Introduction to Helicopter Aerodynamics and Dynamics

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Module No. # 01

Lecture No. # 22

Lead lag dynamics – because, here last time I mentioned to you the hinge offset and the spring; they give the effect of stiffness and normally, the rotating lag frequency square is K zeta over I b omega square plus you can have it M X C G over which is essentially if you have uniform mass, this will become 3 over 2 e bar over l bar where l bar is you know that it is 1 minus e bar.

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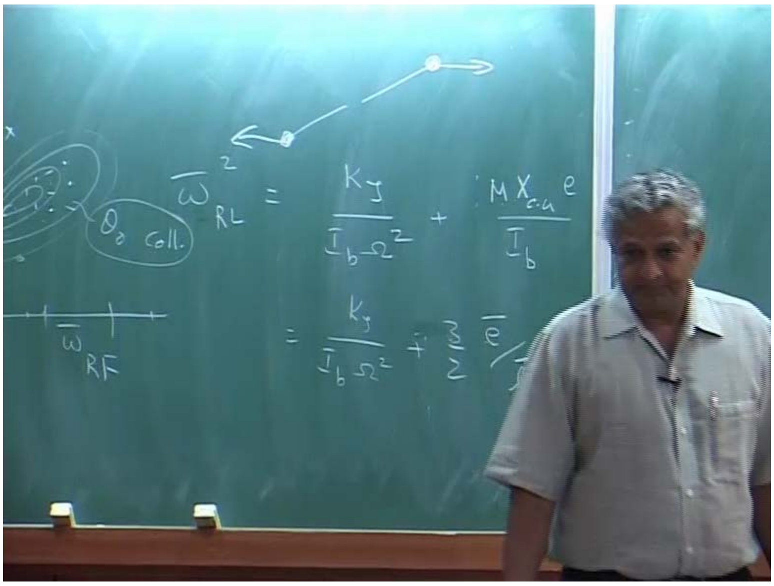

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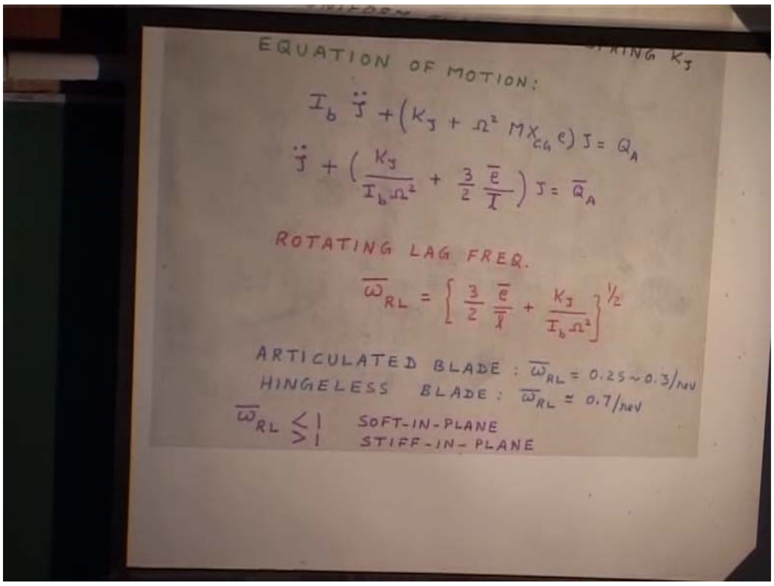

We shall, the choice of the lead lag frequency is in the range of articulated rate blade it will be around the 0.25 to 0.3 because there would not be any spring in the case of articulated blade; only hinge offset only hinge offset.

