left hand control surface you know why it is in this place, far away from the inlet. What is the reason? I can place control volume anywhere right is not it? Can I not put it here somewhere or can I not take this control volume and then do that I can do, yes or no. But if you do that may be analysis will be quite complex. To have a simplified, we have done that and keeping it the one-dimensional flow in mind, because if I used two-dimensional it will be little complex to do that, that is the only thing.

And keep in the mind that we are not considering the unsteadiness; that means, it is not like you know taking a ((Refer Time: 06:28)) by an what we called fighter aircraft, which is be unsteady. It is a level flight what we have considering to deriving in this thrust. Therefore, we have considering the aircraft is moving steadily, and some other assumptions we will be doing, but as you go along, you can note down that assumptions but I would not be stating as done earlier like you know steady flow passes irreversible. You know there is no frictional effect right or Euler flow, all those things I am not stating in a very systematic way, but as I go along, I will telling and some of things I would not be stating very explicitly.

So, why I am placing this control surface here because as I told you, here I can assumed the flow to be one-dimensional, but if I place somewhere here in this zone, then can I play do that. If I do that then the flow will be two-dimensional in nature, because it is a jet flow and tenement will be there, and the flow it is a weak region. So, therefore, it will be difficult to take. So, therefore, I will put it here, where I can assume the flow to be uniform at the exit, and also the other portion from here to there, it will be uniform. And we are assuming that it is this flow is coming with the same velocity with which is the entering in the control volume at the inlet that is the station one and the exit I am saying station two.

And keep in mind that this surface is far away because the effect of because if I take the stream line over here, what will happen it will be you know taking the shape of the thing and then I am having two-dimensional flow. If I take this you know resolve it one in X-direction other is in the Y-direction. So, therefore, it will be two dimensional, if I over here, I will be not considering as I will just say that this y-dimension. Some flow will be coming out, because there is a obstruction, but that will be negligible form, we will see that. And it is placed here, so that the flow can be taking uniform and I am not worried about how much, it is because if you know that actual flow will be little diverging in this zone in the inlet, because a evasive flow will be there. And you know it will be diverging toward that.

So, let us now look at considering steady flow process, the continuity equation becomes what we called rho V dot n d n right and if I take for this control volume sorry first we will take the mass flow rate through the engine. If you look at what is the mass flow rate it is entering if A I is my this area right then I can write down that a how much mass is going out from the engine that rho e v e and this is my area what we called a e right I have shown here A e. So, rho e V e A e minus rho V because it is entering with the rho and velocity V with this and rho V A I A I is the capture area what I called and minus m dot f because some of the fuel is entering.

Because whatever the fuel is entering whatever the mass is entering you will be going out of it right mass is conserved that is the exit mass flow. So, if I want to write down in a mass form, I can say mass flow rate of exit, this is basically if you look at m dot e and this is your m dot I and this is m dot f; that means, m dot is equal to m dot I plus m dot f. You can say this is basically mass flow rate of air plus mass flow m f I can say that right; that means, how much mass flow rate of air is entering into the engine. So, now we will be invoking the continuity equation for the entire control volume what we have done till now, we are considering the mass conservation for the engine. That means, how much mass is entering how much mass is going out and that is the mass leaving the engine is equal to the mass of the air entering into the engine plus mass of the fuel is a very simple one.

Now, what I will do we will be considering in this domain that is you know how much is going out here and how much is going to exist. So, if you look at how much is going at exit that is rho e in this from the engine rho e V e A e, and in this region what you are doing like that is is coming with the same velocity. This again an assumption because nearby it would not be, because of streamline it will be having different values, but however, we are assuming that V. So, that is rho V into area what is that area, total area from here to here of the control volume right cross sectional area is A and the exits area A e. So, therefore, this portion you know if you look at this portion is nothing but a minus a e right. So, are you people getting or not for example, I can say that like this is my A right this is the A e and this is what we called this is basically A. So, what this area is nothing, but A minus A e right that is the through which this is coming.

