Fundamentals of Aerospace Propulsion
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Lecture - 22

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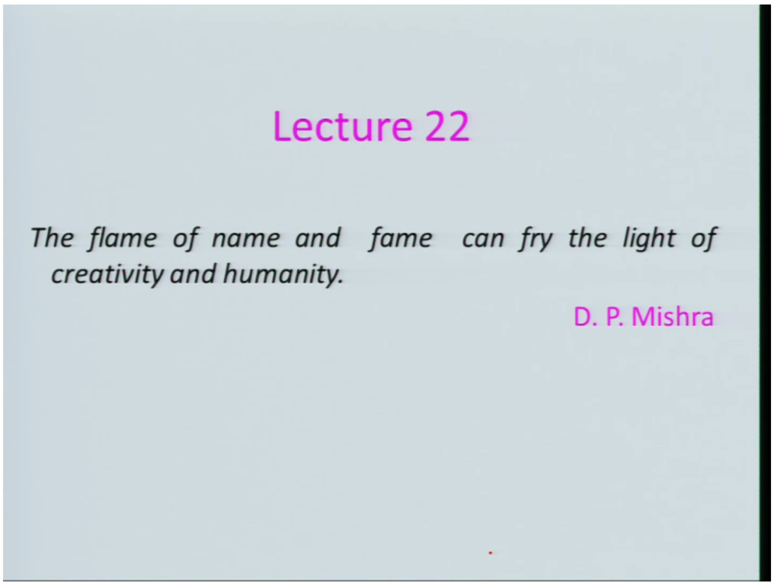

Lecture 22

The flame of name and fame can fry the light of creativity and humanity.

D. P. Mishra

We will start this lecture twenty two and if you recall that we discussed about the what do call combustion wave and which we discussed about both deflagration and detonation. If you remember that we derived expression for a basically, combustion wave on which we learnt about weak detonation and weak deflagration. Which can occur in nature, but however, the strong detonation and strong deflagration would not be occur in nature.

Of course, one can simulate in the laboratory, but generally it is not that easy to simulate as well. Today what we will be doing we will basically concentrating our discussion on the deflagration that is basically the premixed flame. When we talk about premixed flame we always interested to look at what really will be the structure. That means, inside what is really happening because if you recall that in the last lecture we did not really bother about structure of the flame. We assume there is some heat release rate and then we you know use this conservation of mass momentum and energy equation derived the expression, for what we called various properties like pressure density other things.

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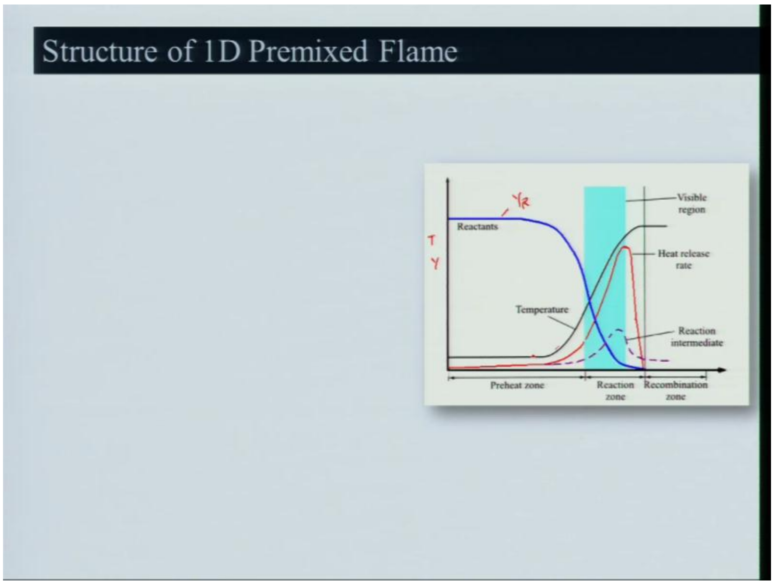

Structure of 1D Premixed Flame

But, today we will be looking at 1-dimensional premixed flame and if you look at 1-dimensional flame can be obtained in the laboratory, but in natural it is quite difficult how that 1-dimensional flame. Of course, if you look at the premixed flame is basically a manmade flame you can assume. As I told you Bunsen and you know founded that flame and of course, one can assume because you can stabilize in a burner and then one can assume it to be 1-dimension. Therefore, similarly in the we can use 1-dimensional burner and then establish a flame which can assumed to be 1-dimensional, keep in mind that here what will be discussing is the laminar 1-dimensional premixed flame

But in nature most of the you know flame will be turbulent in nature and also in the practical situation it will be turbulent. So, therefore, you know it is important to look at turbulent flame, but however, due to complexities and other things we would be looking at because we can relate this laminar flame to the turbulent flame, particular when we will be moving to the caster brain combustor. Then I will be discussing a touching upon that turbulent premixed flame to some extent.

But not really because it is a quite complex subject to be deal in this lecture in this course. So, if you look at what you observe whatever you know from here, here I have plotted here in this vertical axis temperature, mass fraction of you know fuel and oxidizer. If you look at this is basically mass fraction of the reactant you know reactant can be fuel can be oxidizer and of course, there will be a particle.

And if you see this that it is you know decreasing that means, it is consuming its mass fraction of fuel and also you can have a similar plot for the other variation. For the mass fraction of oxidizer it need not to be same, but it will be similar and if you look at what you can observe here that in these zone the gradient is very high. And not only for the mass fraction of the reactant, but also for the temperature, temperature is being shown here right and there is a lot of heat being release in this zone. If you look at this red one heat being release peak value and it receded towards that and of course, there will be some intermediate reaction which will be occurring you reactants, and reaction will be occurring and some of the reactions will be occurring in this region.

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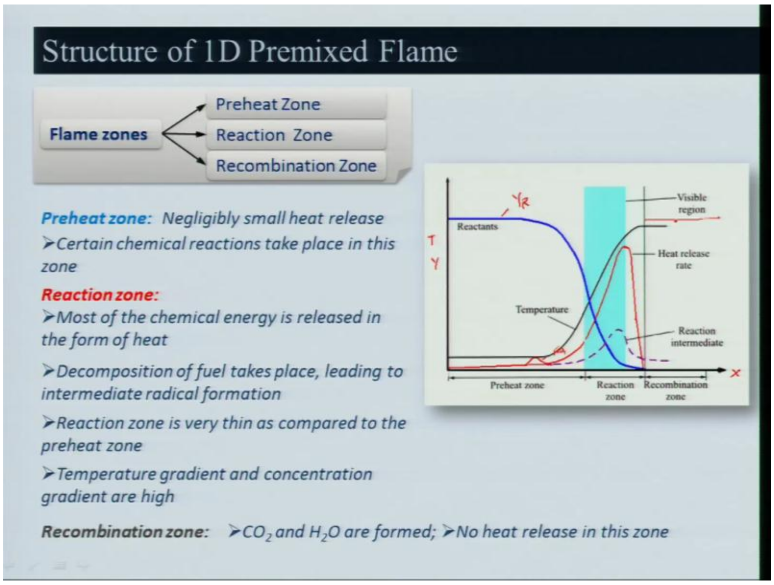

Structure of 1D Premixed Flame

Flame zones
- Preheat Zone
- Reaction Zone
- Recombination Zone

Preheat zone: *Negligibly small heat release*

➢Certain chemical reactions take place in this zone

Reaction zone:

➢Most of the chemical energy is released in the form of heat

➢Decomposition of fuel takes place, leading to intermediate radical formation

➢Reaction zone is very thin as compared to the preheat zone

➢Temperature gradient and concentration gradient are high

Recombination zone: *➢CO_2 and H_2O are formed; ➢No heat release in this zone*

So, if look at the flame can be broadly divided into three categories; that means, flame in the sense flame zone these zone can be broadly divided into preheat zone, reaction zone and recombination zone. What is really happening in this preheat zone if you look at temperature is not really going up, but it little bit you know slowly increasing in this zone because why there is a negligible heat release. If you look at heat release is very, very small in this domain along of course, with that temperature also you know suits up.