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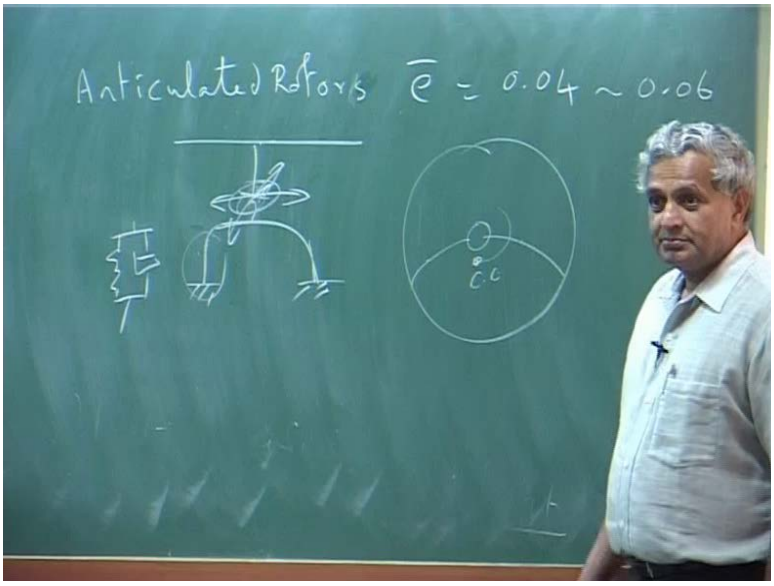

It is always articulated rotors you can have that e bar is of the order of 0.04 something like that around that range 0 4 yeah 0 4 0 6. So, if you have this much you can divide only this term, this term will not be there because there is no spring. So, only this term and your frequency will be 0.25 in the operating non-dimensional like flap; this is 1.09

this is 0.25 to 0.3 but, if you put a spring which is an equivalent for a hingeless blade. So, if you have a restoring spring effect because of the hingeless blade there is no hinge then the frequency is kept at, it goes to 0.7. You try to design to have it around the (( )).

Now, we may add, can I have more than 1? Yes, for some configurations the omega bar R L that is the non-dimensional lead lag natural frequency is more than 1 and some it is less than 1; more than, sorry, more than 1, less than 1 that definition that goes is if it is less than 1 it is called soft-in-plane - it is a name. If it is more than 1 it is called stiff-in-plane. Now, which is better? What is the effect of this frequency on the overall vehicle or the blade because it right now, we have treated only isolated lead lag motion when you combined with flap you are going to have flap lag motion.

In addition, the fuselage can also start moving because fuselage has some motion; that means the coupling of lag with respect to flap motion or with respect to fuselage motion they can lead to instabilities. Isolated lag is fine; that means, just pure lag motion it is stable because the damping is of course, purely aerodynamic damping. In this case, we have not put any external damper there. It is only drag force in the case of flap motion. It is a lift force that provides the damping in the case of lag. It is only the drag and the drag is very small compared to the lift. That is why the aerodynamic force in the lag motion. The drag is very small because of the damping is very small, but alone it will not go unstable. When you couple with a flap for some combination of frequencies that is all, it is not that always.

Some combination of frequencies the blade will go unstable; that is, the aeroelastic instability flap lag instability which is something unique for rotor blades you do not face that kind of an instability in the fixed wing because, you have this bending and this bending, that is all. Whereas, in fixed wing you do not have this bend only you take torsion and bending - out of plane bending here, two bending motions can make the blade unstable.

Now, for you to analyze then you have to solve coupled flap lag aeroelastic problem. I will not go into the details of that. Usually, if you take a stiff-in-plane rotor blade, stiff-in-plane means that lead lag frequency is greater than 1. You keep it around 1.5 or o1.4.

Then technically you can avoid technically you can avoid the flap lag instability, but you have to keep it far away, but if you are not keeping it far away then those rotors that is stiff-in-plane rotors or prone to flap lag instability, but if you keep it around 1.1, 1.2 then it will go on stable so, earlier people did that. I will show later that diagram but, stiff-in-plane blade if you keep the frequency far away. No problem, it will not go unstable now then why we can always have every rotor blade other stiff-in-plane rotor blade, but a problem will be it will have lot of vibrated loads. You can avoid that by keeping the frequency away, but then your vibratory loads will be high then the other question is can I bring it lower than 1. That is soft-in-plane blade anyway, because you are frequency far away from near the flap is around 1.09.

