Introduction to Helicopter Aerodynamics and Dynamics

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Lecture No. # 07

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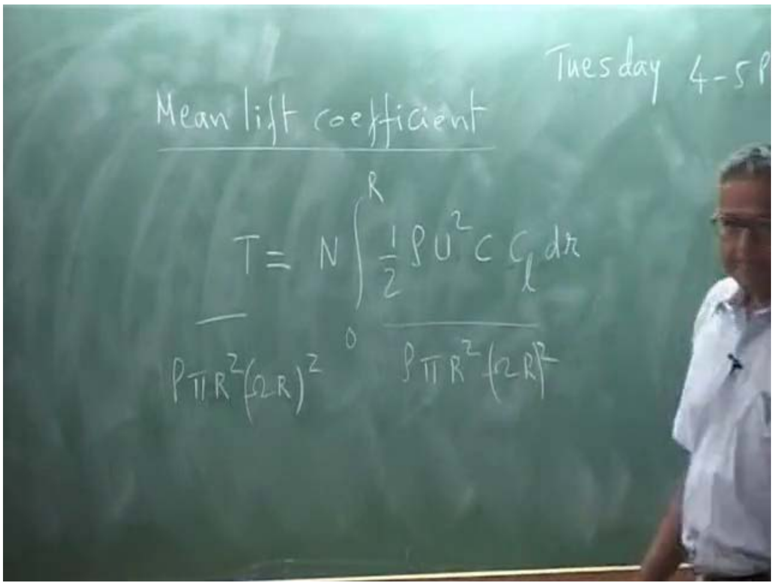

Mean lift coefficient (No audio from 00:26 to 00:32) is just a definition. Mean lift coefficient, because you know thrust which is given by number of blades, lets us say 0 to the whole thing, dynamic pressure half rho U square or U T square whatever you take it. Then, you will have a chord, and you will have lift coefficient, I am not writing it in terms of C l alpha, and angle of attack, I am just giving as a lift coefficient then d r. These are very, very simplistic definition, you non-dimensionalize with rho pi R square omega R whole square, we divide both sides..

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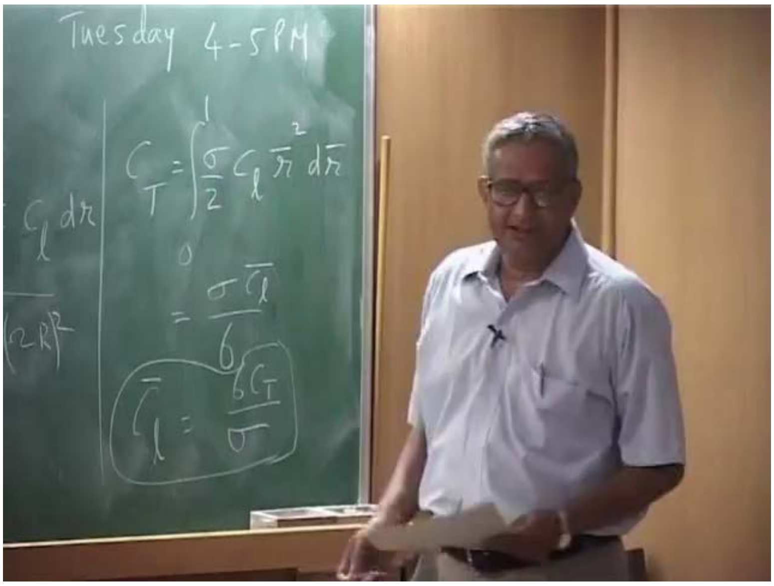

Then you will get, it will be written in terms of C T, because T divided by rho pi R square omega R whole square is thrust coefficient, and here this will become U is basically omega R whole square. So, R over R that will become R bar square, and then you will have C l. And you will get essentially, this is equal to 0 to 1, and this half factor is there, N C pi R that will become sigma sigma over 2 C l r bar square dr bar, that is what, the right side non-dimensionalization. There is, this you must know by now, immediately write it.

This assuming C l is, you take some mean value, some it is a constant everywhere and sigma, any way you are defining with respect to full dimension and then you take out everything, this will become sigma C l over 6. This is the some kind of a mean value because I am taking it although it as a constant value. This will give me C l, you may write it as mean lift coefficient or if you want to put a bar, you can put a bar. So, that C l bar is 6 C T over sigma, mean left coefficient of the Rotor. And you know that, C T over sigma, that is the blade loading and this is given six times that mean lift coefficient. Now, why this is defined is, you will go back and then defined our, if I erase this portion because I use only this section.

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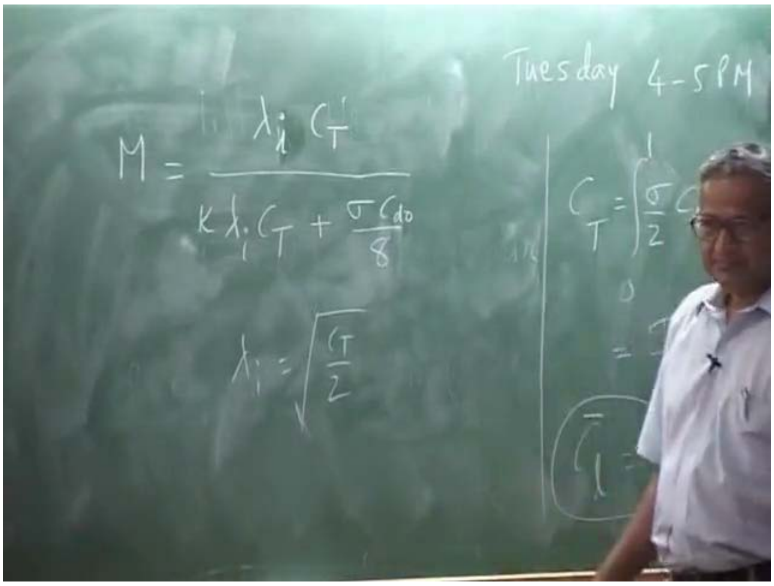

Our figure of merit figure of merit is defined because last class we saw, that is ideal power, ideal power is lambda hover C T because lambda H is or lambda hover or I use lambda i. So, you can take it as lambda i, this is for hover only and then you have a lambda i C T plus sigma over 8 and lambda i for hover is root of C T over 2. Basically, what we are going to replace is, this C T in terms of C l, bar that is the some kind of a mean lift coefficient, mean lift coefficients for a section.

