Introduction to Helicopter Aerodynamics and Dynamics

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Lecture No. # 06

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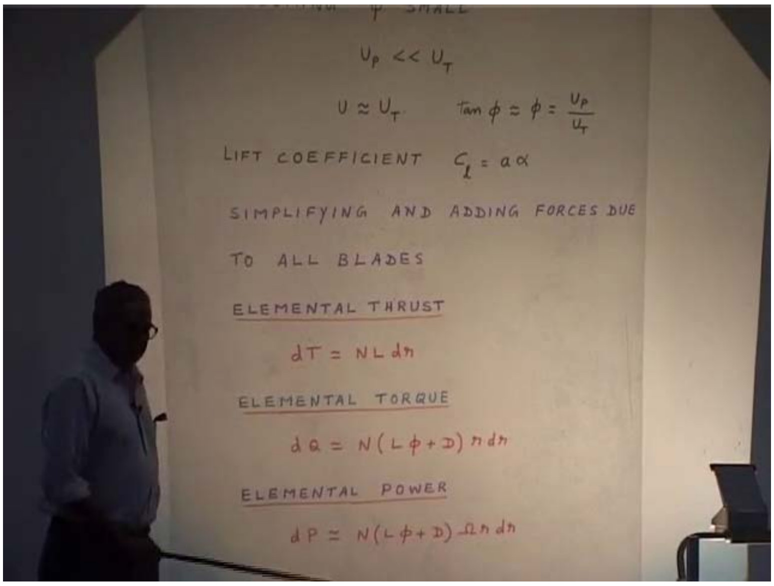

Elemental thrust for linear twist, and ideal twist. Today, what we will do, we will look at these two expressions, that is the torque coefficient, and the power related to the rotor. Now, we have made all the approximations small angle assumption, and the elemental torque is number of blades L into phi, phi you know is the induced angle, and then D is the profile drag coefficient.

So, this has two components; one comes due to lift, another one due to the profile drag of the aerofoil. That is why, this particular term later we will say this is actually induced drag, and this is the profile drag. When you look at the power expression, it is the exactly same only thing is, that is the omega, because power is torque into omega. And again, this has the same component. So, their power required, now as two components - one is due to the induced power, another one is the profile power or profile drag power. Now, there are two components to the power part.

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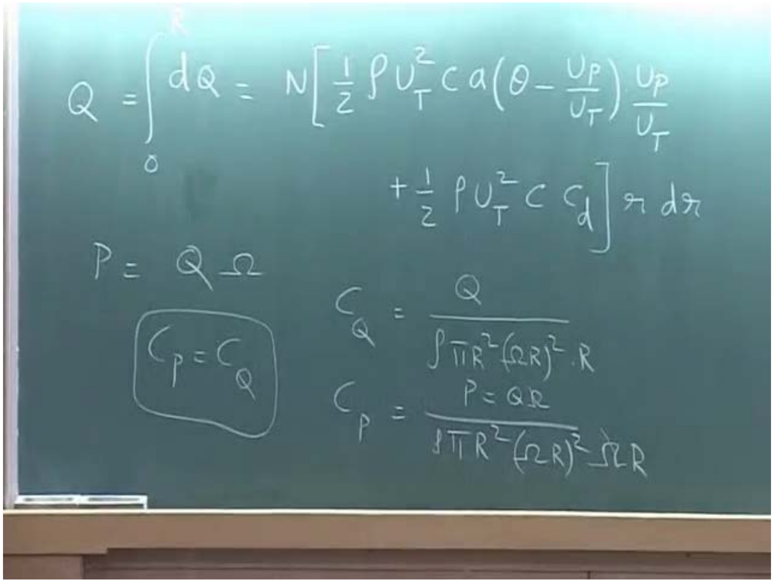

Now, let us write the expressions, after that we will we as usual, we non-dimensionalize the whole expression which is because we take, dQ is N, you are substituting for the lift and the drag, lift is we wrote it half rho U T square chord, and then lift curve slope into theta minus phi. Phi was written as U P over U T into, there is the another factor phi which is again U p over U T plus the drag term, which is essentially half rho U T square C C d into r dr. So, this is my torque, elemental torque due to all the blades in hover. Now, if I want the total torque, I have to go integrate over the length of the blade.

So, I will put 0 to capital R, this is Q and power is exactly just because this is the same expression, there is the omega and omega is a constant. So, power is Q into omega that solve. Now, what we do is, we write the expressions in non-dimensional form. So, Non-dimensional C Q which is the torque coefficient, this is nothing but Q over rho pi R square omega R whole square into r, this is the torque coefficient and you will look at power coefficient, C P is power divided by rho pi R square omega R whole square into omega R, but you know that, power is Q omega therefore, omega omega will cancel out leaving behind, you will simply says C P is equal to C Q.

So, power coefficient, the coefficient not the power is same as torque, power coefficient is same value as torque coefficient that is why, when we evaluate, we never use C Q, we directly go for power coefficient, because C P equal to C Q. So, straight away write all the power expressions. Now, let us non-dimensional lies this, is it clear because power coefficient, torque coefficient both are same for rotor under constant r p m please understand, because omega is constant, otherwise you will get a instantaneous thing.

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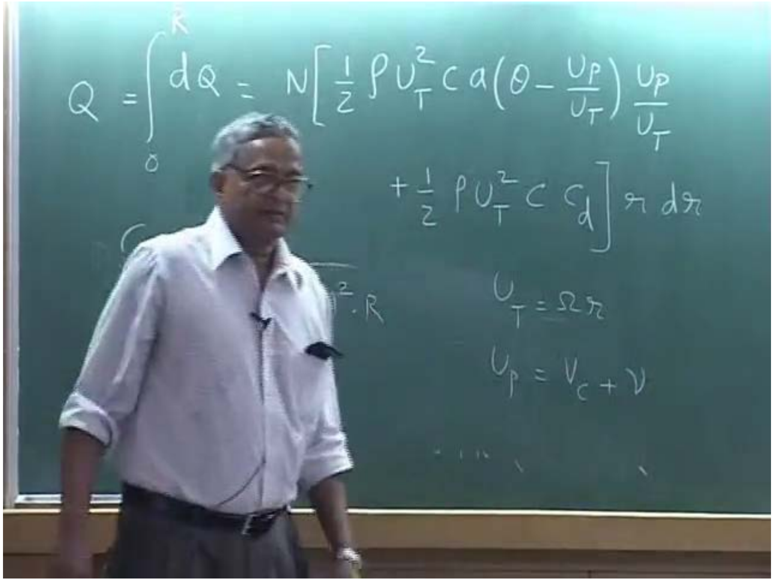

Now, let us write the C Q, you have to divide by rho pi R square omega R whole square into R. So, non-dimensionalize this and you know that U T is omega small r and U perpendicular, if there is a climb, we used it as V C plus nu right. So, non-dimensionalize with respect to capital omega r. So, this will become just r bar, r over r this we call it as lambda. So, we will call, if you take this rho will cancel out then of course, you will get N C over pi R, that will be sigma solidity ratio, you will get in non-dimensional form and I am going to write that in non-dimensional form.

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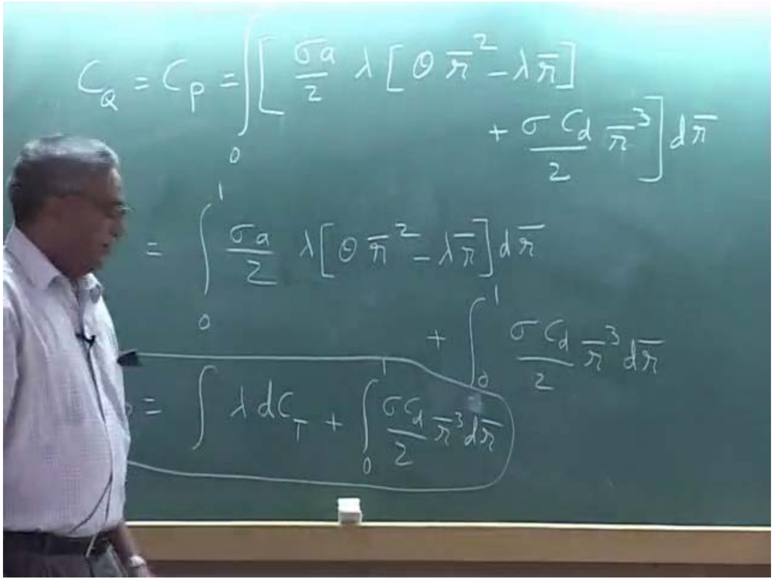

So, I write C Q directly. Since, you know C Q is C P. So, this is in Non-dimensional form value becomes, (No audio from 07:27 to 07:34) sigma a over 2 lambda theta r bar square lambda r bar plus sigma C d over 2 r bar cube dr bar, this is the expression. Now, if you split this term into two parts, we can write it in, this is one term; this is another term, if I write this between 0 to 1 sigma a over 2 lambda theta bar. We had one, what is that, we have to take on r square r bar square minus lambda r bar into dr plus 0 to 1 sigma C d over 2 r bar cube dr bar sorry sorry.