So, negligibly small heat release is occurring and certain chemical reaction do take place in this zone, which we I have not shown may be some of the you know like a species like h o 2 or h o or something will be occurring in this region because of temperature is being little bit suiting up. But however, you know like it is not that if I compare this preheat zone with the life of a human being what it would be nothing really is happening productively. So, for flame is concerned flame is basically what where the exothermic reaction will be taking place if you look at overall in this zone particularly overall reaction, whatever it will be occurring it will be what you call net effect will be nullified. They are neither exothermic nor endothermic temperature is remaining constant, but in this case there will be little bit increase.

So, this will be similar to your what do called student life where you are being prepared to do a productive work during your adulthood right kind of thing. So, but whereas, all these gradient you know gradient is occurring or temperature heat being release in the reaction zone. Of course, the zone in this there will be several species which will be liberated radicals and that will be radiating you know various spectrum of the light which you can see.

For example, C a star o h and then C 2 star and other things ahead I discuss those things you know C a star give you blue and then suit it is yellow and then some purple color all those thing will be coming. So, you will see that and keep in mind this is the reaction zone which is very thin as compared to the preheat zone and in the if you look at; that means, in the reaction zone most of the chemical energy is released in the form of heat. In the reaction zone and decomposition of first fuel takes place and these decomposition really not take place in reaction zone it takes place.

In the beginning of the reaction zone or just before the reaction zone because there will be paralysis product that the decomposition of the fuel like for example, propane, propane will be converted into methane some other things and then goes on like that. So, that will be creating some radicals you know to be what to call be consumed in this reaction zone. And that is very important because whenever I see a flame I always feel it is a life like life is getting you know flame is getting burnt like fuel is being getting burnt.

So, also in life you getting burnt your time you know time is resided as you go on. And you get prepared to contribute for the life which is not being done through education nowadays right, but in principle it should be. Like a flame it prepared the species to be consumed in the reaction zone and then it will release heat and also the light you know. so that it will be people can take help.

So, reaction zone is very important so also the productive life of a human being one should not be really only worried about his food, he should worry about what he can contribute for the others. So, reaction zone as I told it is very thin as compared to the preheat zone and keep in mind that this is the very critical part so, far analysis is concerned and because the gradient is very higher and temperature gradient and concentration gas high as I told you.

And recombination zone this is these zone particularly you know is basically the recombination zone, Where I have shown here temperature should be almost flat there is no change in the temperature along with the x direction if I say this is x direction. Then you know in there is no change, but in real situation particularly for hydrocarbon you know this thing. There will be some recombination like carbon you know dioxide and water will be the formed and some amount of heat being released particularly for hydrocarbon. As result the temperature profile will be increasing to towards the asymptotic value which I have not shown deliberately because you know I want to simplify and then analyze it right.

So, if you look at just summarize the flame zone can be divided into three category; three you know one is preheat zone, reaction zone, and recombination zone. And you should be careful about this because what will be doing we are not bother about this recombination zone. When we will be analyzing and this 1-dimensional flame will be bother about preheat zone and reaction zone.

What is really happening here one has to look at like if you look at the various passes are occurring like if I look at the thermal side you know then there is conduction, there will be convection, there will be also radiation you know heat transfer will be taking place. And some chemical reaction will be taking place and it will be preparing the you know fuel and oxidizer to be burned in the reaction zone. So, of course, in the recombination zone as I told you no heat release in this zone is being occurred for our case, but in that, but whatever the heat be being release be occurred in real situation is very very negligibly small as compared to the in the reaction zone.

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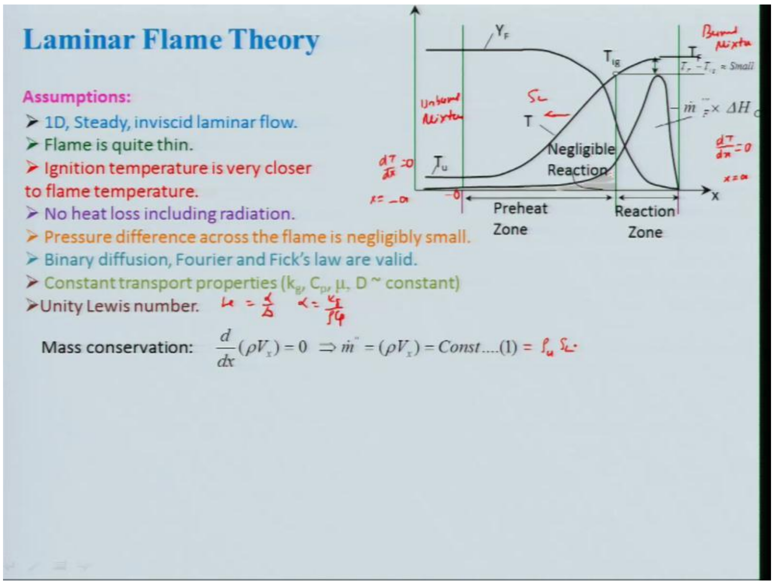

Laminar Flame Theory

Assumptions:

- 1D, Steady, inviscid laminar flow.
- Flame is quite thin.
- Ignition temperature is very closer to flame temperature.
- No heat loss including radiation.
- Pressure difference across the flame is negligibly small.
- Binary diffusion, Fourier and Fick's law are valid.
- Constant transport properties (k_g, C_p, μ, D ~ constant)
- Unity Lewis number. $Le = \frac{\alpha}{D}$ $\alpha = \frac{k_g}{\rho C_p}$

Mass conservation: $\frac{d}{dx}(\rho V_x) = 0 \Rightarrow \dot{m}'' = (\rho V_x) = Const....(1) = \rho_u S_L$.

For the practical analysis proposes which will be doing keeping mind that this is our as I told you is a reaction the fuel flame and if I look at along with x direction when x is very far away. That means, x is infinity what will happened to temperature because temperature profile have shown here temperature gradient will be d t by d x will be 0 right yes or no and this is I am starting from here this is my origin.

So, towards if I go towards you know away from the origin toward the left hand side this will be minus infinity right x is equal to minus infinity. What will be the temperature then gradient it will be again d t by d x is 0. Similarly, for mass fraction of fuel oxidizer you know will be valid this condition will be valid. And keep in mind that we are saying that this reaction, this is the reaction zone where the T ignition will be there which is closer the T f that is again an assumption we are making, but it is not bad.

