Fundamentals of Aerospace Propulsion

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Lecture - 25

Let us start this Lecture 25.

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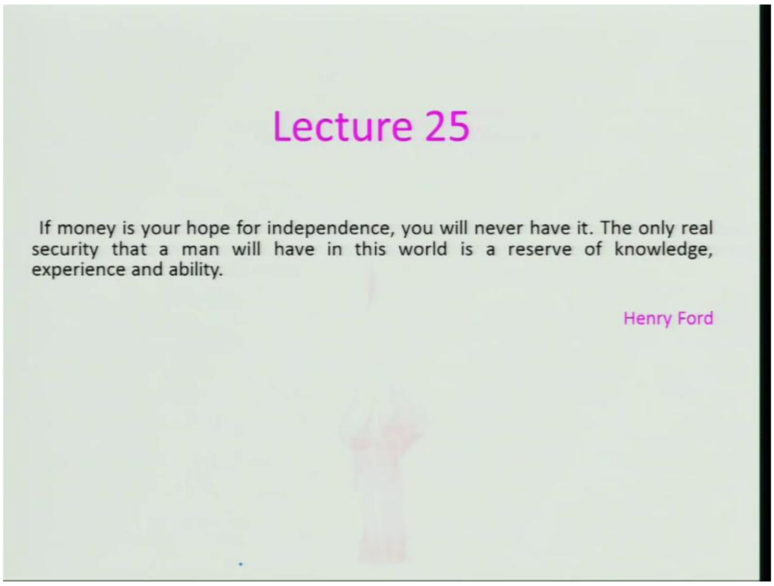

Lecture 25

If money is your hope for independence, you will never have it. The only real security that a man will have in this world is a reserve of knowledge, experience and ability.

Henry Ford

From the thought of person given by industrialist Henry ford, it goes like this, if money is your for independence, you will never have it. The only real security that a man will have in this, world is reserve of knowledge experience and ability; that is very important aspect of education, I do not want to dual upon it let us get into our business, that is what we learnt in the last lecture.

In the last lecture we basically look at, the flame stabilization right and what is a cause is and we defined a term known as velocity gradient, particularly for the bunsen burner. Then we moved into the diffusion flame, various kinds of diffusion flame we discussed but, we will be mostly you know restricting, our discussion to jet diffusion flame and the droplet diffusion, droplet combustion, or droplet diffusion flame.

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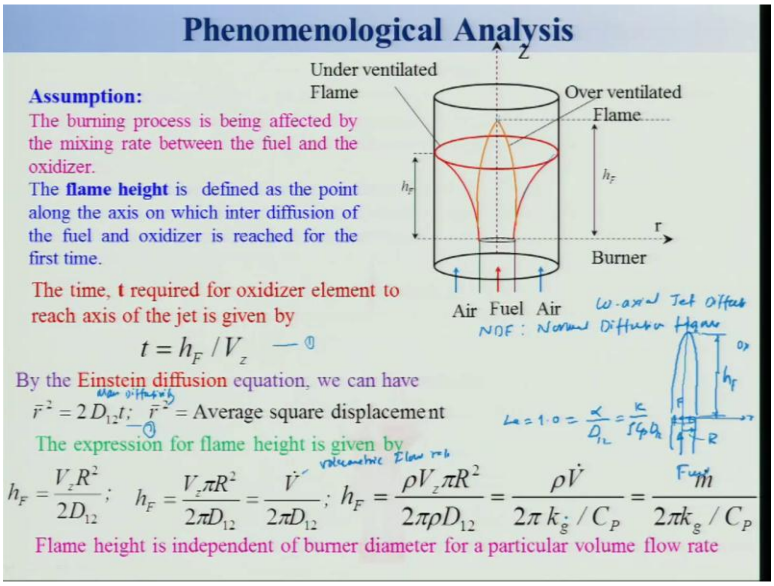

Phenomenological Analysis

Assumption:

The burning process is being affected by the mixing rate between the fuel and the oxidizer.

The **flame height** is defined as the point along the axis on which inter diffusion of the fuel and oxidizer is reached for the first time.

The time, **t** required for oxidizer element to reach axis of the jet is given by

$$t = h_F / V_z \quad \text{— ①}$$

By the Einstein diffusion equation, we can have

$$\bar{r}^2 = 2\overset{\text{Mass diffusivity}}{D_{12}}t; \quad \bar{r}^2 = \text{Average square displacement} \quad \text{— ②}$$

$$Le = 1.0 = \frac{\alpha}{D_{12}} = \frac{k}{\rho C_p D_{12}}$$

The expression for flame height is given by

$$h_F = \frac{V_z R^2}{2D_{12}}; \quad h_F = \frac{V_z \pi R^2}{2\pi D_{12}} = \frac{\overset{\text{volumetric flow rate}}{\dot{V}}}{2\pi D_{12}}; \quad h_F = \frac{\rho V_z \pi R^2}{2\pi \rho D_{12}} = \frac{\rho \dot{V}}{2\pi k_g / C_P} = \frac{\overset{\text{Fuel}}{\dot{m}}}{2\pi k_g / C_P}$$

Flame height is independent of burner diameter for a particular volume flow rate

So, what we will do for this jet diffusion flame, I will take very phenomenological analysis, what is meaning of that meaning is that, if you look at the phenomena and try to simplify. And make some kind of a judgment in a mathematical sense and then you look at not very regress an analysis, but; however it is good enough to give you the what we called some kind of relationship.

Before getting into that, let us look at what we can see is a co axial diffusion flame, this is a co axial jet diffusion flame, what you can see the fuel is in the centre jet and there a annular space in which air is there. Now, if you look at if more amount of air is there as compare to the fuel, you will get a flame surface you know like that which is similar to the simple jet flam that means, simple jet flame means it jet of fuel is swing to the quiescent atmosphere.

But, if the amount of air is less as compared to the fuel, then you will get a flame like this shape which is generally not being, it is known as under ventilated flame and whereas, the other one that, which is generally we see that is the over ventilated that means, you are giving more amount of air as compare to the fuel. But, now this we called it as a normal diffusion flame n d f normal diffusion flame.

Suppose I inter change this fuel stream with the air and air stream with the fuel, I will get another kind of flame which is known as universe diffusion flame that means, like in the fuel code I will give air in the central code and the co or the annular code, I will give fuel just other way around. What will happen it will be different kind of flame and you can manage to get, given a blue flame in case of hydrocarbon.

Where which will be a more brown to the suit formation in case of diffusion flame, it will be allowing in nature, but however, you can get blue flame right if you go for the inverse diffusion configuration. But, we are not going to discuss about that we will be restricting our discussion to the normal diffusion flame and what will be doing, we will be making some assumptions, that we have already stated that here diffusion is more important that mean burning process is being affected by mixing rate between the fuel and the oxidizer not by kinetics that means, it is diffusion control.

And we will be interested in finding out the flame length which have been shown as a h f, how can we define it we can say as a distance along the central line, that is z right on which inter diffusion of fuel and oxidizer reached for the first time, if you look at this is flame tip and this is your inlet. So, these distance is known as the flame height and we will be basically deriving expression for a jet diffusion flame, you know flame height why,. because that is the one of the important parameter that can characterize, the diffusion flame and tally like your burning velocity, you know in the case of jet diffusion flame it is the flame length.

So, therefore, we will be using this phenomenological analysis to derive an expression and see how, you know it is dependent on some parameters. So, what will we do we will be and keep in mind, that we are considering you know a simple diffusion flame like not that I have shown here rather in this way, this is a you know fuel. And it is what we called quiescent atmosphere and there is flame and this height is known as the h f that we will be looking. are you getting that we are now discussing.

Now, if you look at along this z direction this is the z direction and this is the of course, the r direction, then what will be the flame length, the flame length will be can be define basically how much time, it takes for the oxidizer element you know this is oxidizer and this is a fuel to reached the axis of the jet for the first time. If you look at in this case coming with certain time, but in this case the oxidizer is coming, but at the central line it will be coming, it will be taking more time that it will be burn.

So, therefore, we need to look at the time t required for oxidizer element to reach the axis of jet, that will be given by what given by is very simple, what we are assuming the velocity is uniform here like is that number 1 assumption, which need not to be it may be fully developed flow, it may be parabolic you know. But, here we are assuming a uniform flow that is number 1, number 2 is this velocity along with z direction, it will be changing, but we are assuming that let us not worry about it.

We will not change velocity along the z direction along with z, because in the internment will be there I have shown you in the last time, that is what the structure of flame you know all velocity temperature and then mass fraction of fuel and oxidizer, how it is changing at different height with respect to r, we have seen. Because, the internment will be changing and also the flame will be there once it is therefore, it will be changing as well.

But, in this analysis what we are assuming we are saying, the velocity is same along the z direction, which is a very rough or what we called inaccurate assumption. But, we are doing for simplicity, so therefore t will be what h f divide by v z, v z, d z let velocity which is remaining same like you know constant velocity fuel is being moving. But, which is not true, but simplicity we are doing.