Now, if you look at this particular expression, leave out the lambda, this is nothing but differential of thrust coefficient because, if you look at that earlier last class I derived, sigma a over 2 theta r bar square minus lambda r bar dr bar, I wrote it for integral that is C T, is it clear. So, this term, you can write it as simply lambda d C T because differential of thrust coefficient, because lambda is this, if you look at this term last class we got, that is the d C T and this is of course, sigma C d over 2 dr bar. Now, this is C P (No audio from 10:17 to 10:24). We have split the power expression as we did the component due to thrust, the component due to profile drag.

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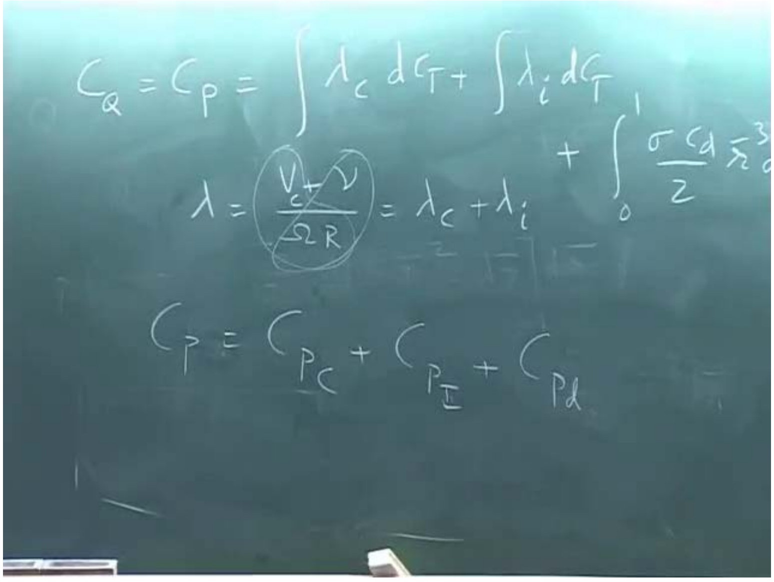

And even this term can be further divided into two parts, because that is, I go back here, you can write this as because lambda, we said V plus sorry V C nu over omega R, this particular term is lambda c, this term is lambda i, one is the climb another one is the induced. So, I can again put it as lambda c d C T plus integral lambda i d C T plus of course, the profile drag (no audio from 11:37 to 11:42) is it clear? Now, you see, how we are really splitting the power required for the rotor into several components only for hover and vertical flight, there is no horizontal flight because that will come later.

Now, I am going to write everything in a symbolic form, very simple. So, erase this C P is C P climb, power due to climb and then power for induced C P I and then C profile drag C P d. Now, the slowly we start the little complications. Suppose, first we simplify then complicate, if lambda C that means, climb is 0 because climb velocity is constant. If you take it out, that is nothing but lambda C CT that solve whatever is the thrust, if you want to lift the weight of the helicopter in non-dimensional form, just velocity into the weight in non-dimensional, that is your climb power then induced, you have to get the induced velocity corresponding to the climb velocity please understand, it is not the induced velocity is same as hover induced velocity that means, we have to learn, how to get the induced velocity in climb, that we will do later. And then, this is the profile drag. So, we will write the whole expression. Now, for hover it becomes, see this integral is what, sigma C d over eight that solve. Is it clear?

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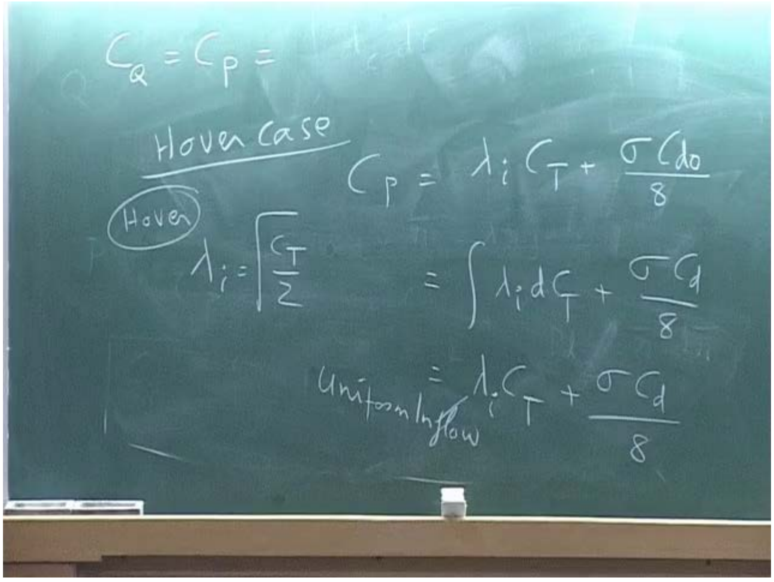

So, I will write the whole thing as, C p I for hover, if I take Hover case, power becomes C p I plus C p profile drag and C p I is nothing but what, if it is the uniform inflow then this is lambda C T. So, we can write it as lambda C T for uniform inflow, lambda means sorry lambda I, I will put it there, uniform inflow and this term is… So, you will have sigma C d over 8 and I am assuming that C d, that is the profile drag coefficient of the blade, every section is constant, because otherwise that also can vary, please understand because if you have different aerofoils, if you have different chamber then the C d also can be a function of span, but for ease of calculation and C d can be function of mark number, please understand because it is not a constant.

Now, in actual calculation actual means in the industry, you do not use this expression, you calculate you get this drag coefficient of the airfoil for various speeds, various angle of attack and then have a table, it like a data table, that is through internal testing and then, whatever is the local angle of attack, what is the local velocity take that value go pick up that C d and put it there, that is how they evaluate it, because if you want to be more precise in your estimation of the power, you have to take the correct characteristic whereas, for the course because it is easy otherwise, it is very difficult.

So, we say everything is constant and we easily integrate from 0 to one and your C p I, this is what you are getting, lambda i C T plus sigma C d, I will put it as C d naught or C d it does not matter, if you want to put a subscript 0, you can put it because drag coefficient normally C d equals C d naught plus some etcetera. So, we will take it as C d naught which is a constant value. So, for this course you will use constant, but here in writing this expression, we assume that lambda i that is the induced velocity in hover is a constant over the full disk.

Then momentum theory has given me lambda i is C T over 2, momentum theory in H over H over case lambda is this that means, I can directly put. Now, you see this is the expression which we got earlier in the momentum theory power when we wrote power coefficient, it is basically C T power three by two under root two, that is the ideal power, which you can never get. In real case, because you cannot say my inflow is uniform, and you will have a non-uniform inflow and then there are some other swirl velocity is there, swirl factor we have neglecting it, swirl in the sense that is actually you are rotating something.

So, the flow also will go parallel to the rotor disk, but that effect is small, but still it is there. In propeller, it is they take it swirl even here you can do, but then how do you get the swirl again through simple momentum theory. I will not go into that, if you want to know I have it, we have done some calculations for propeller blade for N a L. So, that is the that report, we correlated with some experiment which is very good, but in the helicopter blade usually this world takes very less.

So, you neglect that, it is a less power, but then non-uniform inflow all those things are there, that you cannot totally neglected. So, empirically what is done is, they say, let us add of factor there is an empirically factor they add in writing this power expression because of various other reasons non-uniform inflow etcetera.

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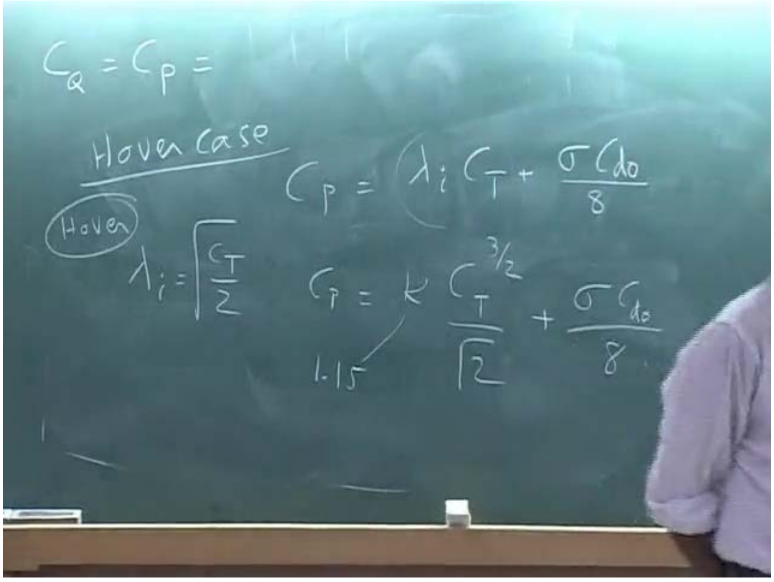

So, you say this is written as some factor k, this is the factor which additional loses because slowly you will realize the loses can be due to several reasons because this k is an empirical factor, but you are adding fifteen percent extra, what are all the various reasons you can say, that part you will so, I will just briefly introduce later then this is sigma C d naught over 8. So, now, you see C p in Hover use this expression. Non-uniform inflow effect then there is a Tip, the Tip is you are integrating 0 to 1, straight away you are taking 0 to 1 in the sense, along the full pane of the blade, but near the Tip, it will not be like, the the real rotors there will be a drop in the lift at the Tip whereas, there our, what does it say because lift is some sigma a over 2 theta r bar square minus lambda r bar, that is perception.