That means T f minus T ig is very, very small this assumption will be using for an analysis, for actually derive the relationship for burning velocity if you look at this flame front will be moving with the S L burning velocity towards unburned mixture. This is unburned mixture right am I right and this side is what burned mixture.

So, we will make a certain assumptions before really looking at the equations and simulating it. So, what are the assumptions which are very simple which we have done 1-dimensional steady in which said laminar flame because we are talking about laminar flame theory? We want to understand and derive the relationship for laminar burning velocity and this burning velocity is with respect to what, with respect to unburned mixture right because it is moving towards the unburned mixture. So, we will take that with respect to it is moving right.

So, an flame is quite thin that is an assumption we are making and it will be dependent on what, the thickness of the flame will be dependent on what that is the thing we should worry about. Will it be dependent on the fuel air you know type of fuel and type of oxidizer or will it be dependent on the fuel air ratio, will it be dependent on inlet temperature will it be dependent on inlet pressure or the pressure. What it would be is one question might be larking in your mind.

So, but we are assuming the flame to be thin, thin of course, we know it is like a surface. So, we will see that an ignition temperature is very closer to that flame temperature. We have this an assumption and no heat loss including radiation we are assuming is it possible there will be a flame. When there would not be any radiation or there would not be any heat loss you know, heat loss I mean we are saying that there would not be radiation. Of course, it is possible provided it is flame you know like dividing of the some species like there are two kinds of radiation one is you know gas phase radiation other is solid phase shoots are there.

But we are assuming that there is no shoot in this flame. So, therefore, that is negligible and there gas phase radiation will be very, very small, but in real situation radiation will be there you cannot really eliminate it or avoid it, but we are assuming it to be having no effect you know on the analysis. Pressure difference across the flame is negligibly small you can invoke that what we have done in the last lecture about combustion wave and look at it that you will find the pressure difference will be very very low.

Therefore, we would not be considering the momentum equation why it will be low because it is a deflagration, but if it is detonation we cannot really manage to you know have this assumption we cannot. So, we would not be considering momentum equation and then keep in mind that we are assuming the binary diffusion which will bigger you know and also furrier and fixed law will be valid because, binary diffusion why we are saying that we will be talking about what we called single step chemistry here. And if it is a multistep chemistry you know which are suppose to I am naturally it will be occurring in the flame. Then this assumption would not be valid for the simplicity we are assuming.

And constant transport properties we are assuming which is not true because the temperature is going from T u will be what, it will be around 300 kelvin and T flame will be around 2200 Kelvin for you know hydrocarbon air flame. So, it is quite a bit so you cannot really have that you know C p particularly will be varying the temperature. So, also the you know diffusivity I mean like conductivity and then mass diffusivity, d is the mass diffusivity and kg is conductivity of the gas will be changing, but we are assuming for the simplicity. And unity Lewis number that we are assuming what is the meaning of Lewis number.

Lewis number is basically this alpha by D the alpha is the thermal diffusivity; that means, alpha is basically K g by rho C p, K g is the conductivity, rho is the density, C p is specificity. That is known as thermal diffusivity and D is same as the d that is the mass diffusivity right That we are assuming one what is the meaning of it that is saying that you know your species equation and the temperature equation or the energy equation will be similar, if it is one I can substitute those value and get that. If I get a solution for one other I can get very easily, but that is again restricted case it is in reality it is not possible, but we will be doing that I mean all the thing.

And the mass conservation if you look at is a one dimensional flow. So, we can get d by d x rho v x is equal to 0 what is the meaning of that. It is meaning is that mass flux is equal to the rho v x is constant, what is the meaning is saying that rho u if i say on then S L, S L is the burning velocity, rho u is the density. So, basically all these analysis what we are doing we are interested to derive a relationship for burning velocity and that comes from the mass conservation. If you look will be basically looking at this burning velocity S L to find out.

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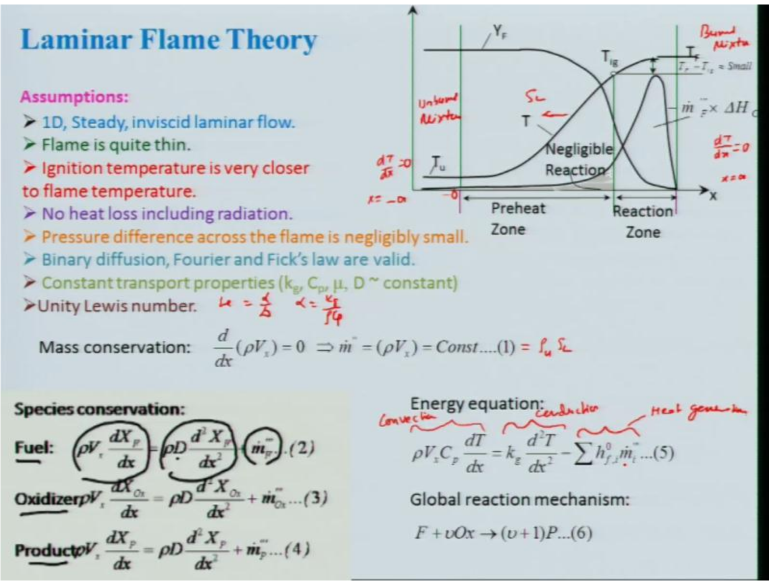

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Mass conservation: $\frac{d}{dx}(\rho V_x) = 0 \Rightarrow \dot{m}'' = (\rho V_x) = Const....(1) = \rho_u S_L$

Species conservation:

Fuel: $\rho V_x \frac{dX_F}{dx} = \rho D \frac{d^2 X_F}{dx^2} + \dot{m}'''_F ..(2)$

Oxidizer $\rho V_x \frac{dX_{Ox}}{dx} = \rho D \frac{d^2 X_{Ox}}{dx^2} + \dot{m}'''_{Ox} ...(3)$

Product $\rho V_x \frac{dX_P}{dx} = \rho D \frac{d^2 X_P}{dx^2} + \dot{m}'''_P ...(4)$

Energy equation:

$$\underbrace{\rho V_x C_p \frac{dT}{dx}}_{\text{Convection}} = \underbrace{k_g \frac{d^2 T}{dx^2}}_{\text{Conduction}} - \underbrace{\sum h^0_{f,i} \dot{m}'''_i}_{\text{Heat generation}} ...(5)$$

Global reaction mechanism:

$F + \upsilon Ox \rightarrow (\upsilon + 1)P...(6)$

So, and as I told you we are using a single step chemistry model for this that is 1 K g of fuel reacting with your nu K g of oxidizer going to the product of nu plus 1 K g is product P. And therefore, we need to look at this species equation for fuel oxidizer and product. Let us consider the species conservation equations as I have animator here for fuel, oxidizer and product and these equations are basically similar.

And let us consider the species conservation equation for fuel which having three terms; first is your convection term that is rho V x d x F by d x which will be equal to the diffusion term. And the mass consumption term consumption of fuel mass consumption of fuel and if you look at this diffusion term is a rho D, d square x F divided by d x square and keep in mind that this diffusivity what we have consider is binary in nature. That means these diffusion between two species whereas, in actual system multi component diffusion would likely to take place.