So, rho V A minus A e plus m dot s and m dot s is will be coming like you know like this in the x direction. This is x direction where assuming that y direction will be very low as you go away you know like that kind of thing what about then, but that is not true. It is an assumption, some component will be there, then clubbing this what we called clubbing. This equation one and equation three we can get because we are interested to find out, which is m dot. So, I can write down m dot s is equal to rho e V e A e plus rho V A minus A e right minus plus rho V A plus m dot f. Because I am taking this left hand side then I am getting that and what I will be doing in place of this m f I will be writing rho e V e A e; that means, in place of this or in place of I will be writing basically now what I will do.

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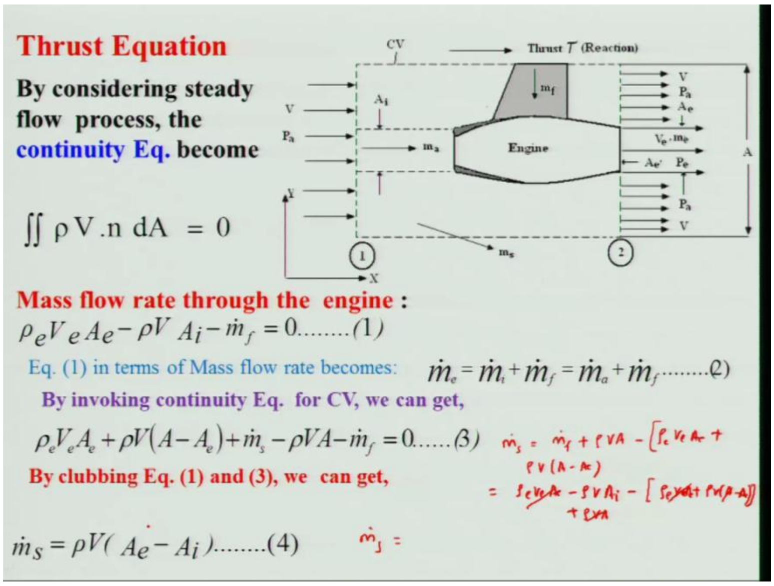

We will just put this in place of m dot f we will you know used this equation one that is nothing, but rho e V e A e minus rho V A i and the same thing as rho e V e plus rho V A minus A e keep in mind that I have left one term here. So, I will add it rho v a if you canceled it out this term will go away and similarly this term will go away with this term. So, what I will get, if I simplify I will get m dot s is equal to rho V A e minus A i, and if you look at I will get m dot s is basically I can get rho v a e minus A i this will be rho into what we called A e minus A i. That means, this amount of mass flux which will be going through this control surface right and then down and goes up like it will be going out because there is burning and we will be using this and find out a expression for a thrust.

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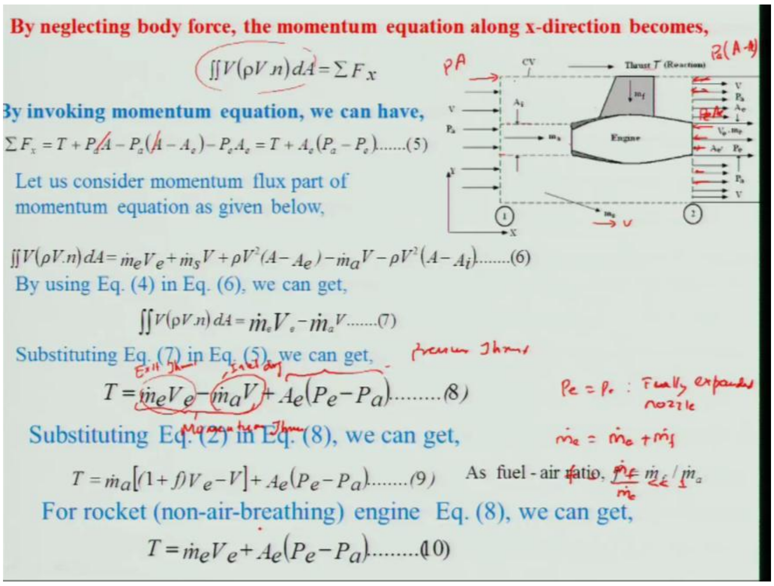