So, do not go near that frequency then soft-in-plane blade you can avoid flap lag instability, but you will have another type of problem which is the ground resonance. I do not know who all came that day for the presentation that is the ground resonance in which blade lead lag motion couple with the body motion and then that will go. It will be very catastrophic actually and you have to avoid that but, the loads are less vibratory loads. So, the question is what kind of rotor system you will choose for your helicopter. If it is usually two-bladed rotors, please note that two-bladed; they are normally stiff-in-plane; they are normally stiff-in-plane and keep the frequency away and you do not get into ground resonance problem this is fine.

But, the loads will be high on the other hand if you go to soft-in-plane yes, but you have a problem of ground resonance but, for you to solve that problem you have to develop coupled fuselage lag dynamics. There is a dominant thing coupling that is pitch or roll motion of the helicopter will couple with the lead lag motion and then the whole helicopter will go unstable. That is called ground resonance it is even before it takes off from the ground, but you also have flap air resonance. So, these are the problems.

Now, you want less load; if you say vibratory load go for soft-in-plane blades; for main rotors of the helicopters, design soft-in-plane, but then they get into ground resonance. So, what you do is you put a damper. Dampers are put in the main rotor for the lead lag motion. Please understand the damper is only for lead lag that will be a either a hydraulic damper or sometime elastomeric damper which is more sophisticated you can put a damper; that means, physically you have to attach and they have to check the damping level high level or how good is that.

But, for tail rotor usually you go to stiff-in-plane, but then keep the frequency away from the flap lag instability zone and you do not have to put a damper. That is why this is a very you know normal question I ask the different people; you always have a damper for the main rotor. Why you do not have a damper? Why split out? Why you do not have a damper for the tail rotor system? Tail rotor systems by design you try have it the stiff-in-plane and then keep the frequency away. But, make sure that the frequency is far away and then you do not have a damper. Absolutely no problem, but loads will be high, but that will be (( )) because, tail rotor loads are not very high. So, you see in the design of and the selection of rotors the frequency placement plays a major role and depending on where you place then you have to add a additional system because the system can go unstable.

You say I have to put one more damper. So, the flap lag instability is one more critical problem and that critical problem comes where you keep this rotor. Fuselage problem is another critical problem that again depends on where this frequency is. So, it is not that when you study helicopters you get introduced to this individual blade motion and their dynamics, but that is not the final analysis or design because, this is just to give you some glimpse of what is the lag dynamics, what is flap dynamics. Next, you will do what is torsion dynamics, each one independently. But, no rotor operates in independent mode and they all are coupled. Now, analyzing flap is very important here. There is not much analysis; I do not do any analysis. I will just say only the frequency because, if you want to analyze then you have to analyze this motion along with either flap or fuselage motion; then will you will bring out some new phenomena.

Therefore, you will find that the amount of material that is devoted to only lag dynamics is very little. Some books do not even talk about that. If you look at Patfield book he will never mention in his whole book lag dynamics. He is all flight dynamics only; but, flight dynamics you will only look at flap motion it is a, it is like what is important for a particular dynamics you use it, but of course, everything is part of it.

But if you want to get a good understanding of the physics of the problem do not try to put everything and then you have no clue. So, you split the problem that is how we say lag. Let me look at it; how the blade will move flap alone. Next, we go to torsion because the critical thing in lag is these two. I would say the 2 soft-in-planes, stiff-in-planes we may call it phrases or something like you know ground because, you have the stiffness of the landing air play a role. The moment you will lift off the ground is on, then it is a different dynamics because that is why sometimes they were you lift it up, but see you this may be (( )). Now, this is supporter landing air support on the ground. This gives this can be modeled by this spring and damper. That means, this can pitch and roll; the physics is different in each case and which ground it is standing that also matters whether it is a concrete ground or whether it is soft ground or it is ice or grass because, what happens is the stiffness changes. So, usually you know whenever they make helipad you see they make a concrete, but you cannot have concrete everywhere, but for normal operation you put it, but then when you want to test it you go and land on ice.