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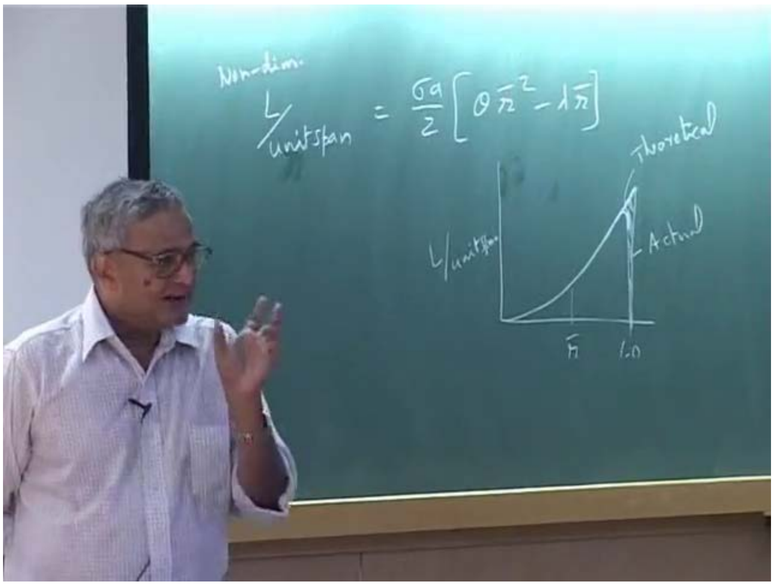

Now, if we plot that expression, I am just going here, because we took that what, lift non-dimensional lift, lift per unit span non-dimensional, if you have that is sigma a over 2, this is what we had that mean, how this lift is varying, it varies as. So, if I plot this unit span versus r bar you go like this, this is 1. Now, you by looking at the diagram you immediately see that, my lift is going up near the route lift is 0 very close, because the dynamic pressure is small. So, this is actually the lift become may be beyond 0.5 you start having a large value, but in actual rotor, this will go and then it will drop, this is the this is theoretical, this is actual because lift at the Tip when there is no (( )) at the end lift is 0 . So, it has to go and then come down to 0.

Now, this particular region loss, you have a lift is large, but drag will always be there and then your inflow can be non-uniform. So, all these effects, this particular thing I think I gave somebody Tip plus someone is suppose to do, it is represented by a last factor because if you want to get real load, you cannot do by momentum theory please understand. You have to use a more sophisticated analysis, if you want to analyze near the Tip, it is very complicated motion because the motion is three dimensional flow will come vortex will go up etcetera that is why, analysis of Tip is very, very complicated as we go along we will see later. Now, you have a expression for power, this is from blade element theory, but you need momentum theory always to get lambda i and this is a uniform inflow.

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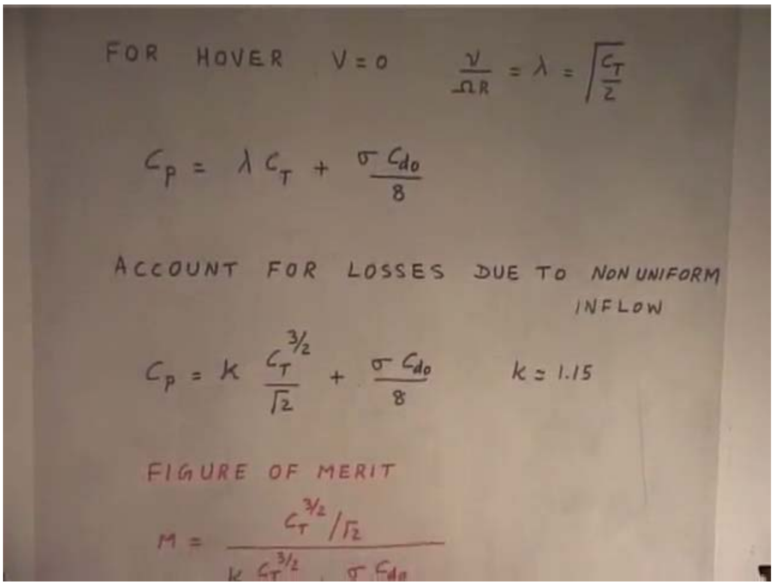

So, now, you have this expression, let us go and write the figure of merit, because you now, got the power figure of merit what was that, ideal power divided by actual power (no audio from 24:40 to 24:50) I will flip this. So, non uniform inflow account for losses that means, when I say non-uniform inflow, you must have procedure to calculate the non-uniform inflow because simple this momentum theory that is not give you that. So, we will learn about that particular procedure next, but before we go let us look at the figure of merit expression. Figure of merit is actual power over sorry this is the ideal power over actual power. Now, you see for the same thrust coefficient, if sigma increases then what happens, denominator is increasing. So, figure of merit will drop.

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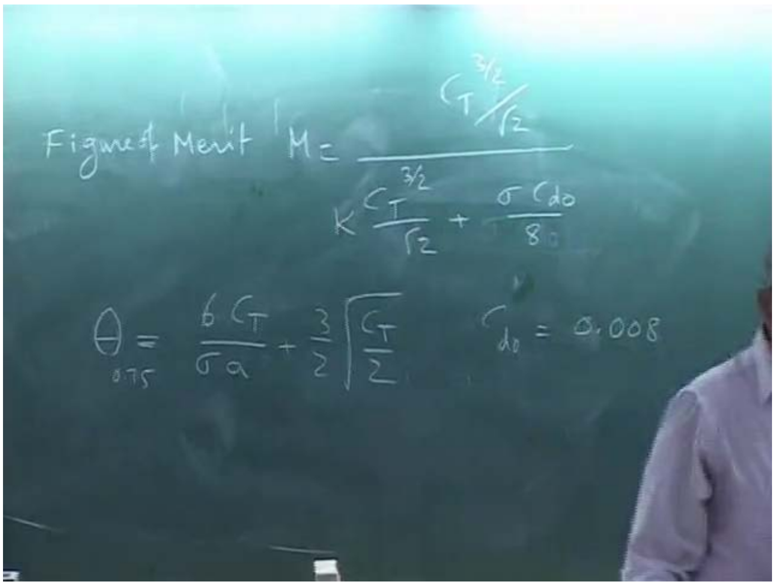

On the other hand, if sigma is small then figure of merit will go up, but last class I mention something that is, you saw theta pitch angle, if you look at that previous theta 0.75 (No audio from 26:15 to 26:25) last class I wrote. Here, figure of merit C T power 3 by 2 over root 2 plus sigma C d naught over 8. Now, let us look at the kind of a paradox or contradiction which you get, if I want a high Figure of Merit same C T, it is fixed that means, sigma must be small, solidity ratio that means, n c o blade area over disk area.

But when I go here, if sigma is small then what will happen, I need a higher pitch angle, but higher pitch angle, if I keep on going then I make land of (( )). So, now it is a compromise, you have to decide what sigma I should use, if you look at most of the rotor blades, you can calculate because, if I will give you a sigma value, I think I have some for the A L H blade.