So, that if you look at the thrust twice keep in mind that this thrust is actual acting over in this direction, but when we take this from the cut from the what we called pot then the reaction to thrust will be in this direction right. So, now, what we will do we will be using this momentum equation and we are neglecting the body force right which is will be very very small to be neglected and we are considering momentum equation along the x direction only. So, we will when we will do that for a steady cases will say this total flux is equal to some momentum flux is equal to summation of the f x the force along the x direction. So, what are the forces acting in the direction one is the thrust right which is along the x direction and what are the other force are acting drag we are not considering right right this is a level flight thrust is equal to the drag.

Am I right or wrong? So, what else other force will be acting on the control volume because we are talk we are using a control volume what are the other forces will be acting it is a pressure force right. So, what will the pressure force in the this surface this will be what we called like P into A all, all surface and same cross sectional area. Similarly, what will be pressure force which will be acting here in the exit that will be P e and A e and what about this annulus area leaving this exist portion if you take it as a circular right. So, what will be there that will be the pressure force which will be acting for in the opposite direction to that right which will be P A A minus A e right which will be opposite direct opposite to the x direction right in that this way. So, then we can write down that the thrust force or the summation of the force along the x is thrust plus P A, A minus P A A minus e right and minus P e A e. So, if you simplify this you know what you will say this will be canceling it out right with this. So, you will get T plus A e P minus P e.

Now, we will be evaluating this term like a momentum flux term for this control volume. So, you will do that then what I am getting I am getting m dot e V e plus m dot s right this is the mass flow rate and which will be moving with the velocity V right because it is far away. So, it will be moving this fuel heat along the x direction right that is an assumption because you are assuming along the y direction that mass flux is negligibly small plus rho V square a minus A e; that means, this amount of you know like which will be there. Because we have considering in the exits and what about this, this also rho V square a minus A e. This term minus what is the inlet inlet is m dot A V in this region minus rho V square A minus A i, because if you look at you know this amount is entering into m dot a into v. But in this annulus region it will be rho V square A minus A I because this is A i, keep in mind that a I need not to be same as a e need not to be it may be same it may not be.

So, if I now put this equation four right into this place right and simplify I will you know it will cancel it out I will get m dot e v e minus m dot A V right and when you substitute this expression you know over here. This is basically equation or equation five like this I will get thrust is equal to m dot e V e minus m dot A V plus A e P e minus P A right. So, if you look at this is the expression for the thrust, and what could you see there are this is basically momentum thrust. And what is this this is you are your pressure thrust and if you look at this is basically is the exits this portion is your exit thrust, and this is what this is inlet drag that is inlet drag. Because that is a acting you know whatever thrust being produced by the jet right is being slower by this inlet drag and one has to reduce it as much as possible. Then what we will gain if my velocity the flight velocity is zero then what I will do I will be only on the land.

But I may get a thrust and keep in mind that this thrust can be maximum values right and this thrust can be maximum when it is fully expanded; that means, if the nozzle fully expand that is P e is equal to P a that is fully expanded expanded nozzle right then you will get the maximum thrust. Now, what we will do this, we will now substitute for m dot e, because we know what is m dot e m dot e is equal to m dot a plus m dot f that we have already derived. You know substitute this value, I will get thrust is basically m dot a plus one plus f V e minus V plus A e P e minus P a.

How I am doing that, because you know like f is basically m dot f by m dot, this is the fuel air ratio by mass in place of m e I am putting I am taking the m a m dot a out. So, I am getting that keep in mind that this is very very small number. Generally, you know it is very, very less than one kind of thing sometime and rather majority places will be neglecting. When we are discussing now I am like on what several places will be neglecting it say that f is very, very you know less than one. Therefore, it will be equal to 1. What will the number, if can anybody tell me, what is the expected? It will be around 25 is to 1, it can be 30, it can be you know kind of thing right; that means, one over 25 right or 30; that means, air fuel ratio will be 30 is to one like fuel air ration will be one over that it is a very very small number.