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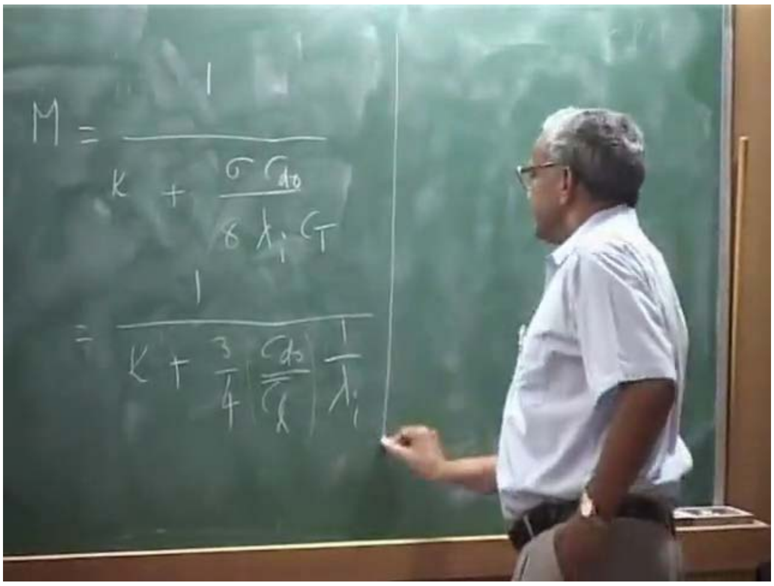

Now, if you take out that lambda i C T divide everywhere and your expression will become let me write it, this is 1 K plus sigma C d naught over 8 lambda i C T. C T, you get it from here, this is sigma C l bar by 6, you put it here, you will get your figure of merit as 1 K plus 3 over 4 C d naught over C l bar and 1 over lambda i. See, these are all modified expressions, here, sigma it does not appear, that is the solidity does not appear. Earlier we said that, if you want to increase the figure of merit, you say you reduce sigma, but if you reduce sigma, your mean angle of attack will go up C T by sigma.

Now, in this form you have C d naught which is the sectional profile drag, C l bar is sectional lift coefficient; you can take it mean value which is the sectional lift. So, what this term, this form tells you is, if a Rotor is hovering you say because the figure of merit, I can improve, if this quantity is basically small, small means it should operate at C l by C d naught at a higher value. C l over C d must be high; if the Rotor operates at C l over C d high value then your figure of merit is also high. Now, based on this type, there are definitions, two definitions which I will briefly tell you the only the definition, that is called the Ideal Rotor.

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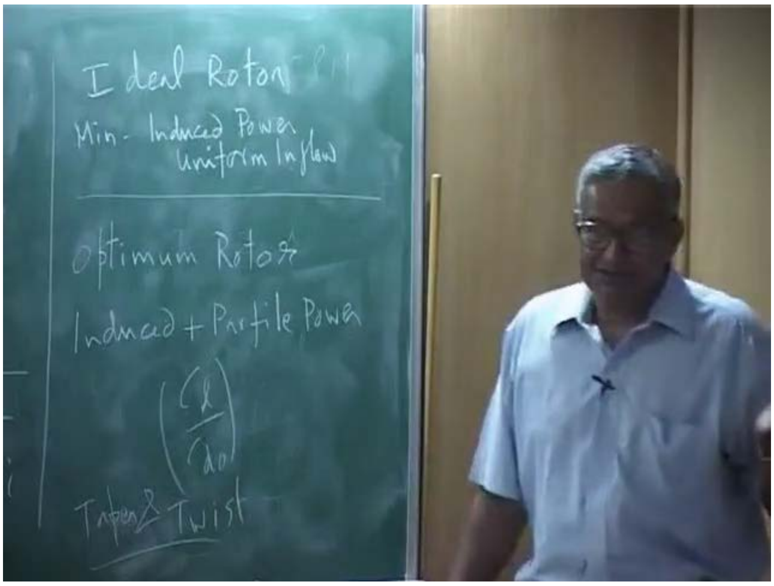

We will not be deriving this, this is just for your Ideal Rotor and the optimum Rotor (No audio from 08:21 to 08:29) these are two definitions essentially. Ideal Rotor is, you have minimum induced power. So, minimum induced power that means, what you will have only when you have constant inflow that is, this implies you have uniform inflow over the disc. So, you may put uniform instead of say constant, I will say uniform inflow and this will give you ideal twist.

Now, you see, this is the ideal case. So, if you want an ideal rotor, you should have uniform inflow, for uniform inflow you have to have a ideal twist, but real helicopters, you know, you are not they are not ideal, because you have profile drag also. You do not have only induced power; you have both induced and profile. So, both are there. So, you have both induced plus profile power.

You have both of them are there and optimum is one which should bring down both that means, one requires nice twist in the sense, the ideal twist, but profile power if you want that means, you want the power to be minimum, but generate good lift that means, C l over C d for a section sectional lift coefficient by C d naught, this must be large, if you have this quantity more than yes because I am reducing my C d naught, that is the basically, profile drag is also coming down, my C l is also high which implies and now, this is aerofoil that is, what is the angle of attack because everything mark number all those things will come into picture in the real rotors.

Now, usually you take that taper and twist. So, you adjust the taper and twist because ideal twist gives uniform inflow, you try to adjust the taper, these two parameters such that you get a optimum rotor. Optimum rotor means, both must be minimum. This you can do it as an exercise or something like that, but right now, I am only telling you that real helicopters. This is only a theoretical part. In real life, you need to deal with this and whatever we have derived, we never bothered about mark number effects etcetera, but you need to have mark number, tip losses many things will come into picture.

So, when they want to go for actual design, they have to take every factor into account that is why, this is just given as a ideal rotor means, I want to minimize that means, ideal twist you give, but that does not mean everywhere, it is operating at a very C l by C d naught is the best, that may require a different condition. So, this is just for your information, that there is some definitions called ideal and optimum rotors. These are all for hovering condition please understand, till now, we have studied only hover condition.

We studied uniform inflow, non uniform inflow please understand, non uniform inflow, whatever expression which we got, that is very good, it is very powerful and it is being used even today. For research, I am not talking about just for (( )) calculations, even in the research, that expression is very good without, that is basically relating blade element, momentum theory in a differential element that solve then we define the power, figure of merit, etcetera.

Now, this is all for hover of course, if you want to calculate the losses, the tip loss etcetera etcetera. Now, I have given you a assignment in which I gave you various four cases of blade, one- no twist; another one, minus ten degree linear twist; another one, minus twenty degree then ideal twist. For this, if you use whatever we have derived, you see, I have five plots when you get all those plots you will know that, what effect the twist really changes. Uniform inflow, we want ideal twist, you will see slowly over, this is what is happening in the Rotor situation, what is the pitch angel at various sections.

So, I want that as a graph because that will tell you because this is actually you will, you have to write a small computer code because you cannot do by hand calculations every time. Write a small code, generate the results, plot them for a given rotary system. Now, we will go to the next flight condition because we will divide as a part of the whole course will go like this, we did the hovering. Now, we say climb and descent in only vertical direction. Once we finish this then we go to forward flight because the forward flight is most complex problem that is why, first we would do hover then you do climb and descent.