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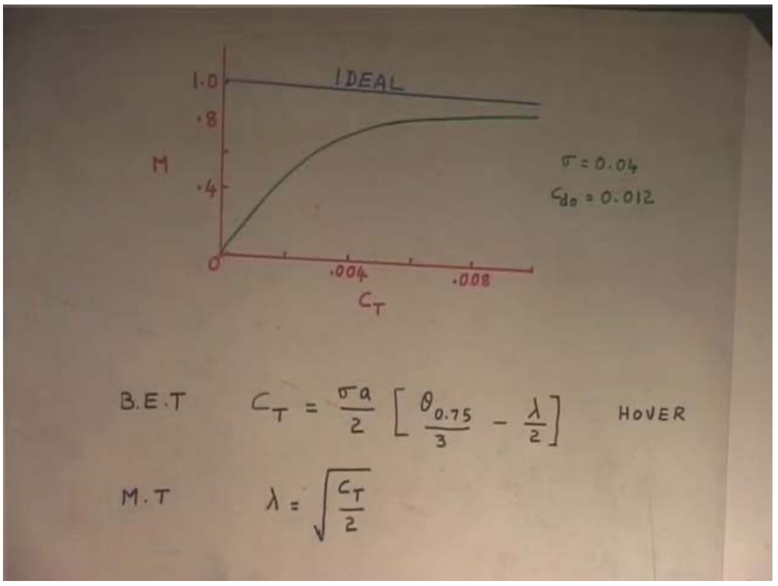

I will just show you that, this is what the curve for I have essentially plotted this Figure of Merit curve for some values. Now, you can see Figure of Merit increases, that is a C T as you increase initially it raises very fast and then it slowly deports asymptotically it reaches 1. Now, you it also tells me 8, if I have a high C T that means, my thrust coefficient is high, I will have a good Figure of Merit because the denominator, if I keep on increasing the C T, this becomes very small that means, I must have a rotor with a high C T.

You know earlier, it requires high power because high C T means I need to have lot of power because you know that lambda it takes. So, this is a the Figure of Merit curve alone if you look at it, it will give you as a tough result which you say, I should choose like this, but then actually you will find that whatever you decide based on Figure of Merit alone will become detrimental your your rotor because in your operation because one is, if I keep on increasing if I decrease sigma, I know that my pitch angle is going up, but if my pitch angle goes up that means, my blade may stall.

So, I cannot make my sigma very small also, just because I want to have a higher m. So, this is the that is why, that Figure of Merit is used only as comparison of two rotors having same C T, it is like same disk loading you try to have and then see which one is a better in terms of the efficiency of power required. So, it not that I keep increasing my C T means then you go most of the helicopters you know that C T is in this range, it does not go to 0 0 8, 0 0 8 is very high C T, 0 1 is too high. So, it is always in the range of naught five please understand, when you look at all these numbers and C d naught profile drag of the blade you may take it as approximately 0 0 8 or six or sometimes, if you want to have a slightly higher value may be 0.01.

So, please understand most of the numbers which we are dealing with here, are small numbers in the non-dimensional form, but this is to give you an idea, how you can estimate the power in a very quick calculation hover power, because you know that initially I showed as you increase the forward speed, the induced power is decreasing. So, automatically say, let me take the Hover condition for my power estimate of course, high speed it may again come to the same value.

So, these things give you a quick estimate, but please understand these are all very good in the sense even in the industrial calculation that is where the designers, the good designer who knows basics strong. You would not do very detailed calculation with a computer program yes you do it that is for actual estimation, but he will know by simple calculated calculations, your results will should make sense, if they do not make sense that means, there is something wrong in your actual calculation in their very detailed thing.

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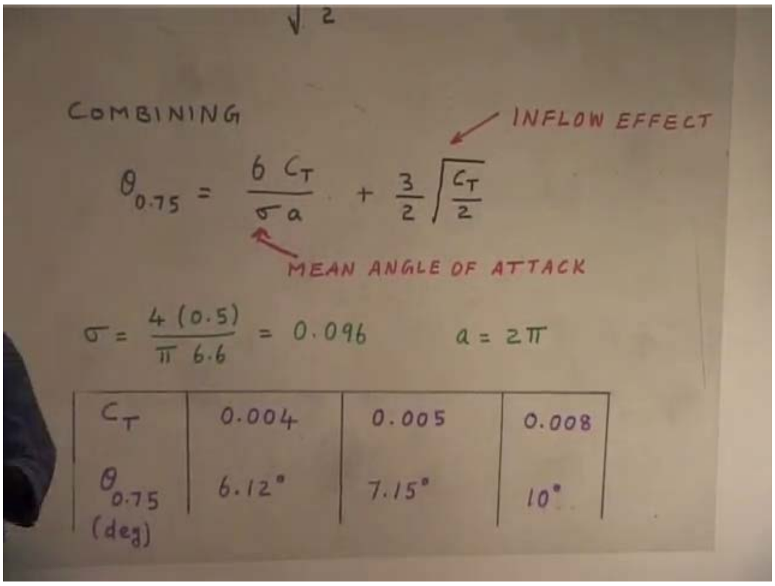

So, that is why, it is very important to understand the various expressions thrust, inflow, figure of merit, theta, C p etcetera all these terms. Till now, we have learned about the power and thrust for a rotor in Hover and using uniform inflow, but now, we will learn about how to get non-uniform inflow and here just for a reference, I took the a realistic rotor sigma you see, and number of blades for 0.5 is the chord, the radius is 6 .6 so, pi R N C over pi R constant.

Now, this value is about 0.096 only one particular rotor has a value which is about sigma point 1.3, which is a high sigma not at all most of them will be around and I use the value lift curve slope as 2 pi and I am using this expression to give you, if I vary my C T how my pitch angle of operation changes, if it is naught naught 4, it is about 6 degrees, but please note all these are in radian then you have to convert it to degree, it is just a simple calculation which I did because you when I give you a homework, you have to do that and naught naught 5 about 7 degrees go 10 degrees.

Now, you see when you are preliminary design, when you are making you do not go and then make a high operational angle because, if you do it actual when it may be still more and start having blade stall more drag more power. So, you try to operate always around five six degrees five six degrees operation, but actual calculation you will say, you will go about eight nine degrees in the Hover case that is detailed calculation I have, when you make in Hover, pilot gives a pitch angle of about eight to nine degrees to the blade, you do not want give twenty degrees or something like that, because in forward flight then you have to consider this as an time varying angle and then you may have stall.

So, all these factors come into picture. So, we have learned just now very simple hover. Now, another important thing which I give before I go further, I wanted to tell you is, we use the word uniform inflow first is, how do I get another I said non-uniform inflow then you will say how do I calculate because uniform inflow, you simply assume lambda i is root of C T by 2. How will I achieve, that is the first question, second question is you say there is no uniform inflow, you will have only non-uniform inflow that means, how will I calculate the non-uniform inflow.

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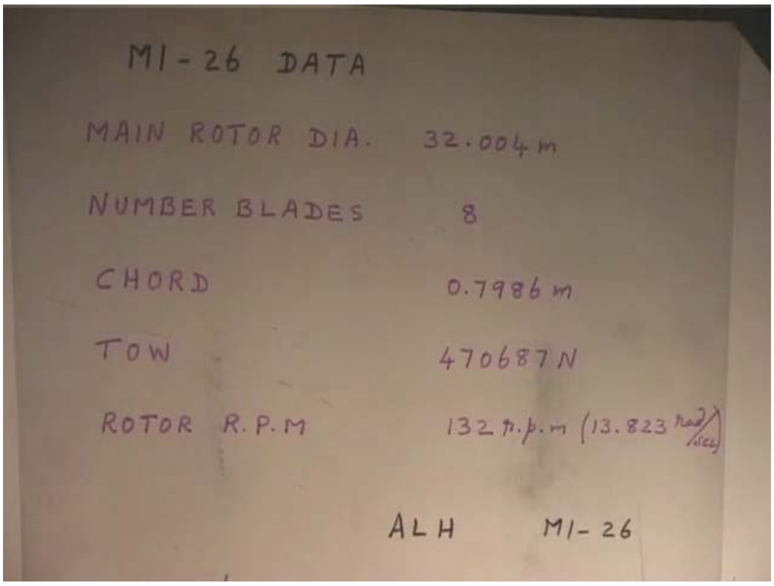

So, now, we learn about, how to calculate the non-uniform inflow. I think, I have some numbers here, which are some kind of comparison before I go just for comparison, before we go to the non-uniform inflow, one is a A L H which is about 4000 kg, this is MI -26 which is about you see takeoff weight, number of blades 8, main rotor diameter 32 meters, please understand it is about 100 feet from here to there, main rotor diameter may be more than that, I do not know because I do not know this will be what 50 feet, one chord, one blade and then rotor R P M 132.