So, now I need to look at the lime with respect to the diffusion, then I will work the Einstein diffusion equation which will be basically the r square is equal to 2 d 1 by 1 2, that means 1 will be for fuel 2 will be for oxidizer this is the diffusivity, d is the basically diffusivity, mass diffusivity, this is mass diffusivity, when you talk about diffusivity one with respect to other you know. So, in this case 2 were taking in real situation, there will several species there will be multi component diffusion, we are not considering all those things.

Now, what is this r square is the average square displacement, because of diffusion the molecule will be moving; that means, molecule will be displacing itself position. So, therefore, that will be average square, what I will do I will combine this expression, if I can say this is 1 and this is basically 2. I can combine these things and get an expression for flame length, that is h f is basically is equal to v z into t in place of t, what I am include you know putting it that is r square 2, 2 by d 1 2, where r is the radius you know air this is the r.

Because, from the you know jet for a pipe for you know orifice, the is r we are seeing that is a maximum 1, it has to penitent because if you look at the flame is coming it has to come from this distance you know. So, these distances is r we are taking that is the maximum, where oxidizer has to diffuse to have a flame height, so we are taking v z r square 2 by d 1 2 keep in mind, that then I what I will do I will say this flame length is v z pi I will multiplied in the numerator and also the denominator, if you look at the pi r square is nothing, but cross sectional area right at the inlet of the jet or the orifice.

So, then that will be volumetric flow rate, metric flow rate and divide by 2 pi d 1 2, so what is saying this h f, h f will be dependent on the volumetric flow rate that means, if the volumetric flow rate of fuel keep in mind that this is single z, that is not co axial z what I have discussed just before that, it is a single z it should from a orifice or pi to an atmosphere quiescent atmosphere. So, it is as a you know v dot of the volumetric flow rate is increasing then h f will increase, this is very important just by looking at simple assumption will you see that how, what we are getting there.

Let us look at instead of volumetric flow rate, can you look at you know mass kind of thing what I will do, I will just manipulate this expression, I will multiplied rho into the numerator and multiplied the rho in the denominator. And of course, the same if you look at the this is nothing, but your volumetric flow rate rho into v dot into nothing, but mass flow rate and I will assume an the Lewis number is equal to 1, what is the meaning this basically alpha by d, d mean diffusivity I can say 1 to 2, what is alpha, alpha is the k by rho c p and d 1 t.

So, if you look at instead of this rho d 1, I can write down k g by c p keep in mind that this is the valid only, if you are assuming Lewis number is equal to 1, this is again an another assumption, which need not to be, then we can see that h f is basically you know dependent on the mass flow rate and keep in mind that k g by c p is non dependent thermodynamic property like temperature and pressure kind of thing.

Because, whereas the diffusivity will be dependent on the pressure, so also the temperature as well, but it is more how it will be dependent, the diffusivity is inversely proportional to the pressure that means, if the pressure is higher diffusivity will be what lower is not, it make sense or not if pressure is low diffusivity low. Because, molecule will be moving if it is the low pressure if it is higher pressure it will be difficult you know it is a very common sense you know.

So, therefore it will only dependent on mass flow rate, now instead of looking at volumetric flow rate and mass flow rate we need to look at the none dimensional number, what we called is as Reynolds number, how does this flame length dependent on the Reynolds number.

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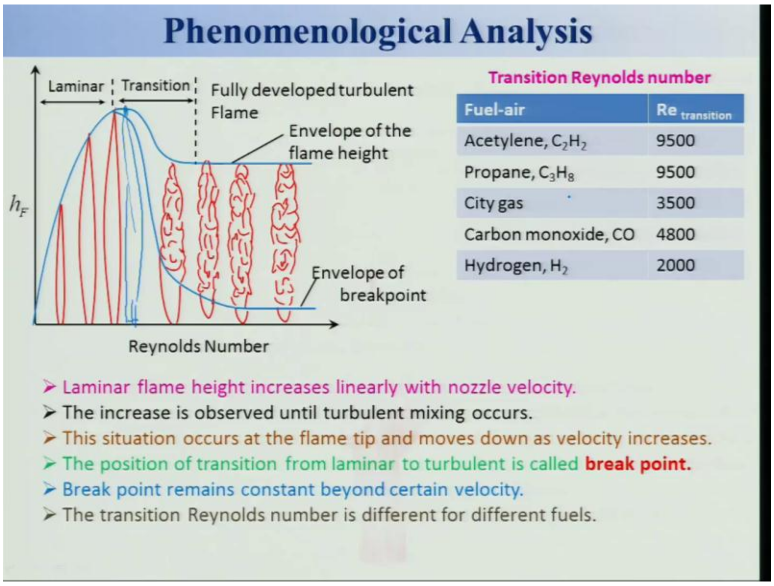

Phenomenological Analysis

Transition Reynolds number

Fuel-air	$Re_{transition}$
Acetylene, C_2H_2	9500
Propane, C_3H_8	9500
City gas	3500
Carbon monoxide, CO	4800
Hydrogen, H_2	2000

- Laminar flame height increases linearly with nozzle velocity.
- The increase is observed until turbulent mixing occurs.
- This situation occurs at the flame tip and moves down as velocity increases.
- The position of transition from laminar to turbulent is called **break point.**
- Break point remains constant beyond certain velocity.
- The transition Reynolds number is different for different fuels.

So, if you look at I have plotted this is taking from the experiment, after that if you look at h f is basically plotted this expect to Reynolds number keep in mind, that what we have done in the phenomenological analysis for the flame length is valid for the laminar. So, if you look at the flame length is increasing with increasing Reynolds number or volumetric flow rate or velocity, you know all those things come into the picture.

So, it is increasing that we have seen, but after that what happens this of course, that is decrease and this zone is known as transition region, at which the tip of the portion you will get some kind of structure is coming the turbulence, will be coming in the pictures and that place particularly here you know and if you look at, If I draw a flame over here you know like this kind of a flame, then you know this portion will be transition kind of things and this point is known as breakpoint.

Because, in which you can differentiate that there is turbulence and as you go the increasing in turbulence, you will find this the upper portion become more turbulent in nature. It would be crisscrossing molecules a random in nature of course, may be you may not be see, but in high turbulence you can even visualize naked eye, but in the low one you may not see particular in transition region.

But, for that you need to use assign photography, so look at the structure you know turbulent structure, so as you go on this break up length this is remaining constant with certain velocity. Of course flame length will remain constant and when you do that which have not plotted here, that if you increase the Reynolds number basically you are increasing, a Reynolds number mean for fixed geometry you are increasing the volumetric flow rate or mass flow rate.

So, then flame will be lifted up lifted from the burner in and then, it will be blow out is occur which I have not shown in this figure, but if you look at that as I told you position of transition for laminar turbulent is known as break point, this is the break point and it goes on and break point remain constant beyond certain velocity. And interestingly this transition Reynolds number is different for different fuels, like for acetylene it is 9 thousand 5 hundred for whereas, the hydrogen it is 2 thousand.

And of course, it varies for propane and carbon oxide of course, city gas you know earlier days in city they were produces gases, I think we will be doing that may be after hundred years, you know all fossils will be exhausted then that is only option to have cities fuel, like you know you produce some biomass gas and other things, that is the earlier people were using.

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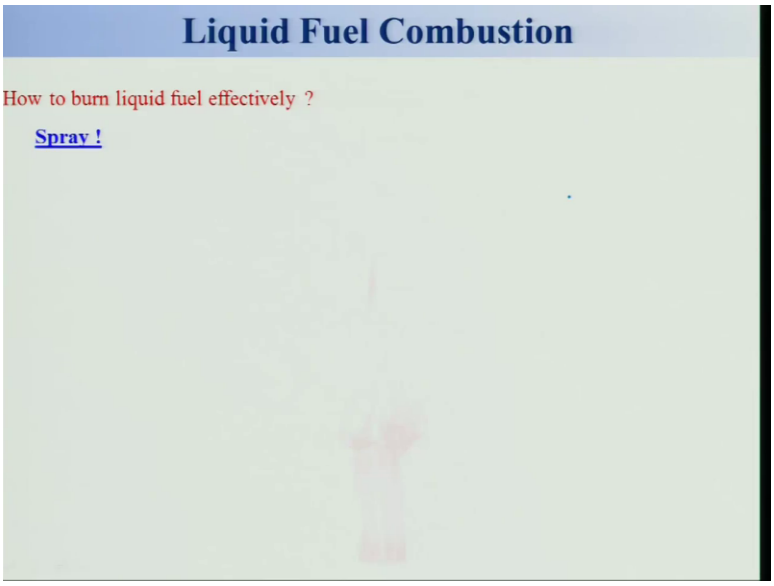

Liquid Fuel Combustion

How to burn liquid fuel effectively ?

Spray !