So, you have to take a helicopter and what is the stiffness of the ice along with this stiffness? They have done go to Himalayas make a ice pack and then make the helicopter land and then do the test make sure that everything is fine, then you come. If it is no, rotor and fuselage (( )) damper; no, that dampers has no use.

See that damper, there is no damper. They do not put a damper do not thing that they put a damper between rotor and fuselage where did you no no no no no you have a isolation system please understand between the rotor shaft rotor rotor shaft to fuselage you have a isolation system. That is to reduce transfer of vibration to the fuselage this instability is not related to that this instability what happens is this can move is a pitch roll rotor I am just showing the rotor blade can bend like this when it bends I am just drawing for two bladed the c g of the system is here and that keeps when it rotates it chants.

That means, that is giving a imbalance or un unbalanced mass which is going around and that combines with the fuselage motion and it is called heavily oscillating and it is very (( )) where the moment the blade touch the ground the blade is gone that is it is a teetering what they do is they make it stiff-in-plane see this happens at a particular combination of frequency matching so, you have it as a stiff-in-plane range.

If you draw the diagram of actually, how the frequency is? Got the frequency of that full dynamics system not just lead lag alone please understand, but it is a, there is something more into that because one is a non-rotating frame, another one is a rotating frame. Now, each blade will be rotating. So, you have to have each blade equations; then, they are getting affected by fuselage motion. So, usually there is something called transformation that is done from rotating frame to non-rotating frame; that transformation is called the multi-blade coordinate transformation.

There is a mathematical transformation; then get it into fixed frame mode in the fixed frame physics is c g is moving that is all; but, you have to know at what rotor r p m operating r p m that really hits it and then you avoid that. But, your rotor r p m always starts from 0 to 100 percent. Definitely, you may cross the resonance point you may cross the resonance point or if you want you shift it out. Best is you put a damper now there are two types of things.

One, you can put a damper right on fuselage; that is in the landing air because some of the helicopters have skid landing. Here, that is why you have to be very careful whether skid has its own characteristics you go to wheeled wheel type is some shock absorber and other thing that will have a different characters helicopter may be same actually a l h. yes, they had a one version which is the skid another version which has this wheel type. Now, wheel type because they had initially some problems and then we have to put a damper increase the damping here tier pressure everything is matches it should be very critical.

So, the analysis you do finally, make sure that during you operational range you do not get into ground resonance problem and one of the way to avoid is put dampers and damper you put, but again please understand damper you simply there put a damper then its weight cost maintenance. Everything gets added and that becomes a very critical component it is not the that is a non-critical. That is why if you want to know the importance of lag motion I would put it the importance of lead lag dynamics of the blade is not in isolated lag dynamics is actually in combination with other modes the importance is come out.

So, these are problems people have solved I have show few things. Here only the basic thing is this is what is soft-in-plane, what is stiff-in-plane. That is all soft-in-plane usually may lead to have a damper you cannot avoid having a damper no flap lag instability. Usually, soft-in-plane you see the frequency these two are soft-in-plane because they are below one and hingeless blade you do not go beyond 0.7 do not go near one.

If you go near one you get into flap lag instability. Please understand if not they what I am saying is when you take the lead lag frequency in the vicinity of flap frequency you will have a resonance problem, but vicinity means, how close are you going to go to 1.09 and no even if it is 1.3 you may get in 1.2 you may get 0.9 you may get. But, that will happen at a higher degree of pitch angle. Sometimes, then you may not operate at that pitch angle. So, you will say, but if you are 0.7 you will not get into that instability you say what they will wrote you take it yes that the basic load itself is less vibrating load.