And this equations will be similar for the oxidizer for example, there will be convection term rho V x d X O x divide by d X is equal to rho D, d square X O x divided by d x square and mass consumption of the oxidizer. And keep in mind that this is per unit volume this mass consumption of oxidizer even the fuel any everything all this term is basically per unit volume.

Similarly, we can get the what we call the product that is the convection term that is rho V x d X p divide by d x which will be equal to the diffusion term and the mass generation of the product. The diffusion term will be rho D, d square X p divide by d x square and the mass consumption of the product we have consider as we have taken in the global kinetic scheme. That is 1 mole of fuel is reacting with nu mole of oxidizer which will going to the nu plus 1 what we called gram of the product.

So, therefore, we will have to solve this equations and to get this each mass fraction of fuel and oxidizer and product. So, energy equation will be having a similar form that is rho V x C p d T by d x this is your what we called convection term and this is your conduction term and this term is your what we called the heat generation or heat release. That means h f i naught m dot i f, i can be fuel in this case oxidizer and product. So, let us look at how we expand this term you now and see this term particularly.

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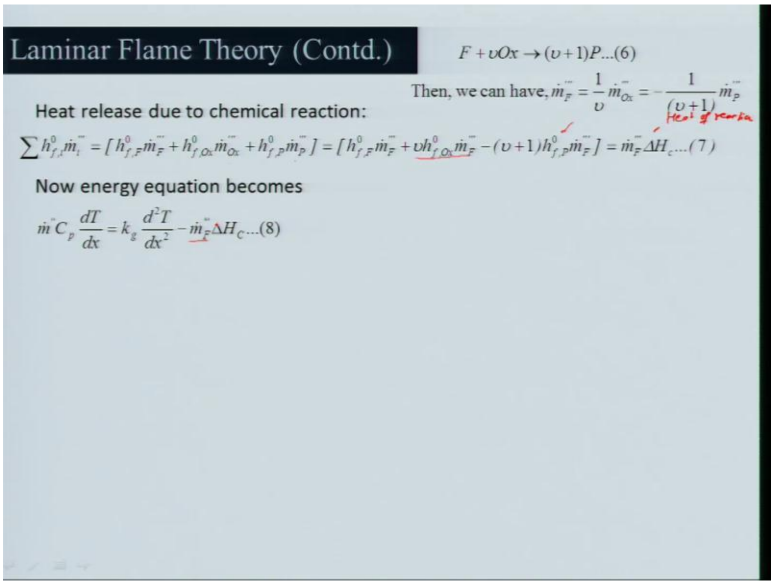

Laminar Flame Theory (Contd.)

$$F + \upsilon Ox \rightarrow (\upsilon + 1)P \ldots(6)$$

Then, we can have, $\dot{m}_F''' = \frac{1}{\upsilon}\dot{m}_{Ox}''' = -\frac{1}{(\upsilon+1)}\dot{m}_P'''$

Heat of reactn

Heat release due to chemical reaction:

$$\sum h_{f,i}^0 \dot{m}_i''' = [h_{f,F}^0 \dot{m}_F''' + h_{f,Ox}^0 \dot{m}_{Ox}''' + h_{f,P}^0 \dot{m}_P'''] = [h_{f,F}^0 \dot{m}_F''' + \underline{\upsilon h_{f,Ox}^0 \dot{m}_F'''} - (\upsilon+1) h_{f,P}^0 \dot{m}_F'''] = \dot{m}_F''' \Delta H_c \ldots(7)$$

Now energy equation becomes

$$\dot{m}'' C_p \frac{dT}{dx} = k_g \frac{d^2T}{dx^2} - \dot{m}_F''' \Delta H_C \ldots(8)$$

So, if you look at we are looking at this 1 K g of fuel reacting with nu K g of oxidizer going to the part of nu plus 1 K g of product. So, we can write down that m dot triple dash F, that is mass of fuel per unit volume being consumed is equal to 1 over nu m dot triple dash O x is equal to minus 1 over nu plus 1 m dot triple dash P. why it minus because this fuel and oxidizer is being consumed whereas, product is being formed instead of righting minus here I have put here.

So, if I do that like let us look at how we can do I can explain this term h f i m dot i I can write on h f i h f F o plus m dot triple dash F and similarly I can write down for oxidizer and for the product. If I substitute these values you know like for oxidizer what it would be m dot triple dash O x will be nothing, but your nu plus m dot triple dash F. So, that is the this term right and similarly form here, I can get to this term if I take this m dot F out what I will get I will get you know h a plus nu h f oxidizer minus nu plus 1 h a product and that is nothing but your heat of reaction this is your heat of reaction.

So, what it eventually this coming that those things will be very easy to look at that I can express in terms of that. So, now what will be using as I told you that we will be only considering the energy equation as the Lewis number equal to 1. Once we solve that we can really look at how this fuel or the oxidizer mass fraction changing with respect to the x axes or along the length of the flame. So, that can be and that is very important assumption, which will be discussing about when we will be in detail. When we have discussing about droplet combustion and that is we will be using one a swap jilt transformation that we will be doing.

Here I am dealing little different way right. So, I want to give both flavor this is more what we called physical and that is more mathematical or mechanical. So, let us look at this energy equation you know becomes m dot C p d T by d x is equal to K g d square d T by d x square minus the react heat release term. What we could observe here what kind of equation it is I need to solve this equation eight, could you observe is it possible I can solve this equation. It is a second order equation or not right and it is a non-linear and couple equation right yes or no can we really solve.

Is it possible to solve with your OD you know you have learnt this OD, ordinary differential equation can I really solve this equation because if you look at m dot triple dash F, how it will be look at ordinary form of equation. That is m dot triple dash F if I look at the even global kinetic you know it will be A f then C f concentration of F. Of course, you will have to in terms of mass fraction if you want to put it you will talk in terms of molecular weight also multiply in to C of oxidizer e power d E by R T.

So, if you look at this term is a very, very none linear term because I should know the temperature, I should know the activation energy, I should know the pre exponential factors. And those activation energy pre exponential factors right will be dependent on what is the fuel air ratio you are talking about then also it will dependent on temperature very highly temperature you know because e power to the you know e minus R by R u t.

So therefore, it is you do not know temperature therefore, you solving this equation therefore, it is quite difficult do that.

So, how do go about we will see will go back to this diagram, if you look at what we are doing I am basically what will be doing to solve this equation because is a quite complex non-linear. And what will be doing we will be basically looking at in these zone in preheat zone which is more predominant here, there will be convection. And if you look at this equation convection and conduction, but; however, the heat release term will be negligibly small because this is very, very small portion.

That will be very, very small portion, this is very, very small portion right. So, we can neglect that and in some zone what will be doing we will be basically looking at in these zone in the reaction zone where we will be neglecting the convection. That means, whatever heat will be release will be carried away by the conduction because the temperature gradient is very high so, the gradient is very high. So, therefore, we can neglect that term so, that is the way which will be looking at it. And I already told you that this is your what we called at x is equal to minus infinity d T by d x 0 and x is a positive infinity that far away from d t by d x will be 0 this is very away from that.

