

Introduction to Helicopter Aerodynamics and Dynamics

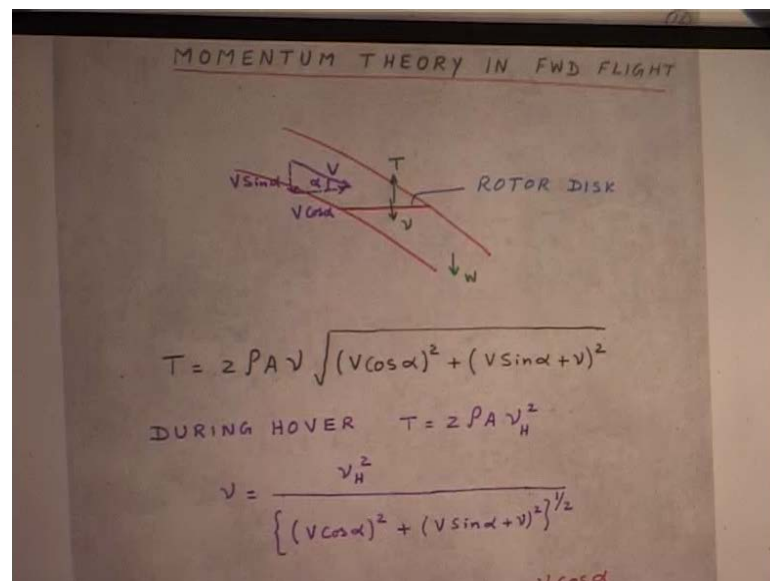
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Lecture No. # 26

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Left last lecture, essentially, we got the expression for the thrust in forward flight, but the rotor disk is in a state of climbing as well going forward. That is why, we put the $V \cos \alpha$ $V \sin \alpha$ and the total inflow through the rotor disk, you will have the component of this v and this is due to the flight speed. So, this is the $V \sin \alpha$, as well as, v .

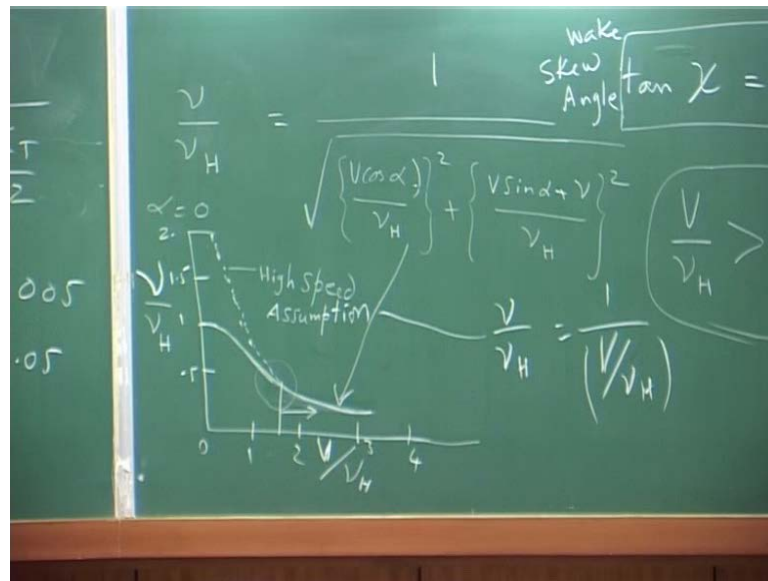
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$$v v = v_H^2 \Rightarrow \frac{v}{v_H} = \frac{1}{\left(\frac{v}{v_H}\right)}$$
$$T = 2 \rho A v \sqrt{(v \cos \alpha)^2 + (v \sin \alpha)^2}$$
$$T = 2 \rho A v_H^2$$
$$v \gg \quad T \approx 2 \rho A v v \cos \alpha$$

Now, usually the expression, that is the thrust, which is written as $\rho A v \cos \alpha$ square plus identical to your similar to momentum theory. The only difference is, now let us plot certain variation of the inflow because in hover it is supporting the same weight of the helicopter. So, thrust is same even if it is climbing forward or moving forward.

So, in hover we write. So, please note, that I am putting a subscript H to denote the difference between the hover inflow and the forward flight inflow. But since the rotor is supporting the same weight, so what you have to do is you equate both of them and write an expression, which is given their v is v_H square or you can, because when you equate both, you will get v is v_H square over this. Ok, straight away.

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And then, you will write this expression.

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So, this is just a modified form of the same inflow expression in forward flight, only thing is, it is non-dimensionalized with respect to the hover inflow.

Now, if you just plot it, because this curve is plotted, how it is done is, I will erase this part because you can plot like this, take it. This is ν over ν_H and on the X-axis you have, this is plotted for, because please understand, it is just an indication, what is the value of α , that is the key. α is the oncoming flow with respect to the horizontal.

Suppose, let us take α is 0, that is easy, if we take α 0, that means, it is, the disk is flying forward straight. Now, you can set α 0, then this is, this term will go of V over ν_H square plus ν over ν_H whole square.

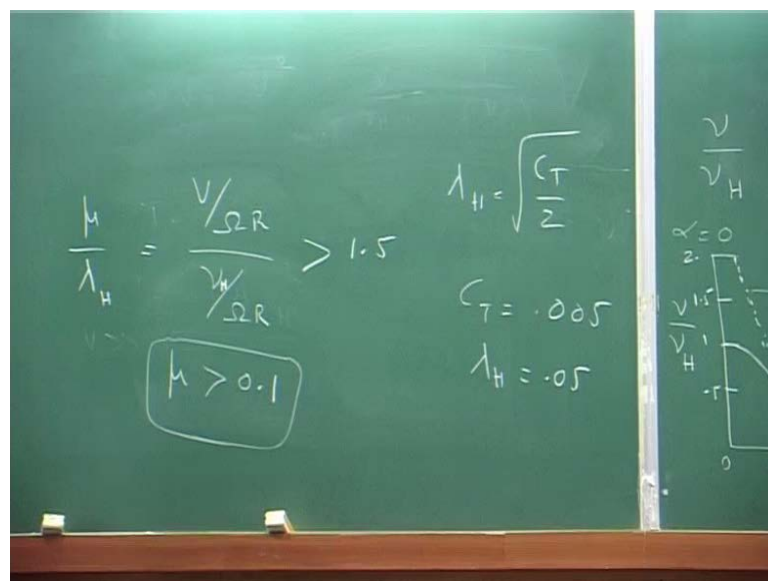
Now, this curve can be plotted, may be 0.5, 1, 1.5 and 2 and here X-axis, you can have V over ν_H , which is 1, 2, 3. So, I am just (()) for α 0. This curve for various values of V over ν_H because α is 0, means this is, this will be, because you know, hover the value is 1 ν over ν_H , the curve will start and go like this. I, may be I am, it is a little sharper, it could be a little; this is the momentum theory in forward, nothing else, how inflow varies with forward speed, that is all.

Now, the another question, which we can have is, because we were asking last class, when the rotor will behave like a wing, at what speed, because we mentioned last class, at high speed the rotor behaves like a wing, in which case, V is much larger than inflow velocity. Therefore, you can write, at, at, for V very large. I am just saying very large, means, how large?