So, now we will be move into the very important things liquid fuel combustion and which will be use in rocket engine and gas turbine engines, so the question is how to burn liquid fuel effectively. Of course, if you look at how our ancestors were burning we are they were using liquid fuel gases fuel was not available at that time, if you look at they used to use what we called weak flame.

And you know candle flame weak flame is similar in nature, it is quite complex, but what is the limitation of that weak flame, you are getting what is the weak flame.

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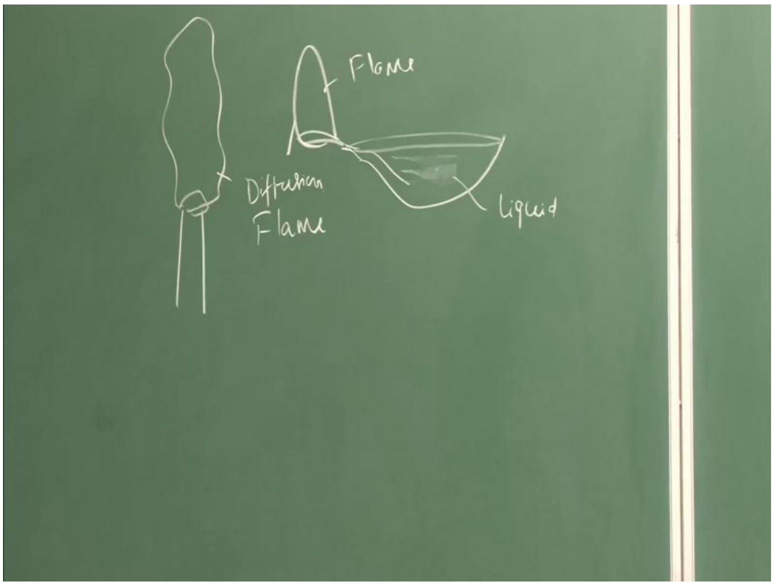

Suppose I will make a what we called you know there is the flame kind of thing, this will be the liquid fuel an earlier days people were having you know ((Refer Time: 17:40)) kind of thing you know this is also a diffusion, now if you look at how to because what is the limitation of this kind of flame, we cannot increase the mass flow, mass flow rate, of fuel we know consumption.

Because, I want to have in gas turbine engine I want burn a large amount of fuel, how I will do about it how will go about it, how can it do that can I do is then I need a more surface area. So, that it will you know do that and it will be very calm, you know it would be compact, so for that what will we have do we will increase the surface area of the liquid fuel. Because, it has to vaporized first, then it will be vaporize then you know combustion, you know it will be mixing with oxidizer then it will be burning.

So, what is the way out the way out that, you produce the spray you know spray you use several places, the what is the common use of spray can anybody tell me.

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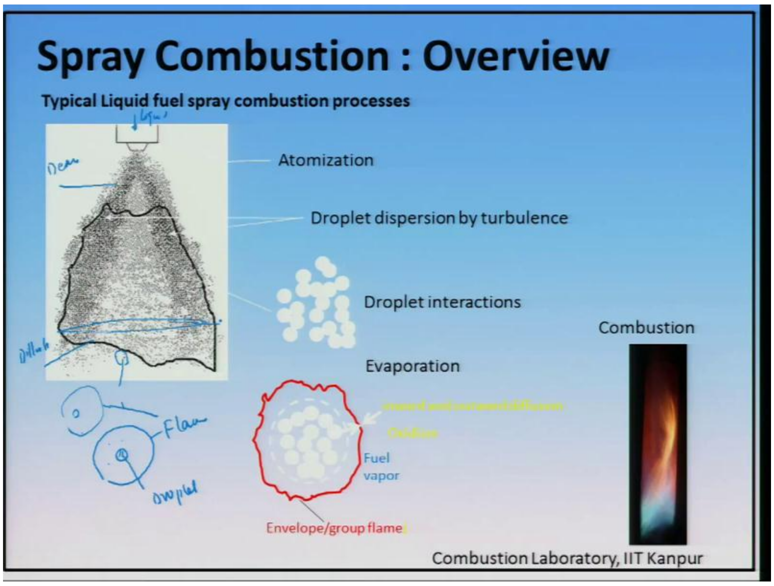

Spray Combustion : Overview

Typical Liquid fuel spray combustion processes

Combustion Laboratory, IIT Kanpur

So, you know there are several if you like even if you go to barber shop for hair cut, you need to use what a spray, sprinkling in the three several places we used a spray, I do not want to get into that it is a quite wide application, what we do we use basically you know we take a liquid and pressurize through, what a small orifice. Such that it will be converted into ligaments and then converted into spray, spray means a small droplet tiny droplet consist of, if you look at this is kind of spray.

So, if I take a this liquid you know any very pass through a small orifice, that is known as pressure you know pressurize or the pressure atomization. Because, you are applying the pressure force you can apply several other forces kind of thing you know, so if I do that I will get a dense spray, here if you look at this is dense spray and the dilute spray it will be here that means, the spray will the several droplet sizes will be there.

And if you converted to the you know what we called combustion, if you do then there is several kinds of combustion can occur, but I will talk about two you know extreme cases one is the let say there is the droplet here, if I take this portion there is a droplet here. And if there is oxidizer and then there will be a flame right and this is your droplet that is one is likely to occur and there might be some places here and there is another droplet, you know like another flame might be there this also another flame, that will occur only when the droplets, you know are quite dilute that means, they are not sink each others.

But, whenever it is a dense population, population of droplets not man, we are very dense populated country you know we do not interact, but we need to interact right. So, therefore, what I am saying then what will happen then this droplets, you know will be together and there might be starvation of oxidizer, you know is happening. We are starve food is not there you know, because too many people to a fed, so similarly droplets are not having oxidizer to get burn.

So, you will be together to get there will be burning that is known as envelope and group flame, what I am saying in our country we need to have a group work that than individual work, that is more important how to survive how to do together, there is a lot of strength unit there is also advantages and disadvantages. So, these are two extreme cases what one can think about, but question arises, how to handle, because this process the spray is a very random in nature and how to talk about for example, if I take a spray and look at here you know in this cross section what will be the droplets size.

In I can take a plane may be use some leaser techniques and used that and get the droplet distribution how it will be, it will be very wide range and it may be form one micron even point 5 micron to may be 5 hundred micron meter micron means micron meter diameter and various distribution will be there.

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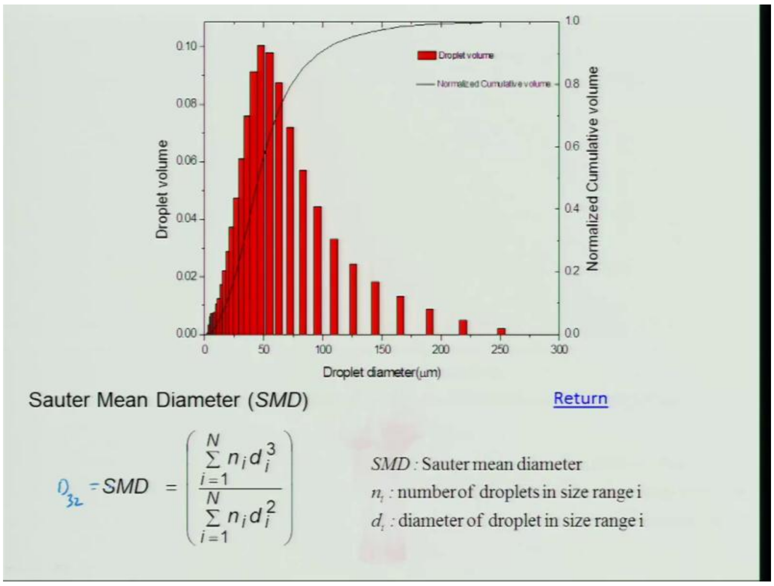

Sauter Mean Diameter (*SMD*)

Return

$$D_{32} = SMD = \left(\frac{\sum_{i=1}^{N} n_i d_i^3}{\sum_{i=1}^{N} n_i d_i^2} \right)$$

SMD : Sauter mean diameter

n_i : number of droplets in size range i

d_i : diameter of droplet in size range i

I will just show you one of this distribution right if you look at is this we have done in the lab and the droplet volume, which will basically you know talk about the how much volume is continent and this is droplet diameter. It is start from may be 2 micron to the 250 micron these are the distribution that means, majority of droplet is occurring around you know 40 to 50 kind of things.

And of course, small amount are there and small amounts are there, but how to handle this a quite difficult task you know are you get hand n also at different distances from the atomizer, it will be different. And keep in mind this is under quiescent atmosphere and in combustor, it will be lot of you know interaction will taking fuel motion will be there, will be also cold flow condition there, will be hot, it will be interacting, there will be product it will be mixed how complex it is, you should imagine I cannot tell you how complex it is when I imagine it is quite complex.

