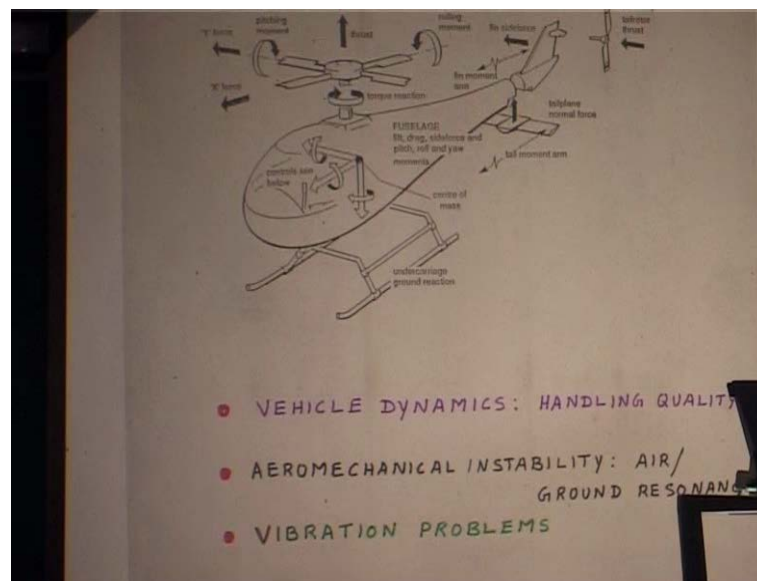


Now, when you want to analyze the rotor blade along with fuselage then what happens is, your blade will get affected; blade motion is affected because of fuselage motion. Because, in all our earlier formulation we said the is hub fixed; hub does not move there is no except forward flight. It is most steadily that is all there is no perturbational motion of the hub. Now, you have a hub motion also you are going to contribute to the dynamics of the blade. So, what was done is they try to classify the coupled rotor fuselage dynamics because, this is the most complex problem and you should have several sequence of coordinates transformation between one coordinate system to the another; to blade rotating, non-rotating, deformed, undeformed because, that kind of a sequential transformation one has to have that is why it cannot be taught in a class, only thing is you will give a glimpse of the complexities and how these problems are tackled.

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So, you have under this heading we try to split the problem into some 3 major categories. See, one is the vehicle dynamics vehicle dynamics means it is like a pilot is sitting, is taking the helicopter or some task or mission for him. He does not bother about how the rotor behaves; only thing is he will see that how the vehicle is behaving. So, that is related to totally the handling quality of the vehicle he will say from starting I have to hover from this point. I have to go to somewhere else do a task come back and during that he should be able to ride the vehicle in a comfortable manner in terms of handling quality then there are certain aeromechanical because this is a word that is used in stability problem. The part of it only thing is you have to make sure that you do not get

into those problems and that is why I put air slash ground resonance. These are 2 types of resonance problems; ground resonance is more catastrophic compared to our resonance. Here also, it is a couple between rotor and fuselage motion.

Now, what type of problem is this? that is, why you have to see in each case how the dynamics itself is changed? And, of course, the last problem is vibration - vibration in the helicopter, not the steady load. Please understand, now this is the vibrating machine - helicopter is the vibrating machine. Only if you sit inside you will know, but if they fly at low speed it is all fine, but when they start flying at high speed then you have to be there basically because, I was fortunate to fly in the **a l h** in the initial protection when they were starting when they flew they will be, you will be **some privacy** sitting at the back the window or the door pane that was vibrating in this much you feel that the whole thing will be shattered which is really scaring because they deliberately did that kind of a loading condition.

So, you have to minimize, but vibration problem always exists in helicopters ever since in the first helicopter was built and even today it is a problem; but, this is being studied for a long time. Industry, they follow certain procedures to reduce the vibration after the helicopter is built because of the simple fact, if you want to know the vibratory load how much is the load that is coming, the estimation of the load from a theoretical formulation to actual reality - that is a big gap because, still we do not know many think we make approximation in blade modeling.

We make approximation in aerodynamic modeling and there are a lot of rotating components in the helicopter. Everything contributes to the vibration and the fuselage is a flexible structure. Now, it is a vibrating structure; now you see in each problem the approximation that is made in modeling and to understand what really happens in the helicopter differ from which problem we were addressing. Suppose if you take I want to study vehicle dynamics; that means, how the vehicle should move when I give a control input, how it turns because pilot has to fly like you have motor cycle you say you want to go at speed your cycle, increase the throttle, but when you increase the throttle it takes it accelerates. Similarly, this also but if it accelerates very fast, very slow, that is how the quality, but that is purely a qualitative statement. The pilot, how he feels comfortable. So, I will come to that point later, but this has to be quantified because one is qualitative **yeah** its good. What it is good means? That it may be good for somebody; it may not be

good for somebody else. So, they try to categorize and then put some kind of a band saying that if it is within the zone to the numbers in general. Pilot that will say it is (()), but in the modeling handling quality is essentially due to if pilot gives a control input how the vehicle responds.

If it is very sluggish then it is sluggish, but if you give a little input it suddenly turns; then it will be very sensitive. So, these are the 2 extremes. So, you have to make it and in the process it should not be unstable that is most important thing the vehicle stability comes into picture, but in solving this problem the fuselage is treated as a rigid body because that is easy, simple. So, treat the fuselage as a rigid body with its own inertia properties mass and all the inertia tensor, but you have to consider the rotor blade in modeling the rotor blade usually I am telling about its not mandatory to get a good understanding the blade is modeled only for flap; only flap motion of the blade is included just to get initially today of course, with the computing power and modeling you know you can have flap lag torsion, but if you have too many things finally, you will not know what is happening why certain things are happening. So, to have an understanding they said we will take first flap motion, but flap dynamics whether they consider the full flap dynamics are they do some approximation now this is again purely based on the dynamics of flap because we learnt when we did the flap motion the flap responds very quickly to control input. The time lag is very less that is, you know one quarter of the revolution. Your given input it takes the time constant we did since it is very fast. So, they say any input the pilot gives the rotor immediately tilts; there is the time delay is less.

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$$F(t) = \underline{a_0} + \sum_{n=1}^{\infty} \underline{a_n} \cos n\omega t + \underline{b_n} \sin n\omega t$$

quasi-static FLAP

$$\left\{ \dot{x} \right\} = [A] \left\{ x \right\} + [B] \left\{ u \right\}$$

So, you can assume it as though it is a quasi static approximation; it is a word right now. You take it as a word because quasi static even though the blade dynamics is it has all the motion, but you neglect all of them. You say input output steady value if I give this much theta naught what will be my or theta naught theta 1 c theta 1 s anything. What will be tilt of rotor disk like? What you solved in the trip problem which we quite a few and gave up, but a 2, solved 2 students. Now, this is the approximation you make and then solve like your aircraft stability problem you have the helicopter stability problem, but only thing is here the pitch and roll are coupled. So, even though in the aircraft terminology you longitudinal stability; you do separately and then lateral separately you do here you can do.