So, we can neglect that and this expression if I you know in the case of rocket engine this will be V will be zero right and then I will get this zero and then I will get the thrust expressions as m dot e V e plus A e P e minus P A right. We will be discussing little bit more about when we will get into rocket engines right may be I invoking this expression again and talk about the some other things. So, what is the difference between non air breathing and air breathing only that inlet drag will be there and expression is similar right.

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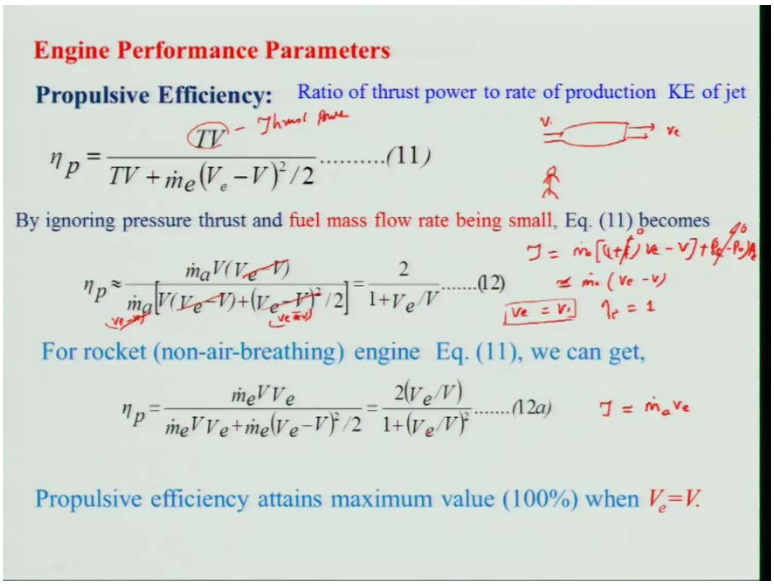

Now, we will be looking at engine performance parameters what are those there are several efficiency will be defining. So, first we will discuss about propulsive efficiency and what is that that is basically ratio of thrust power to rate of production of kinetic energy of the exit jet. Because you are basically that is why called it jet engine, you know or a jet proportion in some places people talk about it we used because the kinetic energy and then from that we will be getting the thrust power is a ratio.

So, mathematically if you want to look at you can write down thrust power means thrust into velocity that will give me thrust power. This is nothing but your thrust power and the thrust power plus the amount of energy, which is going out in the exits and keep in mind that this is what this portion is you know kinetic energy which is being what we called being going out. You know the mass into this change in kinetic energy is going out. So, this if you look at we are looking at with reference to what with reference to the earth right if you look at this if I look at a kind of vehicle you know like let say this is a vehicle engine which is going this is A V e and it is entering with V.

So, there is a proportion over here and with reference to that how much V e changing this mass, you know that will be changing m dot V e minus V whole square that is the mass which will be changing. So, by ignoring the pressure thrust and keep in mind that what will be doing till now we are not considering whether the pressure thrust is ignored or not.

Now, we will be ignoring, we will say that it is fully expanded nozzle; that means, P e is equal P A in the thrust expression and fuel mass flow rate being small; that means, the f you know is negligibly small. From that then what we will do the thrust expressions become because what we have look that that is m dot a one plus f V e minus V plus P e minus P A into A e. We are saying this is zero and this is also zero, f it is not true, but we are saying for the simplicity, then I will get m dot a V e minus this will be is equal to m dot a V e minus V, actually this is approximately equal. And this is my thrust and V is the velocity that is the power and then m dot A and I will be using the same thing V, V minus C and this is the whole square because I am taking in place of m dot e I am using basically m dot a.

So, I will take it out and if I simplify this thing you know if I take A V e minus V out of this place what I will get, I will take this V e minus V. So, this will cancel it out and this become V e plus not V e minus V right divided by two and if you just simplify that you will get basically two plus V e divide by 2. What it is indicating, it is indicating that propulsive efficiency will be dependent on the exit velocity, and flight velocity ratio. And when it will be 100 percentage when V e is equal to V right; that means, exist velocity is equal to the flight velocity n p will be one that means 100 percentage propulsion is quite a great you know is it possible we can get that.