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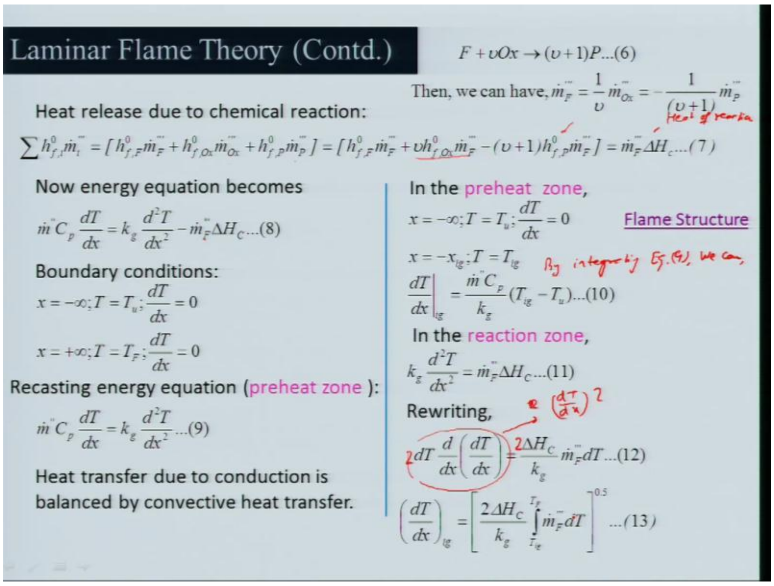

Laminar Flame Theory (Contd.)

$$F + \upsilon Ox \rightarrow (\upsilon+1)P \ldots(6)$$

Then, we can have, $\dot{m}'''_F = \frac{1}{\upsilon}\dot{m}'''_{Ox} = -\frac{1}{(\upsilon+1)}\dot{m}'''_P$

Heat of reaction

Heat release due to chemical reaction:

$$\sum h^0_{f,i}\dot{m}'''_i = [h^0_{f,F}\dot{m}'''_F + h^0_{f,Ox}\dot{m}'''_{Ox} + h^0_{f,P}\dot{m}'''_P] = [h^0_{f,F}\dot{m}'''_F + \underline{\upsilon h^0_{f,Ox}\dot{m}'''_F} - (\upsilon+1)h^0_{f,P}\dot{m}'''_F] = \dot{m}'''_F \Delta H_c \ldots(7)$$

Now energy equation becomes

$$\dot{m}'' C_p \frac{dT}{dx} = k_g \frac{d^2T}{dx^2} - \dot{m}'''_F \Delta H_C \ldots(8)$$

Boundary conditions:

$$x = -\infty; T = T_u; \frac{dT}{dx} = 0$$

$$x = +\infty; T = T_F; \frac{dT}{dx} = 0$$

Recasting energy equation (preheat zone):

$$\dot{m}'' C_p \frac{dT}{dx} = k_g \frac{d^2T}{dx^2} \ldots(9)$$

Heat transfer due to conduction is balanced by convective heat transfer.

In the preheat zone,

$$x = -\infty; T = T_u; \frac{dT}{dx} = 0$$

Flame Structure

$$x = -x_{ig}; T = T_{ig}$$

By integrating Eq.(9), we can,

$$\left.\frac{dT}{dx}\right|_{ig} = \frac{\dot{m}'' C_p}{k_g}(T_{ig} - T_u) \ldots(10)$$

In the reaction zone,

$$k_g \frac{d^2T}{dx^2} = \dot{m}'''_F \Delta H_C \ldots(11)$$

Rewriting,

$$\left(\frac{dT}{dx}\right)^2$$

$$2dT \frac{d}{dx}\left(\frac{dT}{dx}\right) = \frac{2\Delta H_C}{k_g}\dot{m}'''_F dT \ldots(12)$$

$$\left(\frac{dT}{dx}\right)_{ig} = \left[\frac{2\Delta H_C}{k_g}\int_{T_{ig}}^{T_F}\dot{m}'''_F dT\right]^{0.5} \ldots(13)$$

So, and what are the boundary condition if you look at of course, already talked about it T is equal to T u x minus infinity d T by d x 0, x positive infinity it will be t is equal to T F d T by d x 0. And as I told you we will be looking at this preheat zone in case of preheat zone this will be 0, in this case this will be 0 in the preheat zone. For preheat zone this will be 0 therefore, you will get m dot C p d T by d x K g d T square by d T x square.

That means the whatever you know the conductivity in you know like heat is coming from the reaction zone it will being you know balance between the conduction and convection, through conduction is coming and it is convicted away right that is the balance. So, it and if you look at for the preheat zone the of course, these are the condition has to be valid that is of course, at x will minus infinity T is equal to T u d T by d x 0 x is equal minus x ignition you know that I have showed you at that point T is equal T ignition.

See basically you want to d couple you know this equation and solve divided rule basically, but you are landing in one problem I do not move this T ignition I do not know you know you are trying to solve, but you are landing in problem. So, how to do that we will see it in later on. So, if I what we called integrate this equation what I will get d T by d x ignition is equal to m dot C p K g T i g minus T u right, I am just integrating this equation. By integrating equation nine, by integrating equation nine we can get d T by d x right because the other term it is 0. So, that is going away right and then you are getting m dot C p K g T ignition minus T u right.

So, in the reaction zone however, what will happen in the reaction zone what we will be doing we will be basically, looking this convection term will be negligibly small only the conduction and the reaction will be there. It is in small means it is small as compare to the contribution of the heat release heat and whatever being conducted you know that way you cannot really say is not there, but we are just assuming. Then if what I will do I can write down here multiply d T here I can multiply d T right if you look at what is this term.

This term will be coming as what we called it will be coming as d T by d x whole square of course, provided if I multiplied this time left hand side 2 and this time right hand side 2 then you will get this term as that. So, then if I just do this I will get d T by d ignition is nothing, but 2 delta H c by K g integrated m dot F over the T ignition to T F because this range is you know T ignition is one side and other side will be T F right we know that as a result you will get it.

So, but still you know use this expression over here equation ten and thirteen we can club because this is T ignition and this is d T by d x ignition right this equation will be same as. This right hand side of equation ten will be same as the right hand side of equation of thirteen right then we will do that. So, that we are avoiding this gradient temperature gradient at ignition point so that is the way we are doing.

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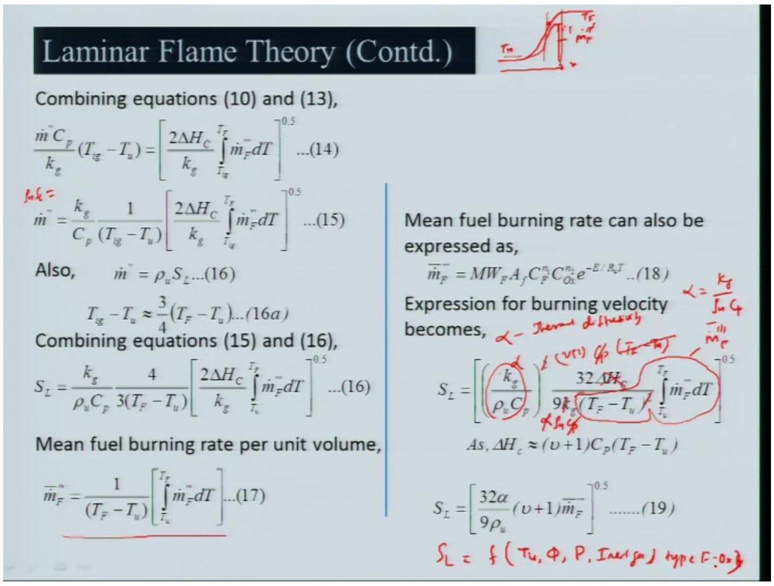

Laminar Flame Theory (Contd.)