Then you know that this coupling is quite strong you cannot avoid that coupling. So, you have to take the coupling and this is used for all control stability characteristics of the vehicle. Now, these equations, I will just write the equation I will not write anything beyond you can put it like this is the standard. You all may know $\dot{x} = Ax + Bu$ is the control input x are the states which you may say states are your u, v, w 3 velocities 3 angular motions, but who will give me the matrix A and the matrix B ? now, this is where the fill starts; one is you theoretically you can estimate why it is like approximation. Only you can do you take the rotor loads transfer the rotor loads the fuselage and then check for a small change. What is the change in the moment and force etcetera? These are the system dynamics. System dynamics which includes flap motion is here, but if you

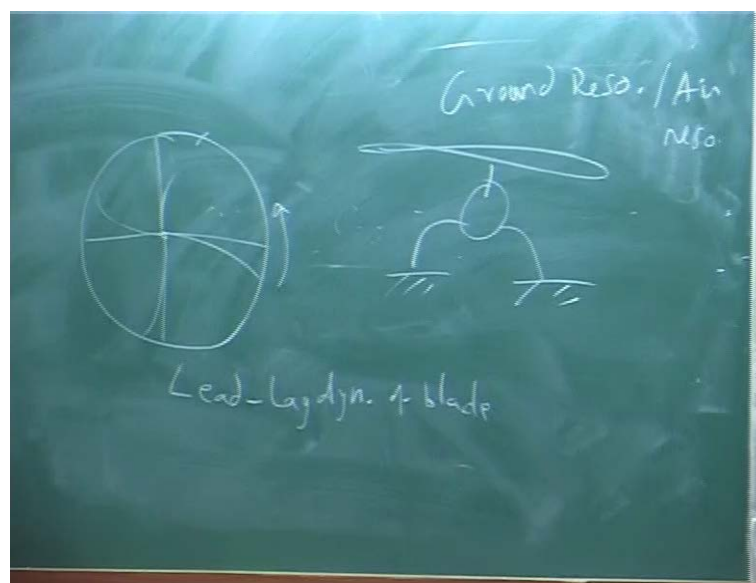
want to treat flap separately then you have to add that equations. That is why you use quasi-static flap and link how flap is affected because of fuselage motion in aerodynamics. So, loop and then you get and this is the control derivatives these are you may call it stability derivatives or system matrix. I will call it, you formulate this and they will vary from every fly every flight means every speed this matrix is different A and B are different. If you hover it has one value if you go to a little forward speed it will have one value it will go to another speed it will have another value. So, this entire formulation is actually called linearized stability analysis; that means, if you are flying at that speed if there is a disturbance what happens that is all can study, but he still does not tell you if I want go from hover to some speed, what should I do? That pilot will (()) you and put you will go keep on going and then you will stop that transient part. You cannot capture them because once your hovering you have a matrix here you give an input around that point vehicle.

Vehicle will start moving the movement vehicle starts whether the a is still valid or not this matrix suppose if it goes to 5 meter per second speed 10 meter per second speed 30 40 meter per second; that means, every speed the value is (()) matrix a is different. So, this is a transient part that is a very tricky problem to solve. So, this is the type of equation you flap alone do you say and then try to estimate the vehicle estimate the vehicle handling quality and then they say theoretically this is the range after that flight test because you are the matrix A and B are also theoretically estimated, but how could they are do you know the inertia of the vehicle properly mass inertia I_{xx} I_{yy} I_{xz} that have to be estimated if you make a error there it will have different characteristic and if you make a aerodynamics approximation it will give you something else final proof is when you flight. So, what they do they do flight test ask the opinion, but the flight test they will put sensor pilot will give an input they will see what is happening then based on that this is the another field. I do not know what kind of field you estimate again that is the parameter estimation you try to do parameter estimation of these from flight test (()) and then you again adjust which is a separate study I will not get into that, but here you study stability control that is all he is the vehicle stable because this equation.

You all must have learnt in your some course without this at least this part we will come then may be next class or I will start something that is the periodic. For periodic system

if A is constant matrix here suppose if a is not constant then what you will do that we will learnt next class I will teach you a little bit of that this is what happens in helicopter, but people do not of course, research yes, but if you go into 2 complicated thing then you will be lost completely. That is why understand simple they done that you built your inside into how the vehicle behaves now. The next problem is the aeromechanical instability which is again a coupled rotor problem if you take ground resonance just for the sake of ground resonance for simple problem.

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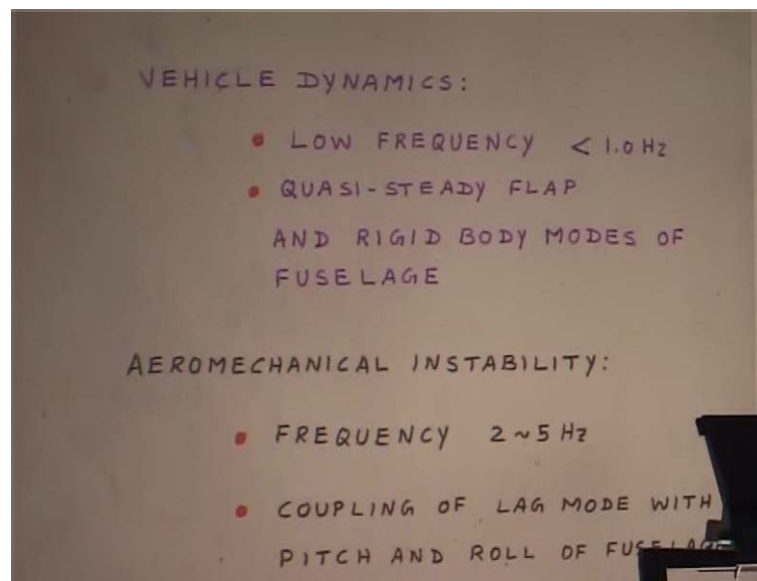


Ground resonance is the helicopter is like this it is on a ground standing in this problem what is more important of course, the fuselage can pitch and roll, but what motion couples the fuselage motion is of the blade what dynamics of the blade which couple is suppose you say this is the blade the lead lag motion lead lag dynamics of the blade.

This is important if you want to solve ground resonance or air resonance. So, you see the physics is different here you take flap here you take lead lag because with lead lag you can predict this of course, you may say if you are why should I not include flap if you want to include you include, but you cannot neglect flap. That is what I am telling you have to have flap dynamics in the problem because without **flap** you have to have lag dynamics in the problem. If you do not have lag dynamics you may not get that instability, but you can have flap lag everything that is for more accurate prediction of the instability the frequency the damping etcetera. So, that I will show some result

because this is what I did some 20 years ago 25 years because this we correlated with a experimental data and fortunately, we got good correlation that is how my equations became kind of (()) right equations. You may call it correct equations then onwards people started getting that is not a problem. Now, you see one flap is needed more important there you go lead lag is important. Then, you come to vibration problem all are important flap lag torsion everything is important and the fuselage in these 2 cases can be treated as rigid, but vibration problem fuselage is cannot be treated as a rigid structure it is a vibrating structure. So, now, you see that problem is the most complex problem. Because, you have to take flap lag torsion dynamics; now in the range of frequency also there is a difference between this problem, this problem and this problem what frequency range these dynamics happen.