You will find out the thrust becomes (C_T) , only this term $V \cos \alpha$, but now when α is 0, that is, the disk is flying horizontally, this term will become 1. So, my thrust is this expression. Now, you equate these two because thrust is, it is supporting the same weight of the helicopter. So, you will have $\mu \nu H$, sorry, ν into V is νH square. So, you will have..., you take $1 \nu H$ this will lead to..., very simple.

So, you see, I am making an assumption, that is, flying at high speed and if you plot this curve, this is the exact, whatever momentum theory says, this is, if I plot this curve, this is again, ν over νH 1 over v , this is like a 1 over x type of curve, that 1 over x type curve will come, it will go, this is directly high speed assumption. This is high speed, this curve because you know, that as V become 0, this going to become asymptotically infinite, but in general, the deviation will start V over νH greater than 1.5. So, this is, now your question is V over νH , you can make this assumption greater than 1.5; the high speed assumption is fine. Assumption of high speed, that is, I do not bother about this complicated, but this is only for very basic level.

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Now, let us look at in terms of non-dimensional quantities, non-dimensional quantity you will have, because V over ωR , this is nothing, but approximately, you take it the $V \cos \alpha$, α is 0 we have taken. So, this is basically, μ over λ hover. Now, in non-dimensional, usually forward speed, rather than specifying forward speed, you put it in non-dimensional forward speed, which is in terms of advance ratio.

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The image shows handwritten mathematical derivations on a piece of paper. At the top, there is a small diagram with a downward arrow labeled 'w'. Below it, the following equations are written:

$$T = 2 \rho A v \sqrt{(V \cos \alpha)^2 + (V \sin \alpha + v)^2}$$

DURING HOVER $T = 2 \rho A v_H^2$

$$v = \frac{v_H^2}{\left\{ (V \cos \alpha)^2 + (V \sin \alpha + v)^2 \right\}^{1/2}}$$

ADVANCE RATIO $\mu = \frac{V \cos \alpha}{\omega R}$

TOTAL INDUCED VELOCITY $\lambda = \frac{V \sin \alpha + v}{\omega R} = \mu \tan \alpha + \lambda_i$

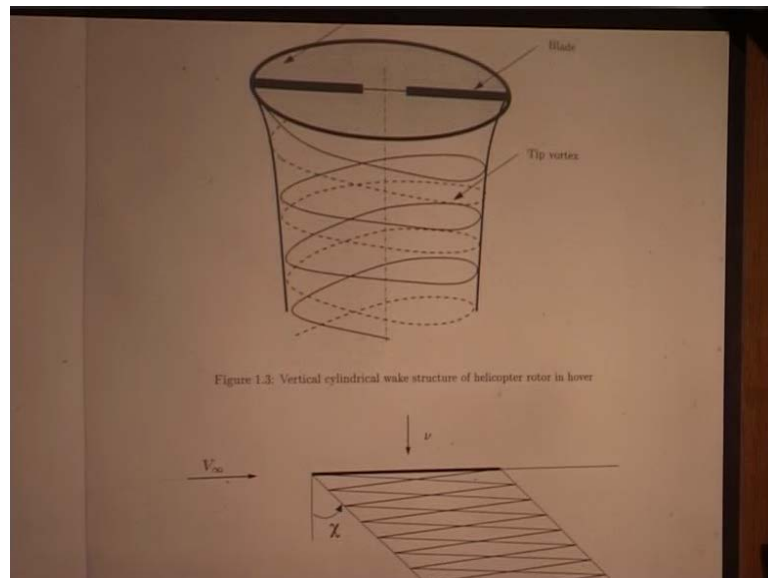
$$\lambda_i = \frac{C_T}{2 \sqrt{\mu^2 + (\mu \tan \alpha + \lambda_i)^2}}$$

Because we defined last time, advance ratio is $V \cos \alpha$ over ωR , we took it the velocity in the disk divided by (ωR) . Now, for what non-dimensional value you can make the assumption of almost the wing, sorry, the rotor acts like a wing.

Usually, λ_H is a hover inflow, we know λ_H is C_T by 2 and if C_T is, I am saying up 0.05, then λ_H is 0.05. Now, if you take 0.05 into 5, 0.075. So, usually they say, μ greater than 0.1, greater than 0.1, then you can use the high speed assumption for inflow. The wing, basically the rotor behaves like a circular wing, reasonable, but in between 0 to 0.1 in the forward speed.

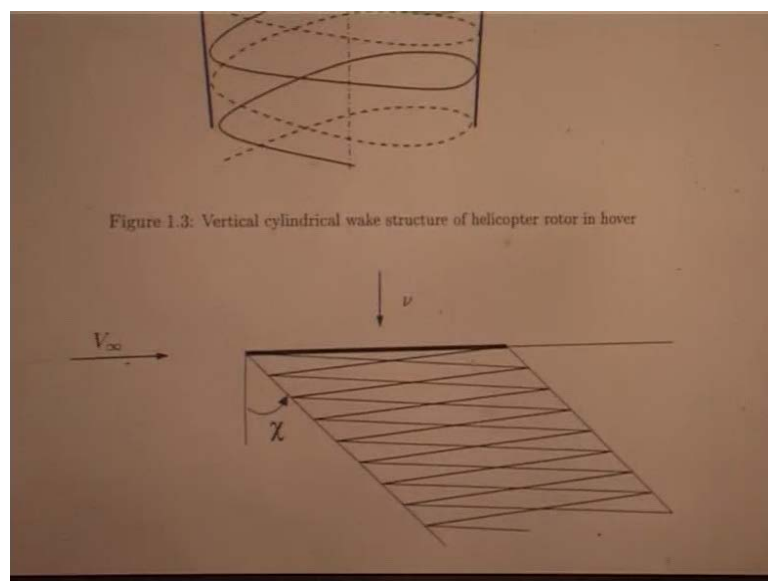
Because this is forward speed advance ratio in the range 0 to 0.1, but μ is greater than 0, yeah, in this range, usually it is called, rotor is called in transition.

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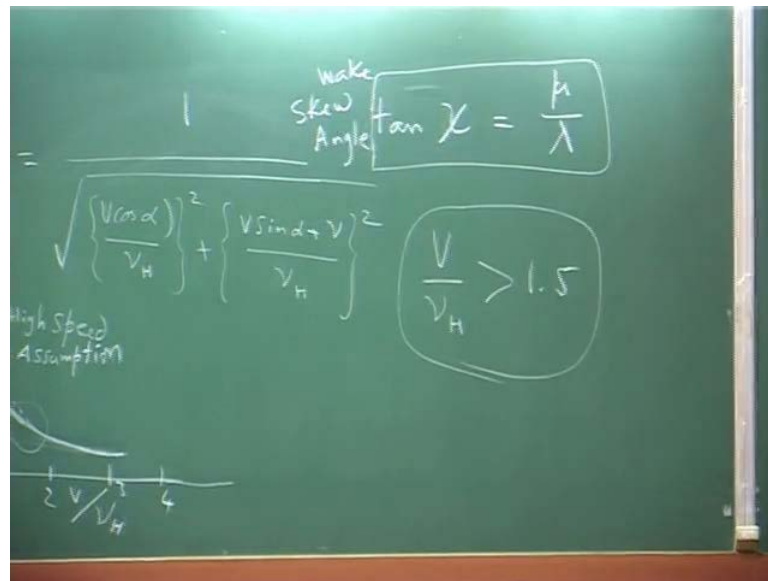


The time the rotor wake, I will show, because one, you start the forward speed, now if you look at this, what really happens is, initially the rotor wake, wake in the sense, whatever is the disturbance, which is coming out from the rotor blade, the lifting theory or anything, the wake comes below the rotor straight. But once you start moving forward, the wake gets, it is in a skewed cylinder.