So, now for the rocket engine or the non breathing engine equation 11, we can have right we can get the similar way that is m dot t V into V e because in the rocket engine what we are saying, we are saying that m dot approximately equal to m dot V e. Because it is a fully expanded nozzle, we are assuming P e is equal to P A, and there is no V, flight if you look at no air is entering into the engine, am I right? So, my thrust expression will be m dot simply V e right. So, I putting it here, and if I simplify, I will get 2 V e divide by V divide by one plus V e by V whole square. If you look at it is similar, but not same that is the difference that is this is square is coming in the 1 plus V e by V square and here there is another term coming from the numerator, so there is the difference.

And in this case, if I want to find out when it will be the propulsive efficiency will be maximum that will be again V e is equal to V, am I right? Because if I say this is 1 and this 1 2 by 2, it becomes 1 1 means hundred percentage more than you cannot because how much energy will be giving. You are all getting that in the or converting into thrust power is it really possible? That means whatever energy I will giving, I am just taking I am just utilizing it to converting. If you can do that that will be wonderful I already discussed that you will get maximum propulsive efficiency when V e is equal to v right exits velocity is equal to flight velocity.

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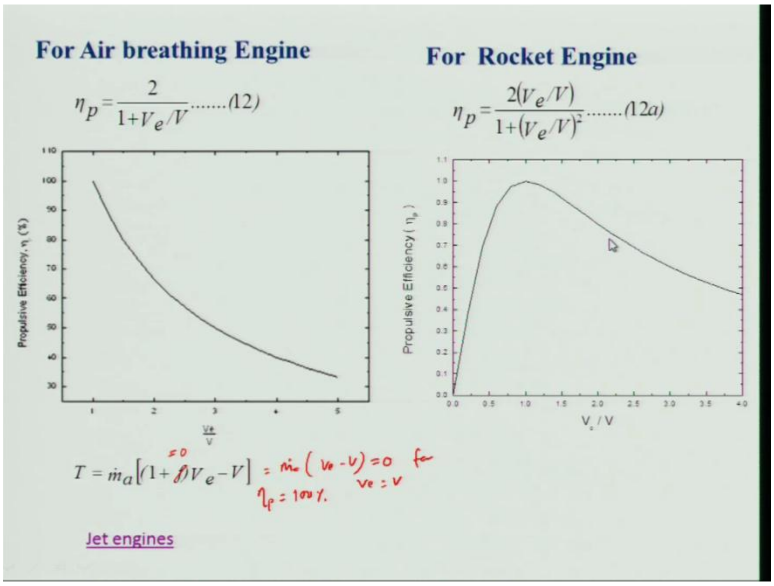

Now, let us look at for a air breathing engine, and little bit talk about the efficiency, what is happening, how it is changing with the velocity ratio that is exits velocity and flight velocity ratio. Exits means exist from the engine and flight velocity, of course, in this case air, breathing engine the velocity with which the air is entering into the engine. So, if you look at, if I look at this propulsive efficiency the V will find at when V e is equal to V 1, same you are getting 100 percentage after that it is decreasing; that means, when the exit velocity will be go on increasing for a particular flight velocity. What will happen propulsive efficiency decreasing that means, if I want to fly an air breathing engine, for a particular flight velocity then if I go on increasing V and interval because the propulsive efficiency is coming down.

But if I want to always we want to maximize the propulsive efficiency am I right, but if I maximize, what happens if V e is equal to v and what happens thrust will be zero, if I say this is you know zero, so approximately zero or in compare to so I can say that thrust will be zero m dot a V e minus V is equal to zero for V e is equal to V. But propulsive efficiency will be 100 percentage for this condition that means I am having trouble, if go one extreme size I am getting 100 percentage efficiency thrust is zero. If I go other way around that is you know V e is far greater than the V right I might get higher thrust if V e is greater than V far more better, you know v can be negligibly small flight velocity then I am getting the maximum thrust right, but may propulsion what I will have to do. It is like that what I started this if you are too materialistic or what I called you know thinking immaterialist then you will be in big trouble that what we are today and if you say that I do not want materialistic then you cannot survive. So, you all take a middle part and on based on these thing there are several kinds of engine people have developed right jet engines.