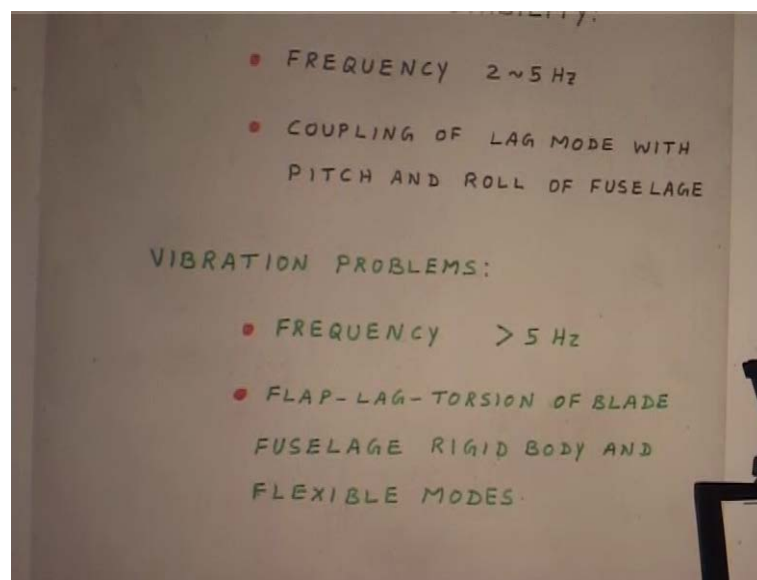
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If you take vehicle dynamics, this is I put approximately less than one hertz it is a low frequency because, usually a human being cannot give input. Suppose you say you want oscillate somewhat is the maximum frequency you can oscillate human beings. **human beings** I am not talking about a machine human being what frequency you can say some inputs about suppose you say you want to have a collective input you want to keep moving what frequency you can do maximum no 5 10 is very high it is about 2 2 less than 3 hertz around 2 to 3 hertz is a maximum you can do and also vehicle dynamics. Sometimes, you may have a slightly higher frequency do not bother about that in general it is a low frequency phenomena, vehicle dynamics low frequency of course, this I

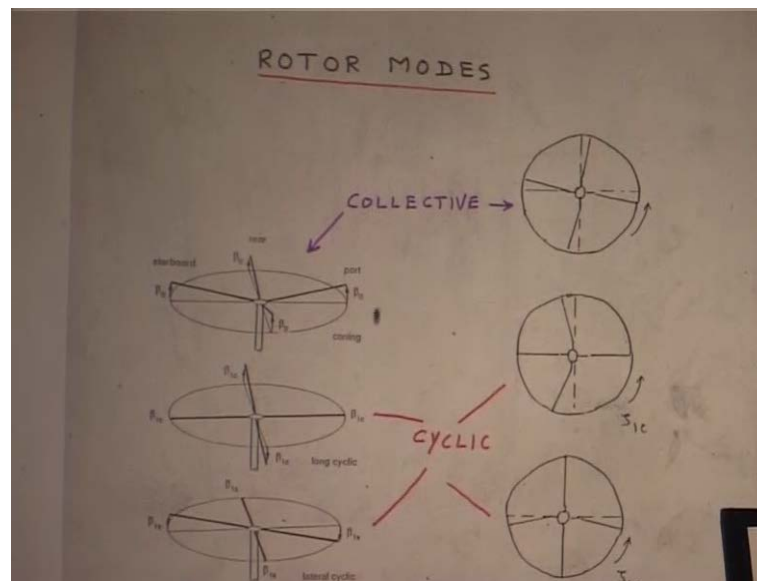
mention quasi steady flap quasi steady quasi-static you can take and rigid body modes of fuselage fine aeromechanical instability frequency is in the range 2 to 5 hertz because that is related to what is the lead lag frequency and usually the lead lag frequency falls some other than that is a transformation please understand and this range is slightly different from this of course, you may find something in that range also one or 2 modes may be sitting there in this range of frequency, but mainly the numbers are low you never have this some 0.2 hertz whether you will have a instability of lead lag **sorry** a ground resonance no not that low frequency you will not have, but you may have a vehicle dynamic mode 0.2 hertz 0.3 hertz something like that.

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Then when you go to vibration problem it is greater than 5; that means, usually it is (()) it is a lot of frequencies will be there, but they will say 20 hertz 40 hertz 80 hertz like that it is a anything more all here you need to have flap lag torsion of the blade aero-elastic full aero-elastic modeling of the blade fuselage first they take rigid body then you take flexible modes; that means, fuselage is treated as both it can move like this you can also shake now this is how they you can split, but now you may asked why if I develop completely flap lag torsion fuselage dynamics full equations can I not use it for all this problem yes you can use it there is nothing wrong, but then for ease of understanding from design aspect by what is which mode is critical it is easy to isolate that part then understand that part otherwise you say I will make everything and then give you on total bunch of result nobody will know what to tinker with.

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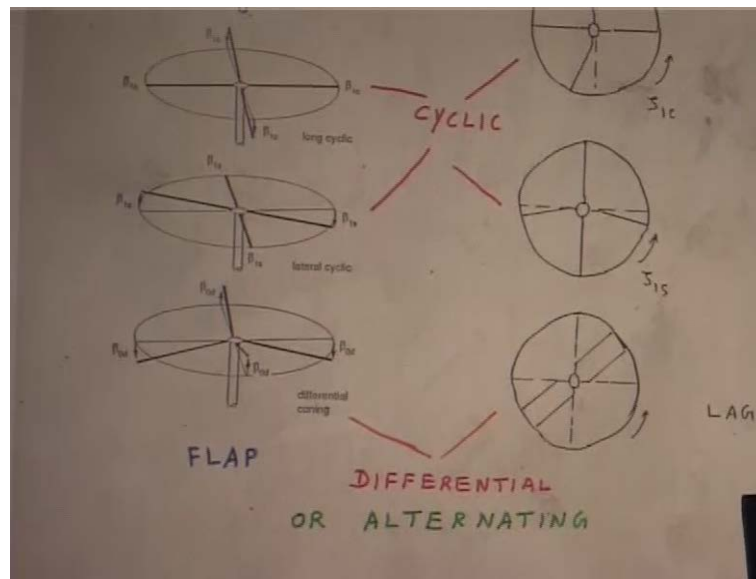


Now, I will give you a little bit about something related to because we are talking about rotor fuselage; fuselage is non-rotating, but rotor is rotating how the dynamics is couple because for a person sitting outside the rotor is like a disk. Now, you say it is like a disk how the disk is moving not how individual blades are moving the motion of the disk to motion of the blade **yes** there is a relation, but as a disk from a non-rotating frame I do not bother which blade is rotating. So, long as the rotor disk keep it (()) I know that is going to charge my helicopter. So, there is something called, I will tell you the modes how they split just for ease of understanding this is called when all this is a flap this is lag I have taken a 4 bladed system. If all the blades go up by the same flap then it is called collective then you know that all the blades are going to go like this see it is not that they are going to be steady. Please understand, what we solved in earlier problem is steady flap now this flap itself is going to do like this and this longitudinal this will oscillate and lateral it will. This is the dynamics of the rotor system now there is a all of them move simultaneously by the same amount that is called collective flap. Similarly, here I put collective lead lag you can have same, but the terminology is collective torsion also you can have, but geometrically representing is a bit difficult. That is why I put lead lag flap then when you go to cyclic you say these you start from one blade one blade has gone up then number 2 blade does not move number, 3 goes down and number 4 does not move same thing is shown here. Number 1 has gone back because this is the

direction of rotation gone back number 2 not move number 3 has gone forward and number 4 (()) this is one way another way is just tilt by 90 degree.

So, this is one c this is one s, but you cannot split these 2 they will always be coupled even though for diagram explanation this is how it is given, but you cannot couple these 2 modes there **sorry** decouple they are always couple and then the last one if you have 4 blades. Now, this is a very tricky thing you said all your putting 4 blades. Suppose, if I have only 2 blades, what will I have? There is a another situation from when I look at it from outside if there are only 2 blades in the rotor system what do I call collective we can understand both the blade go up another one is one blade going up another blade going down and that is this one blade is up the next blade is down and the next one is up and then this is down. So, it is like in a flap 2 blades are down 2 blades are up similarly here if you see lead lag this is back this is forward this is back this is forward this is called alternating mode or differential mode differential or alternating.