So, you say it is all right otherwise, you have to put a damper there if you want to make it soft-in-plane then you may land up with some other problem, but even stiff-in-plane please understand is not you make it 1.2 you have to place it far you see if you make it 1.2 you are going to have instability that instability is flap lag instability.

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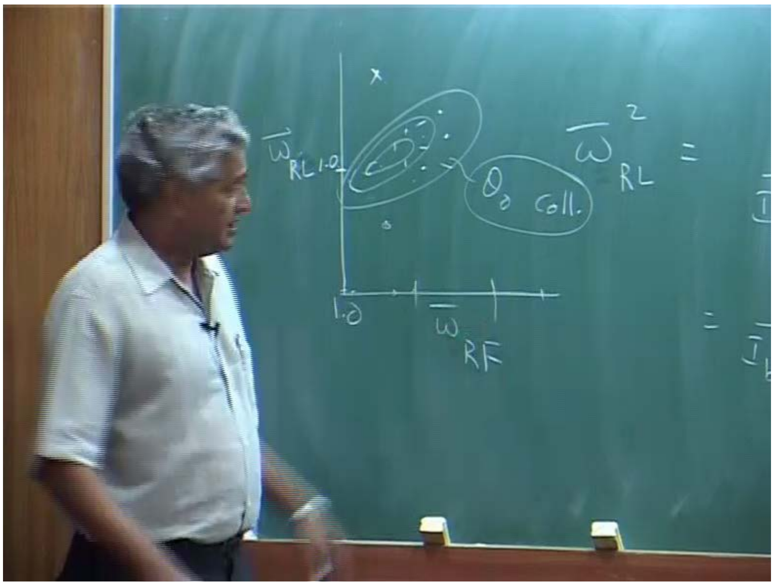

So, usually flap lag instability has I will just show a diagram; it is usually I think this is a which is flap, which is lag I think this is omega lag I think this (( )) me. I will just check that because, I want to make a hung a wrong diagram this is just for your yeah.

Here you have the instability zone, but it be drawn very crude approximation this is the rotor flap frequency, rotating flap natural frequency, rotating lag natural frequency and there will be some kind of a curves like this. Something like these are each one corresponding some pitch angle theta 0 collective.

The curves will be this is like a bubble. This is called a bubble diagram; name itself is bubble what happens is you operate at different collective angle. Of course, you have to have aerodynamics please understand this is a aeroelastic instability diagram. I will go into the detail later when I will see whether we will go into that because if you have a large pitch angle and any combination of frequency within this in this bubble any combination of here this is around 1.0 and it starts from 1.0 and goes.

Here, it is 1.0 lead lag if you have a point here suppose if this is say some 0.3 radian theta naught 0.3 this is about 15 degrees if I keep the blade 15 degrees and operated any point here inside is unstable in the sense any combination of flap lag frequency which lies within this zone the blade will go unstable.

Suppose you, I am not operating at 15 degree I operate at 0.2 that is 10 degrees then this bubble will come; that means, if you are outside is bubble for less pitch angle you are stable like that you have small bubble zone. But, normally in the rotor blade you will know what angle you will operate and this diagram is drawn this is a standard famous diagram and this of course, a lock number all those things will come you can draw several curves of this type and what not people will say is let me go you know omega flap is around 1.09 or something if you know this point you cannot take it too far unless you introduce that what is that flap pitch coupling (( )) straight go in the lag mode either you are here which is a soft-in-plane below one or take it very far.

So, that you are not going to hit this bubble region, but soft-in-plane you avoid flap lag instability if you are far away, but you get into ground resonance, but the loads are less stiff-in-plane far away you avoid flap lag instability. But, you have a high vibrating load weight reduction I am see, load means the sense not the mean load vibrated load you may put a damper then in that case because your tail boom tail boom with the lead lag can get into you will get it to that instability because that is like a ground resonance they supports with the that is what happened in the tilt rotor.