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When you go in a skewed cylinder, as you increase the speed, this angle is going to keep on changing. So, that is why, in this range of 0 to 0.1. Let us write this as a tan from this diagram, tan chi, this is called the wake skew angle; tan chi is basically nu over lambda wake skew angle.

And if we apply this rule, we said, that nu over lambda H, approximately 1.5. If you take it, usually the angle of that becomes, see, 0; when nu is 0, tan chi is 0, that is straight. When nu is very large, the wake is swept and this angle is almost 90 degree. If you go around advance ratio 0.1, then that angle is close to around 60 degrees. So, in the range of 0 to 60, that is the transition, in the sense, the wake is not completely swept behind the rotor disk, it is not straight below also, but it is very close to that and then, it is skew.

And this really complicates the problem because the wake is close. When the blade travels, the wake from the front blade will close to the rear blade, usually you will have high vibration, etcetera in that transition zone. Normally, helicopter you do not fly in this. Of course, you have to, when you increase the speed you will be going in that, but you do not try to operate fully flying.

Later, when you go to the power curve, if you want to fly at a minimum power, minimum power point is actually beyond this 0.1, it is around 0.15, somewhere around that. But if you want good range, then there is another, which is still further. So, you find that this zone, where it is 60, 0 to 60, you have lot of variation in the vibration, but the

important thing is, the rotor behaves like a circular wing and I said, aspect ratio is only very small. And as a result, there is a lot of variation in the inflow.

Our assumption of uniform inflow over the entire disk is not really valid because you have done one homework, even in the hover case it is not, but in forward flight it is still never. Now, how do we get the varying inflow in the rotor disk? But please remember, this equation or the earlier expression, which I showed you, they are used to get the, this, this relationship, please understand.

This is induced flow, ν over ωR . This I wrote last time, this is essentially the equation, which you use it to get the, but please remember, here it is non-dimensionalized with respect to ωR , tip speed. Here, when we did, we did the, this curve non-dimensionalized with respect to hover inflow. Of course, you can divide by ωR , both of them, then it will become λI over λH . Here you can write it as μ over λH . So, these are different ways of...

(())

This curve, see this curve is essentially, I wrote the high, the high speed assumption, that is, v over... high speed. Alpha is, see alpha is small, you can take it because cosine alpha is almost, but it is the high speed assumption and if I plot this curve, please understand, I have plotted this with alpha 0, exactly this is the curve, this continuous line.

Whereas, when I plot this, is almost come close up to 1.5, then only it starts deviating. This is, this curve, this is, this curve with alpha 0.

(())

This one, no, this is not high speed break.

(())

Why, I can fly horizontally, that is not high speed. Please understand, high speed is when V is much larger than ν , it is not nothing do with alpha. Please understand, alpha does not define the high speed, it is the velocity defines the high speed. Alpha defines the, how, whether you are climbing and moving forward, that is all. I can have alpha 0 and the disk can go like this, is it clear?

That is, based on that assumption, you draw this curve, but later you realize that things are slightly different. This is just for understanding; that is why, your inflow quick calculation.

If you want to know if the velocity is non-dimensional, more than 0.1 mu, then you can take it my inflow calculation is pretty quick. But you have some question?

(())

No, no, this is, no, this is V, V, this is nu over nu H.

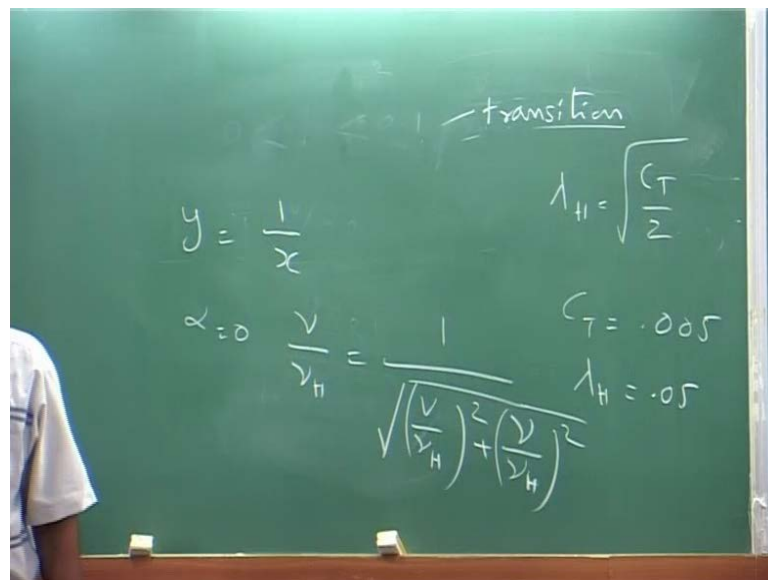
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Yeah, when it becomes low, it shoots. This is nu, this is y, when it is 0 what happens?

(())

Yeah, high speed, V is higher, this is the x. I am plotting a curve Y equals 1 over x.

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1 over x curve will just go like this. What, you guys have a doubt? See, I am just plotting Y equals 1 over h 1 over x, how this curve will be?

(())

Yeah, V is higher only, when v is higher, v is higher means, this expression is same as this expression; same means, they give almost same values, is it clear.

See, this is exact, as far as in a momentum theory. I do not make an assumption that μ is much smaller than $v \cos \alpha$ or anything like that, only thing is. α equal to 0 because I have to assume them, α , I took it, that 0 value and I am getting inflow purely from this equation, that is, when α is 0, my inflow will be...

But this I cannot directly solve because it is in both sides, I have to do iterative, is it clear. Because μ over μH is on right hand side also, otherwise what you will say, I will square it, make and then as a, what it is, become a 4th, 4th order and then, try to get the roots.

That is different thing, you can do it, but then when you do, you do a meaningful root, otherwise you just take this, this is plotted; is it clear or doubt?

(())

Wait, wait, wait V should be...

(())

Higher, yes, V is higher means, V are definitely, it is greater than 1.

(())

Dotted line, when V tending to 0, it is going...

(())

Yes, high speed assumption is only on this side I am saying that, but is it still clear or not clear? I am plotting two curves, one curve with high speed assumption, another curve no assumption, straightaway, directly from here; it is the momentum theory. Now, I plot this curve, this curve is this.