Combining equations (10) and (13),

$$\frac{\dot{m}'' C_p}{k_g}(T_{ig}-T_u)=\left[\frac{2\Delta H_C}{k_g}\int_{T_{ig}}^{T_F}\dot{m}_F''' dT\right]^{0.5} \quad ...(14)$$

$$\dot{m}''=\frac{k_g}{C_p}\frac{1}{(T_{ig}-T_u)}\left[\frac{2\Delta H_C}{k_g}\int_{T_u}^{T_F}\dot{m}_F''' dT\right]^{0.5} \quad ...(15)$$

Also, $\dot{m}''=\rho_u S_L ...(16)$

$$T_{ig}-T_u \approx \frac{3}{4}(T_F-T_u)...(16a)$$

Combining equations (15) and (16),

$$S_L=\frac{k_g}{\rho_u C_p}\frac{4}{3(T_F-T_u)}\left[\frac{2\Delta H_C}{k_g}\int_{T_u}^{T_F}\dot{m}_F''' dT\right]^{0.5} \quad ...(16)$$

Mean fuel burning rate per unit volume,

$$\overline{\dot{m}_F'''}=\frac{1}{(T_F-T_u)}\left[\int_{T_u}^{T_F}\dot{m}_F''' dT\right] ...(17)$$

Mean fuel burning rate can also be expressed as,

$$\overline{\dot{m}_F'''}=MW_F A_f C_F^{n_1} C_{Ox}^{n_2} e^{-E/R_u T} ..(18)$$

Expression for burning velocity becomes,

$\alpha = \frac{k_g}{\rho_u C_p}$

α – Thermal diffusivity

$$S_L=\left[\left(\frac{k_g}{\rho_u C_p}\right)\frac{32\Delta H_c}{9k_g(T_F-T_u)}\int_{T_u}^{T_F}\dot{m}_F''' dT\right]^{0.5}$$

$$As, \Delta H_c \approx (\upsilon+1)C_p(T_F-T_u)$$

$$S_L=\left[\frac{32\alpha}{9\rho_u}(\upsilon+1)\overline{\dot{m}_F'''}\right]^{0.5} \quad ....(19)$$

$S_L = f(T_u, \Phi, P, \text{Inert gas}, \text{type } F:Ox)$

So, combining this equation ten and thirteen we are getting m dot double dash C p by K g T i g minus T u equal to route over 2 delta H c K g integrated over m dot triple F d T it is integrated over T ignition to T F. So, if you look at still we are not solving because we need to integrated this you know integration that it is not really possible because you do not know that temperature, how it is varying with temperature and the T ignition is also not known.

So, you will have to do some approximation for that and of course, I can simplify this m dot dash is equal to K g by rho C p 1 over this just a rewriting this equation fourteen in this fashion nothing more than that. So, we know that m dot double dash is nothing but rho u S L from the continuity equation. So, I can write down over here you know I can write down rho u S L is equal to this right. So, then I can really look at combining this equation fifteen and sixteen I can get S L is equal to K g by rho u C p because rho u has gone over here and rest of the thing same as a that of the equation fifteen.

Now, what we need to know, but still you know that here I have introduced T F minus T u in place of T ignition and T u, what we are assuming that T ignition minus T u is approximately 3 by 4 of T F minus T u. If you look at what I am doing this is my T ignition this x direction this is your T u and T F. What I am saying T ignition minus T u right this portion is nothing but three-fourth of T F minus T u right and that is fair enough to say that, but this is ad-hock.

There is nothing sacrosanct about it, but we are looking at flame temperature profile from experiment in approximated in doing that. This may not be valid for you know limit mixtures it may be valid for the, what we called stoichiometric where the reaction will be govern by the activation energy. So, therefore, there will be valid whereas, for limit mixture it cannot whole good nor even for hydrogen air where the activation energy is very very small you know it would not be valid. So, this is the limitation of this kind of analysis.

So, then still we are having a problem with this m dot triple dash F d T by d F, but we can assume that mean fuel burning rate per unit volume will be you know m dot F triple dash. That means, this term I am saying that is nothing but m dot F triple dash average. Average over the region what will happening because its changing you know this much burning is changing in the region from here to here. If you look at this zone it is changing it is like you know coming over here this is your m dot F it is changing reaction rate in other words right.

But here we are saying no, no we are not bother about integration will say it average what is that average that will be approximated as the what we called a reaction rate Arrhenius from of reaction rate. What is that molecular weight of the fuel A f C f n 1 C oxygen n two e to the power E by R u t. And if I know this values A f e and then concentration of that which will come from the where what is the fuel air ratio I can estimate these things, provided I know this temperature right and that temperature will be doing some average one kind of things right

So, expression for burning velocity that means, that is equation sixteen becomes what like if you look at what I will do I am taking this K g by rho C p inside. So, therefore, it will be square inside of square route 0.5 is nothing but square route and then I am taking also 4 and 3.

So, 4, 4 it is 16, 16 route 2 is 32 and 3 square is 9, I have taken inside 9 and K g is already there and T F minus T u whole square because this term I am taking inside, inside the square route so it will be square. But we can you know like look at this delta H c and whether we can express in terms of temperature that can be approximated as delta H c is apportion nu plus 1 C p T F minus T. So, what will be doing instead of this I will be writing down C p nu plus 1 T F minus T u in place of this?

So, what will happen then this T F will be cancel it of the only one T F will be there and this portion right this what is would be this will be m dot triple dash F average this equation yes or no. So, I can write down and if you look at what is this 1 K g by rho C p that is nothing but alpha thermal diffusivity and if you look at K g and there is a C p.

So, if I look at what is alpha, alpha is nothing but K g by rho u C p. So, C p by K g will be you know if you look at this is alpha square alpha I can write down basically this K g will be alpha rho u C p. So, in place of these I can write down as alpha rho u C p this C p will cancel it later on that this rho because this is rho basically so this will be canceling it out and rho u will be there.

This is nothing but you’re what alpha square right alpha. So therefore, I will get S L is equal to 32 divided by 9 rho u nu plus 1 m dot F triple m this is very aligned you know expression, but if you look at how many approximations. We have done and what is the sanctity of those approximations one us to worry about, but however, we are getting a close form solution right and that is we can see. So, from these what we can see that burning velocity will be dependent on the nu. That means, fuel air ratio right nu is the stoichiometric ration in our equation it will be dependent on thermal diffusivity right; that means, properties K g rho u and C p.