But, how to handle that, what we do we basically look at average droplet size there are several ways of averaging it, but we will be using sauter mean diameter average which is also known as you know d 3 by 3, what is the meaning is that n I, n I is the number of droplet inside this, suppose I will take something may be you know 45 to 40 55 micron, in that how many of droplet will be there, I will make it bean right the way the garbage people collect they collect in bean are you getting a point.

So, you will have to make a bean kind of things and diu is the droplet of size average size, you can assign and this what it indicates n I d I q, what it is indicates it indicates the volume, volume means its related to the mass been, you know consumed a mass of the fuel is being consumed. And what it is govern by that there is a area, that is a surface area n I d I square, because surface area will be influencing what evaporation wave.

Because, surface evaporation is a surface phenomena, so therefore, in combustion we used the 3 by 3, which is known as sauter mean diameter, if I know for a spray that means, then it will be easier to handle, what is the s m d a sauter mean diameter it represent the average you know droplet size of a entire. Spray which will be containing several droplets of different size different you know range.

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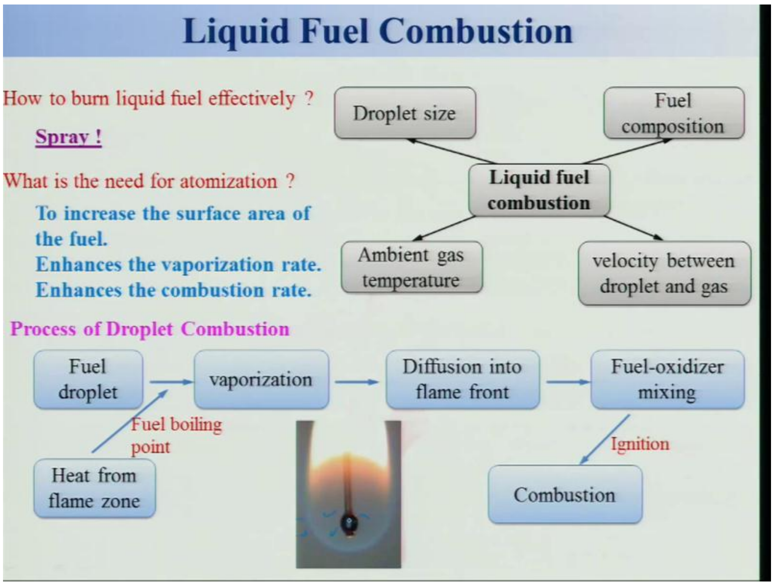

Liquid Fuel Combustion

How to burn liquid fuel effectively ?

Spray !

What is the need for atomization ?

To increase the surface area of the fuel.
Enhances the vaporization rate.
Enhances the combustion rate.

Process of Droplet Combustion

Droplet size

Fuel composition

Liquid fuel combustion

Ambient gas temperature

velocity between droplet and gas

Fuel droplet

vaporization

Diffusion into flame front

Fuel-oxidizer mixing

Fuel boiling point

Heat from flame zone

Ignition

Combustion

So, what we will do what is a need for atomization I have already told, but I will just to repeated to increase, the surface area of the fuel enhance the evaporated vaporization rate and enhance the combustion rate unless, otherwise it is vaporize it cannot really combustion cannot really take place. So, that is a idea if you look at liquid fuel combustion is quite complex and it will be dependent on the fuel composition, that is one of them droplet size and its distribution, it will be dependent on ambient gas temperature and pressure.

And it will be also dependent velocity between droplet and gas, keep in mind that each location it will be different, it is not that it will be 1, because depending on the situation in the aerodynamic in the combustor it will be different. So, we need not get to the spray combustion which is quite complex, but however we will be looking at the droplet combustion, which is simple in nature and we will be looking at under quiescent atmosphere only.

But let us look at a droplet which this is a figure, I have taken from my student in my lab which I produced if you look at this droplet and there is a flame around it and how it does happen, what are the process involve in that, suppose I will put a droplet hanging from here, small droplet which is of course, order of millimeter 2 m m, 1 m m or 1 point 5 m m it is not like micro size what we will see, then if I do that I need to ignite means I need to give some initial energy.

So, what will happen the energy will go and vaporize the gas and when is vaporize gases, so what will happen that from this you know like a zone, if there is flame being established then the flame heat will be going to the from the flame surface from the flame to the droplet surface. And it will be you know vaporization will be talk taking place, because it is rich boiling point and vaporization means gas will be going out you know all those places.

Gases is going out and oxidizer is coming and then diffuse into the flame and fuel oxidizer will be mixing and then the flame surface is being formed and that depends upon, you know where it is coming due to the diffusion passes, that is the one thing and other thing is that this will be this heating, that means droplet will be evaporating if it is evaporated the droplets surface, you know if it is evaporated, it is also diameter go on sinking at certain point there would not be any fuel to be burned and it will be extinguish

So, if you look at this is quite a complex process and then it becomes self extern as long as flame is around the droplet, so now we need to look at this droplet and analysis, it how an diameter changes with respect to the time that is our objective. Because, it is very important to know that.

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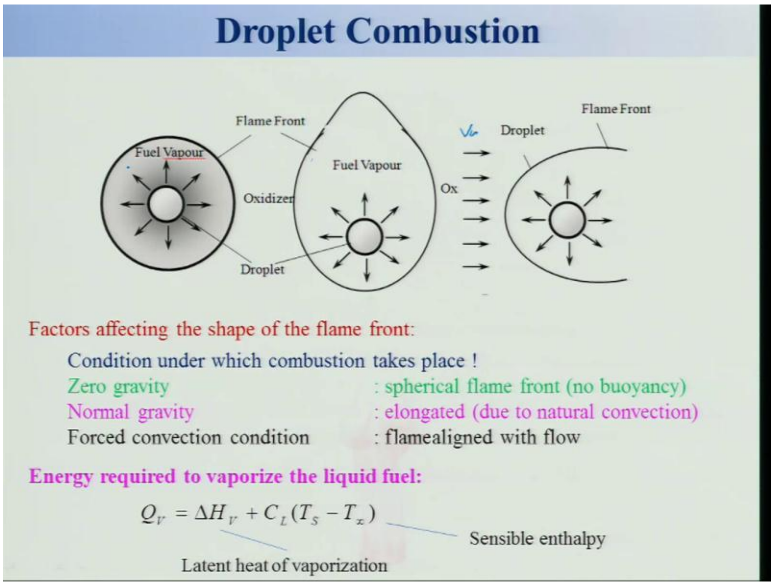

Droplet Combustion

Factors affecting the shape of the flame front:

Condition under which combustion takes place !

Zero gravity : spherical flame front (no buoyancy)

Normal gravity : elongated (due to natural convection)

Forced convection condition : flamealigned with flow

Energy required to vaporize the liquid fuel:

$$Q_V = \Delta H_V + C_L(T_S - T_\infty)$$

Sensible enthalpy

Latent heat of vaporization

So, let us look at a three situation one is droplet is a spherical flame other is a droplet around that along get at flame, when there is a velocity if I look at these this is thing flame is coming that way. So, the three situations will occur that this will occur you know when the spherical flame, this is spherical flame and under zero gravity, that means, there is no gravitational effect or gravity is very 0, very small it tending toward 0 rather we have done some work.

So, on that we have a droplet towers and conduct some experiment and we will get almost not spherical like this, but almost spherical far better than the along get that one, which is due to the natural convection. Because, this is coming the gas will be lighter it will go away and then you will get what I have the figure I have shown from my lab, the droplet combustion that is basically under natural condition.

And force convection is where which will be you know in real situation will be occurring, that there is a velocity which is going on flame will be like that, we would not be discussing about it, but we will be discussing about this droplet combustion. So, let us look at energy required to vaporize in liquid fuel will be basically q v is equal to delta h v and c l t s minus t infinity, where delta h is latent heat of vaporization and this portion is the sensible enthalpy, keep in mind that this c l is the specific heat of the liquid fuel and t s is the surface temperature, t infinity is the very far away or ambient temperature you can say far away from the droplet that is the t infinity.

So, what will we do we will look at this you know spherical flame, front around the spherical droplets and that is corresponding to the 0 gravity condition and look at, whether we can analysis for that we will have to invoke

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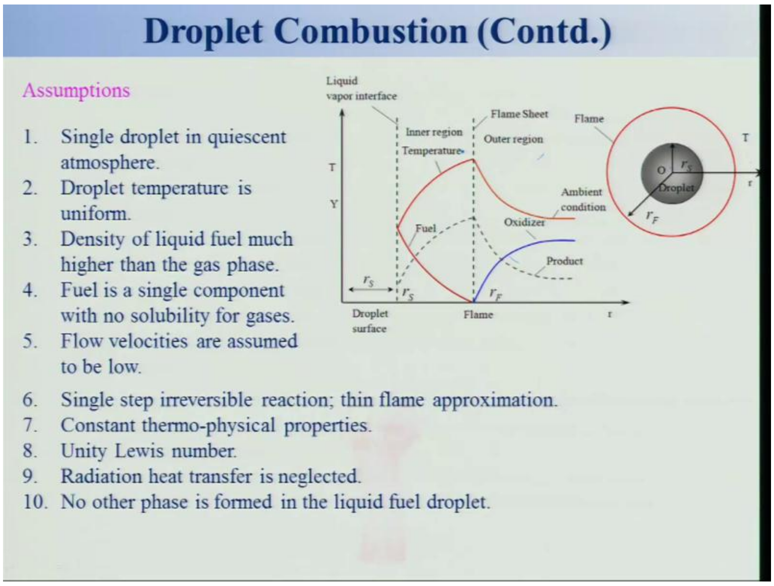

Droplet Combustion (Contd.)