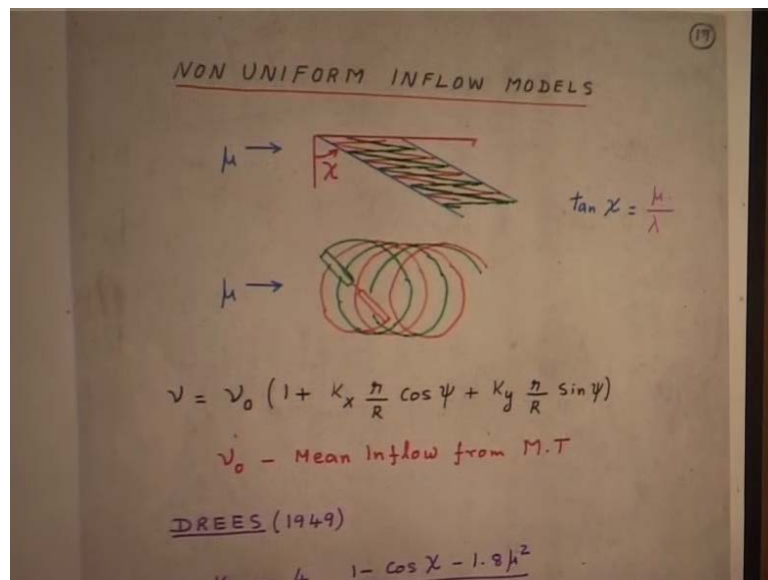
When I make a high speed assumption, that is, this curve, now at what point I can make the high speed assumption, at what velocity? That was the question, the velocity at which you can make the high speed assumption.

You are not making lot of error is when V over νH is greater than 1.5, that corresponds to, if you take νH as, sorry, λH is root of $C T$ by 2, so you get this. So, V over νH , you can non-dimensionalize as μ over λH . It is greater than 0.075 precisely, but you normally take greater than 0.1, 0 to 0.1 forward speed, which is the advance ratio, forward speed divided by tip speed. If it is in the range of 0 to 0.1, you say, that is transition, is it clear.

But during the time, that region, how the wake, wake is close to the rotor and it is of course, getting skewed. As you increase the speed, the wake goes back. So, that is why the assumption of ((C)), because if you want to get quick answer what is the inflow, you cannot make that assumption in this zone, beyond that it is ok, is it clear, alright.

Now, the question is, rotor behaves like a circular disk, now circular disk, we in this, we assume, that the inflow is constant everywhere, but which is not true. There will be substantial variation in the inflow over the radial as well as **Azimuthal** location. Then, you have to have a theory, but this is, I will briefly give a, this is only history, what we are going do is we will follow it.

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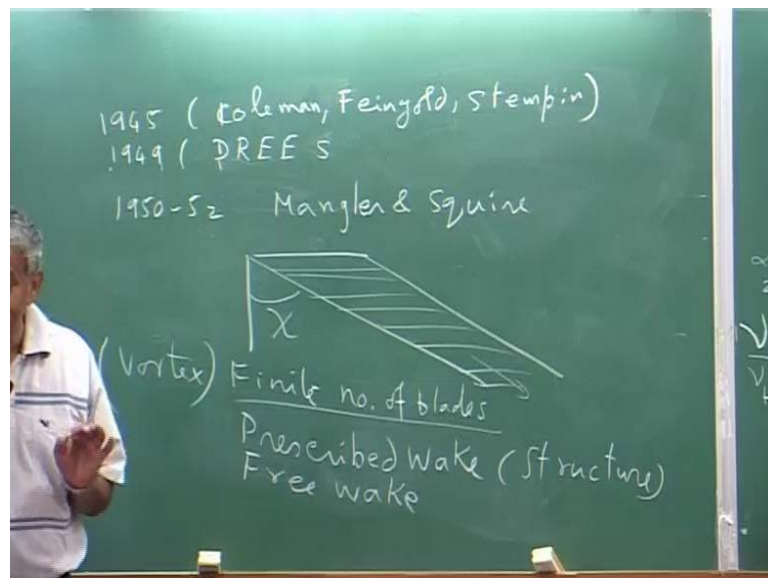


Well, I am going to show, this is how the wake, I have just drawn for two blades, how they keep moving behind because each blade will give out **import x**, as well as, you will have a ((C))x also. I showed you, you will have trailing as well as **shed**.

But if you make the assumption, that my loading is constant that means, my circulation γ is constant over the span of the blade. That means, you will have only tip vortex and the tip vortex is what is shown here. So, you will find, they will come in each blade, will give and they may go and interact with the other blade. So, you will have lot of helix inter point initially.

Now, I am just giving you a brief history, how do you get the inflow? So, it is, it is a very interesting part, what you do is, you assume actuator disk. Actuator disk **win**, it has infinite number of blades and every blade is giving out. That means, it is almost like a skewed cylinder, but assume, uniform loading γ is constant, that means, it will be like a lot of wake semi-infinite skewed cylinder. With this assumption, I will just write the, because I will not go into the details of this formulation because **these are all a little...**

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So, this is actuator disk first, that means, infinite number of blades momentum, similar because if you want to get momentum theory, does not give you the very variation here.

Please understand, you do not start using the differential momentum theory what we derived in the class. That λI , which is the function of the blade pitch angle, at every section we had, that σA over 16 , etcetera, right, do not use that here.

So, you assume, the wake is, this is the cylinder, that is all. So, you have, using this kind of, you find out this wake structure, you assume uniform loading, which means γ is constant.

How it became like this? Purely, because of the forward speed. So, this was done in 1945, that is, by Coleman, Feingold and Stempin. 1945 they used this, they got some closed form expression, I am not, I am leaving it, just giving in terms of a closed form expression for the inflow variation along the rotor, only in the 4 and half direction. Then, in 1949, I think that is 49, yes 1949, he made a slightly different assumption. He said, that hey, I am not going to assume uniform loading, but my loading is going to vary sinusoidally. So, he had a trailing as well as shed vortex that means, his circulation is constant along the blade span, but it varies with Azimuth location. That means, you will have a trailing as well as shed vortex, with that he made, this is called Drees model. He again got the inflow variation on that.

Then, in 1950s, using the potential flow theory, the 49, 50, 52, that period, but please note all of them are actuated this theory. 1950, 52, I will put it; this is Mangler and Squire using potential flow in a pressure field. So, this is like a disk, it is the potential flow assumption and then, get the pressure difference across it and he solved with, he got a series expression for inflow series, long series. Now, these are all till 50. Simultaneously, slowly, the computational capability because they all got close form, please understand, closed form solution. This assumes actuator disk, but that is not the real situation. Real situation, you have number of blade 2 or 3 or 4 or 5, something like that.

So, you have vortex theory, similar to fixed wing, lifting line. But vortex theory, you need to have, this is also vortex theory only, but it assumes infinite number of blades, whereas here finite number of blades; vortex theory, finite number of blades, which is a realistic situation. But then, what is the structure of the wake?