What you are showing that the amount of problem they had seen; he showed that they, but tilt rotor program for your information it is round up you (( )) just 2 years back they had this problem because the rotor is rotating the wing can vibrate it gets into all sorts of instability problem of course, it took 50 years what he said is to 1955 started a whole project

So, the weight point of view I do not want to get into where this rotor is in the rotor I do not think you save tremendously a lot of weight as a tail rotor. Of course, I have I do not know may be few kgs, you may save few kgs. That is all instead of 1.5 or 6 which you have kept may be I will bring it down a little bit fine make sure that you are away from the bubble and your pitch angle is never going to hit that see this bubble will keep on expanding of course, this is based on very simple calculation not be stall model and other things, but if you use a more systematic analysis, a bubble may become like this for another pitch angle, but if you know that you are never going to operate. At this, more angle of attack for pitch angle of the blade its fine because the question is this is all done in hover condition all these calculations.

Now, suppose you say I am going to do it on the side-ward flight climb flight or a descent flight various types of flight conditions usually heavily loaded rotor. Please note, heavily loaded rotor its damping is high normally because the rotor thrust is large; but, the lead lag damping you do not know flap damping is high usually lightly loaded rotor yes it can get into these problems.

So, it is a very tricky thing because, these results were generated only for hovering rotors not the descending rotor or a climbing rotor. What will happen? Make sure that you are far away; I do not know tail rotor you do not have any damper that is sure right? that is all; that means, it has to (( )) that is all very simple. Because, it cannot be a soft-in-plane tail rotor no because, otherwise unnecessarily you will put one more damper than its own life. All those other maintenance, every other problem, it is better to avoid as many components as possible. But, only problem is you cannot do it; sometimes you have to put a damper now this much is enough for lead lag motion, isolated lag. But, isolated lag is stable. Please note, that if I treat only lag dynamic it will be stable because there is little bit of damping that comes from the aerodynamics. Of course, if you want to put a damper - external damper yes it will be stable, but the external damper is put in only to avoid this problem flap lag instability or ground resonance problem.

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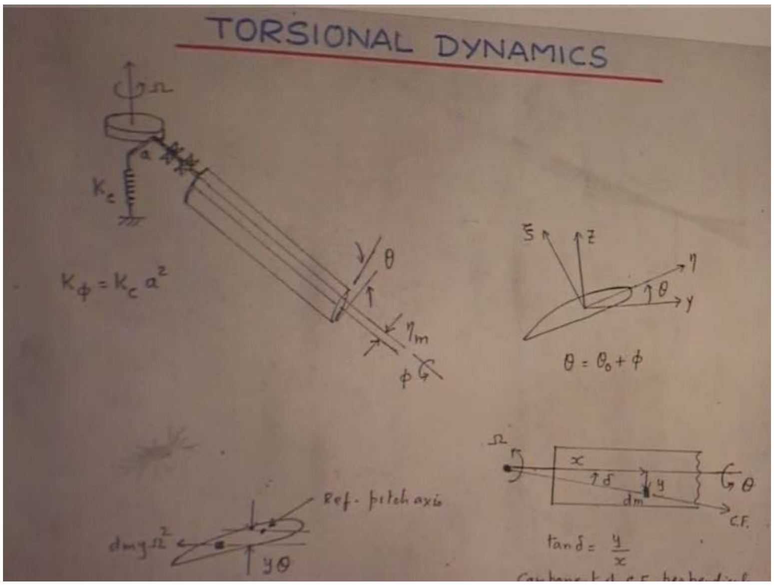

Now, let us take the next case which is the isolated torsional dynamics; but, if the little interesting; but, I will if you want I will go through the whole very simple model, please understand, very simple model. The blade is rigid blade straight and it has a pitch bearing and you have a control rod because that is the one which moves up and down to give the pitch angle change.