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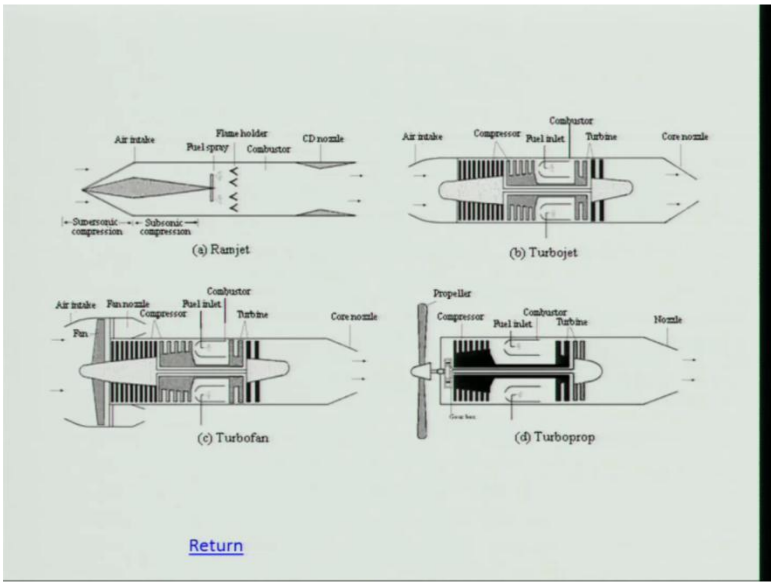

What are those that are basically what are the jet engines you know I based on this concept what are the engines I you are aware that is the ramjet turbojet turbofan and turboprop. So, if you look at as I shown you systematically we are again visiting I am not really discussing about that, but what really is happening in the ramjet ramjet well be flying at the very what high Mac number and what happens to the exit velocity.

It is also equally high right; that means, in this case what I will get my propulsive efficiency will be very low because if I want to fly at high Mac number then the nature allow give a higher thrust otherwise I cannot can I for the same kind of drag you know I cannot. So, therefore, here the propulsive efficiency is very low and as you go on to the turbojet it is you know propulsive efficiency lower than the in higher than the ramjet, but thrust will be lower than the ramjet.

And turbofan thrust will be you know medium kind of thing propulsive efficiency will be quite and if you to turboprop propulsive efficiency is highest, but the thrust will be very very low. So, you will be at a slow moving a something we use the turboprop engines right. So, we will be discussing about that, but let us look at now the rocket engines. How this propulsive efficiency will be varying with V e by V. How will be, it will be varying will it be varying that the similar way that is goes on decreasing certainly no. If you look at this expressions, you will find the propulsive efficiency is increasing, you know from the zero because I can have you know V e or V may be very very high as compared to V e right; that means, I am going to toward zero, but in reality it is not possible.

So, it will be of course, from this expressions is being plotted here. So, it will be goes on increasing till it reaches V e is equal to V you know the velocity ratio becomes one and after that it decreases right. Generally, it is not possible to maintain this V e by V is equal to one right because it will be all accelerating. So, therefore, people try to operate in this region where propulsive efficiency will be very high instead of this region people do not operate although sometime during the static you know during the intact may be sometimes, but that is a very small time.