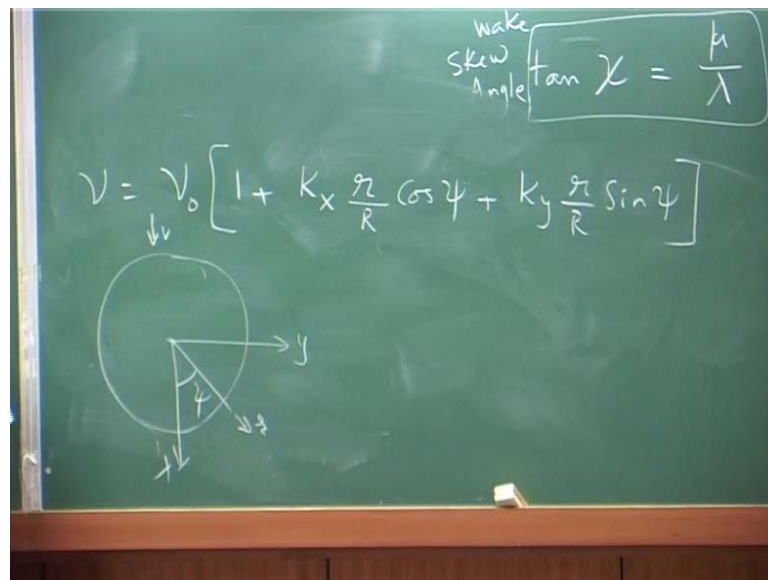
This is the structure I have assumed, that means, I will take something similar, like this. Structure is decided, only thing is, I do not know the strength and this is the prescribed wake analysis, prescribed. That means, you prescribe the, basically prescribed wake, wake structure, but another one is free wake. That means, you allow completely, that is, wake can interact with another wake and finally, it will come to its own equilibrium for a

given condition. These are all computationally more involved and of course, prescribed wake is reasonably, you know, better than free wake; free wake computationally, very, you know, time consuming. But these are research fields, which people have been working on.

Now, all of them, please understand, this is very, very interesting, one is finite number of blades, this is infinite blades; they got in the 50s, these are all even. Now, it is going on, people have developed models, some people use it, but some people use even this. We use this and these results have, industry, they may have some models, if they want, they will use this part if it is available, otherwise these are good, absolutely no problem, only thing is, more and more accurate determination of vibratory loads. Yes, then still it is the question of debate, where I go and then improve my model. So, I am talking about only inflow modeling, nothing more, inflow modeling in forward flight steady case.

Now, in this course, what we will do? I will briefly mention what these are, how all of them ultimately is represented, wake skew angle, that is all these. What is the only parameter is wake skew angle.

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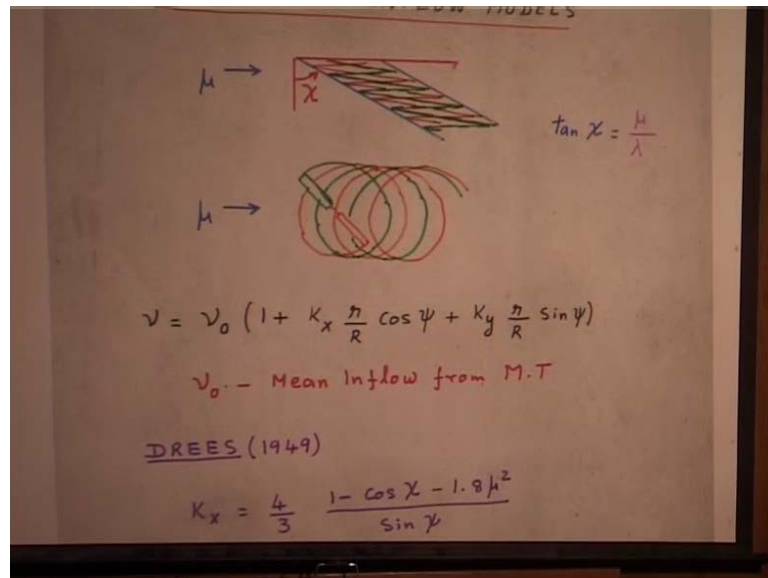


Because you write your nu... k x, k x..., alright. Here, what is this nu naught? So, this is my rotor disk and this is my, this is the traction of flight, this is x-axis, this is y-axis. This angle is psi and this is the radian, that means, please understand, here psi represents the Azimuthal location in this disk r over R. And basically, r represents wherever in the

distance, nu naught is some kind of a mean value, mean inflow. But please, now I am using one more symbol, nu sub-zero, but this is in forward flight. That means, inflow is basically the value you obtain from the momentum theory. So, that is this value.

So, now, you nu 0 is this quantity, though I write is nu because this assumes uniform inflow. Therefore, nu 0 or in other words, if it is a, you can use this expression lambda I if you know C T directly. So, even though I put I, it now I am changed to 0 lambda I into omega R, that becomes nu naught. So, please understand, there are different symbols which may be confusing, but the idea is, you have to understand clearly, that what we are using; is it clear?

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Because you assume there is a uniform inflow, which you get it from momentum theory, this is, I obtain these two quantities: k_x , k_y represent essentially the variation with respect to r and ψ and that is all. And what are these? This I will write it, I wrote one of them, but I will give the other one, what just for reference.

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1945 (Coleman, Feingold, Stempin)

$$k_x = \tan \frac{\chi}{2} = \sqrt{1 + \left(\frac{\lambda}{\nu}\right)^2} - \left|\frac{\lambda}{\nu}\right|$$
$$k_y = 0$$

Drees

$$k_x = \frac{4}{3} \frac{1 - \cos \chi - 1.8 \mu^2}{\sin \chi} = \frac{4}{3} \left[(1 - 1.8 \mu^2) \sqrt{1 + \left(\frac{\lambda}{\nu}\right)^2} - \left|\frac{\lambda}{\nu}\right| \right]$$
$$k_y = -2 \mu$$

He obtained is k_x , please understand, this is $\tan \chi/2$ and k_y is 0. That means, this is not there, this term is $\left(\frac{\lambda}{\nu}\right)$, this term is there and this given directly in terms of $\chi/2$, which is also given by this $\tan \chi/2$, you know, this you can get it because $\tan \chi$ is ν over λ . But please understand, in this the λ includes $V \sin \alpha + \nu$. Do not think, that, that is only ν because this is the total λ , please understand; this is not only the $\nu V \sin \alpha$ you take.

Now, you see, I define in terms of μ and λ μ is in the plane of disk λ , is normal to the disk, that is all you follow, these are the two quantities.

Now, Drees model, he gave $k_x = \frac{4}{3} (1 - \cos \chi - 1.8 \mu^2) / \sin \chi$, which is written as $1 - 1.8 \mu^2$ square root of $1 + \lambda/\nu$ whole square minus $\left(\frac{\lambda}{\nu}\right)$, you can say λ/ν over ν , that is enough. And then, k_y , he also gave k_y , which is -2μ . But please understand, k_x , this is value only forward flight, you cannot apply it for hover because ψ is 0, otherwise you will get infinity only for forward flight. So, here, he says k_x is 0 for hover, that you have to take it.