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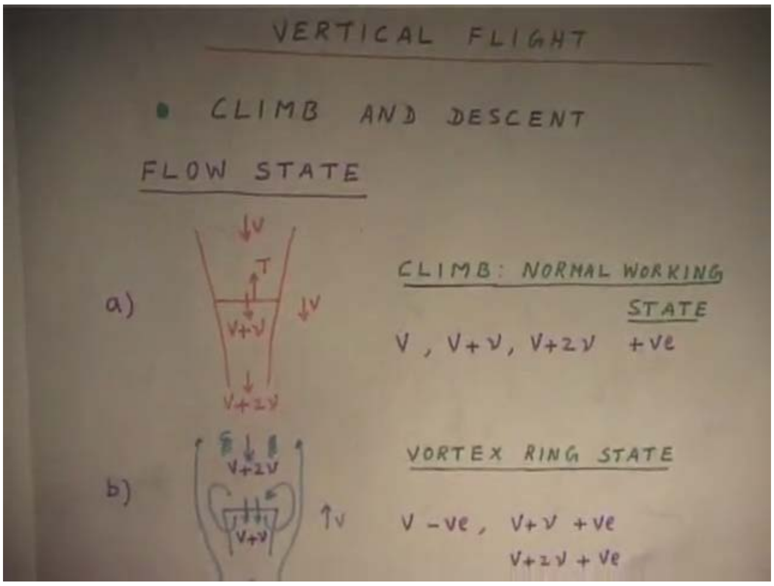

Today, I will introduce the basics of climb and descent whatever derivation we can give, but there is a lot of discussion on the climb and descent itself. Then, we say vertical flight either it can go up or it can come down, that solve. You say the helicopter is hovering, you are climbing. Now, this particular diagram, there are four diagrams, I have shown. Each corresponds to a particular flight (( )). The reason, this is split into that is for ease of understanding and finally, we will derive some expressions which I will be deriving and we will use them.

Now, the climb which is the also called normal working state, because we use the same definition what we used, that is this is the Rotor disk. Rotor disk is supporting the weight of the helicopter and the climb and descent are please note uniform speed that means, it is not accelerating or decelerating nothing, it is going with steady speed. So, that is why the condition here, I have plot is steady speed, if you are sitting on the Rotor what you will feel is, the far field upstream, you will see the wind is coming with a velocity V and everywhere because they you are moving.

So, the wind is going down, this is outside the slip string, velocity is also down, but the rotor is doing some work and that is basically increasing your, you will derive for this case also. At the rotor disk now, it is V plus nu, that nu is the induced velocity, V is the velocity due to the motion of the disk or the rotor and then far field downstream. I put V plus 2 nu, but then we can we will prove that, that is actually 2 nu because V plus w you will write it is like. So, this is. So, here it is.

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So, you have a… here it is V plus w, this is V and here it is, but for this condition, but I made the assumption please understand that that nu, this is the induced velocity is constant and it is uniform over the rotor disk, it is uniform everywhere and this is the assumption which we make and then w is also far field downstream, it is like this. Now, if you look at this diagram, this diagram tells the velocity of the flow inside the slip stream everywhere it is in the same direction, everywhere it is going downwards.

So, this is the first part when you are climbing, but only thing is the value of nu that may vary, because that depend, this is not the value at hover, the induced velocity will vary depending on flight condition, hover it will be root of C T over 2 lambda i, that is the hover condition, but once you are climbing the inflow nu over omega R, that is different value for that, you will get an expression now, after that.

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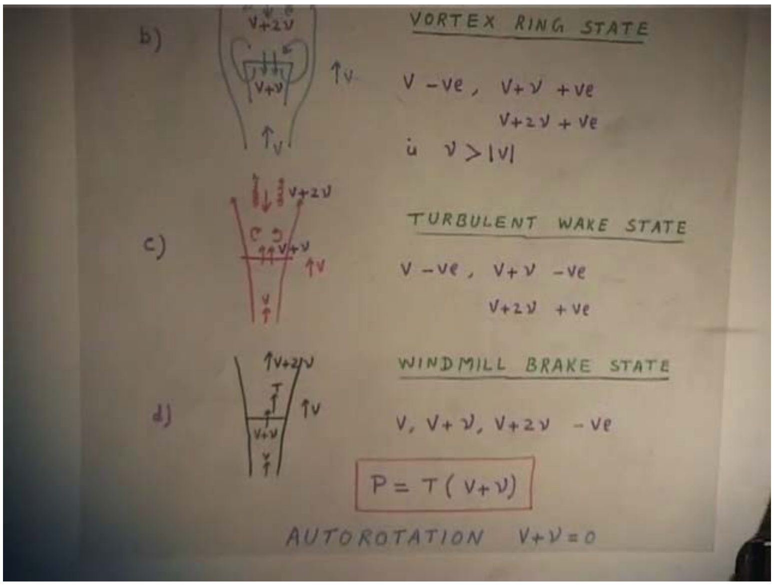

So, this is this particular condition of the rotor is defined as normal working state, here; why normal mean flow is everywhere same inside the slip stream and the outside also it is in the same direction. Now, we will go to descent condition, but descent is split into three regions depending on the kind of flow, what we say is, first it is hovering, it as just started descending it as just started descending that means, what I am reverse everything, the flow V from there, if you are sitting on the rotor disk far field down actually, down means that is upstream actually, velocity is coming towards you, because you are moving down.

So, it is equivalent to velocity coming, but then at the rotor disk because the rotor is still supporting the weight of the helicopter, which is equal to the T for it to support, it has to push the air down, that is how the rotor is doing the work in pushing the air down, that is how it is staying. So, at the rotor disk, the rotor is pushing the air down, but from the far field, the flow is coming up you understand; that means, this flow here and these are coming up, but if you look near, you have just started descending; that means, the velocity V capital V is still, if you say small because you have a inflow.

If you are hovering, you will get a value of inflow hovering, you started descending and your decent velocity is less than the inflow that mean, inflow is large, this is less though I put the symbol V plus nu because V is actually opposite, nu is down, this is up, but nu is large. So, V plus nu is more I have put this direction down, please understand I have put the direction down and then again I simply blindly use the momentum theory that far field downstream is two times what happens at the rotor disk. I will put this also, V plus 2 nu, I just put it please understand whether it is right, wrong, that is a later part.

Now, you see what happens is the flow is coming, this is pushing, this is called the… you will find one flow is coming, something is going down, it will go around the, this is the rotor disk you will find the big vortices will form. And then, they will once the strain becomes the vortex become big then it will detach and it will go up, again another one will form that is why, these things will; that means, the flow inside the slip stream what you have, it is not in all in one direction, it is going to be a mixed flow. This is the condition when you are descending down into your own vehicle this, because you are pushing there and you are going into that, in the sense the flow is coming up decent velocity.

This will create lot of vortex around the rotor, we called vortex ring state, but you see V is negative, negative in the sense because it is coming up, but V plus nu is positive down, V plus 2 nu is also positive which is down whereas here, all the quantities are positive, everything is down. Here one is up, other towards down. Now, you see in this state, the helicopter will have lot of vibration because the vortex will come and detach. You will sit in a helicopter when the pilot is actually coming down, comes vertically down, this shown as vertically down even sometimes when he comes at some angle. You will find all thing, that lot of vibration will be there.

Now, this state the flow is still downwards at the rotor disk. Now, let us increase our descent velocity a little more, when you increase the descent velocity more, V is negative, but V plus nu; that means, at the rotor disk, my inflow is down, but the descent is more, the net value is still positive up; that means, my nu is less than V. So, at the rotor disk, I am now having the flow up, but far field far field is still, because that is two times nu, I am always taking it as an assumption that will be down.

So, at the rotor disk I have reached, but you see now, between this state and this state V plus nu positive; that means, the flow at the rotor disk is down, here, V plus nu negative flow at the rotor disk is up; that means, there is a condition at which there is no flow at the disk, there is no flow.