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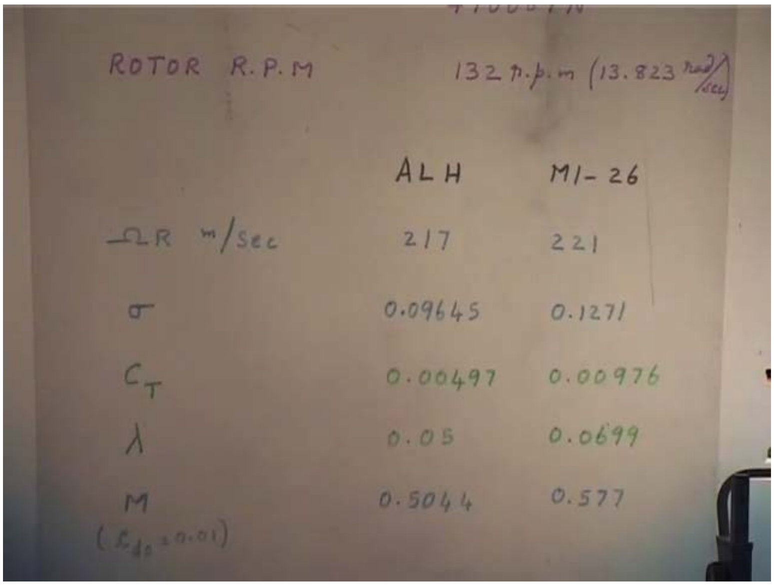

Now, you see the Tip speed of both these helicopters, the weight loss is totally different, why I want to introduce here is, so that you have a appreciation of the numbers in non-dimensional form, but industry for comparison yes, but then when you actually design you can tell your manufacturer, I want a non-dimensional thing of this, no you have to give actual dimension. So, you need to know that number also Tip speed is 217 meter per second 221, solidity see this is 096, 0.12, thrust coefficient this is a high.

And then lambda inflow, lambda is this is the hover inflow and then the figure of merit parallel flows even though the rotors are of different class, different weight everything, but when you look at the non-dimensional numbers, they come reasonably they fall in one area in the non-dimensional level, this is just for information only because you can collect the information from any two helicopters and then start comparing them. So, if you collect several helicopters then you will be able to have an idea of how these numbers will…

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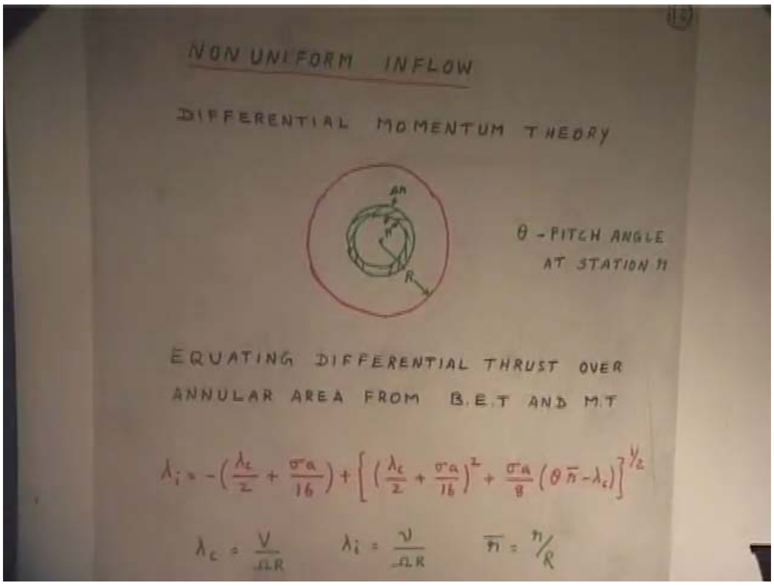

Now, we will go to the Non-uniform inflow calculation, I will just I have written final expression here, but we will describe it very systematically. I will go through the derivation here, because that is important. See, what we have is, this is called differential momentum theory, you may ask whether it is valid that is a different question. You take the rotor disk now, take a annular area annular area at a distance lower case R and with a thickness dr, delta r this is my annular disk, what I will do now is, I will start comparing the thrust generated by this annular area using momentum theory and using blade element theory please understand.

You may ask, what is that? Momentum theory you said, it is for the entire disk, we used a slip stream and then we assume that some uniform inflow everywhere, we calculated we got the expression. Now, I am saying as though the slip stream consist of several concentric cylinders with the different radius and I take a small elemental area of the rotor disk, I neglect everything else, I assume this particular annular area is not affected by either the flow inside or outside that, this is an assumption please understand. This is an assumption I make and I simply, blindly please I blindly use momentum theory there is no logic in this, the logic is finally, it matches very good with real life situation that solve, if you want to do very detailed.

So, that is why, there are several people come up with ideas, it is like this. Momentum theory is based on energy, you simply apply instead of for the whole disk, I take a small region I apply over only that region that mean, what that region you have isolated from the rest of the things and you assume that nothing is, that is not affected by anything outside. This is an assumption you make and then you simply start using it. Now, the proof, what you are doing is all not correct comes, if you get the result, if it does not match with the experiment to far away then you say this assumption is not good, but if you find this is much closer, it is reasonably good. Then, you say may be this assumption is fairly good.

And that is how, otherwise you have to do, I told you vortex theory prescribed wake, free wake, vortex theory you do even there are approximation in that theory, but that is a little bit more physics based, I would say because you say, I say there is a vortex strength and then try to calculate the strength whereas here, you simply apply, what you did for a whole disk to a small area, that is the first assumption you are making then you will get from here see now, we will go step by step.

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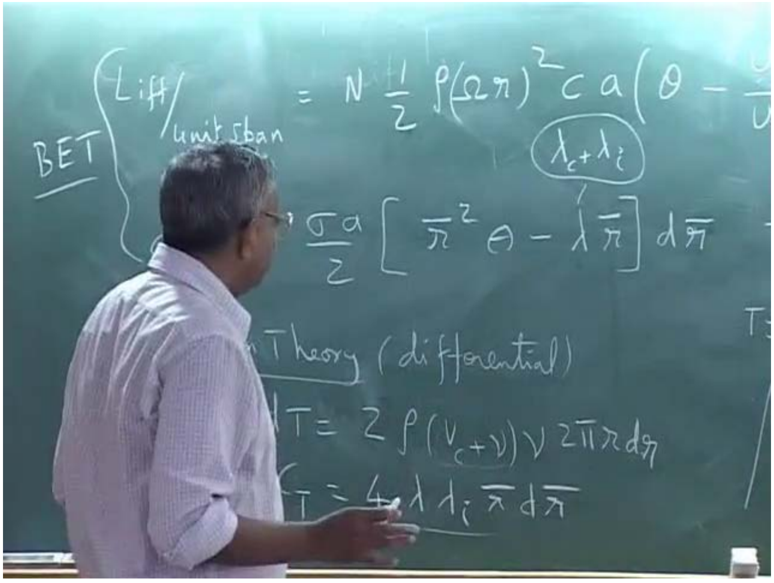

If I take the blade element, blade element we know that lift per unit span, this is due to all the blades you have to take, that is N essentially half rho omega r because this is what dynamic pressure chord lift curve slope into theta minus U p over U T into dr sorry per unit span I will I can eliminate this, per unit span this is the lift which we derived earlier. I can non-dimensionalizes, this if I non-dimensionalize, I am going to get that expression which is written there on the other side of the board.

So, I will get d C T you divide rho pi r square omega or etcetera then you will get lift per that is d C T this will be sigma a over 2 into r bar square theta minus lambda r bar, but this is not per. So, you have to take the dr bar because this is the thrust developed elemental thrust develop over dr bar due to all the blades that is why, sigma comes in. This is from, blade element theory got it. Now, you say momentum theory, momentum theory, we are going to now use differential thrust d T, this is I will write momentum theory.

So, this is please note, this is from blade element theory what we got. Momentum theory, differential I am going to put differential, what is my area, area is 2 pi r dr. So, I am going to get differential thrust is, if you go back your equation sorry momentum theory, we wrote thrust is equal to mass flow rate. So, we wrote 2 rho A, this is for hover, if it is climb then you will have mass flow rate will add the climb velocity also right now.

Let us take it only for, because I have used climb also here, if you had climb then this will become 2 rho area of the disk V plus nu because this is the rho A V plus nu is the mass flow rate because the climb velocity plus the induced, this term is the mass flow rate, change in velocity is to nu. This, we will prove it in the next class. This is how, I write my thrust. Now, I am saying this is this expression this is for Hover, this is for climb, hover and climb you may call it global momentum theory, global means I take the entire area area of the entire disk. Now, I say I am not want to take the entire area, I take only the annular area.

So, I will write here, 2 rho V climb Plus nu into nu, what is my area, 2 pi r dr, if follow what I have done and this is the differential momentum theory. Now, you may ask actually I will tell you now, why I have to take the annular area, why cannot I take only a small arc dr d theta, you can when you can do that actually, that is used in industry, if you want even the (( )) variation that is again assumption that is, all these are very simple assumptions you make and then try to get the inflow value.

Now, you have a expression for this, you can non-dimensionalize this divided by as usual rho pi r square omega r square, if you non-dimensionalize you will get d C T. This is from momentum theory, you will have d C T will be because you have 2 and 2 will have 4, I will write that 4, this term is lambda because V C plus nu over omega R is lambda and then this divided by omega R is lambda i then, r bar dr bar because non-dimensionalize rho pi r square omega capital R whole square.