But please understand, these models, this model is very good; Drees model is a good model. How do you know, whether it is good or bad, that we will come later. See, when I define, it is a good model. Of course there are better models are now, we can use a better approximation, which have come, which we use it in our thesis. Of course, we have option to use this, but please understand, in all these models if you want to use, you still

have to go and get this nu naught from momentum, that relation you have to get that. So, you, I will give you, I just gave a brief, no I would say the overview, because if you want little bit more detail, now it is part of one of the my student's PhD thesis, no, it is like a history because even the reviewer says, said it is pretty neat, good. Just inflow calculation, how it all started, but this is 45, this is 49, 50, but helicopters flew around 39 to 42, but people were struggling to get expressions.

But today, of course, computational capability has improved, but if one wants to use, he has to develop his own computational tool to get an inflow, but some are good physical models and they are able to get results, which are only after getting this you realize, if I use this model I will be able to get certain results, which are resembling the flight test, that is the key. Ultimately, the whole thing is, whether these are valid or not valid, purely depends on how would the results match with the experiments. So, I leave at that part.

Now, we will go in this course, please understand, we will just use this expression. So, if I say Drees model, you know, that this is the Drees model. So, this is, this is sometimes people use, it is old, but Drees model is reasonable, but in all our formulation, please understand, I do not include this because it makes simpler, because most of the expressions, which would derive in the books, whatever you get, it assume uniform inflow, is it clear, which is a gross assumption.

But if you want to get some closed form results in the reasonable length of mathematics, basically algebra, then make an assumption. It is, my inflow is uniform over the whole disk even though it is not correct, is it clear, because this you have to know, because there is this is not the end of it, this is a good approximation for realistic problems, but this not just sufficient. We need to take more time variation also into account here, that becomes, those all started later and today it is at some stage, where we can model the time variation also in an approximate version time variation of inflow.

Please understand this is a constant. When I say constant, it does not vary with time; when I say uniform, it is uniform everywhere; that is all. But uniform, but it can be time varying, that is one model, it can be uniform constant, it can be just constant, but need not be uniform, that is what this model is. But then, the most general is its time varying and it is not uniform and that is the much better approximation. If you want to get some more refined, I would call it more refined results, you need to have that models, but those

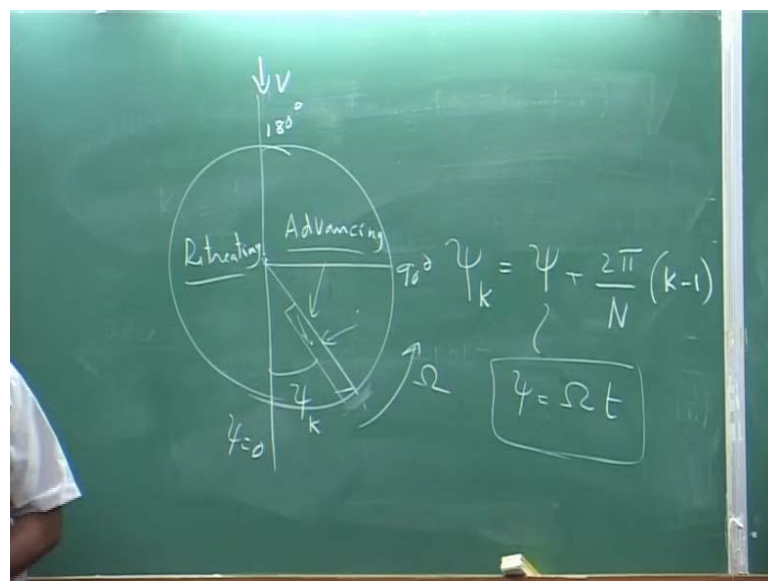
models are time consuming. You cannot put it, you know, simple expression, you have to do computationally, that is why, for this course, because we need to get the loads because hover we learnt, forward flight we need to go and then analyze.

What do we analyze in forward flight? First is, you say, I want to fly at that speed, what should be the control because you need to fly? Whether you were able to achieve equilibrium, equilibrium of forces and moments? Very simple, for the body, for the helicopter to fly at just, take level flight, that is the first. Second one is, if there is a disturbance, is it stable? That is the stability part, that is the flight dynamics, but even the first part, which is called level flight or anything, it is called a trim or equilibrium, but the trim equilibrium part itself is a aero-elastic problem.

Then, you will slowly realize, in forward flight things become much more complicated and then, you start analyzing only some narrow portion for a specific. It is not, that you include everything in your (()) and then analyze it, this is what happens. Now, we got an expression for inflow in forward flight good.

Let us now go back, actually start, we learnt momentum theory, how do we apply because you know, that this is for you to get this nu naught, you need to know C T and the C T was given here. Please remember, C T, C T is given here, lambda I to C T expression is right there. Now, I will close this here; let us go back to the rotor blade.

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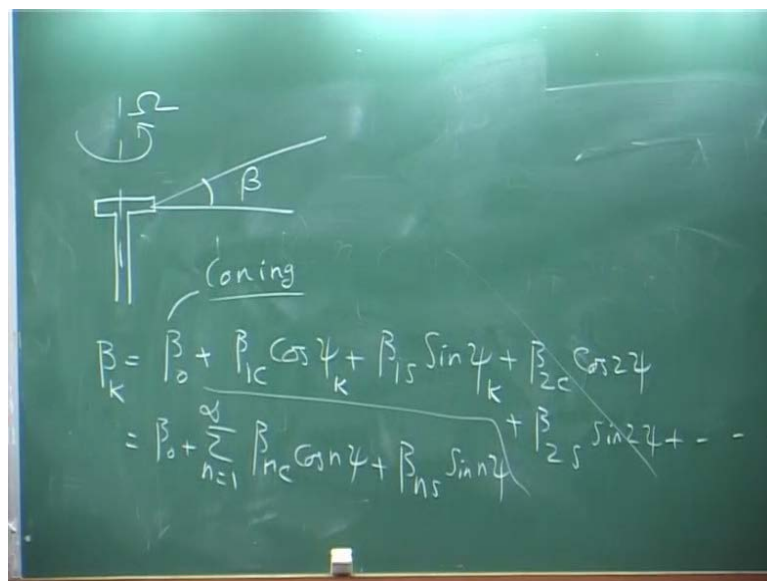
So, you got a, we said, in forward flight because of time varying, this side is more velocity, this is advancing and this is retreating; advancing side more velocity, retreating side less velocity, oncoming flow, I am neglecting radial flow.

But later we will see how we very crudely take the radial flow. Radial flow means, because this flow if I resolve it, one will come here, another will go, this is along the span of the blade, this is like a swept wing flying forward. That means, there is a flow over the surface not along the aerofoil card wise, but in the radial direction, it neglected or we can take approximately some, that we will come to that later.

Now, you know, that this blade is going to experience load and it is going to, we will consider very simplistically, this velocity varies, we know that, sine psi. Now, the lift is going to be, because it is a square of the velocity, so you have harmonic variation of lift. So, the blade is going to go up and down.

Let us consider only the flapping motion, flap. When you say flap that is the out of plane motion of the blade only; out of blade motion. How it will vary because you know, something is sinusoidal or I would say not say periodic, therefore you expect, that flap motion is also periodic, but we will call the flap motion by the symbol beta.