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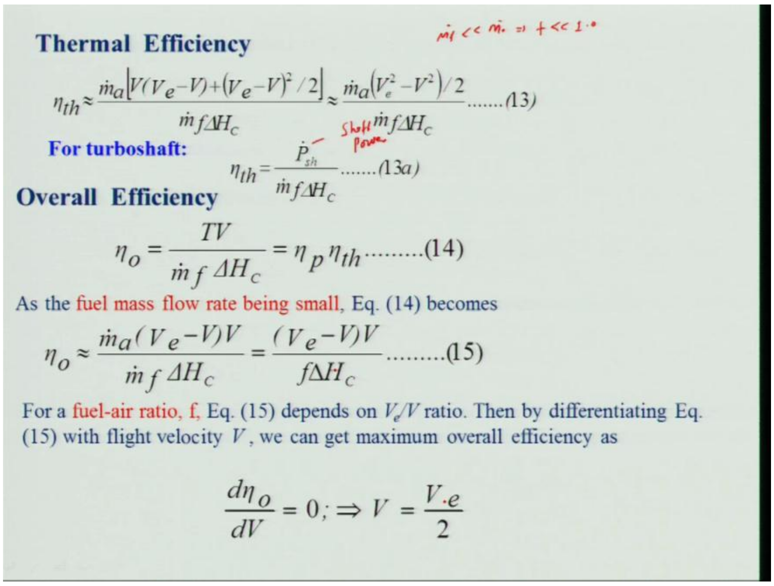

Now, we will be looking at thermal efficiency, because this is about propulsive efficiency which is tell you how good your engine is giving providing the what we called thrust power. But now we need to look at right how good it is in converting the fuel or the you know chemical energy into the kinetic energy, because here what we are using change in kinetic energy we are using to get the thrust power. So that means, this is we can say that the thermal efficiencies ratio of the kinetic energy into the the amount of energy or the fuel being burned and that means, total chemical energy. If I just simplify this and then if I expand this one you will get that is equal to m dot a V e minus V two whole square this is nothing, but your kinetic energy change in kinetic energy divide by m dot f delta H c right.

So, keep in mind that here what are the assumptions we are thinking, because I am not using same as equal because same assumption we are doing the nozzle is fully expanded; that means, there is no pressure. You know thrust form the pressure different change and also the m dot f is very very much less than the m dot a or f is less than one very very less that assumption we have made. But however, you know for the turbo shaft engine, you know we cannot really talk about that kind of things because the flight velocity would not be there, so what we will be looking at we will be looking at the instead of change kinetic energy we will be looking at the shaft power. Because in the tub shaft engine you know we used the shaft right.

So, therefore, I can used P, this is basically shaft power and m dot f delta H c m dot that is the fuel you know consumption rate into H c is the heat of combustion. And keep in mind that we are assuming the combustion efficiency which one which we are not considering in this case. But in real situation it would not be there. So, now overall efficiency will be what, overall efficiency will be basically the propulsive efficiency and the thermal efficiency. In other words, it will be the thrust power ratio of the thrust power and the amount of energy being consumed or being you know like used for that that is the thrust power m dot f delta H c. And you will find that you can derive this I mean which is very obvious that propulsive efficiency into thermal efficiency.

Now question arises, how it will be varying this overall efficiency with respect to let say velocity or velocity ratio. So, what we will do we will again assume the same assumption that a f is very very smaller the fuel mass flow rate is being small. So, the equation fourteen becomes you know like this what I am doing m dot a V e minus V, I am using the thrust. And I will find out V e minus V into V f delta H c. And if you look at for particular fuel-air ratio or for a particular kind of fuel, you will find that that it is dependent on the V e and V that is exit velocity and flight velocity.

And what we will do we want to find out what is the overall efficiency of a propulsive device or engines that is more important than that of propulsive only propulsive efficiency thermally. So, for that what we will do, we will differentiate this equations with respect to V the flight velocity, and we will find that is equal to zero. If you do this algebra will get when the flight velocity is half of the exit velocity. We will get a overall high maximum overall efficiency, which is you know generally the case being used particularly for your Boeing kind of series.

And we will see also why this long range commercial or the rather passenger aircrafts you know is being operated at Mac number of what, what is the Mac number it is being operated, point?

Student: ((Refer Time: 41:27))

Professor: No, 0.85 kinds of things, or .8 kinds of thing. Why it is so this is the one of the reason, but another reason, I can talk about it, I think I will stop over here. And then we will discuss these things in the next lecture. And some of the more like performance parameter like static thrust and and other things, and range all those things we will be discussing about…