Now, you see I have two expressions for the differential thrust coefficient, one from blade element theory, one from momentum theory. Now, I simply relate relate means just equate both. Now, you see this lambda is you know that, this lambda is lambda C plus lambda i and here you have lambda lambda i, equate both of them then collect all the terms. Now, let us see when I equate, I erase this part, I erase this completely then I will write this after that you will go to that.

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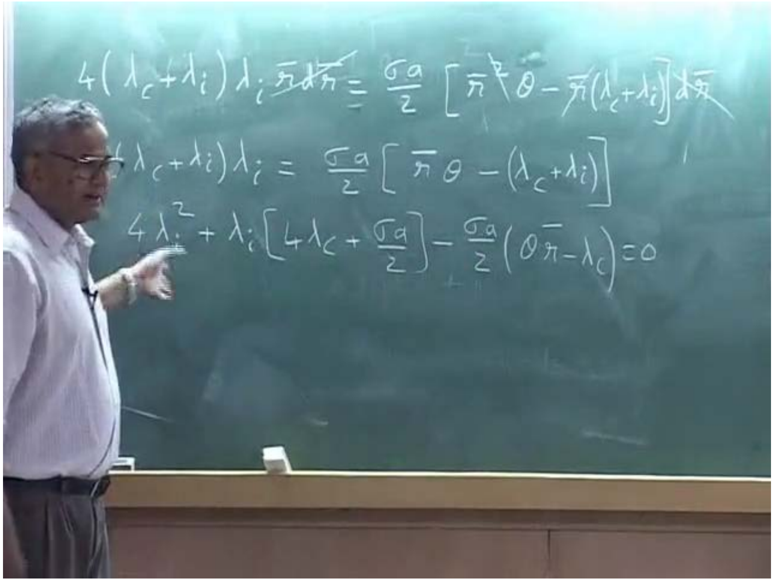

So, you will have 4 this lambda I am using it as lambda climb plus lambda i lambda i r bar dr bar, this is equal to you have your sigma a over 2 r bar square theta bar sorry r bar square theta minus r bar lambda C plus lambda i dr bar, cancel out then you will be left with the simple equation which is 4 lambda C plus lambda i into lambda i equals sigma a over 2 r bar theta minus lambda C plus lambda i. Now, this is nothing but a quadratic expression in lambda i. So, you will write this as 4 lambda i square plus lambda i 4 lambda C plus sigma a over 2 minus sigma a over 2 theta r bar minus lambda C equals 0, this is the quadratic equation in lambda i.

Now, you see the interesting part, this is the induced velocity and I have the local blade angle pitch angle is there and number of blades is there, lift curve slope is there, all these factors are there that means, if I solve you write because this is a quadratic equation, you can write lambda i equals minus all those things and that is what is given here.

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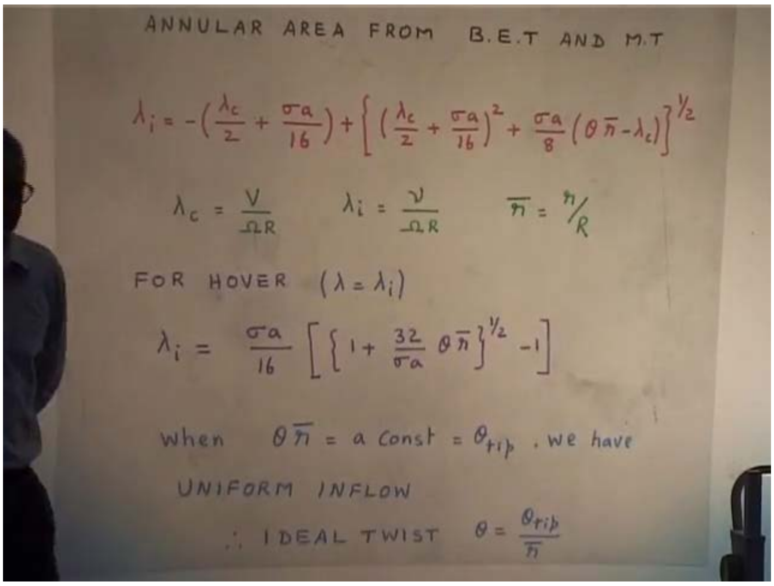

So, you can note down this expression because this is the this is the now, you see minus B, this is the term plus or minus root of B square minus 4 a C divided by 2 a and then you divide by that factor 4 etcetera you will get that expression, but you know that minus B plus or minus 2 roots are there, but minus root then you get a negative value for inflow which you cannot have. So, from physics you say I must have only the positive value for the root all right and that is why, you put only the plus sign. Is it clear? Now, this lambda i, if you look at it, this is a function of r bar that mean depending on my span location inflow can vary.

So, this is the please understand is a powerful expression in the sense, it is widely used in the calculations even in research, even we use this. This is very, very important expression; we do not use the uniform inflow usually. Uniform inflow is to gross on approximation, but it is good it is alright, it is easily you can calculate and then show, but you always take non-uniform inflow through this expression. Now, I just want to reduce this, this is with the climb, if I make lambda C is 0 automatically the inflow in hover.

So, I will is it clear because that is how the quadratic equation is solved and then you get the root, this equation you get that value. Now, let us write just for Hover case that mean, lambda C is 0, here, lambda C is 0, lambda C is 0, you will get lambda i is you bring this term first because lambda C is 0, lambda C is 0, everything is 0 that is why you will have sigma a over 8 is a common factor, you can take out that sixteen also that is alright.

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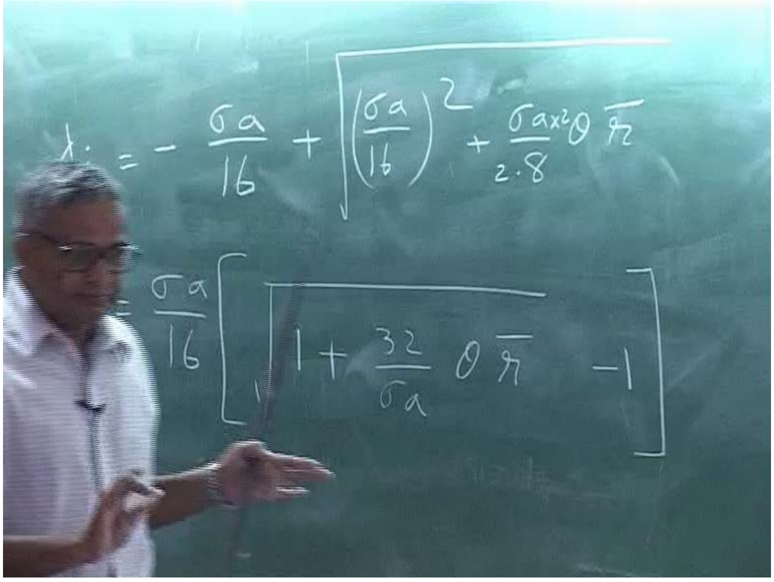

So, you will write that that is simplified, lambda i becomes minus sigma a over sixteen this is the first term and then the second term becomes what, sigma a over whole square plus over 8 theta r bar. Now, you take out sigma a over sixteen outside that means, here you are essentially multiplying by 2, when you take it out, you will get open a bracket put this term first, this will be one plus because sigma a over 2 what is that, when I take out square, I will be left with one more term. So, that term will become that 32 over sigma a theta r bar minus one because that minus one is this is the term.

Now, you look at your, but is it clear now, what you do is you see my inflow is a function of angle and r bar, if I want lambda i to be constant everywhere then if this quantity is a constant, constant means uniform inflow. So, I will say for uniform inflow if I want I need that is lambda i is a constant that mean, theta r bar should be equal to some constant. So, theta r bar equal to some constant. This, you write it as because when r bar is equal to one, that is the tip. So, the constant is nothing but the tip.

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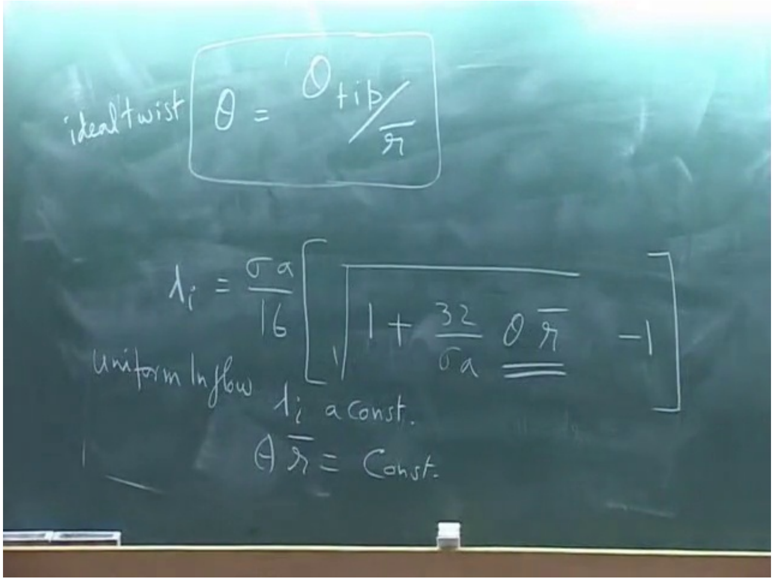

So, your theta becomes theta tip over r bar. This is the ideal twist. Now, remember earlier I mention, why the twist is, if I have this kind of a twist because r bar is r over capital r, I will get uniform inflow you follow, now, if I get uniform inflow, so what, that is another question. Uniform inflow helps, if you go here, if I substitute for that particular expression. Let us, get that thing I just want to relate because there are very interesting ideas in this.