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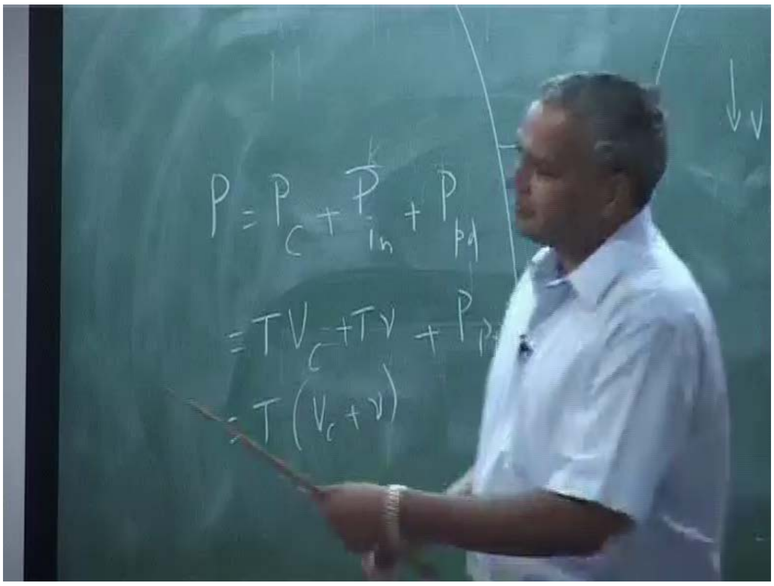

Now, we had seen earlier when we define the power, we wrote three terms power, P climb power climb, power induced and then power profile drag, this is thrust into velocity climb, this is thrust inflow plus of course, this you leave it you P P D that let it keep it as it is, this is T into V climb plus nu.

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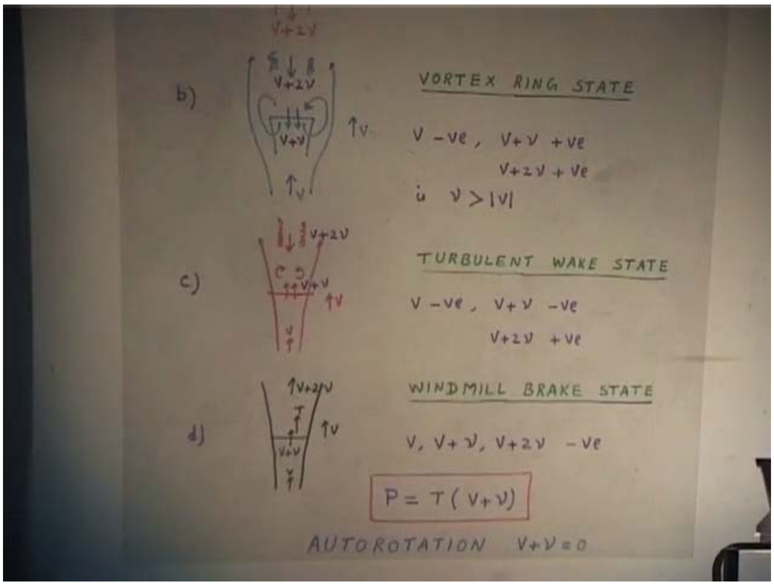

And I am not used the subscript here, V is the velocity. Now, when we climb plus nu or V plus nu that velocity term when that is 0; that means, my power is 0, but I am still supporting the weight please understand, power means the induced power. I do not supply any power, I do not require power, but I will able to support my helicopter that is between these two states that is the boundary line, because V plus nu positive, you come here, V plus nu negative.

And in this state you see, the flow here is up, here also is up, but this flow is down, you will have circulation, but you will in the wake, wake means above the rotor and that is called turbulent wake state that particular condition and vibration will not be as high as this state. Here, you will have lot of vibration, low frequency vibration you will see, whole thing shaking, if you sit inside then you will know in the helicopter then you will see this is shaking here, it may not be as sever, but still the flow is turbulent in the wake.

Now, you extend you descent still faster and that is the flow here here here, everywhere is in the same direction V, V plus nu, V plus 2 nu. Now, V plus nu is up, thrust is down; that means, actually you are generating power from the flow to the rotor, earlier you supply power to the rotor to support the weight and the power you supply to the rotor to support the weight slowly will decreases with descent velocity and at a particular condition where V plus nu 0, you do not need to supply any power, but you will support the weight please understand.

Now, you descent faster, you will start generating power and that is the windmill condition. That is why, the windmills what you have because there the wind is flowing, it is generating power, you need to generate power from the wind you do not rotate the rotor then you are supplying power actually the wind itself supplies the power to rotate it and then that rotation is taken for electricity generation. Now, you see, this is called WINDMILL BREAK STATE. Here, the flow down at the rotor disk far field everywhere in the same direction. This is, you see similar to the client only thing is everywhere the flow is same.

Now, these are basically the states for any rotor operating in climb or descent, this is just for very simplistic thing. Now, I have written here, just for the sake of understanding, this is the induced unclimbed, the power required for climbing and induced, when that power is 0, that is called auto rotation, but actually you will find in real situations auto rotation it is a little different, that we will learn that, because this you have to know, that means, I do not require power to support my weight, but that does not mean that I am hovering please understand, I am coming down.

So, the loss of potential energy is actually converted into the kinetic energy spinning. Suppose, if you more is converted it will start spinning faster. You do not want that situation that is why, this condition is very, very important in helicopters because it is called auto rotation. In case of engine failure because you are rotating, engine fails because of it is inertia what will happen is it will start, it will still rotate, but then the drag force is acting, you are not supplying any power then what will happen, if you do not do anything, if the descent is not there then rotor will come to a stop.

But what you do is, you immediately start descending, when you start decent then the rotor spin, but you do not want the r p m to drop 0 and then pickup like you remember, I drop the seed those coming down very fast, once it picked up, it was going very slowly right. Now, this particular condition of auto rotation is very important in helicopters because this is one of the design requirement and they also go through some training. In case of engine failure, how do you control the vehicle, you design, but you come, but you will come with a velocity V down that is not a very small number, but it is reasonably, but when he comes near, he will be given certain instructions how to really save the vehicle.

They do that, but it not that they come and do every day, they will do the auto rotation only at some altitude just for a training, but you can save the complete vehicle when they come near the ground, there is something called a flat, they will again go up that instructions once you, once I derive something more then you will understand, this is what, it has to do, but it is a very, very interesting phenomena and we will show that, a rotor in auto rotation acts like a parachute essentially. It is like a parachute.

It is almost as efficient as a parachute of the same time only thing is parachutes are really big, but rotors are not that big, but it is good and you can save the vehicle in case of engine, it is not that, you will lose the (( )). Only thing is, there are restrictions, how fast you will learn, where you will learn, how far you go, there are many several questions related to auto rotation, but we will not get in to all the details, but we will see this. But the key part is, we need to know, how to calculate inflow right, because we said, I shown all this 4 flow states, but I have to get the inflow through the rotor disk, an expression as a function of my velocity of climb or velocity of descent.

So, we first assume, but please note now, what we derive today may be next class. In this state momentum theory is valid because everywhere the flow is same direction. This state momentum theory is valid, you can get the inflow, but these places, there is no theory please understand. You do not have any theory to get, what is my inflow at that time in vortex ring state or in turbulent wake state, what will be my inflow. Autorotation is actually the boundary between these two, this is this is called the windmill break state.