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Let us take a very simple case, the blade is flapping and the flap, let us say, this is the hub and blade is I call it beta. But this is one approximation I am showing because this is

rotating. Later, we will see how to represent the blade. Now, let us say, the flap motion because it is excited by a periodic load, any periodic function you can represent by a Fourier series. Therefore, I am representing the flap motion also as a Fourier series and so on, so on, so forth, which is, write it as $\beta_0 + \sum_{n=1}^{\infty} \beta_n \cos n \psi$ plus. So, my flap motion is a Fourier series. So, we got inflow, which is the time varying function, in the sense, sorry, not time varying, azimuth varying. Now, you have flap motion which is represented like this, which has all the harmonics, but what we will do is in this course, we will say, we are not going to be bothered about this 2nd harmonic onwards.

But please, I would like to caution you, 2nd harmonics is important when you go to slightly high speed; high speed means, I am talking about 0.3 non-dimensional μ , about 0.3. Then, you will find the 2nd harmonic content is more than the 1st harmonic content, but (C) for the present, assume we neglect everything. So, there is a $\beta_0 + \beta_1 \cos \psi + \beta_2 \sin \psi$, only three term approximation. Please understand this is an approximation I am making for the motion of the blade, physically we will look at it, how it really represent. What it really means, this is β_0 is for one blade, but if I want to look at all the blades and if I index them, then I will be giving k , β_k , this is where, you know, this is the ψ_k . If what (C) ψ_k becomes $\psi + 2\pi$, which is the number of the blades. So, if this is the 1st blade, this will be, ψ_1 is nothing but ψ_2 , but this is ωt .

Now, what happens, each blade is represented as Fourier series because we have to know each blade is independent, we are not (C) teetering rotor, it is an independent. This blade behaves, that blade behaves, but all of them have the same expression that means, when it rotates, all the blades execute identical motion. When it comes to a particular value of the azimuth, will have only one value, it is not, that each one is doing independently when it goes round and round. Now, you see, this is the blade β_0 means, it has gone up, that is, all the blades have gone up by the same amount and this is called the coning; all of them have gone up same value.

Now, when you look at (C) c , $\beta_1 c$ represents what? If $\beta_1 c$ is positive, that means, this is my ψ_0 position, this is 180, this is 90, this is 90 degree, this is 180. That means, when the blade comes back to this point, if this is the (C) , this will be like this, it will do like this, at the back it will go up, when it comes to the front it will come. So, when it

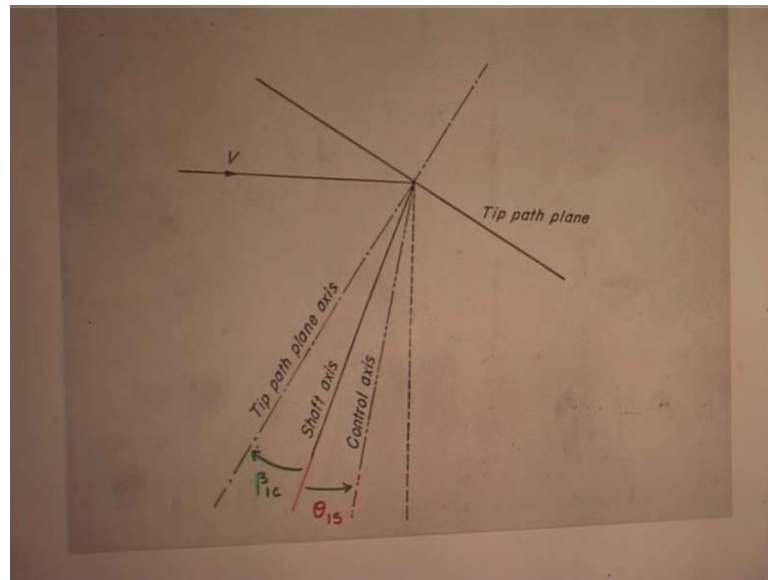
rotates it will do like this, that means, the disk is tilted in the forward direction. So, one is, I took the disk up then I tilted in the forward, but $1s$ represent the tilt in the, because here it is up, here it is down, that is, tilt in the lateral direction.

Now, essentially, what this represents is these three quantities only. The three quantities, the orientation of the disk, disk in the sense, you are outside, you are looking at the tip path plane because all the blades are going round and round, the tip path plane of the rotor disk is like a circular disk. How the circular disk is, circular disk, it can go up, tilt (()), it can tilt, but when you go to higher harmonics, these are all like warping of the plane. It is like, you know, corrugated sheet, how they will have it is, that motion. Now, you see, that the complex representation of that plane, I do not look at the warping; I look at only the beam path. So, the rotor disk, now please understand, without this you cannot fly the helicopter, you need flap because the flap is the one, which is tilted, your thrust vector, whatever is there, you tilt the thrust vector this way, then you fly forward; you tilt that thrust vector in the other direction, you fly. The tilting of the thrust vector is obtained by flapping.

Now, how do we achieve the flapping? Because pilot gives only pitch angle change, you know, that collective pitch angle, all the blades experience the same change in pitch, that means, thrust will go up, more weight, more lift, but when you do cyclic, swash plate, cyclic plate because pilot sticks left side or front, is basically tilting the swash plate, as a result, is giving a cyclic variation in the pitch angle. The cyclic variation of the pitch angle is responsible for tilting the disk because one side increases more lift, other side lifts. So, it kind of tilts, but there is a dynamics involved because it is not just simple because the blade is (()) go.

Now, pilot collective and two cyclic for one main rotor. So, he gives, he can control these three technically. Now, the question is, you said, like this pilot is giving a pitch angle in one frame and you defined your λ μ in some, we put, we drew a line rotor disk, this is the rotor disk line, what is that disk? Is it the tip path plane or is it the hub plane or is it some control plane, what is that plane? Now, you see, you can define several reference axis and this will complicate the problem. I will briefly tell you, there are different types of frames of reference because when I said flap, flap is with respect to what plane? Is it with respect to...

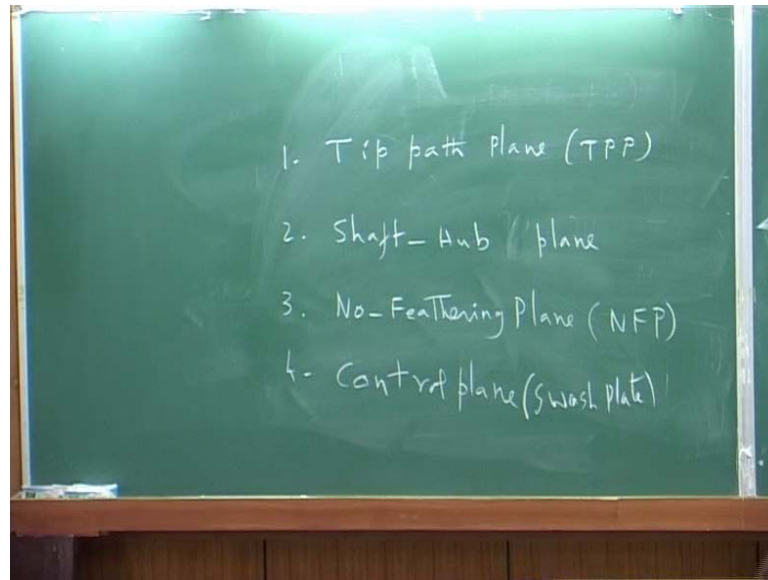
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I will briefly show one diagram and just to indicate, that very simplistically, do not bother about all these things. This is the tip path plane; tip path plane means, you draw a plane, which actually, the blade is going only in that plane no matter where it is, it is going in that plane. The root, do not bother because the blade can flap up, that is why, you are not bothered about the root part. You are looking at the tip of the blade, how it goes round, all the blades will follow the same path because this is the same Fourier expression. These are assumptions we are making because all blades perform the same motion.