Assumptions

1. Single droplet in quiescent atmosphere.
2. Droplet temperature is uniform.
3. Density of liquid fuel much higher than the gas phase.
4. Fuel is a single component with no solubility for gases.
5. Flow velocities are assumed to be low.
6. Single step irreversible reaction; thin flame approximation.
7. Constant thermo-physical properties.
8. Unity Lewis number.
9. Radiation heat transfer is neglected.
10. No other phase is formed in the liquid fuel droplet.

The equations for before doing that let us look at the structure what will be getting, if you look at there is we are assuming that is a sink spherical 1 number 1 and single droplet temperature in quiescent atmosphere, there is no velocity nothing. And droplet temperature is a uniform that means, if I take a liquid from here this is center o and r s is there droplet diameter at any incent of time. And of course, this red one is the flame and uniform temperature in the liquid fuel that means, in the droplet.

Is it possible, it is not really possible that means, along with this the droplet or temperature remain same its not possible, but we are doing that, this is an assumption to simplify density of liquid fuel is much higher than the gas fuel. Of course, that is because you know the density of liquid fuel will be around 8 hundred you know 50 k g per meter q kind.

So, which is quite higher than the your air or the gas the fuel is a single component with no solubility for gases right single component, if it is a what we called happen it is happen, but if you look at a t f aviation turbine fuel can it be single component fuel certainly no its contents several of them. So, you cannot called a t f is a single component fuel.

So, flow velocities are assumed to be low that is an assumption again and single step irreversible reaction is being assumed and keep in mind that we are also assuming thin flame approximation. If you look at this is oxidizer and this is fuel and they are you know meeting and reacting instantly as soon as meet at the surface. So, this is known as flame surface and keep in mind that this fuel and this is on the outside of the flame and temperature increases and also it reaches, at the higher value of flame surface and then decrease.

And keep in mind that there is inner region and this is of course, outer region in a region of flame from here to there and this is an outer region. So, an constant thermo physical property if you look at the temperature is changing. So, for c p will be changing with respect to radial direction, but we are not considering that we are considering c p k g like k g in sense that conductivity for the gas phase and that is constant.

As we are an unity Lewis number alpha is equal to d diffusivity, that we are saying radiation heat transfer is neglected that we are doing, but which you cannot. So, no other phase is formed in the liquid fuel only the liquid fuel and keep in mind, that we are not looking invoking any equation or solving for the liquid phase only gas phase we will be looking. And very important one that we will be assuming that at the density of the liquid is very higher, as compared to the gas density then it will be residing at a very small rate

So, it is almost quasi steady step, quasi steady means it is changing the droplet diameter is swanking, but it is at a very slow rate . So, therefore, we are assuming quasi steady assumption, which is a you know not there very explicitly in this keep in mind this is very important assumption

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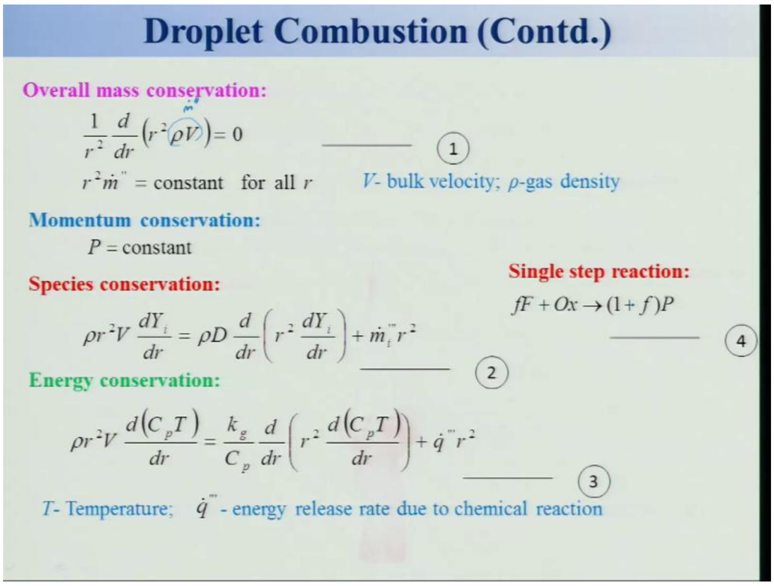

Droplet Combustion (Contd.)

Overall mass conservation:

$$\frac{1}{r^2}\frac{d}{dr}\left(r^2 \overset{\dot{m}''}{\rho V}\right) = 0 \quad \text{———} \quad ①$$

$$r^2 \dot{m}'' = \text{constant} \quad \text{for all } r$$

V- bulk velocity; ρ-gas density

Momentum conservation:

$$P = \text{constant}$$

Species conservation:

$$\rho r^2 V \frac{dY_i}{dr} = \rho D \frac{d}{dr}\left(r^2 \frac{dY_i}{dr}\right) + \dot{m}_i''' r^2 \quad \text{———} \quad ②$$

Single step reaction:

$$fF + Ox \rightarrow (1+f)P \quad \text{———} \quad ④$$

Energy conservation:

$$\rho r^2 V \frac{d\left(C_p T\right)}{dr} = \frac{k_g}{C_p}\frac{d}{dr}\left(r^2 \frac{d\left(C_p T\right)}{dr}\right) + \dot{q}''' r^2 \quad \text{———} \quad ③$$

T- Temperature; $\dot{q}'''$ - energy release rate due to chemical reaction

So, this what will we doing we will be looking at overall mass conservation equation for the on steady step quasi steady step, that is 1 over r square d by d r r square rho v, which is nothing, but if you look at this is nothing, but mass what we called these things. So, we can write down r dot m dot triple dash there is mass flow rate is equal to constant, because see this can be this basically is nothing, but your this is your m dot triple double dash mass flax rate, what is the meaning, meaning is that at higher radius mass flax will be smaller it is a obvious thing.

Because, of the mass is going out as it is going away from the droplet surface, so it will be reduce, because surface area is increasing going by square right. So, and v is the bulk velocity rho is the gas density, that we are saying momentum you will see I have not discussing, but you will see one dimensional, it will be constant kind of thing this from the deflagration point of view and that is valid only for the one dimensional steady flow.

Species equation if you look at rho r square v d y by d r this y is the mass fraction of the species, I species it can be fuel it can be oxidizer it can be product and this is a convection term. And this is rho d, d is the diffusivity and this is r square d y I by d r and this portion is m dot I triple dash r square is nothing, but your what we called due to the chemical reaction the mass being generated.

So, energy equation if you look at it has similar term, that is the convection this is your diffusion term and this, is the reaction term and here as I told q dot triple dash energy release rate due to the chemical reaction. And we are considering the single step reaction that is f moles of fuel reacting with 1 mol of oxygen going to 1 plus f product, keep in mind that I d little other where around in the case of premixed flame here it is different.

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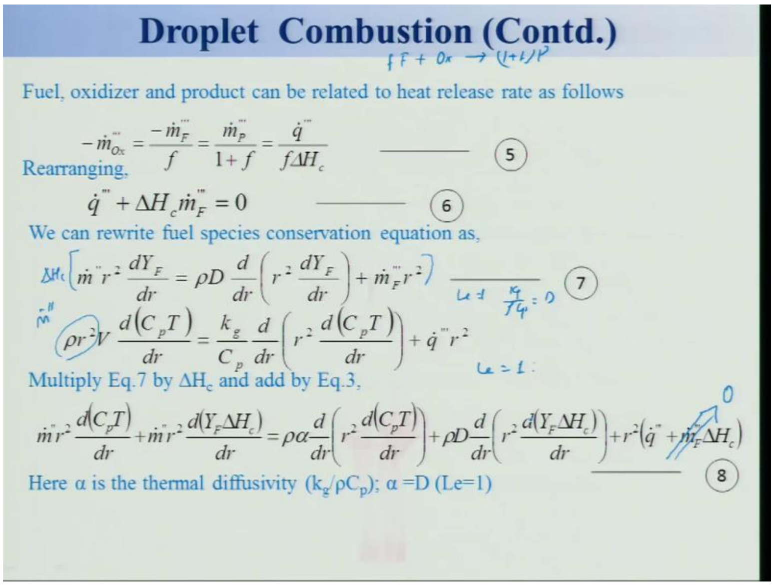

Droplet Combustion (Contd.)