It will be dependent on also on the temperature that is the reaction and density is there means it will be dependent on pressure as well. So, we will see how it is varying and right so that means, S L if you look at S L is a function of what unburned temperature. It will be equivalence ratio it will be pressure and what else also the, if it is the additive is there I can say inert gas you know will be seen that.

What else it can be type of you know fuel air mixtures because this m dot triple dash will be different for the different fuel air like hydrogen oxygen, hydrogen air, methane air, methane you know oxygen or propane air it will be different. So, it will be dependent on also the type of you know fuel oxidizer system. So, these are the things will see that.

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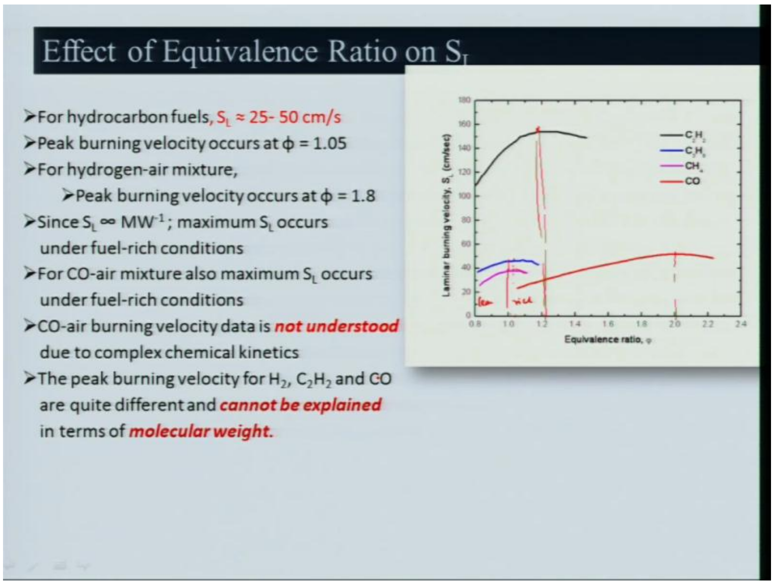

Effect of Equivalence Ratio on S_L

- For hydrocarbon fuels, $S_L \approx 25\text{-}50$ cm/s
- Peak burning velocity occurs at $\phi = 1.05$
- For hydrogen-air mixture,
  - Peak burning velocity occurs at $\phi = 1.8$
- Since $S_L \propto MW^{-1}$; maximum S_L occurs under fuel-rich conditions
- For CO-air mixture also maximum S_L occurs under fuel-rich conditions
- CO-air burning velocity data is ***not understood*** due to complex chemical kinetics
- The peak burning velocity for H_2, C_2H_2 and CO are quite different and ***cannot be explained*** in terms of ***molecular weight.***

Then let us look at effect of equivalence ratio; that means, these are the all if you look at this is the C H 4 four air and laminar velocity in the vertical direction in the horizontal direction equivalence ratio is being plotted, equivalence is fuel by air divided by fuel air stoichiometric. If you look at this will be methane air you know is having this line and it is decreases on the lean side because this is your stoichiometric. So, this is lean right this is the rich and of course, if you are if you look at propane air right then it will be little higher values, but it will be similar in nature.

So, if you look at for hydrocarbon air you know 25 to 50 is the peak value because this is around 37 kind of thing peak value. This is around you know 40 42 even some mixtures will be there 50 kind of, but however, acetylene you know this C 2 S 2 is acetylene is also an hydrocarbon, but it is exception. Where it is having a very high velocity high burning velocity, but its peak value does not occur at the stoichiometric. This occur not realistic stoichiometric, but little bit around that 1.05 around that it occurs which were for general item methane, propane, butane, pentane like that, but for except acetylene here. Where the peak value here will be occurring sorry will be occurring in somewhere this side.

And as I told you for hydrocarbon air it will peak a burning velocity occurred at 0.015 and hydrogen air mixture which I have not shown in this diagram it is very high values, peak burning velocity is occurred at phi is equal to 1.8. And similarly for of course, for the C O air you can see phi peak value occurring around two kind of thing. And this is another very important thing you should know that S L is proportional to the molecular involved. That means, molecular weight you know what we called increases the burning velocity will decrease.

And that is the reason why maximum you know S L burning occurs and fuel-rich condition that is one reason what people have talked about. As I told you C O air mixture the maximum occurs on the fuel-rich condition and C O air burning velocity data is not really understood due to the complex kinetics still it is remaining. You know why it is so what is the reason how it can be why it is occurring this way right. And peak burning velocities for hydrogen C 2 S 2 and C O air you know occurs as the reside cannot be explained in terms of molecular weight. So, still yet to be you know explode and found out why it is occurring this.

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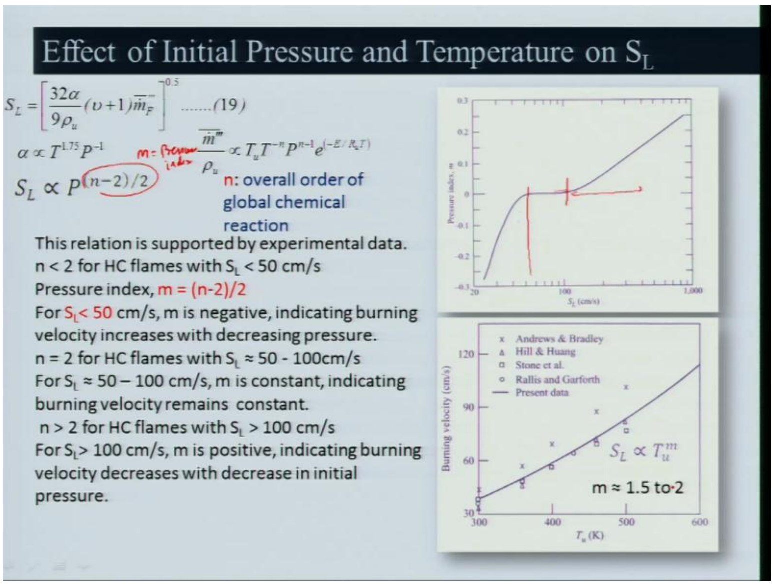

Effect of Initial Pressure and Temperature on S_L

$$S_L = \left[\frac{32\alpha}{9\rho_u}(\upsilon+1)\overline{\dot{m}_F'''}\right]^{0.5} \quad ....(19)$$

$\alpha \propto T^{1.75} P^{-1}$

m = Pressure index

$$\frac{\overline{\dot{m}'''}}{\rho_u} \propto T_u T^{-n} P^{n-1} e^{(-E/R_u T)}$$

$$S_L \propto P^{\boxed{(n-2)/2}}$$

n: overall order of global chemical reaction

This relation is supported by experimental data.
n < 2 for HC flames with S_L < 50 cm/s
Pressure index, m = (n-2)/2
For S_L< 50 cm/s, m is negative, indicating burning velocity increases with decreasing pressure.
n = 2 for HC flames with $S_L \approx$ 50 - 100cm/s
For $S_L \approx$ 50 – 100 cm/s, m is constant, indicating burning velocity remains constant.
n > 2 for HC flames with S_L > 100 cm/s
For S_L> 100 cm/s, m is positive, indicating burning velocity decreases with decrease in initial pressure.