Now, that is the tip path plane, then you have shaft, the shaft axis and the hub, that is the hub plane. Now, the hub can be, this is the shaft axis, the shaft axis need not be perpendicular to the tip path plane because the shaft axis can be like this, the blade, the tip path plane can be like this. That means, shaft axis is another one; tip path plane if you want to take that is another axis. Similarly, you can define one more axis, which is called the no feathering plane, no feathering plane, NFP.

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So, I will write here some three, four planes. So, one is tip path plane, this we call it TPP. Then, another one is shaft...

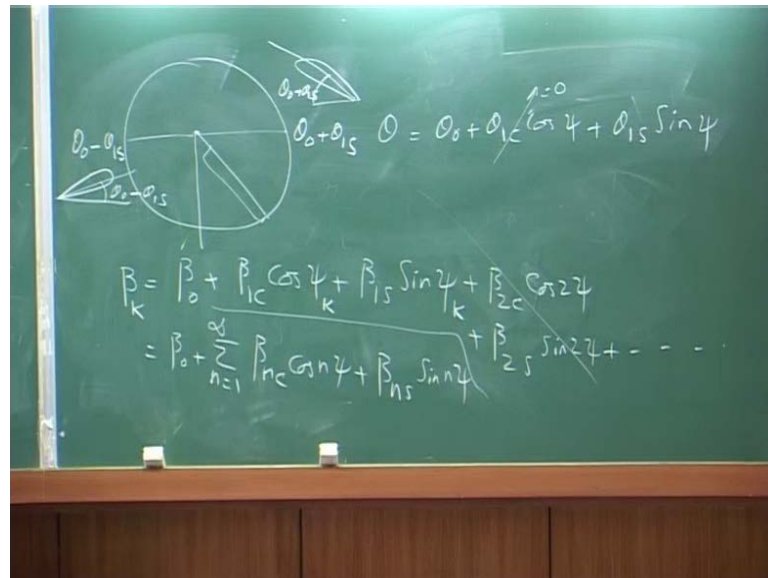
This we call it hub plane because the shaft, hub is perpendicular to the shaft and the shaft is attached to the helicopter. So, the helicopter tilt, shaft also tilts. Then, you also define no feathering plane, this is NFP.

Then, you can have one more plane, which is control plane. Now, the, what is the control? You have to know what each one of them is and you can choose any one of the planes for your problem, but it actually is, everything has its own complexity, but finally, you choose one. That is why, in the text book, some books, old books if you see, you really do not know what they are using unless you are very clear about. Later, after more and more of understanding, oh this is what you see, this is what tip path plane you have agreed.

What happens if you are in that plane? There is no flapping because the blade is only in that plane, blade will not come out of that plane, so there is no flapping in tip path plane, agreed. Because you have a plane, as a blade tip is going in that plane means what? There is no flapping with respect to that plane, but the pitch angle of the blade can change as it goes around in that plane, is it clear.

Now, you choose no feathering plane. In this plane the pitch angle of the blade is constant in that plane; the pitch angle of the blade is a constant. This is, sketching is, maybe we can with computer, three-dimensional we should be able to make it. I can briefly describe you with this diagram, a very simplistic diagram.

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That is, you take this, our theta is what, theta naught, this is what? The pitch angle, let us say this is 0, just for simplicity. When it is 90 degree, the angle is theta naught plus theta 1s. When it is here, it is theta naught, that means, the same aerofoil. If you look here, it will have a large pitch angle, which is plus theta 1s; when you come here, it will have minus theta 1s, less pitch angle. But you want to get a plane in which the blade has theta naught only, that means, what you do?

If you tilt this plane up like this, what will happen? That means, this will go up, here it will come down. So, you will find, that the pitch angle here will reduce, here will increase till you tilt it, such that the pitch angle is same both sides. That is what is shown here, the theta 1s, just for indication.

Now, this is same as, then the, I mentioned control plane. What is control plane? Control plane is basically the plane of the swash plate, but please understand, swash plate I mentioned here because swash plate is there for these helicopters, swash plate plane.

Now, if you do not have any other additional mechanism, please understand, if you do not have any other additional mechanism to introduce pitch in the blade because sometime you can have some coupling, which you will learn later, or you can put a trailing edge tab and that will twist the blade. Your swash blade may be here, trailing edge tab may be twisting, but if you do not have any additional mechanism to, **twist**, change angle of attack of the blade, these two are same, you follow.

So, here we have several, four planes, whether all of them can be same or all of them cannot be same? In certain specific situations, you will find two planes, may be same, otherwise each one is independent. Now, whole point is, what reference axis I must choose for defining my motion of the blade defining because I have to get the aerodynamic load. Please understand, I have to get the aerodynamic load, I have to define my inflow, you follow.

You have so many planes your hanging around, so what we choose in this class? Hub plane, but the complexity in hub plane is, it will have both flapping as well as feathering. Feathering is basically pitch angle change; your blade will have both flapping as well as feathering, that means, you have to take both of them into account, it becomes, expressions become more complex. On the other hand, if you choose this plane, there is flapping only; feathering, if you choose this plane, there is no feathering, but there is only flapping.

So, one of them can be made simpler, but here, you have to consider everything, but this is more systematic and you will not make any mistake. And most of the research we use this plane because that is very systematically you can develop, because these planes are fixed with respect to the helicopter, whereas these are all not fixed with respect to the helicopter, you understand. Now, we will use only hub plane.

Now, the whole point goes, what is my advance ratio? What is my inflow? Everything is referred with respect to hub plane. So, this is the shaft, this is the hub, which is 90 degree. So, you define what is the velocity, in the plane of the hub, normal to the hub, that is all. In the plane of the hub is your forward speed, normal to the hub is your inflow, that is all.

But in this plane, the blade may be like this, it may be twisted, does not matter, but when you go and calculate aerodynamic load on the blade, please understand, you have to define proper coordinate system and this is where you have to have a very systematic development.

If you follow the systematic development it is clear to you because if, most of the, actually all books, material, they do not give the systematic, they will give final expression. They will say take this, $(())$, actually they make lot of approximations in getting those, unless you are sure what is being done you will say, why it is taken like this.

So, what I thought, in my course, in this notes also, which I am giving, we will follow pure, like basic mechanics, like dynamics, we will define a coordinate system, we will define a position vector. Then, we will get the velocity and then you we will get the acceleration, we will define every point.

And then, we will say, at every stage it will become complex, but we will say throw this, throw this, make approximation and then you will get a neat expression, which is to the what you see in some other publications, books, they make lot of assumptions.