$fF + Ox \rightarrow (1+f)P$

Fuel, oxidizer and product can be related to heat release rate as follows

$$-\dot{m}'''_{Ox} = \frac{-\dot{m}'''_F}{f} = \frac{\dot{m}'''_P}{1+f} = \frac{\dot{q}'''}{f\Delta H_c} \quad \text{———} \quad ⑤$$

Rearranging,

$$\dot{q}''' + \Delta H_c \dot{m}'''_F = 0 \quad \text{———} \quad ⑥$$

We can rewrite fuel species conservation equation as,

$$\Delta H_c \left[\dot{m}'' r^2 \frac{dY_F}{dr} = \rho D \frac{d}{dr}\left(r^2 \frac{dY_F}{dr} \right) + \dot{m}'''_F r^2 \right] \quad \text{———} \quad ⑦$$

$Le = 1 \quad \frac{k_g}{\rho C_p} = D$

$$\overset{\dot{m}''}{\boxed{\rho r^2 V}} \frac{d(C_p T)}{dr} = \frac{k_g}{C_p} \frac{d}{dr}\left(r^2 \frac{d(C_p T)}{dr} \right) + \dot{q}''' r^2$$

$Le = 1:$

Multiply Eq.7 by ΔH_c and add by Eq.3,

$$\dot{m}'' r^2 \frac{d(C_p T)}{dr} + \dot{m}'' r^2 \frac{d(Y_F \Delta H_c)}{dr} = \rho\alpha \frac{d}{dr}\left(r^2 \frac{d(C_p T)}{dr} \right) + \rho D \frac{d}{dr}\left(r^2 \frac{d(Y_F \Delta H_c)}{dr} \right) + r^2 \overset{0}{\cancel{(\dot{q}''' + \dot{m}'''_F \Delta H_c)}} \quad \text{———} \quad ⑧$$

Here α is the thermal diffusivity $(k_g/\rho C_p)$; $\alpha = D$ $(Le=1)$

So, you should keep that in mind it will be similar So, if I look at this fuel oxidizer and product from that reactions, you know I can write down. Because, what I have written down there that is m of fuel plus oxidizer is going to one plus f, but so I can write down this minus m dot triple dash o is equal to minus m dot triple dash f, f is the stoichiometric coefficient of fuel is equal to m dot f dash m dash triple dash p one plus f. And similarly for energy equation write down like q dot ripple dash f plus delta h, this is very important one which will be using right for analysis.

Because, if you look at the equations you know what we have seen energy equation or species equation, there is none linear terms and none linear terms is basically comes from the reactions, So, if I form this if I take this q dash triple dash f delta s c and the fuel I will get q dot triple dash plus delta s c m dot triple dash f is equal to 0, this is very important statement, what is saying that total is equal to that means, whatever heat being release this basically coming from the heat of reaction into mass flow rate of fuel, so one is minus, so therefore it is you know 0.

Now, we can rewrite this species conservation equation, as you know in the similar way like we have seen, so this is none linear term as I told and I will take the energy equations. As it is I can take this you know instead of you know we have used here, what we called Lewis number is equal to one is equal to alpha you know sorry is equal to 1, so therefore, k g by rho c p is nothing, but diffusion alpha by d.

So, I can put that together and putting this, now what I will do now I will multiply this what we called equation by delta s c and add this together, this equation 7, I will multiply by delta s c and then add together, what I will get this expression, you know for this what we called this is nothing, but here what this is nothing, but your m dot triple dash, we have done from continuity equation. So, then this equation will be delta s c, s c is not changing with respect to no heat combustion does not change in radial direction.

So, you can take it very easily and then this one and you will get, what is this if you look at this equation is 0 that means, what we are doing, we are now manipulating mathematically keeping in physics in contact, such that it will be 0 none linear return as gone now. And what we are doing we are also saying that Lewis number is equal to one, so therefore, d is equal to alpha, so we will do that, so I can club this together and club this together easily on left hand right hand side.

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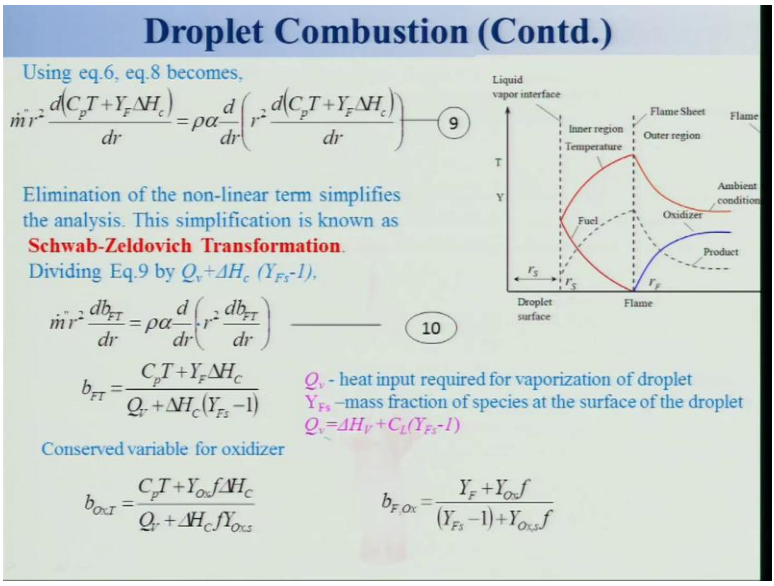

Droplet Combustion (Contd.)

Using eq.6, eq.8 becomes,

$$\dot{m}'' r^2 \frac{d(C_p T + Y_F \Delta H_c)}{dr} = \rho\alpha \frac{d}{dr}\left(r^2 \frac{d(C_p T + Y_F \Delta H_c)}{dr} \right) \quad ⑨$$

Elimination of the non-linear term simplifies the analysis. This simplification is known as **Schwab-Zeldovich Transformation**.

Dividing Eq.9 by $Q_v + \Delta H_c\ (Y_{Fs} - 1)$,

$$\dot{m}'' r^2 \frac{db_{FT}}{dr} = \rho\alpha \frac{d}{dr}\left(r^2 \frac{db_{FT}}{dr} \right) \quad ⑩$$

$$b_{FT} = \frac{C_p T + Y_F \Delta H_C}{Q_v + \Delta H_C (Y_{Fs} - 1)}$$

Q_v - heat input required for vaporization of droplet

Y_{Fs} –mass fraction of species at the surface of the droplet

$Q_v = \Delta H_V + C_L(Y_{Fs} - 1)$

Conserved variable for oxidizer

$$b_{OxT} = \frac{C_p T + Y_{Ox} f \Delta H_C}{Q_v + \Delta H_C f Y_{Oxs}} \qquad b_{F,Ox} = \frac{Y_F + Y_{Ox} f}{(Y_{Fs} - 1) + Y_{Ox,s} f}$$

So, if I will do that then I can write down you know m dot r square d c p t plus y f delta s c divided by d r is equal to rho alpha, because in place of d I am putting alpha, because Lewis number is equal to 1. So, I can write down this term, now it has been simplified the none linear return as gone away in case of premixed flame, if you remember what we did, we look that physically and we say, that in one case the diffusion is you know like reaction is more important and other cases reaction is not important in the preheat zone, then we only convection and the conduction you remember that premixed flame, how did you do the analysis and in the reaction zone reaction and the conduction.

So, this elimination of none linear terms simplify that analysis this simplification is known as schwab zeldovich transformation keep in mind that premixed flame can be done in the similar way. So, also the jet diffusion flame right, but I have used two method one is the mechanic you know mathematical way of fuel handling other is physical physics way of handling or physical way of handling.

So, both are similar in nature you should keep, so then if I divide this equation by q b plus delta s c y f s minus one where q b is the heat input required for the vaporization of droplet and y f s is the mass fraction of species at the droplet then we have already talked about these things term.

So, if I will do this then I will get m r two d b f t p r is equal to rho alpha d by d r r square v f t and this is basically the conserved variable this is b is known conserved variable how it is we will see that c p t and y f c right and if temperature increases you know v y f will decrease. So, let us look at it here if you look at temperature is increasing here then what happens to y f mass fraction is goes on decrease is a conserved.

and similarly for other things; that means, I have clubbing to here and solving this conserved variable b. So, conserved variable for oxidizer it will be v o x t c p t plus y f f delta s c you can used again equation of temperature you know what we called energy and oxidizer you can arrive at this.