Usually, the windmills operate like this, because a windmill do not go anywhere, whether the wind is blowing that solve, it rotates right, because they are stationary, but wind blows from somewhere at the rotor disk and then far field back. You understand they generate power because windmills are kept for power generation. Now, that particular flow condition corresponds to this type, is it clear, this are you, what you are saying is, am I not generating power here, yes you are.

But this is in the helicopter you operate, if you come with the descent velocity, you normally helicopters, I will tell you, the moment you come to this condition, V plus nu 0 that mean, the power required to rotate the rotor is 0. You do not need engine power of course, if there is a profile drag you descent with that that we leave it, assuming take it as a ideal condition, you do not need engine power.

Suppose, you start descending faster than that condition what you are doing is, you are generating power when you start generating; that means, the rotor will start going faster, but your blade there is a limit of in the design. You will have centrifugal force, what acts at a particular R P M, I told you right at the beginning in helicopters your rotor omega is a constant. Suppose, you keep increasing your omega what will happen, your stress will go tremendously because for most of the blades for the operating omega, the root stress or the root the centrifugal force is of the order of hundred thousand Newton.

Now, you double it, it will become four hundred thousand, that solve the whole blade will start flying all over the place you understand. So, you do not try to design faster, the pilot is instructed to come only to that, he is not going to generate power in the helicopter, whereas windmill is a different operation. This is just to indicate that the kind of flow patterns what exists, if the Rotor is descending like this, he will never come to this state and this state.

Helicopters will not try to operate in this way, but you will find the momentum theory or whatever theory which we are going to is only valid here and here, they are not valid actually there is no theory. So, momentum theory is not valid, but we need to have some theory otherwise, how will you calculate. So, that we will derive then there is an approximation made mostly in test and then they will draw the diagram and say they take it as some straight line something like that and I will say, use this for autorotation that solve.

If somebody says, what is the inflow you will just say, take this value that solve. You cannot calculate the inflow in autorotation. You have to have an approximate theory even some publications are there, experimental test, try to fit a curve through that and then come up because you see, when the flow is like this, what theory you will have, because it is all going one flow is coming, another flow is pushing down, everything is mixed, lot of vortices big big things and flow is highly mixed flow.

Now, why it is shown like that is, what happens, a flow is coming from the bottom right and here at the rotor disc, flow is going down, it is trying to push the air down then what the flow will naturally what will happen, it is like some crowd of people coming towards you, here, you are ten guys you are trying to push them up and you are stronger in pushing them out then, what will happen they will just go around you, precisely what is that, go around you and they will go, that is why, this is put like this, but then again here they are coming. So, the flow will again come inside.

See, in all these things actually the flow is like this, that is why, even if you fan you have, if you take the fan and put it close to the ceiling. You will not get any air even try, I do not know whether you have a fan or not put something very close to the fan above some surface. You will not get any air in the ground because basically, it takes the air takes it up and then pushes it down. That is, how the flow goes and it is idealized by these kind of a now, is is it clear what we will do is, we will first derive the inflow, assuming that that expression is valid for all conditions, please note that I am making an assumption.

And then, we will plot that result on a curve and that is called the generalized or you may call it universal inflow diagram and that diagram is used for most of a practical purposes. So, first we take the a situation of the climb because climb is that is why, you see climb condition is always very good and well behaved and your theory is good and windmill theory is fine, but in between it is not there, but then you still use that is the, I would say that is the beauty, beauty in the sense, because you do not know anything. So, you have to use something, that is what the bottom thing.

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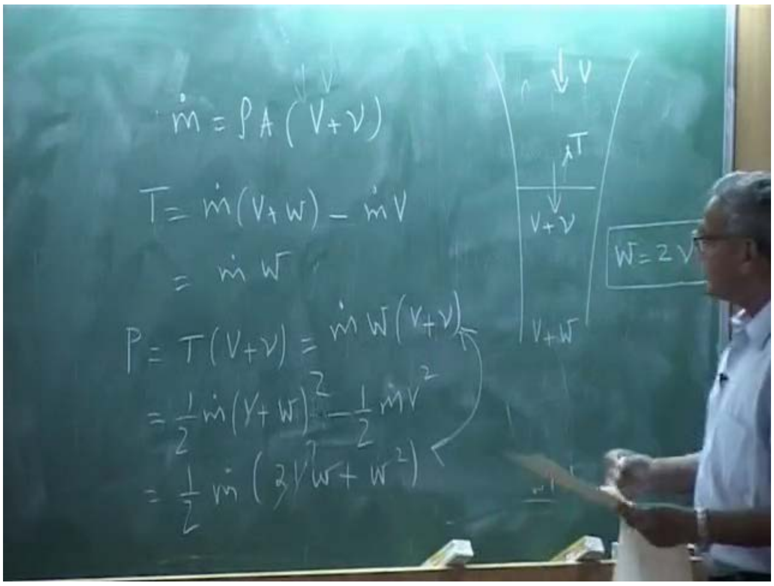

So, now let us take the climb condition, which is identical what what we derived earlier. That is, this flow is coming with V, here it is V plus nu, here V plus w and thrust is T. Mass flow rate we use the same, mass conservation, momentum conservation and energy conservation that solve, only these three basic laws of fluid mechanics. So, m dot mass flow rate is rho A V plus nu because V plus nu. Thrust is change in momentum from initial to final or final minus initial that is, m dot V plus w minus m dot V because this is the final momentum, initial momentum because these are rates. So, that is nothing but the force.

So, this will be m dot w. Now, you take power power is induced power, please understand. This is because, this is the momentum theory, we do not have profile grifile track nothing, induced power P is thrust into V plus nu because thrust is the force acting and V plus nu is the velocity there. This is change in the kinetic energy here, kinetic energy is half m dot V plus w square minus half m dot V square because final kinetic energy minus initial kinetic energy. Now, you substitute, you simplify this, you will have basically what, half m dot 2 V w plus w square.

Now, substitute m dot here, and then you will have, may be I erase here, this is what, m dot is you put this, this is also what, m dot w V plus nu. Now, you see m dot V w, m dot V w they will cancel. m dot w, m dot w will cancel, leaving behind the condition that, w is 2 nu that solve because this, you equate these two, I am substituting thrust is m dot w into V plus nu this is in terms of kinetic energy, this is thrust into V plus nu. Now, when I equate, I get w is 2 nu which is identical result as what we got in the hover and you will find even in descent, it is going to be the same that I will show, but only the thing is the sign will change a little bit there. Now, what is my thrust and the power, thrust is m dot w, m dot is given here. So, I may be I erase this.