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Now, for a 4 bladed rotor I have represented 4 types of blade motion and each one looks a particular rotor mode **rotor mode** collective rotor mode cyclic rotor modes alternating rotor modes you got it now if I have more blades if I have 3 then you can have only collective this cyclic and this cyclic which mean longitudinal 2 cyclic only you can have because this cannot be there this will be existing only when you have even number of blades because otherwise that mode cannot be there now if you have eight blades

because they are or you have 5 blades then 5 blades what will you will have then you say this one you will have 2 cyclic modes then you can go to eight blades like that this is now you see this is some kind of a transformation please understand transformation between individual blade to rotor modes. So, blade mode rotor mode this is a mathematical transformation because each blade will have one flap motion if you have 4 blades 4 will be flapping; that means, if 4 blades have 4 flap motion means in the fixed frame I must have equal number of degree of freedom. So, it is like if I have 3 here also 3 because it is one is to one it is a mapping kind of a thing because here this dynamics is 4 flap modes (()) blade I will have 4 here those 4 are shown here for a 4 bladed system

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MULTI-BLADE COORDINATE
TRANSFORMATION

$$\alpha_M = \frac{1}{N} \sum_{K=1}^N \alpha_K$$

$$\alpha_{-M} = \frac{1}{N} \sum_{K=1}^N (-1)^K \alpha_K \quad (\text{for even } N \text{ only})$$

$$\alpha_{nc} = \frac{2}{N} \sum_{K=1}^N \cos(n\psi_K) \alpha_K$$

$$\alpha_{ns} = \frac{2}{N} \sum_{K=1}^N \sin(n\psi_K) \alpha_K$$

Where $n = 1, \dots, L$ and,

$$L = \frac{N-1}{2} \text{ for odd } N$$

Now, this transformation I think I have I will just show you the transformation and why it is done what is the role this is called multi-blade coordinate transformation this is the one of the names this is also called Fourier, but not Fourier transformation it is the Fourier coordinate transformation this is not normally used in everywhere only some the motor people know electrical, some group they will know what this transformation is and usually, this is very peculiar only to rotors. Other people will not know this, but this is called some places Fourier coordinate transformation not Fourier transformation, please understand that why I do not want to confuse it. I put the word multi-blade coordinate transformation here the general alpha.

Alpha is a general flap or lag or torsion any model that is the degree of freedom of a blade. So, α_k is first blade flap second blade flap k can vary from 1, 2, 3, 4, 5, as to many number of blades that is why I have given here k represents how many induction 1, 2, 3 n is the number of blades.

Now, you see when I add all the motion of the blade that is β_1 plus β_2 plus β_3 plus β_4 divided by 4. It is like a mean value of flap motion that is the collective here I do minus 1 to the power k first blade will have negative, second blade will have positive, third blade will have negative. Then, this is called the alternating mode. I wrote only for even and only even number of blades that transformation will come; then when you go to here $\alpha_n \cos n \psi$ I have given this particular thing you will come back $\cos n \psi$ $\alpha_k \sin n \psi$ α_k ; this is 2 over 2 over 2 over 2 here this end is defined by this particular l n can take this n lower case n can take the value 1 to capital L where l is in terms of the number of blades l equal n minus one over 2 for odd n ; that means, if you have 3 blades 3 minus 1, 2; 2 by 2. So, n can be only one, but if you have 4 blades again you go for e one blade it is N minus 2 by 2. So, this is again 1; but, if you have 5 blades; 5 minus 1 by 2 that is 4 by 2, 2.

So, you will have $\alpha_1 \cos \alpha_2 \sin \alpha_2 \cos \alpha_2 \sin$ and that n also will come here. that is the second cyclic modes this is the transformation, but please understand in this α_k is the individual blade degree of freedom and this is the rotor degree of freedom you may say what is this looks because, why there is very $(\)$ thing I am actually doing in all these things a summation over all the blades **right**. Why do you have to **summit** of over all the blades? Because, you know that at the hub it is not one blade effect it is the effect of all the blades. So, I have to add all of them and that is the one which goes to the fuselage and when you derive your fuselage equations you will find that summation is always there because all blades you have to sum up at every instant; what is the load then? That summation you can use to transform from individual blade to rotor degree of freedom. This transformation will come out nicely; of course, this also has another advantage; but, you may say I should have to do this. Replace it if you want you can do, but this gives a good insight to the problem.

Now, if you know this rotor modes please understand if I know the rotor mode, can I get the individual blade mode? Individual blade motion **yes** that is the inverse transformation which is given here.

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$$\alpha_{nc} = \frac{2}{N} \sum_{K=1}^N \cos(n\psi_K) \alpha_K$$

$$\alpha_{ns} = \frac{2}{N} \sum_{K=1}^N \sin(n\psi_K) \alpha_K$$

Where $n = 1, \dots, L$ and,

$$L = \frac{N-1}{2} \text{ for odd } N$$

$$L = \frac{N-2}{2} \text{ for even } N$$

The inverse transformation is

$$\alpha_K = \alpha_M + \sum_{n=1}^L (\alpha_{nc} \cos(n\psi_K) + \alpha_{ns} \sin(n\psi_K) + (-1)^K$$

Last term will appear only for even N.

Now if you look at this, this looks like a Fourier series because any alpha k is some a naught plus summation this. Like a Fourier series, it is not a Fourier series because when you do Fourier series please understand, what is that your a naught, a 1, a 2 and then b 1, b 2, all those, they are all constants because the periodic signal is written in a sum of harmonic that is orthogonal functions that is the Fourier transformation. But, the coefficients are constants in the Fourier transformation because Fourier transformation if you say a signal, you will write f of t is some a naught plus summation a cosine n some n omega at plus b n sin n omega right n, but these are coefficients corresponding to each frequency and the omega is fundamental period of that signal.

Here these are constants and it goes from 1 to infinity and these functions are orthogonal functions if you integrate over 0 to 2 pi to any multiplication they are 0; that is why this expansion. It is a very nice representation of a periodic signal, but this is a same representation for a even non-periodic signal also like a random signal. Random signal you simply say it repeats after infinite time, that is all. Now, the summation will become an interval. Whereas, this looks similar only difference is this alpha m n c alpha they are functions of time; they are functions of time they are not constants because, the rotor disk can do like this can do like this can tilt this way. So, it is similar, but not same; the form looks somewhat and then one last term will be there this is whenever you have a alternating mode you have to add that time. Now, this is the transformation; now this you can use it to solve all the coupled rotor fuselage every problem.

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• PERTURBATION FLAP EQUATION IN ROTATING FRAME

$$\ddot{\tilde{\beta}}_i + \frac{\gamma}{8} \dot{\tilde{\beta}}_i + \bar{\omega}_{RF}^2 \tilde{\beta}_i = 0 \quad i = 1, 2, 3, 4$$

(centrally Hinged Spring Restrained)

Collective

$$\ddot{\tilde{\beta}}_0 + \frac{\gamma}{8} \dot{\tilde{\beta}}_0 + (\bar{\omega}_{RF}^2) \tilde{\beta}_0 = 0$$

1-cosine

$$\ddot{\tilde{\beta}}_{1c} + 2\dot{\tilde{\beta}}_{1s} - \beta_{1c} + \frac{\gamma}{8} \dot{\tilde{\beta}}_{1c} + (\bar{\omega}_{RF}^2) \tilde{\beta}_{1c} + \frac{\gamma}{8} \tilde{\beta}_{1s} = 0$$

1-sine

$$\ddot{\tilde{\beta}}_{1s} - 2\dot{\tilde{\beta}}_{1c} - \beta_{1s} + \frac{\gamma}{8} \dot{\tilde{\beta}}_{1s} + (\bar{\omega}_{RF}^2) \tilde{\beta}_{1s} - \frac{\gamma}{8} \tilde{\beta}_{1c} = 0$$

It has, I will just briefly tell you, suppose I take a perturbation flap equation I put 0 centrally hinged spring restrained played motion just for the sake of it and the aerodynamics have added here and rest of thing I am setting 0 this is a perturbation motion this is the please note perturbation flap equation centrally hinge they say now if I running from 1, 2, 3, 4. Now, I add all of them I get collective; collective will look identical to this because beta naught this is only thing is here it is 0 when you go 1 cosine and 1 sin they are coupled because this is a part of the because I will not go into the details of why I bring out this particular aspect is to introduce some concept you will find 1 c equation will have 1 s similarly 1 s equation will have 1 c; that means, these 2 are coupled.