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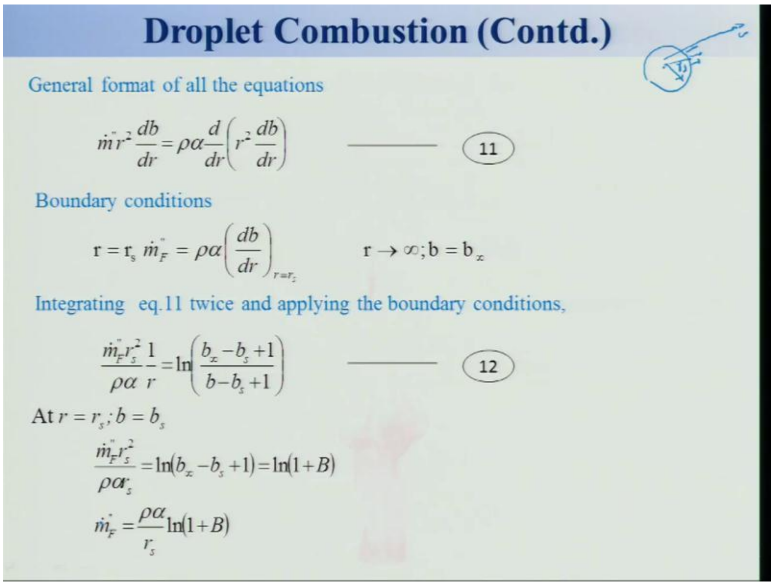

Droplet Combustion (Contd.)

General format of all the equations

$$\dot{m}'' r^2 \frac{db}{dr} = \rho\alpha \frac{d}{dr}\left(r^2 \frac{db}{dr}\right) \qquad \text{———} \qquad ⑪$$

Boundary conditions

$$r = r_s\ \dot{m}_F'' = \rho\alpha\left(\frac{db}{dr}\right)_{r=r_s} \qquad r \to \infty; b = b_\infty$$

Integrating eq.11 twice and applying the boundary conditions,

$$\frac{\dot{m}_F'' r_s^2}{\rho\alpha}\frac{1}{r} = \ln\left(\frac{b_\infty - b_s + 1}{b - b_s + 1}\right) \qquad \text{———} \qquad ⑫$$

At $r = r_s; b = b_s$

$$\frac{\dot{m}_F'' r_s^2}{\rho\alpha r_s} = \ln(b_\infty - b_s + 1) = \ln(1 + B)$$

$$\dot{m}_F = \frac{\rho\alpha}{r_s}\ln(1 + B)$$

Similarly, if you take fuel and oxidizer you can arrive at also similar form I am not going to discuss, so what will we do we will be solving this equation basically right and in a generalized form. So, the general form will be looking like that m dot r square d b by d r is equal to rho r alpha d by d r r square d b by d r. So, what will do we will basically boundary conditions what will be r is equal to r s that is the surface or droplet surface is equal to m dot f is equal to rho f d b by d r; that means, gradient will be depending on the droplet surface.

Because, if droplet surface is there then you know at the surface there will be some flax will be going that will be based on what vaporization rate, these are the flax that means, there will be some gradient which will be dependent on how much fuel is getting consumed per unit area are you getting my point. So, therefore, there will be gradient and this is at r s right and this r s is being changing with respect with time keep it that in mind when r is equal to infinity, if I take this you know somewhere here infinity are right then b is equal b infinity.

So, equation one if I integrate twice and applying this boundary conditions, so I will get m dot f because this is mass flax of the fuel r s square rho alpha l by one by r is equal to l n b infinity minus b s plus one divide by b minus b s plus one right. So, and when r is equal to r s b will be p s right. So, and if b is equal to b s and this will be basically is you know 0.

So, that become m dot f r s square rho alpha r s is nothing, but l n b infinite minus b s plus one nothing, but one plus b right that we are defining. So, this I can write down m dot double dash that is the mass flax rate of the fuel whether mass flax rate of the fuel is equal to rho alpha by r s l n one plus b what it say it is saying that it will be dependent on this b right.

It will be dependent on rho and it will be dependent on the thermal diffusivity; that means, k g by rho c p of course, right and r s that is dependent on the diameter of that right the mass flax will be what we called will be reduce as you goes on decreasing the r s. Now, it will be what we called r s will decreases, so that what will happen this will increase, M dot r s that why droplets are very smaller the droplet you know vaporization rate will be more combustion will be better.

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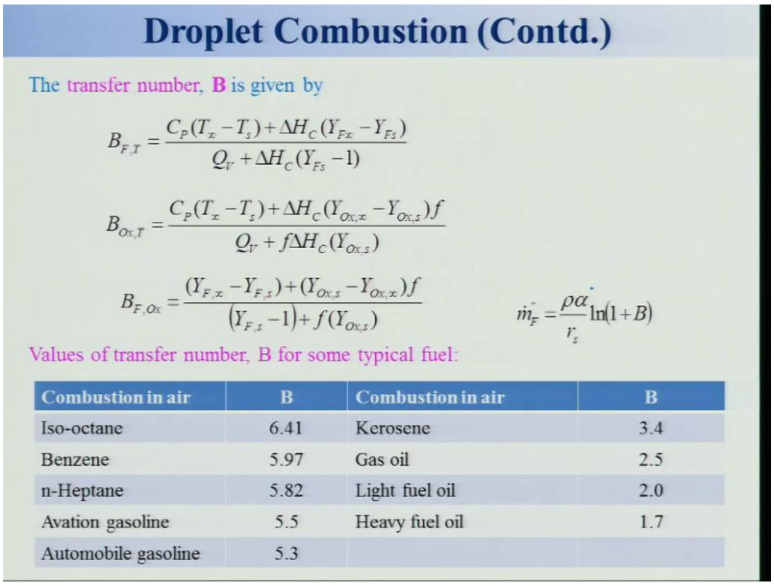

Droplet Combustion (Contd.)

The transfer number, **B** is given by

$$B_{F,T} = \frac{C_P(T_\infty - T_s) + \Delta H_C(Y_{F\infty} - Y_{Fs})}{Q_V + \Delta H_C(Y_{Fs} - 1)}$$

$$B_{Ox,T} = \frac{C_P(T_\infty - T_s) + \Delta H_C(Y_{Ox,\infty} - Y_{Ox,s})f}{Q_V + f\Delta H_C(Y_{Ox,s})}$$

$$B_{F,Ox} = \frac{(Y_{F,\infty} - Y_{F,s}) + (Y_{Ox,s} - Y_{Ox,\infty})f}{(Y_{F,s} - 1) + f(Y_{Ox,s})}$$

$$\dot{m}_F'' = \frac{\rho\alpha}{r_s}\ln(1+B)$$

Values of transfer number, B for some typical fuel:

Combustion in air	B	Combustion in air	B
Iso-octane	6.41	Kerosene	3.4
Benzene	5.97	Gas oil	2.5
n-Heptane	5.82	Light fuel oil	2.0
Avation gasoline	5.5	Heavy fuel oil	1.7
Automobile gasoline	5.3		

So, the transfer number that two b is called as transfer number and some book it is known as Spalding numbers because Spalding is a person who gave this number this formula shows the c p t t infinity minus t s plus delta s c y f infinity minus y f s, f s means it is the mass fraction of fuel at the surface y infinity means mass fraction of the at the infinity which will be eventually zero right no fuel will be there at infinity radius.

So, q b delta s c y f s minus this is generalize it is been done. So, b oxidizer will get similar terms and b fuel and oxidizer will get similar terms. So, values of transfer number b will be phase upper typical fuel you can see, you can see that these are the numbers 6 point 4 1 is ISO octane whereas, the heavy fuel oil is 1 point it is not really varying much you know in between it is varying.

Now, as it is natural logarithm you know, if you look at this l n b r, so that means, if there is increasing 10 times of this number, which is not the case then what will be increase in mass flax keeping everything same. It will be only two times kind of things roughly, you can look at that and then may be, because the logarithm is not really changing much, but where the properties will be changing very important 1.

And what will be doing how to choose these properties of alpha and rho is very important particularly in the analytical solution like this where temperature is changing from three hundred to 2 thousand 2 hundred 3 hundred Kelvin.

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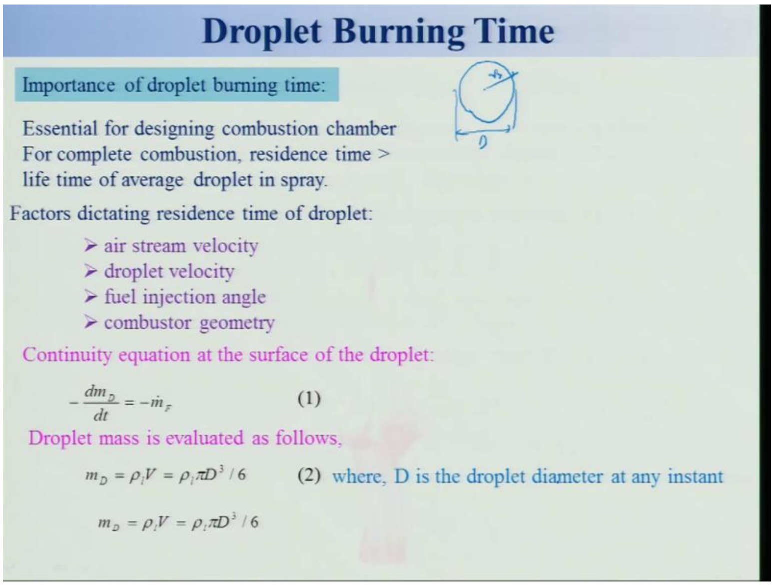

Droplet Burning Time

Importance of droplet burning time:

Essential for designing combustion chamber
For complete combustion, residence time >
life time of average droplet in spray.