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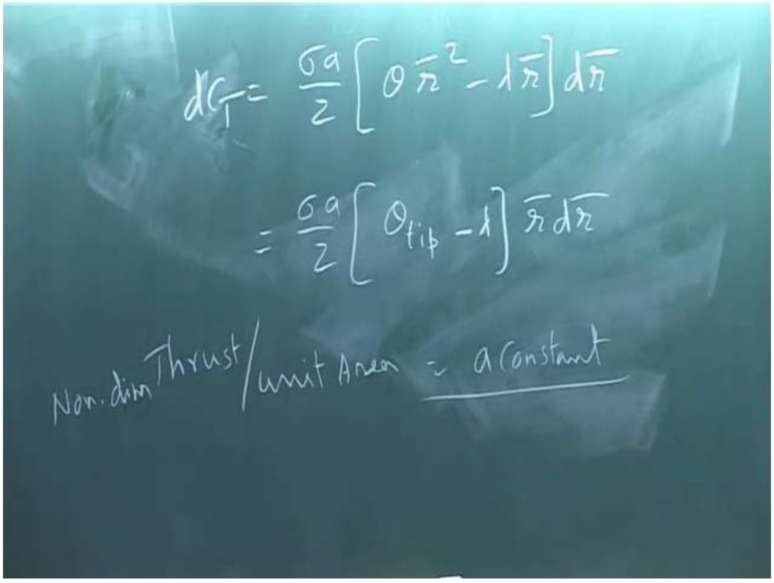

This is a non-dimensional lift, if I write it, non-dimensional lift, I may or you can put it d C T also, I think d C T comes because this is not a Non-dimensional, This is now, d C T is this, if I substitute theta has theta Tip over r bar then what will this become, this will become theta Tip r bar one of the r bar will cancel out right and here lambda r bar. So, I will have lambda r bar. Now, what is the area of the angular, disk annular area is 2 pi r bar dr bar that mean, I am saying lift per unit area or thrust per unit area, you can call it thrust per unit area, area is a Non-dimensional please note.

I am using Non-dimensional thrust, this is the thrust, if I divide by 2 pi r bar dr bar that means, what r bar dr bar will go off and this is a theta tip and my lambda is uniform inflow right that mean, I get this for uniform inflow, Non-dimensional thrust per unit area or in other words this is the, which means my loading, I distribute the load uniformly please understand, because it is not that some area I load more and some other area I load less, it is like my entire weight of the helicopter is uniformly distributed over the rotor disk. In other words, equal area support equal weight which is a good design because you do not want to put some heavy load in one place and very lightly loaded in some other region.

So, you see the uniform inflow has lot of implication that is why; you have to try to achieve that uniform inflow. In the earlier days, people were not making a twisted blade, but nowadays all the blades are twisted twisted blade, but of course, they are all uniform cross section because from manufacturing considerations, most of the blades are made uniform cross section. Now, you see, this is the ideal twist, but manufacturing ideal twist is a big difficult. So, you now know, I am not going to take the twist till r equal to 0.

So, I stop at may be twenty percent of the blades span, I stop because my aerodynamics section usually starts around 0. 2 5 0. 2 5 of the radius, so, I can twist only up to that point. Now, that is why you now know, why that non-uniform inflow calculations are done and from there only, you can show this. Uniform why, what type of twist, ideal twist otherwise this is the this is the proof, but then for this proof, you need blade element theory, you need momentum theory that is differential momentum theory, these two thrusts you equate and then get the inflow, put it there you see, if I want uniform inflow my blade must be twisted.

This is all for hover, please understand this is all for hover. There are certain other important points, I just briefly mention two of them now, before I leave you. I mention that that tip region, the lift as what we saw, it goes like a parabola to the tip r r square, but the near the tip region, the lift is actually 0, it is not a high value so, there is a drop.

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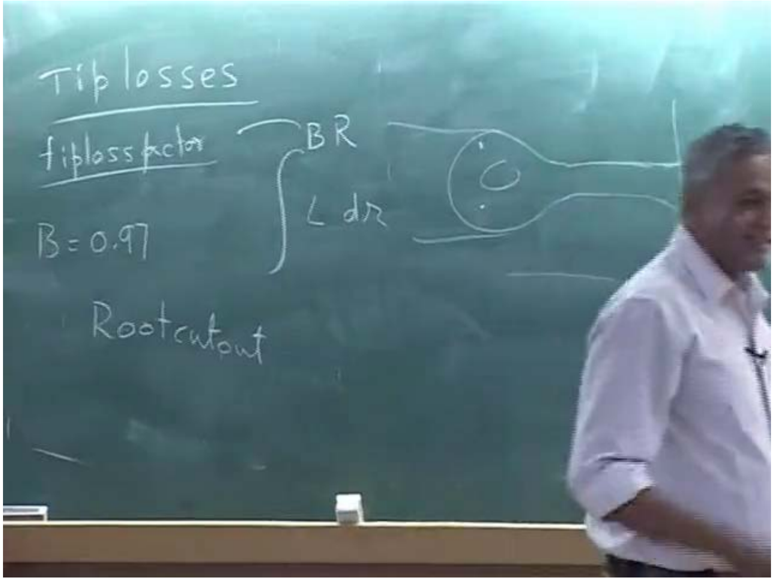

Now, there is a correction you have to give this is called the tip losses tip loss, because the flow is going to be three dimensional flow there, because even, if you take a vertex then it will go up, but if you really want to precise calculation, you have to use a vertex theory, but earlier (( )) formulated, he used a two dimensional vertex way because number of blades become important because the tip, how the wake will go, how many blades are there, everything is important.

So, he made a theoretical calculation and then he came up there are several people used to different derivations, I will not go into the details, what this final conclusion of that is, instead of integrating that lift dr from 0 to 1, do not go to 1, you go to 1 is actually non-dimensional as B, B is a tip loss factor because somebody I given this this we has to B is tip loss factor and it is less than 1 which is, what is the value is they found that, if you use a factor B equals 0.97 for the lift part, drag part you take it till the end, but you integrate your lift only up to 0.97 of the radius, do not integrate till the end they find that this expression is good, because that lift whatever you theoretically predict is very good.

And now, industry uses most of our calculation also we use tip loss factor, but if you really want to do a very detailed tip for those who are interested in aerodynamics, you can take the real tip and you have you can do line vertex or a sheet vortex then really calculate the quantity because the flow near the tip is very complicated because it gives to noise, it gives to drag there are so many other factors, but now industry uses different aerofoils because near the tip they do not use the same aerofoil, they try to use a thinner aerofoil because the drag will be higher.

So, to reduce the drag, they make a thin aerofoil near the tip, but that tip section, if you really see 0.95 to 1 only in that zone they will change , rest of the section it will be same. Now, the research part of it is essentially refining further and further your analysis capability and improving the performance of the rotor and there is one, which is the best tip there is no answer today, because I I can court from one of the person from industry, he said that there is no optimum tip.

Till now, nobody knows every industry uses its own tip because they will not give any details out and there is one which is called as I think (( )) they put berp some British experimental rotor program that is called the berp B E R P, that tip, if you see the blade it will be like this, the tip will be like a paddle something like this in a very funny form. Some people may use like this, some people may use like this, there are millions types of tips, but now, if you want to really do for every tip experiment. You know, it is good actually, I thought that we have a test ring, we should be able to manufacture different different tips, shapes just make an attachment and do it just like a future research.

Then, you will have your own wealth of data to say, what tip is good, but then predicting that with your analysis, that is still the gap is quite a bit, if you want to really go to find values otherwise gross values, I am telling you otherwise you can you will not be able to fly the helicopter, we make lot of assumptions, but still the mean values, what is the thrust, what is the pitch angle, that is all whatever this simple theory is good enough for you to give all those things that is why helicopter flies, but if you go to more complicated things, I want to improve.