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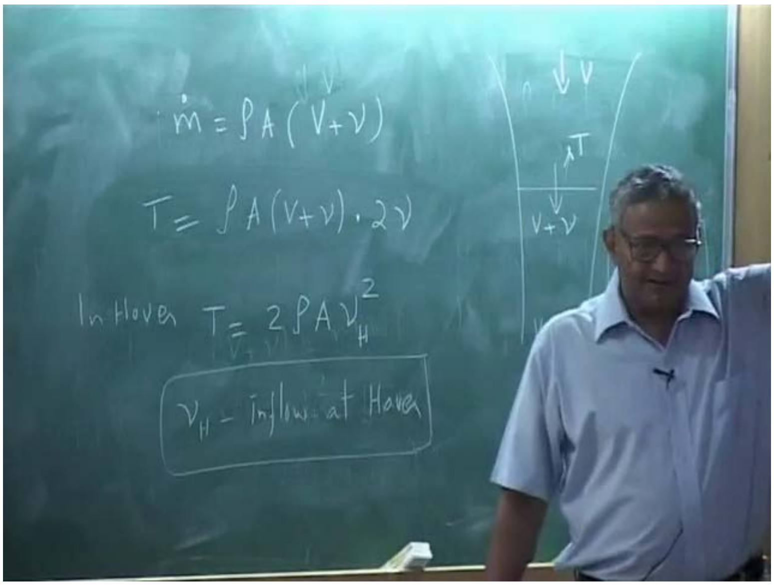

So, my thrust is rho A V plus nu into2 nu that solve. V is the velocity of climb. I take it V positive means, it is up; if it is negative it is down, but I cannot use this expression immediately, I will use later I will derive for the descent separately. Now, you will write this equation in terms of hover inflow, because thrust is equal to the weight of the helicopter. So, in hover T is what, 2 rho A this is that. So, it will be I am going to use a symbol nu H, nu H is hover inflow, inflow at hovering condition. So, you say nu H is inflow at hover because I want to non-dimensionalize the equation.

Now, you see I can equate both because I am supporting the same weight please understand whether I am hovering or I am descending or I am climbing, my rotor supports the same weight therefore, this T is equal to this T. So, if I use that expression rho A will go of, two will go of, you will be left with what, 2 rho A.

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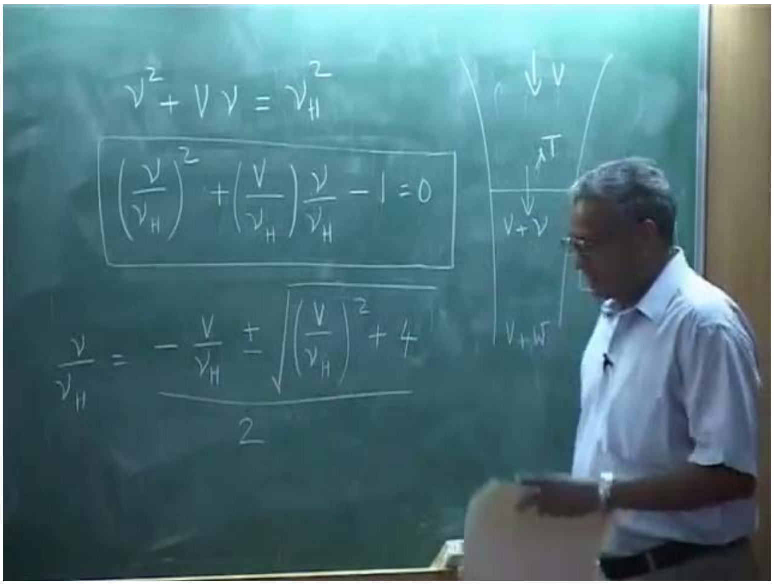

So, I will have an equation, nu square plus (no audio from 48:43 to 48:52) you divide by nu H. You will have this

(No audio from 49:01 to 49:30)

You have a simple quadratic relationship inflow basically non-dimensional with respect to hover inflow. Now, I can write a solution of this equation that is, nu over nu H equals minus I will have plus or minus square root of V square is (no audio from 50:21 to 50:29) this is the value of the, but divided by 2. Now, you would take out simplify this because you take the 4 outside and then this will be V by this 2 2 will cancel out and here, you will put V by 2 H.

So, you write the equation in this fashion, but which route you should use it, because which route you should because this quantity is a positive quantity because V over V is positive, nu H is positive this is actually positive quantity and this quantity is more than this quantity. So, if I use a minus sign all of them were negative, but you know that my induced velocity is positive; that means, I must use the positive root of the expression. I cannot use the negative, because negative, if I put my inflow is going up.

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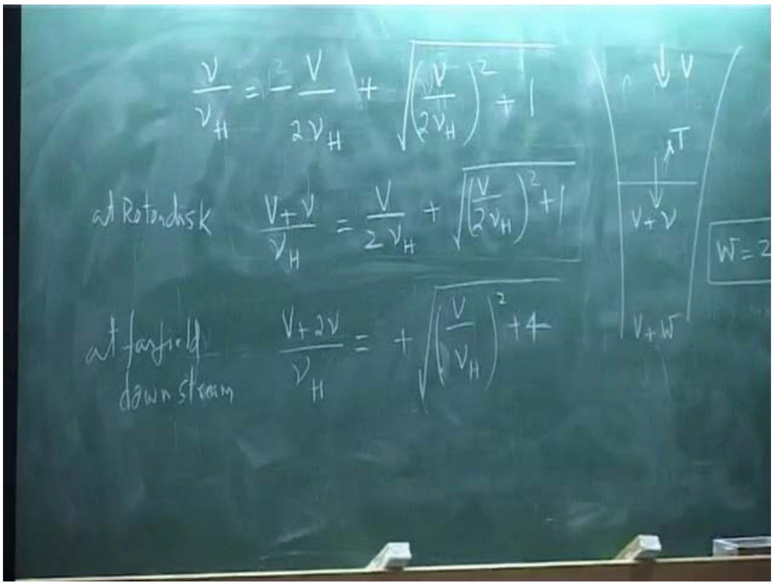

Now, we will write our inflow variation nu over nu H incline is minus V over 2 nu H plus square root of, (no audio from 52:10 to 52:19) this is the inflow. Now, if I want, what is the net flow at the rotor disk, net flow that is V plus nu. So, at rotor disk, my flow is V plus nu over non-dimensional because I am. So, I am adding P plus nu H P over this value. So, I will add the same thing here, this will become now, V over 2 nu H because the minus sign will go up because I am adding plus V by nu H plus root of same thing V over 2, if I want at far field that will be V plus 2 nu.

So, I know nu, I will be multiplying by factor here 2 and then adding V. So, when I multiply that V over nu H will cancel out. So, what will happen is at far field far field downstream. You will have V plus 2 (no audio from 54:11 to 54:20) whole square plus 1 that solve. So, this is these are the three expressions for climb case.

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no 4 that is sorry sorry I am sorry, this will be here, I am sorry that I have to multiply by 2. So, the 2 inside when I take it 2 will go of, it will become 4, V over this 4. Now, you see this gives actually when we plot you can plot V over nu H on the y axis and nu sorry nu over nu H on the y axis and V over nu H on the x axis, when V is 0, you will get actually, value is 1 hovering flow.

As you keep increasing your climb velocity please understand as you keep increasing your climb velocity, you will find this term is more than this and your value will start decreasing decreasing decreasing, when P over 2 nu H is quite large you are assuming, very large means then whether you add 1 or not it really does not matter square root, you will approach asymptotically to low value, 0 value technically very high infinite speed, it will become infinite climb speed it will become…

So, you will find that, the inflow through the rotor disk reduces when you are climbing because basically, you know that the mass flow rate you have increased. You increase the mass flow rate, I have to support the same weight. Since, I have increased my mass flow rate, my velocity can be less that solve induced flow and this same concept will come in forward flight also. So, you will find your inflow, inflow mean nu please understand, that value keeps decreasing as you increase your climb velocity.

Now, let us look at the descent part because the descent part is a little usually little confusing because the flow is, we assume this condition then we will apply everywhere because I am taking because you know that my nu is is this clear because I am erasing this part. So, this expression later we will use it for plotting.