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The image shows a whiteboard with handwritten equations. At the top, under the heading "Collective", is the equation $\ddot{\beta}_0 + \frac{\gamma}{8} \dot{\beta}_0 + (\bar{\omega}_{HP}^2) \beta_0 = 0$. Below that, under "1-cosine", is $\ddot{\beta}_{1c} + 2\dot{\beta}_{1s} - \beta_{1c} + \frac{\gamma}{8} \dot{\beta}_{1c} + (\bar{\omega}_{HP}^2) \beta_{1c} + \frac{\gamma}{8} \beta_{1s} = 0$. Under "1-sine", is $\ddot{\beta}_{1s} - 2\dot{\beta}_{1c} - \beta_{1s} + \frac{\gamma}{8} \dot{\beta}_{1s} + (\bar{\omega}_{HP}^2) \beta_{1s} - \frac{\gamma}{8} \beta_{1c} = 0$. Under "Differential Mode", is $\ddot{\beta}_{-M} + \frac{\gamma}{8} \dot{\beta}_{-M} + (\bar{\omega}_{HP}^2) \beta_{-M} = 0$. At the bottom, it says "EQUATIONS IN MULTI-BLADE COORDINATES" and "(NUMBER BLADES N=4)".

That cyclic modes of the rotor are coupled collective is independent. In this case that it will very simplified case and of course, differential mode or alternatively mode that is also independent, but whether this will always be like that no for a simple case of hover it will be like this these 2 are always couple, but when you go to forward flight you will find everything may come coupled all of them will be coupled.

Now, why we look at this particular equation is you can this is a dynamic equation this is a you can write it as $m \times \text{double dot} + c \times \text{dot} + k \times \text{equal to } 0$ and you can solve the eigen values is a vibration problem when you solve the eigen values they are nothing, but the frequency and damping, now what happens to that?

So, here the equation in multi-coordinates number of blades can that is what I have given here what happens to the frequency. This you know this is a $m \times x \times \text{double dot}$ you know $\zeta \omega$ and its dot something 2ζ and then ω^2 . So, you immediately say hey this is my frequency in collective mode and this is related to damping. Whereas, when you go here you cannot immediately say this is my frequency because it is a coupled mode you can solve for the frequency and damping because you do the matrix equation you can solve it I hope you know it 2 degree of freedom system then alternating you know immediately when you look at that this is what is a interesting part I am writing directly the answer.

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EIGENVALUE COMPARISON

<u>Rotating System</u>	<u>Non-rotating System</u>		
	Mode	Damping	Frequency
All blades have same eigenvalues	Collective	$-\frac{\gamma}{16}$	$\sqrt{\bar{\omega}_{RF}^2 - (\gamma/16)^2}$
Damping: $-\frac{\gamma}{16}$	Differential	$-\frac{\gamma}{16}$	$\sqrt{\bar{\omega}_{RF}^2 - (\gamma/16)^2}$
Frequency: $\sqrt{\bar{\omega}_{RF}^2 - (\gamma/16)^2}$	Cyclic (a)	$-\frac{\gamma}{16}$	$1 + \sqrt{\bar{\omega}_{RF}^2 - (\gamma/16)^2}$
	Cyclic (b)	$-\frac{\gamma}{16}$	$-1 + \sqrt{\bar{\omega}_{RF}^2 - (\gamma/16)^2}$

(a) is denoted as the high frequency flap modes (or progressive flap)
 (b) is denoted as the low frequency flap mode (or regressive flap)

If you look at this I will give you here rotating system non-rotating system; that means, the frequency of the blade and the damping of the blade in flap mode because this is just an example for flap mode in the rotating mode all the flap equations are same. So, the frequency is root of omega bar R F square minus gamma by 16 whole square this is the rotating natural frequency and the damping is gamma by 16 with a minus sign that is every blade will have the same value, but when you go to the non-rotating you look at the modes first sake collective mode collective mode has the same frequency as this and same damping differential also will have the same frequency and same damping, but when you go to cyclic modes the frequency will be one of them because there are 2 frequency you cannot say this is for this, this is for this, because, this is couple mode you will find 1 plus this and minus 1 plus this suppose if you say the rotating flap frequency is 1.09 **1.09** rotating flap frequency. That is, this number I am taking it as 1.09 when I look at the cyclic mode I will have 1 plus 1.09 and then minus 1 1 will be 0.090.

Similarly, if you go to the lead lag mode lead lag mode this can be around 0.7 if it is 0.7 you will have 1.7 and 0.3 that is these are non-dimensional frequencies. So, you will suddenly find the 2 different frequencies sitting there in the rotor mode please understand this is what causes the ground resonance problems and coupling because you suddenly from the fixed frame you see 2 different frequencies appearing and the frequencies this is a normally measured they are denoted as I always say high frequency low frequency these 2 collective differential is known; but, when you go to cyclic in some places books

will be progressive and regressive when you are first time you go and attend a conference and then listen suddenly people say oh regressive mode regressive lag mode what is the regressive lag mode what is the progressive flag mode these are terminologies which imply some meaning attached to that and, but that meaning is a little bit more shuffle that is why I always say based on high flap frequency low flap frequency, but then you may call what is progressive regressive that is due to some phase they will say.

Now, a progressive mode can become a regressive mode a regressive mode can become something like that. So, that is why I do not want to use the word progressive and regressive, but in general people normally use regressive mode as the low frequency mode, but if you go by precise definition of regressive means what sometimes it can also be a progressive mode. That is why I do not want to in my all writing of publication everything we always a high frequency low frequency either it is flap or lag.

Now, this frequency will couple with a fuselage motion the fuselage pitch and roll frequency need not be near the flap frequency or the lag **sorry** lag frequency, but if usually you will find pitch and roll frequency of the helicopter on ground it is about 2 to 5 hertz somewhere around that you will find these things can come this particularly the low frequency mode will come close to the fuselage frequency and that causes resonance. Now, you say, oh can I shift that? You can shift it. You shift that then you will have some other loads problem. That is why tail rotor blades it is a you remember I told you that is called stiff-in-plane soft-in-plane.

Now, if you take a stiff-in-plane blade it will be already 1 point 7 something like that 1.7 1.8 then when you have that large value it will become 2.7 0.7 of that order. Whereas, when you take 0.7 it will be 1.7 and 0.3 this is how the numbers and this will happen for all whether it is flap or lag or torsion every mode it happens depending on its own frequency that is why you find in the fixed frame you will have all sides of frequency coming in usually you will not know where it is coming from because you know my blade is only having this frequency and here I find some other frequency sitting; there you will in one of the experiments actually that was very interesting thing there was one frequency which was coming experimental measurement, but you put standard aerodynamic **2** cases if this I taught I should (()) experiment was done with 2 types of blades one with match stiffness I hope you remember what is match stiffness means I

mentioned in the class. Another one none matched. So, if the question is there in the exam then you will keep turning your notes no I am just sake matched and non matched everything is same only this difference was there experiment was conducted stability was analyzed everything. We have our equations it is matching very good with the experimental data in one case only change is the other experiment. There was a change in the match stiffness which you can also incorporate in your mathematical model we incorporated in the mathematical model, but the results we are not getting it matching with the experimentally.