Factors dictating residence time of droplet:

- air stream velocity
- droplet velocity
- fuel injection angle
- combustor geometry

Continuity equation at the surface of the droplet:

$$-\frac{dm_D}{dt} = -\dot{m}_F \qquad (1)$$

Droplet mass is evaluated as follows,

$$m_D = \rho_l V = \rho_l \pi D^3 / 6 \qquad (2)$$ where, D is the droplet diameter at any instant

$$m_D = \rho_l V = \rho_l \pi D^3 / 6$$

So, therefore, we will be doing some average I will just quickly look at what is our interest our interest is to find out the droplet burning time because the time. Because, the time which is required you know to such that all vaporization combustion take place, before it is exited out of the combustor, otherwise it is the wastage you know and my efficiency will be decrease will be decreasing.

So, essential for designing combustor and complete combustion residence time of the fuel element must be greater than, the life time of average droplet size or sometime people used a lager droplet size in the drop in the spray, but generally you know like that depends upon the designer, what you wants. So, factors dictating residence time what are those air stream velocity droplet velocity fuel injection angle combustor geometry in real situation.

But, now what will be looking at in the single droplet, so we would not be really talking about all those things right and continuity equation at the surface of the droplet we will be looking at what is that as I told you that it is basically surface you know r s. So, what I am saying is d m by d t is nothing, but m dot m there is change in mass of the droplet with respect to the time is nothing but, so I can consider that m d is nothing, but what rho l v right density of liquid into v and that is nothing, but rho l pi d q by 6

And what is this, basically the d d is the droplet diameter at any instant of time that is at any instant it may be changing as it is residing.

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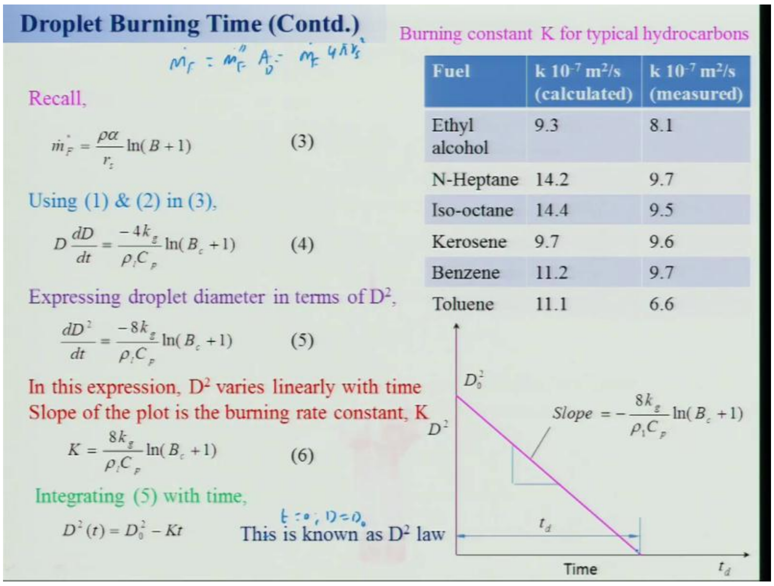

Droplet Burning Time (Contd.)

$\dot{m}_F = \dot{m}''_F A_D = \dot{m}_F 4\pi r_s^2$

Recall,

$$\dot{m}_F = \frac{\rho\alpha}{r_s}\ln(B+1) \qquad (3)$$

Using (1) & (2) in (3),

$$D\frac{dD}{dt} = \frac{-4k_g}{\rho_l C_p}\ln(B_c+1) \qquad (4)$$

Expressing droplet diameter in terms of D^2,

$$\frac{dD^2}{dt} = \frac{-8k_g}{\rho_l C_p}\ln(B_c+1) \qquad (5)$$

In this expression, D^2 varies linearly with time
Slope of the plot is the burning rate constant, K

$$K = \frac{8k_g}{\rho_l C_p}\ln(B_c+1) \qquad (6)$$

Integrating (5) with time,

$t = 0,\ D = D_0$

$$D^2(t) = D_0^2 - Kt$$

This is known as D^2 law

Burning constant K for typical hydrocarbons

Fuel	k 10^{-7} m^2/s (calculated)	k 10^{-7} m^2/s (measured)
Ethyl alcohol	9.3	8.1
N-Heptane	14.2	9.7
Iso-octane	14.4	9.5
Kerosene	9.7	9.6
Benzene	11.2	9.7
Toluene	11.1	6.6

So, then I can write down what I will do I will have to used this m dot f into rho l r s and l n b plus 1 and using this equation 1 and 2, 3 I can get basically, what I am doing you know if I look at this m dot f. M dot f will be what m dot dash f into area of the droplet and what will be m dot f into 4 pi r s square is not it. So, you can find out basically it will be may be pi d square kind of thing right will it be right something will be coming.

So, now if you do that this portion then you will get d d by t, because we are talking about that one right and then expressing the droplet diameter will get d d square by d t minus 8 k g rho l c p l n b c plus 1. Because, the b c is corresponding to the combustion. So, in this expression d square varies, linearly with respect to the time and scope slop of the burning is known as constant.

So, this is a k values right if I integrate this equation 5, I will get d square t is equal to d note square minus k t, because at initial t is equal to 0 t is t is equal to 0 d is equal to d note, keep in mind that I can get this expressions. So, this is known as the d square law this is very important concept, this should be understand that is and this constants, if you look at varies you know this is calculated one and these are the measured values, you can say it is changing from 8 point 8 to 6 point any kind of value.

Of course, now if I plot this d by time you will find I will get slop and this slop corresponding to the burning rate constant. Because, I know d is varying with respect to time right and this, also for combustion and similar thing you can get for the vaporization as well and this will give me the time required d t, you know for the complete combustion.

Because, here d is zero at this time this is the lifetime of the droplet, you can say because at when d is equal d become 0 there is nothing s h. So, this time we should provide for the in a combustor designing, so that I can get the length kind of thing.

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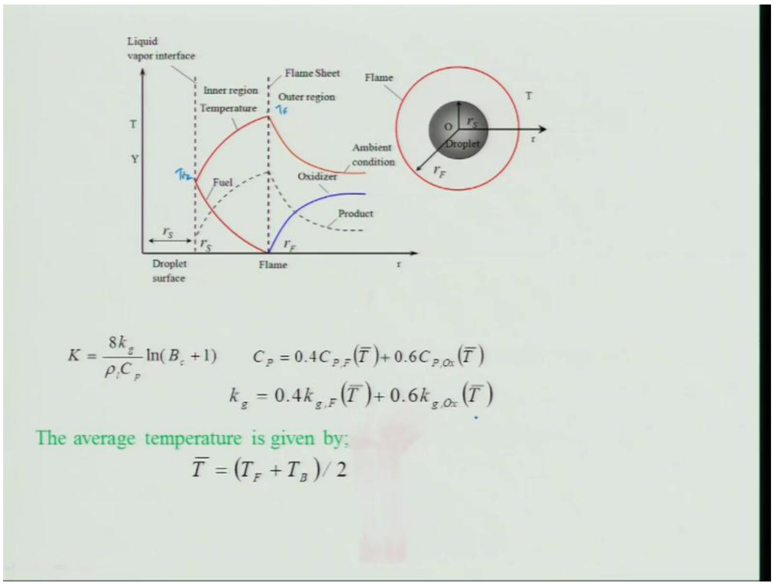

$$K = \frac{8k_g}{\rho_l C_p}\ln(B_c + 1) \qquad C_p = 0.4C_{p,F}(\bar{T}) + 0.6C_{p,Ox}(\bar{T})$$

$$k_g = 0.4k_{g,F}(\bar{T}) + 0.6k_{g,Ox}(\bar{T})$$

The average temperature is given by;

$$\bar{T} = (T_F + T_B)/2$$

So, I will just tell you that how to determine you know c p values because if you look at k g is the very important, so also the c p, because c p is changing of temperature t here and also here it is having peak value 2 thousand for this, our analysis we will be using this. But, several book have used different 1 that point 4 c p average e t average, what is average temperature is basically t f plus t b divide by 2 t b means this is t f right and this will be t b boiling point temperature of the liquid at the surface, divide by 2.

We are taking and then k g also point 4 k g f t plus point 4 k g of f t by 2 keep in mind that this is an ad hoc, so which will be looking at it, so when we are solving problem we will be using the same 1. But, of course if it is given the problem different one you can look at it. So, we will be visiting this droplet combustion particularly when we will be looking at the liquid rocket engines hopefully.

And with this we will stop over here.