So, let us look at effect of initial pressure and temperature on the burning you know what we called burning velocity. So, if you look at S L, S L is basically we have derived that 32 by 9 alpha by rho u nu plus 1 into m dot F triple dash average. What it is saying that is if I look at this alpha, alpha is basically proportional to temperature power to the 1.75

and inversely proportional to pressure. That means, if pressure increases alpha will decrease you know that way.

So, and whereas, m dot F triple dash rho u it will be P power to the n minus 1 because the P n comes from the what you call the concentration and minus 1 from the rho u and of course, temperature dependence is there. Now, if I look at this S L, S L is the what we called will be dependent on this n values n is the overall order of global chemical reaction, n is the overall order of reaction you know like kind of things.

So, then S L is proportional to P power to the minus n minus 2 divide by 2 right because why it is coming 2 is a square route therefore, divided by 2. So, this relationship is supported by the experimental data, these n values will be varying for reaction like hydrocarbons and depending upon fuel air, mixture ratio and other thing. So, let us look at how it is being supported and these value this is what we called is basically m, this is known as pressure index, pressure index. That is m being plotted pressure index m is plotted here with respect to S L.

When the burning velocity is in certain range you know in this range if you look at this range what you are getting, you are getting that pressure index is negative. That means, it will occur when m is less than 2 then m will be negative. That means, it will be not second order reaction, but rather it will be may be 1.0 you know 5, 1.4 or something like that n, n is 2 means what second order reaction right, n is less than 2 means it is less than second order reaction. Keep in mind that it is not eliminator reaction this is a global reaction and in eliminator reaction you really cannot get this fractional values that is also important.

So, then what you are getting for hydrocarbon flame n is there and which will be occurring less than 50 kind of things you know 20, 30, 40, 50 this is 50. What you see the m is negative what is the meaning of that; that means, when pressure is what we called decreases what will happened the burning velocity will increase. Why because this is a negative value right if pressure is decreases then S L will increase or if pressure will increase then S L will be decreasing so this is the term where.

And whereas, n is equal to 2 then what happen to m, m is almost remaining constant if it is constant you know right it is 0. That means, the burning velocity is independent of what pressure burning velocity will not be changing with respect to the pressure and which will be occurring in this region. Where, m is equal to 0 and that is occur only when n second order reaction that means, reaction is second order.

But however, in the real situation it would not be in the sense I cannot say it would not be, but in certain range it may occur, but in certain range it will be different right. So, when n is greater than 2 for hydrocarbon flame particularly when S L is greater than 100 in this region, then what happens that m is positive. So, therefore, the burning velocity decreases with decreasing in initial pressure. So, what you can draw a conclusion, you can draw a conclusion that the burning velocity will be varying with respect to pressure depending on the order of the reaction or the depending on the system what we are talking about right.

So, let us look at how it is varying if you look at this variations and other thing you will find that temperature is basically you know affecting the burning velocity. With increase in temperature the burning velocity is increases which I have shown here for methane air flame. And the m values you know will be varying from 1.5 to 2 kind of thing that is the relationship you can get. Generally, it increases with respect to the inlet temperature.

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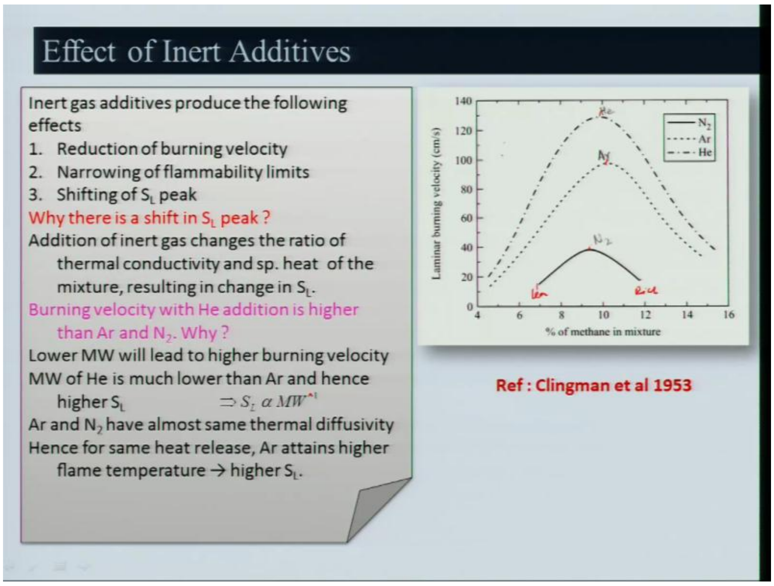

Effect of Inert Additives

Inert gas additives produce the following effects

1. Reduction of burning velocity
2. Narrowing of flammability limits
3. Shifting of S_L peak

Why there is a shift in S_L peak ?

Addition of inert gas changes the ratio of thermal conductivity and sp. heat of the mixture, resulting in change in S_L.

Burning velocity with He addition is higher than Ar and N_2. Why ?

Lower MW will lead to higher burning velocity

MW of He is much lower than Ar and hence higher S_L $\Rightarrow S_L \alpha MW^{-1}$

Ar and N_2 have almost same thermal diffusivity

Hence for same heat release, Ar attains higher flame temperature → higher S_L.

Ref : Clingman et al 1953

So, let us look at effect of inert additives like if I look at this the 3 additives one kind think of for a methane air flame and laminar burning velocities plotted with respect to the percentage of methane in the mixture. You can see that when it is nitrogen the burning velocity is very very low around 30 peak value is around 36 or 37 meter per second, but when you will go for grouping down both the lean side and the reside. This will be lean side you know lean and this is the rich mixture kind of thing.

And similar observation you can get that burning velocity is changing with the percentage of methane in mixtures for the argon and helium, but as your you know sub substituting this nitrogen with argon and helium the burning velocity is increasing. But the peak values also is not in the same place it is shifting toward the right that means, more fuel. So, if you look at this observation there is a reduction in burning velocity when you are coming from helium to argon to nitrogen and narrowing this flammability limits.

Of course, this is the mixture where between which you know burning velocity will be there you can have combustion beyond which there would not be. So, it is narrowing as you are coming from helium to the nitrogen and shifting of the peak you know burning velocity I have already discussed, but why this is there because addition of inert gas changes the ratio of thermal conductivity to specific heat.

You know like it is burning velocity is coming alpha the thermal diffusivity right. So, this is a similar thing is occurring resulting in the changing the burning velocity. And burning velocity with helium addition is higher than the argon and nitrogen, question arises why because there what will be happening the molecular weight will be you know changing because these are all heavy. Nitrogen heavier than the argon, argon is heavier than helium you know so with this you know molecular weight being what we call reduce therefore, burning velocity is increasing.

Like burning velocity proportional to the M W minus 1 if you look at that is coming from your reaction rate, you remember that the reaction rate m dot triple dash F average M W there is route over. So therefore, it is coming from that and argon, nitrogen have almost same thermal diffusion and hence the same it will argon at higher flame temperature than the higher S L. So, therefore, you will getting that. And with this will stop over and will continue discussion in the next lecture.