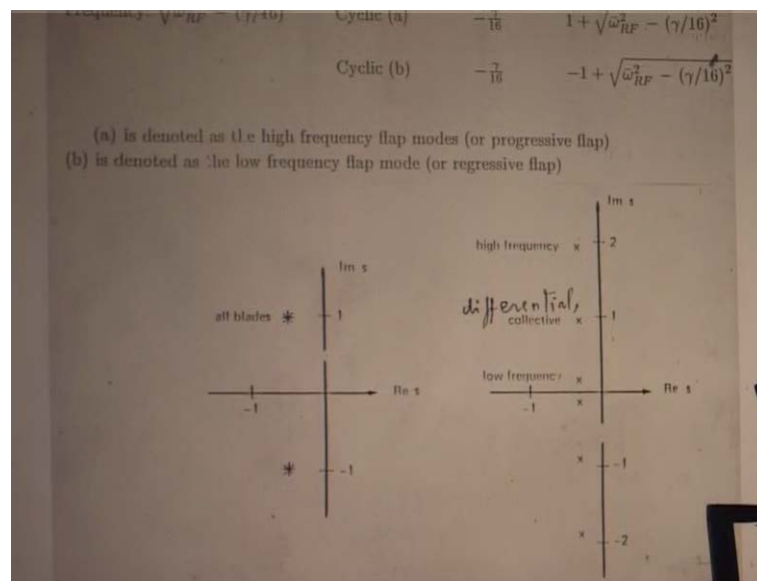
So, you say one set of equations representing the dynamics of the couple rotor fuselage system; one particular configuration of the blade it is exact in the sense very good correlation. Just one change in the parameter which I can also incorporate in my equations (()) in the results then this is what is going on because, this is something like that is where the lack of understanding of what is the reason for this. Because, then you realize we have our model see that was an experiment that is not cheating model you make approximations you may 1, 2, 3, 4, 5 and then derive then you find this model is valid for both cases in terms of representing them, but one is giving result and another one not. So, it was a big debate and the finally, let us what is the effect you change in the aerodynamic model.

Then it is what was introduced is in the inflow model please understand inflow a dynamic inflow model was introduced; that means, the inflow is also changing when it is shaking; that means, you should have a model for that he said constant inflow uniform inflow everything we use now you say I want the time varying inflow please understand how do I get a time varying inflow then you say let us, it is all again very if you see the physics is very beautiful inflow is related to thrust. That means, if I change my thrust my inflow varies simple similarly my pitching moment I will calculate at the hub pitching moment role moment. I assume a like a (()) model inflow, but I will say my inflow is time varying. So, I simply relate change in trust to this change in dynamics to this and then some mass effect has to come that came from very interestingly it came from airships the dynamics of airships that was done long time back by (()) **I think 1940s are this all are very old paper forties or twenties munk m u n k it is a very interesting thing** airship is a ellipsoid. You know what is ellipsoid? Ellipse you rotate about any one of the axis major axis or minor axis, it is a major axis.

Now, if you freeze the major axis to 0 it will be a disk **right** circular disk. He developed an equation if an airship moves what is the difference between the airship motion in air and then aircraft motion in an air because, when you have airship it displaces large volume when it moves it also takes the surrounding air with it. Suppose, if you take a balloon you see hit it just see it does not go very nicely it just gives a (()). Actually, the inertia balloon when you tap it is not the balloon inertia balloon plus the surroundings air inertia added. That is what the apparent inertia effect; now he developed one for that mean air mass is added into the **your** physics. So, that was he develop he if you squeeze everything it will give you some approximate number and just you take that number put it in this model you run it and invert then you were able to capture those frequencies; that is what I am telling you see then they.

So, aerodynamics of something else affecting my frequency in getting in the couple rotor fuselage dynamics and you are able to get that, but it is a relation of so many hanging things brought together to solve, but you may say these all are (()). What is, there is no solid proof. Solid, yes some of the things are not really if you say momentum theory is valid for steady flow is it valid for unsteady flow do not know, but then you say if there is a perturbation change in thrust change in inflow that is all you related you do not give a proof later people started working little bit more and more and then think that is why I am saying is sometimes the experiments throw.

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So, what I have this is what the root locus plot which I asked you real part imaginary part of the roots. So, you put the value all of them have same frequency and same damping in the collective mode whereas, in the rotating frame they all are same when you go to non-rotating frame they split you will have a collective differential here high frequency mode there low frequency mode there of course, this is a mirror image because plus or minus always complex roots. So, you get suddenly more frequencies and these are responsible for certain instability I will not go into the details of the problem I simply skip the whole thing I taught I will show directly to some stability because 2 important things are there in the helicopter why it is unstable you will not be able to answer.

Now, but after I briefly explain that you will say, oh may be this reason that is all because helicopter is a unstable vehicle because, I would like to show some particular **yeah**.

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The image shows handwritten mathematical equations on a piece of paper. The equations are organized into three sections: COLLECTIVE, CYCLIC, and COUPLING PARAMETER.

COLLECTIVE

$$\left[\begin{array}{cc} \bar{\omega}_{RF}^2 - 1 & \frac{\gamma}{8} \\ -\frac{\gamma}{8} & \bar{\omega}_{RF}^2 - 1 \end{array} \right] \begin{Bmatrix} \beta_{1c} \\ \beta_{1s} \end{Bmatrix} = \frac{\gamma}{8} \begin{Bmatrix} \bar{\theta}_{1c} \\ \bar{\theta}_{1s} \end{Bmatrix} + \gamma \left(\frac{\theta_0}{3} - \frac{\lambda_0}{4} \right) \begin{Bmatrix} \dot{R}_y \\ -\dot{R}_x \end{Bmatrix}$$

CYCLIC

$$+ \begin{Bmatrix} \bar{\theta}_y \\ -\bar{\theta}_z \end{Bmatrix} + \begin{bmatrix} \frac{\gamma}{8} & 2 \\ -2 & -\frac{\gamma}{8} \end{bmatrix} \begin{Bmatrix} \dot{\theta}_y \\ \dot{\theta}_z \end{Bmatrix}$$

COUPLING PARAMETER

$$S_c = \frac{\bar{\omega}_{RF}^2 - 1}{(\gamma/8)}$$

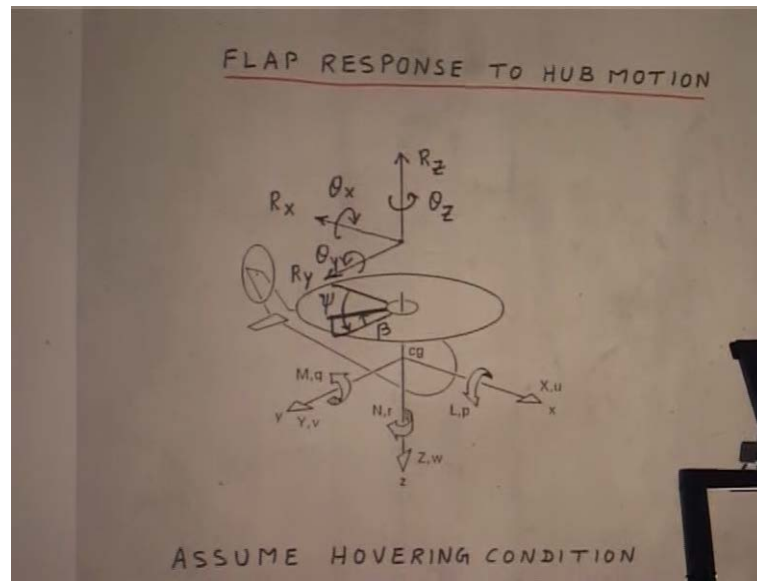
Below the coupling parameter, there are two more equations:

$$\begin{Bmatrix} \beta_{1c} \\ \beta_{1s} \end{Bmatrix} = \frac{1}{S_c^2 + 1} \left[\begin{Bmatrix} S_c \\ 1 \end{Bmatrix} \bar{\theta}_{1c} + \begin{Bmatrix} -1 \\ S_c \end{Bmatrix} \bar{\theta}_{1s} + 8 \left(\frac{2C_T}{\sigma a} + \frac{\lambda_0}{4} \right) \begin{Bmatrix} S_c \\ 1 \end{Bmatrix} \dot{R}_y \right]$$

$$+ \left(\frac{2C_T}{\sigma a} + \frac{\lambda_0}{4} \right) \begin{Bmatrix} 1 \\ 1 \end{Bmatrix} \dot{R}_x$$

When you were dealing with the handling quality of the helicopter, I will show the just I think very simplistically. This is what I said is the flap in the non-rotating frame how it is affected by various inputs because the dynamics of the problem if you want to know I will show one picture then directly that equation.