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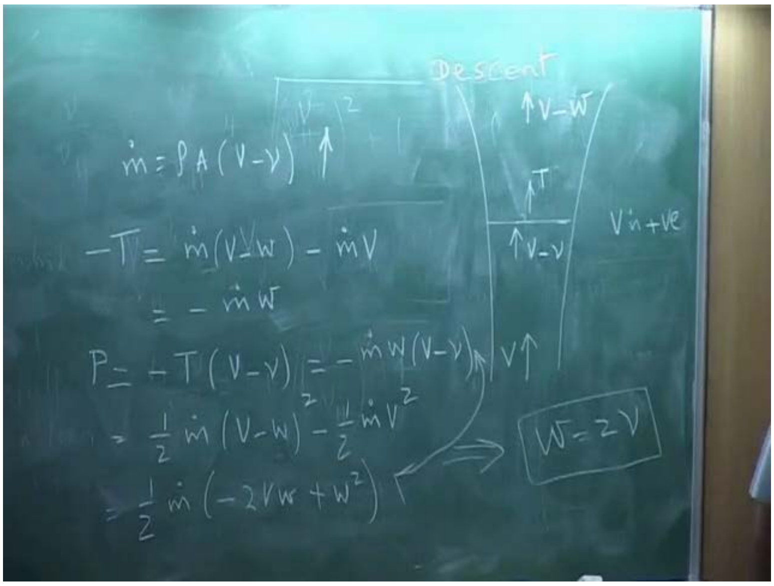

Now, let us take the descent condition. (no audio from 57:47 to 57:57) descent you know that, this rotor is supporting thrust, it is supporting the weight, thrust is always up, weight is down. Now, your flow here is coming V please understand, I have changed the direction and at this point, I am putting because the rotor to support the weight it has to push the air down; that means, my nu is downwards, V is upwards.

So, I am going to write like this, V minus nu because nu is down. And then here, it is again w is down because you are pushing air down; that means, there it is coming I am using please understand, some book see, I do not think this is given in all the books and I find that the way I have written is much clearer in terms of the sign, in terms of the description, but finally, result is same everywhere.

So, you see V is positive, I am taking positive here, later I will convert that, but when I say positive means in this case, the flow is coming up because this is for explaining the diagram I am showing. Later, I will I can change the sign convention, this is just to because otherwise, because there is some slight change in the terms, is this clear because it is coming you are increasing actually this flow is going down. So, the velocity here, decreases velocity here further decreases, but everywhere V is I am plotting it up.

Now, my mass flow rate m dot is rho A V minus nu, but this up please understand mass flow rate is in the upward direction, but what is my thrust, this thrust is acting on the rotor, but I am writing the equation for my fluid. So, my thrust acting on the fluid is minus T change in momentum final minus initial, final is V minus w minus m dot V please understand is it clear because m dot V is the initial, m dot V minus w is the final and this flow is going by my thrust which is actually acting on the fluid is opposite, flow is this way, this is acting that is why, I put the minus T.

Now, this will give me the value minus m dot w. Then you have to write the energy is the power, power is no T equal to m dot w it will become finally, you get T equal to m dot w. Now, the power is because the thrust is down, flow is up. So, minus T (V minus nu) please understand, because the force is acting down velocity is up on the fluid here, that is why, this is and of course, the this is equal to the change in energy. Change in energy is the final minus initial which is half m dot V minus w Whole Square minus half m dot V square again, if you simplify you will get, half m dot minus 2 V w plus w square.

Now, again here you will be substituting, minus T is minus m dot w, put it here. So, you will have minus m dot w into V minus nu. So, these two are equal, because this is equal to this. You will again find because this is plus m dot w, m dot w go nu is actually you will get again this leads to w is 2 nu you follow, again it will give the same condition, because you equate because m dot w V this is minus, minus both will cancel out and this is plus, this is also plus you will get w is 2 nu.

So, whether it is up, down, forward it does not matter, the far field induced velocity due the rotor disk is actually double of what happens at the rotor disk. Now, you look at your thrust because you got this expression, my thrust is here, I will be substituting this quantity here. So, let me put the thrust I will write here, but what is my T, T is maybe I, can I erase

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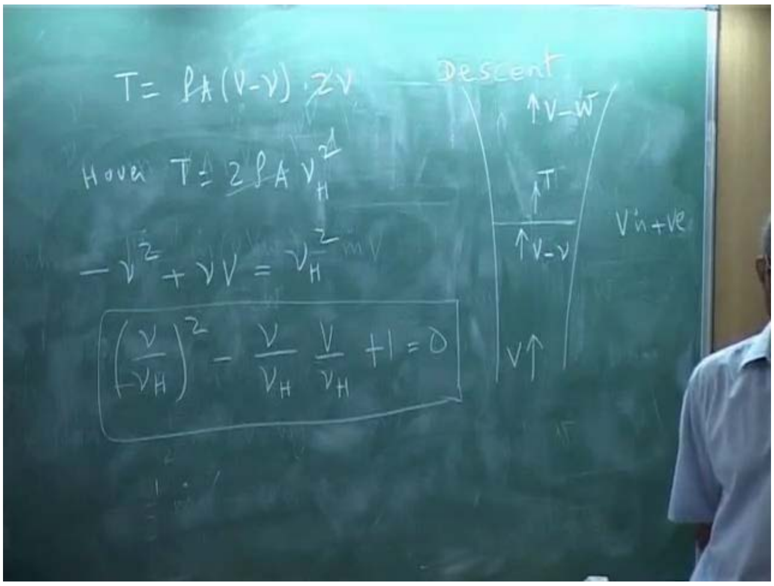

My thrust is because from here, m dot w, m dot is given here. So, rho A V minus nu, w is 2 nu into 2 nu. So, now, I will erase all this part, but please understand here, V is up, nu is down as per this. Our hover condition as usual in hover, this is what, T is 2 rho A nu H square combined both of them then, you will have this will go off. So, you you will get minus nu square plus nu V equals. So, you divide everything you will get minus of nu over nu H whole square plus nu over nu H, V over nu H or in other words, you can change the sign make this plus, this is minus, this is plus.

Now, this is the equation for (no audio from 1:06:46 to 1:06:53) descending. Now, let us get the roots, the roots of this equation are that is why, there is a slight change between climb and descent that is, what I am I wanted to say here, because here in the climb case you are this as plus, this was minus, whereas in this case it is changed, that is why, this is a little.