Then more detailed study is required that is why, tip loss is one, another one is root cut out, I put only here, I did not put, the blade aerofoil does not start from 0, because you have a hub and your blade root section will be something like this, because you have to take put it up a bolt attach etcetera and the hub will come and the aerodynamic section will start only after some distance that means, all this portion there is no lift, you will get only drag. Now, you have to take that also in your calculation, but in all our calculation because it is easy, you integrate from 0 to 1 0 to 1, it is easy you write an expression and everything is compact which is good, but for a little better calculation, more realistic calculation I would say you take a root cut out.

And the root cut out, you call it as some e offset, you can call it root offset. So, this is the root cut out, this is the tip loss factor in calculating for thrust, drag, you take the everything, is it clear. Now, I thing, I will send you one assignment because I am preparing it, I will send by e mail, you make calculation I want you to generate, I will give you the data only these equations there is nothing more, but for a rotor then you plot the graph, how they vary with various twist expressions, but you have to trim the, trim means C T must be balanced. So, you have to do some iterative calcu you have to write code, but do not copy one fellow write everybody friends, but I want finally, the Figure

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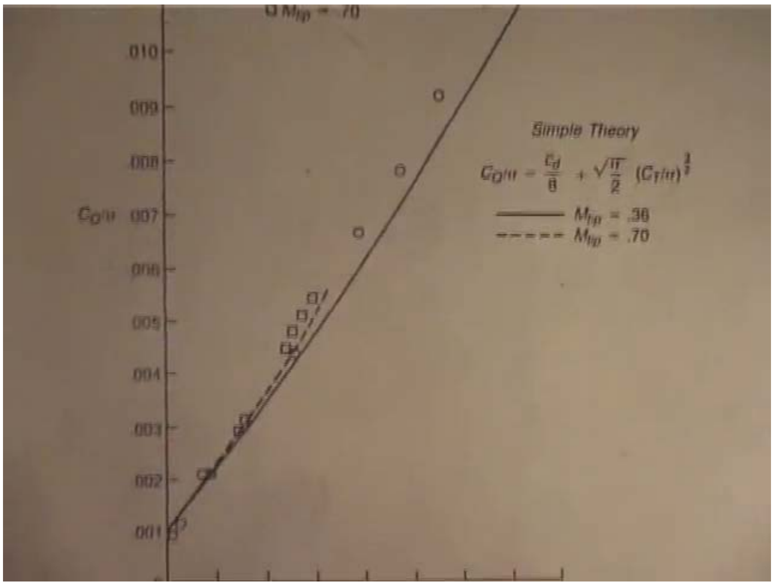

Now, since you asked I will show you one diagram, this is the curve C Q, C Q is C p torque coefficient C Q over sigma, this is C T over sigma power three by two because that expression, what was that we had.

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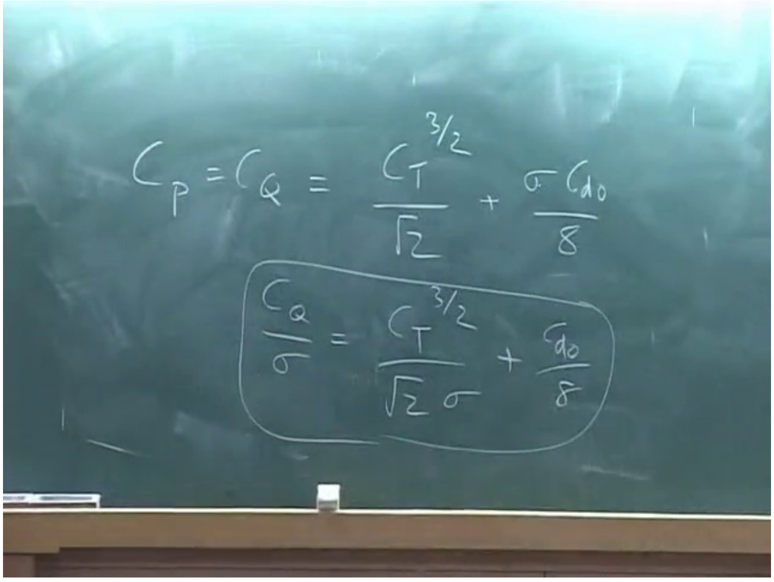

See, you had C p is C Q which was C T power 3 by 2 root 2 plus sigma right. So, you divide by sigma, if you divide by sigma this will be (no audio from 1:13:51 to1:14:02) and C d naught over 8, if I plot C Q by sigma and this by sigma, this is a straight line C T power 3 by 2, and this line is, this line (no audio from 1:14:20 to 1:14:25) C T by sigma power 3 by 2, this is 3 by 2

(( ))

No no this is an error that solve. It should be

(( ))

usually C T you you compare rotors of similar class, rotor means similar weight class, do not write above fully from, but even then we compare two helicopters with a different weight class figure of merit is not treated very seriously in actual design of course, if it is below five or 0.5 or other thing is a bad rotor.

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Solidity is very important because solidity gives you, I told you C T by sigma, C T by sigma is blade loading which is also blade pitch angle that is why, simply saying sigma does not make much, but saying C T is because thrust coefficient that means, am I in the helicopter class, because it is usually in the range of point naught naught 5, but C T by sigma sigma by itself, if I give you you can take a propeller plate and give a sigma value.

It does not specify whether it corresponds to a helicopter or a propeller or something like that, but C T by sigma that is why, C T by sigma in last class also that is, it has so many meaning, in the hover case you know that, it is approximately the pitch angle. Now, if C T by sigma keeps on increasing that means what, the mean pitch angle you are increasing, mean pitch angle increase means you can increase up to some point after that blade is going to stall that means, you can go up to that C T.

Suppose, you design a helicopter, how much because you do not design only for that particular weight even though you design for four thousand kg, you know sometimes you may have to carry a little you know, the capability, but this factor in hover is yes, you were relate to stall same thing and happen to forward flight also, but if I simply represents C T by sigma for a helicopter, C T is fixed because the weight is known, rho pi r square omega rho is known, sigma is known that means, C T by sigma cannot vary for a helicopter please understand, but C T by sigma as a parameter, if you say that is the non-dimensional blade loading or indirectly pitch angle, what is the capability of the rotor, how much C T, I can really take it before the rotor becomes, one way is stall another way, it is vibrating there are several factors, another one is a hub load increases.

So, you can set various criteria and you take the rotor and then you say, I am taking into the extreme of the rotor because internal we can do, keep taking to the extreme. And you say beyond that my pitch link because this have it was define they get the pitch link load because the pitch angle is the moving and the pitch link load is directly depended on torsional moment, because when the blade stalls, torsion moment comes this is a dynamic load, please understand dynamic stall means, it is accelerating, your oscillatory load will increase which is related to fatigue life.

So, they say where will I start having large oscillatory load. Now, large means what large. Now, you have to define some number. I have my limit of infinite life metals and then I will say three times the oscillatory load, if I get that some situation that mean, I have reached my limit I cannot go beyond that. So, this is how they try to come up with some kind of a limit of the rotor because in certain operations C T by sigma everything, but the pitch link load will go up to such an extent then pilot will not be and some time the vibration will go to high value, he will not be able to operate the weight that is why, it is very interesting to see you know, because this is based on my discussion with the industry people. What is the best helicopter, how do you say, a helicopter is best or rotor system.

He said that, if my operation is limited by power then it is the best helicopter. In the sense, I do not have engine power that is, I am not able to fly more otherwise everything is wonderful, but invariably please understand. Most of the helicopters, it is not restricted by power. It is actually restricted by suddenly vibration will increase at some speed beyond that you have to sit in a helicopter to see the vibration. I am telling you, noise will go up pitch link load will go up, pilot will find it difficult to control even though, I have power in my engine please understand.

I cannot take the vehicle beyond, because I cannot control the vehicle, vibration is very heavy then you say so, what sets limit is not the engine power, it is the other characteristics really is setting the limit of your helicopter you follow, and this is where still problems are there. It is a very challenging problem you will see, but know it is really beautiful, so many factors you have to consider, you say I am everything I can satisfy. So, I can fly, if you give me more power I can take it.

So, power restricted helicopter is the best helicopter, but invariably it is the other restrictions which will set, you cannot go beyond this speed because beyond that speed, if you go the whole thing will shake like hell, because you have to sit there because I once went a sat in a product, your window fan and everything you know, you cannot hear anybody inside because pilots talk to them only through wireless even though, they will be sitting by the side.

Noise is terrible, that noise you have to open and then hear it because I I came out for about, I would say two to three hours, I did not hear anything absolutely nothing was, it is it is you do not hear and then the vibration, if you see the whole side they were vibrating this much amplitude this much that means, if it is here goes this this huge you thing that because you are you are the (( )) whole thing is going to blow up, take up, that is the level of vibration. Now, imagine it is not the power; power is there, if you want to fly, you fly, if you fly you blow the vehicle. So, these are restrictions which will come out. So, you have to make sure, I reduce these tensions. I think with this, we will close today.