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This is the picture for the dynamics. The blade is flapping the fuselage is all these motions I have to derive the complete equation hovering condition and I will not bother about that part because this (()) is a mess. What I want to show here is just the how my rotor cyclic mode $1c1s$ influenced by θ_1c1 I put a bar is basically a pilot input if the variation in that how they will θ_1c1 will affect both $1s$ again will affect both this is r y dot which is the fuselage longitudinal **sorry** lateral motion r_x longitudinal motion and then θ_1c1 double dot y in the sense about the y axis which is like pitching acceleration. And, this is the roll acceleration angular acceleration and then this is the pitch **sorry** there is the pitch rate this is the roll rate.

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$$\begin{aligned} \begin{Bmatrix} \beta_{1c} \\ \beta_{1s} \end{Bmatrix} &= \frac{1}{S_c^2 + 1} \left[\begin{Bmatrix} S_c \\ 1 \end{Bmatrix} \bar{\theta}_{1c} + \begin{Bmatrix} -1 \\ S_c \end{Bmatrix} \bar{\theta}_{1s} + 8 \left(\frac{2C_T}{\sigma a} + \frac{\lambda_0}{4} \right) \begin{Bmatrix} S_c \\ 1 \end{Bmatrix} \dot{R}_y \right. \\ &\quad \left. + 8 \left(\frac{2C_T}{\sigma a} + \frac{\lambda_0}{4} \right) \begin{Bmatrix} 1 \\ -S_c \end{Bmatrix} \dot{R}_x \right. \\ &\quad \left. + \frac{8}{\gamma} \begin{Bmatrix} S_c \\ 1 \end{Bmatrix} \ddot{\theta}_y + \frac{8}{\gamma} \begin{Bmatrix} 1 \\ -S_c \end{Bmatrix} \ddot{\theta}_z \right. \\ &\quad \left. + \begin{Bmatrix} S_c + \frac{16}{\gamma} \\ 1 - S_c \frac{16}{\gamma} \end{Bmatrix} \dot{\theta}_y + \begin{Bmatrix} -S_c \frac{16}{\gamma} + 1 \\ -\frac{16}{\gamma} - S_c \end{Bmatrix} \dot{\theta}_z \right] \end{aligned}$$

DYNAMIC INFLOW MODEL

$$[M] \{\ddot{\lambda}\} + [L]^{-1} \{\dot{\lambda}\} = \begin{Bmatrix} \text{perturbational thrust coefficient} \\ \text{perturbational roll moment} \\ \text{perturbational pitch moment} \end{Bmatrix}$$

Now, you see all of them will affect my flap now when this is affected hub loads are changed when the hub loads affected fuselage motions is changed.

So, this particular thing there is one important which they always say coupling parameter that is I give one input if I give longitudinal cyclic I find I get a roll also this is related if you see $\omega R F^2 \text{ minus } 1 \text{ divided by } \gamma \text{ by } 8$. Now, you would have seen this expression in all your beta naught beta 1 c thing and that is the coupling parameter.

Now, you see if you change your flap frequency higher you increase the coupling tremendously that is the reason why flap has to be at a specific value because it influences everything your flight dynamics is completely influence by flap, but you say lag does not influence well **yes**, but that dominant you look that is the dominance of each mode and then the lock number which is the gamma.

So, you see both of them affect the coupling.

So, actually what happens is as we go at some speed if pilot wants to give a pitch motion actually the helicopter will roll and it is not what you get the unintended motion is much more than what is intended then what you do because when you give this is suddenly roll then you would start correcting that and it will give then when you correct that this will start doing you will have a tremendous coupling between this motion.

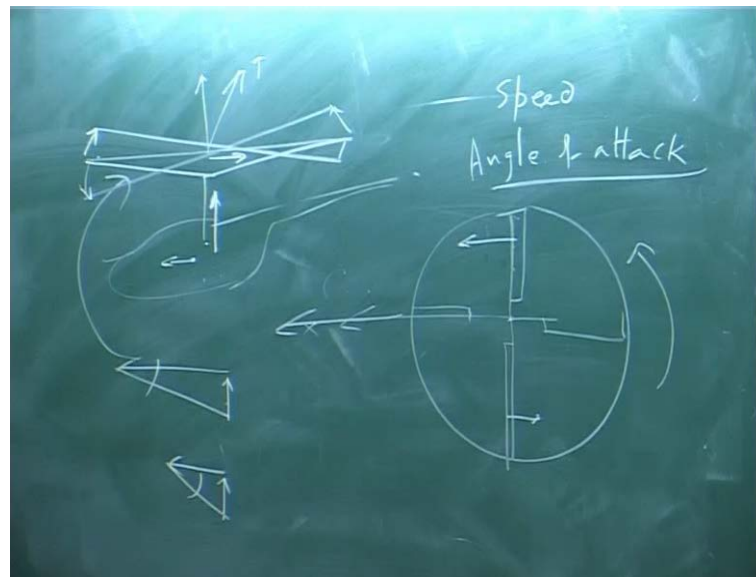
Sometimes you will get the undesirable motion much more than what you intended further.

So, these coupling one has to be you should pilot should get used to that number one in the design you try to minimize that is all you cannot eliminate suppose if you say a centrally hinged blade ωr of the 0 good, but then centrally hinged blade will not have a good control characteristic because you cannot get a control moment because your $c_{m y}$ is $c_{m x}$ you got that is related to same $\omega R F^2 \sin^2 \theta$.

So, this is how on one hand if you say I do not want to coupling then you lose something else control.

So, this has to be adjusted now I that I will show you some important stability characteristic which is like suppose you say the helicopter is hovering take it and suddenly the helicopter moves a slight motion or there is a wind anything you can take it if it moves like this just is called speed stability if you are suddenly if you go what happens one side I will say this is the very interesting thing there are 2 aspects which are speed stability angle of attack stability and something like that speed stability

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You are your helicopter is you are hovering and you find there is a small some disturbance a disturbance can come even if you do not move air can come assume that air is coming then what will happen if this is the rotor disk and this is your forward

direction this half of the blade what will it will experience the increased velocity **right** this will experience decreased velocity pilot is not changing any control please understand is hovering with the fixed angle collective everything is fixed disturbance one side lift increases then what will happen this will start no not role that is what the it is not it will first flap because flap dynamics has to happen what will happen this will start flapping up flapping up means this blade will go where it will flap.

You know the phase these at the centrally in 90 degree or it is empty at something like that **right** it will go and flap somewhere when it flaps what happens this will come down; that means, your rotor tip path plane what happened tip path plane has tilted back when it tilted back the trust vector what happens what if the t goes when the t goes back what happen this will start noose up **right** and when they starts noosing up this force now what will happen it will also give another rear word component then what will happen.

It will start going back when it start going back what will happen this will start tilting this way then it will noose down then again you go it will do up.

So, it will start doing like a pendulum and this is unstable.

This motion is a unstable motion.