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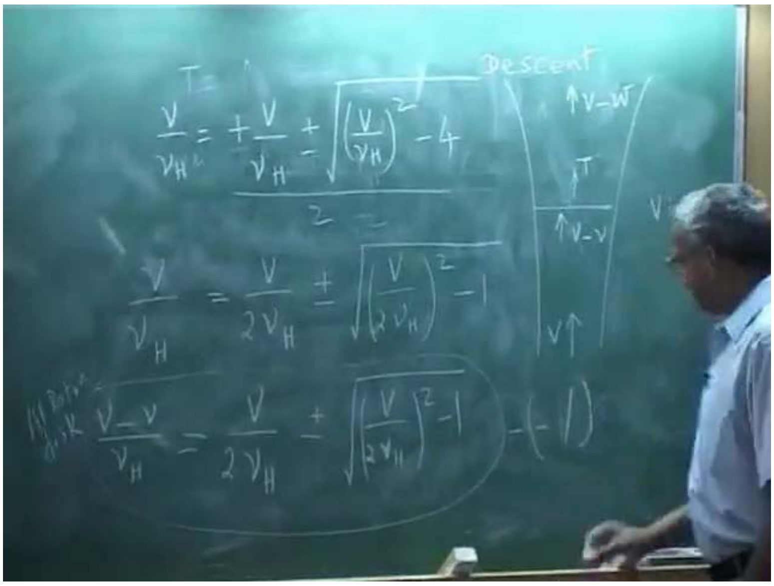

Now, let us write nu over nu H is minus; that means, minus V over minus V minus and minus, this will become plus plus or minus root of V square minus 4 divided by two, if I simplify this. Now, let me write the root and that is nu over nu H in descent is V over 2 nu H plus or minus (no audio from 1:08:28 to 1:06:38) here, which root should I use because both will give you positive value, you follow is it clear because you will get because this is same, you are subtracting 1.

So, therefore, this quantity is less than this and whether you add plus or minus sign, you are going to get nu over nu H is positive. Now, what root should I use, that is a first question, you will have the both roots another thing is, this will become imaginary when V is less than 2 nu h; that means, there is no root. My if my descent velocity is less than 2 nu H, I cannot have, but when it is valid; that means, my descent velocity should be more than 2 nu H ,more than 2 nu H is what, descent velocity, V plus 2 nu H should be positive.

This is positive in the sense here, it is minus mean V plus 2 nu because I have taken V is a positive. So, V plus 2 nu positive which means this equation is valid only in the here, it is not valid in these places, but I will get a root, is it clear or you have some confusion because you find you know that only when V is greater than 2 nu H, but the velocity should be much larger than 2 nu H then this is positive and you will have a root otherwise, it is imaginary you do not have you cannot plot that curve also, but both roots are valid, implies that velocity must be much larger than two times the induced velocity at the hover rotor disk.

So, what we will do is, we use this conditions to try to write our flow velocities and then may be, I will right that now at the rotor disk at the rotor disk is actually you have to add what, V V plus nu, but actually V plus nu means what, I am having what, V minus nu at the rotor disk far away V minus 2 nu. So, I must have V minus nu at the rotor disk and then faraway I should write the other 1. In other words, I can write it in this passion either V minus nu, I am please understand, I put a minus sign here, minus sign minus sign everything which implies I may get what, this will be V over 2 nu H, this is again minus or plus that will be same.

So, I can put a plus or minus, it does not matter (( )) V over 2 nu H minus 1, this is at the rotor disk at rotor. Now, please understand very simple. Now, I can go and change the sign of the velocity term, this velocity term, minus sign then minus means what, I am taking negative that is like a descending; that means, if I put V minus sign because here, in this expression V is always positive please understand positive means, it is descent here. If I put a minus V then what will happen that V becomes a positive quantity. I can always take that as the positive. So, you will find this expression just for the sake of plotting only.

I will put V as minus then I will change all everywhere minus sign and then you will get V plus nu you will get it, but in that case you know, V negative I have to go, my velocity is decreasing means, descent velocity I must have V negative value because if I put here, I have taken V positive for descent, I will write minus V of minus. I can do it there and then this minus V, I will simply call it as you follow; that means, when the velocity is descending, I will put a minus V, if it is climbing because you will find that expression I get, I do not have to keep changing the curve. It is like one and I will show the diagram then you will understand may be, if it is confusing, I will tell you again.

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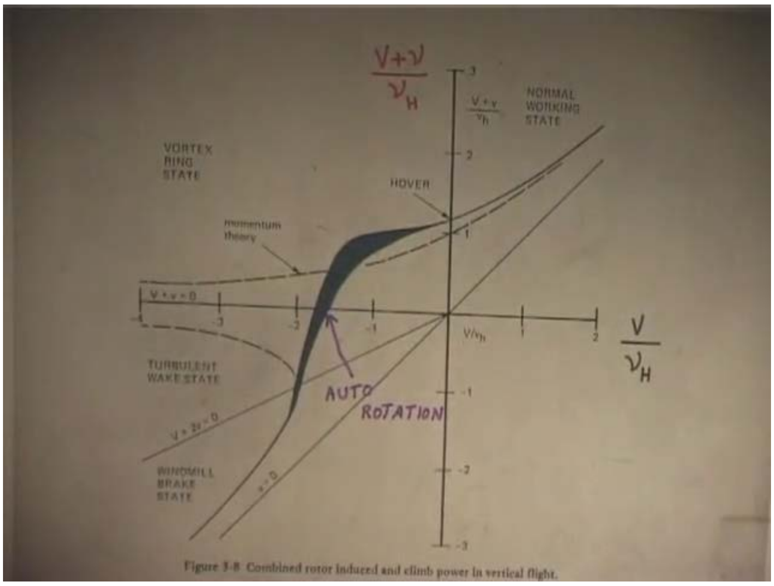

You see, V over nu H positive means, I am climbing when it is negative means, I am descending. So, for me to plot here, when I want to plot the descent curve, I must take this expression and then substitute V positive values here, you understand. Instead of that, if I put minus of V then that V if I put a minus sign that is actually descending. So, you will get the curve, I will draw this curve next class using these equations, may be, I will give the same equations and then show you, it take this part; that means, you take this equation otherwise, what happens is one set, you can use that for the entire zone.

Here, I will just want to check this curve, V plus nu over nu H positive I have taken because I will show two curves one is this way, one curve is nu over nu H verses V over nu H, that is this you follow this part, only the induced velocity another curve is V plus nu over nu H that is the velocity at the rotor disk as a function of climb and descent because descent is this side, climb is this side. And this curve, this particular curve is called the, you know universal inflow diagram, but I will show the other diagram the next class, I will plot using these equations.

We will plot the equations and then show that, this is what is really given as the inflow and then all your autorotation everything comes from here. And the region is split into several parts normal working state, vortex ring state, turbulent wake state, windmill brake state. you will see the curve continuous line only here and here, that is the solution is valid in this zone, it is just a extrapolation of whatever route which you have obtained you are simply drawing that route you follow even though, it is not valid, you will simply use that route, that is what is being done in the inflow diagram.

And then, just it is red, it is black patch it is by experimentally, you try to get a inflow, you can do that and then they are all plotted as the some patch. So, you approximate is basically an approximation please understand of the inflow in the region, vortex ring state to turbulent wake state. Just draw like a line simply and that equation is, what you used for autorotation curve because you do not fly here because windmill brake state, you do not want to go that generate power.

Actually, you are more interested only here, V plus nu 0; that means, the rotor flow at the rotor disk is 0 inflow, total flow is 0, that is what my autorotation that means, I am more interested in this zone, but I do not have root, I do not have the solution will give me this curve and this curve. We will show that later, next class. I will plot that, this is just for the passing, I wanted to show this diagram, we will come back again and show how these two are plotted, climb, descent. Then, you will see this is what the whole thing is about, in terms of inflow curve for climb and descent and then please it is non-dimensionalized. So, that it is applicable for any rotor and this is what is used for autorotation evaluation. I think, I will leave you now. We will come back to the next class.