So, what you do is you, but this you cannot take it only pitch motion please understand the helicopter will not just pitch about one place it is like this it will go like this like a pendulum it will oscillate about some virtual point, but this is the speed stability because what you give you give a forward motion in the sense a disturbance that will actually make the helicopter noose up and the noose up thing plus there is a tail then it will start going back and it will and it can do both pitch and roll both sides of motion this is very important stability derivatives which you causes the blade a helicopter to be unstable there is one more term which is called the angle of attack stability please understand this is speed stability another one is angle of attack stability.

Then you may have what is that is a little different in the sense helicopter is flying forward steady speed when it is flying forward you find suddenly the helicopter gets a slight disturbance in the vertical direction vertical; that means, that is equivalent to variation in inflow **right**.

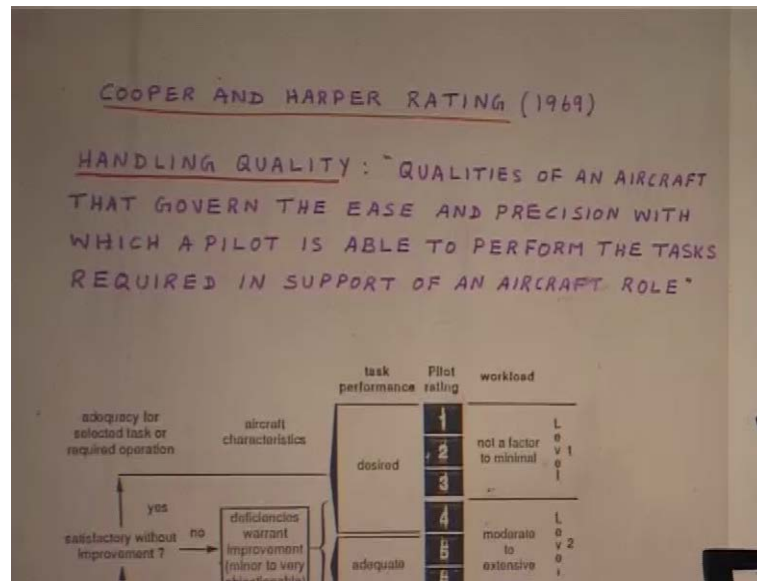
The inflow variation here also it will change here also it will change, but in this side this is a retreating the forward velocity is less here the forward velocity is more therefore, the change in angle of attack and one side to this side is different please understand this is angle of attack stability. You see very varies it is the settle thing you are flying forward this side advancing side velocity is more this is retreating velocity is less.

Now, you say there is a slight motion of the helicopter or some gust is coming some wind is coming something you may because always there is a disturbance and that is in the normal to the board which is like an inflow inflow is same throughout the disk because of this motion of the helicopter, but the affect is advancing slight because you have a large velocity I may have a same inflow in the other place same inflow the angle of attack change is different because of the change in angle of attack difference this will start flapping and that will again do role motion pitch motion anything.

So, these 2 are most important stability derivatives, but it does not mean only these are there are lot of other stability because then entire matrix of a every entry is a stability derivative, but the critical entry is which are which influence the flight dynamics more is this and speed. So, these are the, but the physics it is a quantify we have to get it; that means, that depends on what speed you are flying how to calculate then it will depend on whether the blade is stalling whether where is not stalling all sorts (()).

So, this is and it is a very very important thing I will just show a few because this is for information to most of you.

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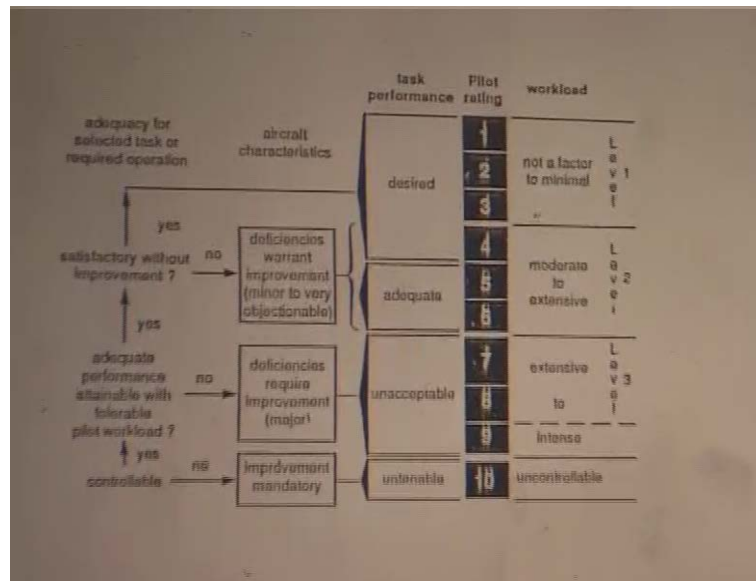


There is something called a Cooper Harper rating. I do not know whether it was introduced to you anywhere?

Yes no no at least that is good.

So, that you guys you can know something this is the handling quality it is the description what if that if qualities of an aircraft it does not matter a helicopter aircraft any flying vehicle that governed the ease and the precision with which a pilot is able to perform the task required in support of an aircraft roll; that means, is ease ask to perform a task and how accurately and precisely you can perform. See, in the world they have competitions also they say go around like this and then various maneuvers.

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See, it depends on the pilot's skill and it depends on the capability of the vehicle. Sometimes, the vehicle may not have a capability pilot may be good another one is vehicle may have a capability, but the pilot cannot do ultimately we have to get use, but the vehicle has certain capability only. Now, how do we improve and at the same time you reduce pilot work load?

So, here they actually give with this I stop. It is a qualitative rating by the pilot please understand sometimes when you start you find it is difficult once you have learnt then you say it is not it is its easy. So, the same pilot same (()); when you start learning bicycle riding you will say and this is a really crazy I how everybody is riding I cannot ride it. But once you have learnt it is easy. So, the same person depending on experience you will say oh this task is (()), but normally they classify as 1 2 3 4 then 5 6 7 8 9 10 this is basically level one level 2 level 3 and then complete intensity you just do not bother about this.

So, any aircraft or vehicle which is built you will evaluate it is desire all aircraft characteristics **yes** it meets here you see somewhere here deficiencies, but warrant improvement minor to very objectionable here deficiencies require improvement and major here improvement mandatory; that means, it is you just cannot.

So, you see they always ask a question adequacy for selected tasks are required satisfactory without improvement if he says yes go up. So, you see it starts with is it controllable.

First, if he says no go and change your aircraft first is controllable if he says yes, but adequate performance attainable with tolerable pilot work load if he says no you go deficiencies require improvement if he says yes is it satisfactory without improvement if you said you have to improve means these things war and improvement minor to sometimes very exceptionable if he says yes then he say adequacy for selected task are required then this is the aircraft which is a desired then they will say r this aircraft is good, but this who desired a test pilot that you are the test pilot role it is all what if he says no it is no because you do not built and then you do not ride it if you built and ride then it give yourself will feel you are building something and somebody else asks us to write.

This is a very tough that is why aircraft after design it goes through that is why p t 1 p t 2 you know technology demonstrator t d 1 t d 2 all these you find everything has modification **modification modification modification**. Finally, you will say this is (()) you cannot do because in some cases I tell you in the helicopter I have some details they had to change the rotor shaft itself height completely because they found that it is vibrating too much and that is the major design change which means it is like a new project.

So, it bent through the whole thing. So, there are also questions from HAL side there was in the beginning somewhat I call you may call it a give a report, but the final decision is they will take you say what you (()). So, like that there are questions some questions require are you going to change the entire blade entire blade design is; that means, you were starting from day one go through. So, if finally, it is like pilot has to accept and the vehicle.