You assume blade is rigid; that means, it is not torsionally soft; first assumption you make and this control system stiffness k c with the offset a because there is a linear spring here there is a linear spring if you put, if you have a hinge then you know that this into this distance is the angle because if you take in terms of angle that will be controlled into a square that offset then, you can have barring torsional stiffness you can add have put everything into equivalent control stiffness.

Now, what is made dynamics there is a spring blade is rigid, but into the two barring we are like it is free to rotate. But, the restoring force is given by the spring when it rotates suppose you keep it at a pitch angle in this case you do not treat the blade as a line you have to treat the blade as a three d body this lateral distribution for just a simplicity case I will show if this is the blade I am drawing like this I put one heavy mass here and a heavy mass here; just it is spinning then this is my axis right. This mass will get a centrifugal force in some particular peculiar direction. That direction is given here; see this is the center; this mass is offset from the axis (Refer Slide Time: 33:00). So, it will get a force like this and the incline in the direction and if it is up also it will get in some other direction you follow because dynamics is a little bit tricky to understand. That is why if you formulate systematically you will see what is my moment; then, a component of the centrifugal force lag like this because, if it is horizontal this force will go; that force will go. A component will be like this; they will if you tilt it up you will get a component in the horizontal. They will try to give the moment; it will add to the stiffness of the dynamics. Now, how do you develop the equations? I will just start that point so that you will know that this is a little bit complex and there is the very interesting thing.

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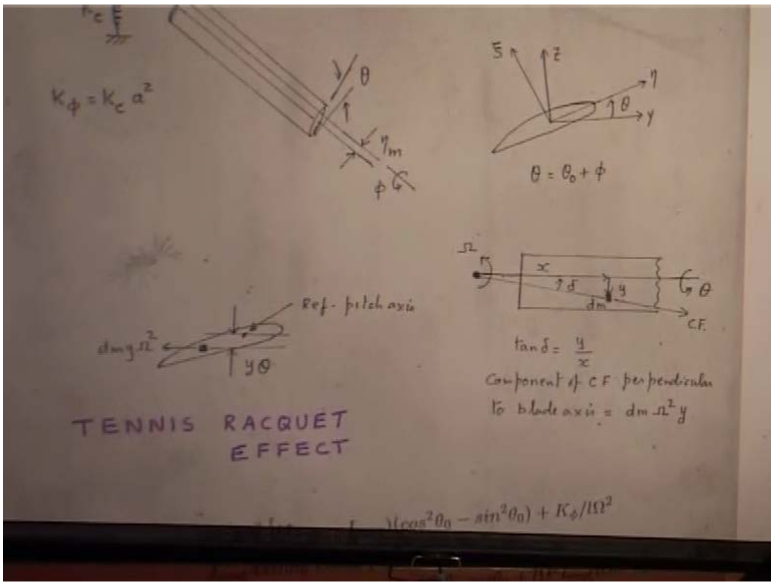

That is why there is a word which is called the tennis racquet effect later I will do because, tennis racquet is bit and he is holding it to air and keeps it at an angle. Then, you serve; see if you rotate it will try to straighten because of the two centrifugal forces. They will try to come and when he hits the ball, you will not know which location he is hitting.

Listen when he is swinging it may turn a little bit, but then where it hits you do not know. Initially, he may keep you may think it may come a little away. So, this is a terminology they use a tennis racquet effect I will just give the derivation and this is the reason why you will see control system. Stiffness plays a major role in when the blade is operating. See, we did the, I gave you the test data test data for I think 5 degrees or 16 degrees, 6 degrees, 900 r p m. Your matching when the r p m went up you find that the value you need to have pitch angle with 6 degree you are getting 60 newton or 61 newton but, what was observed was only 52 or 53 newton. But, why that big difference? You will… slowly that may be due to this reason I will just… because this derivation I will just show because, this you know slightly interesting part I will…

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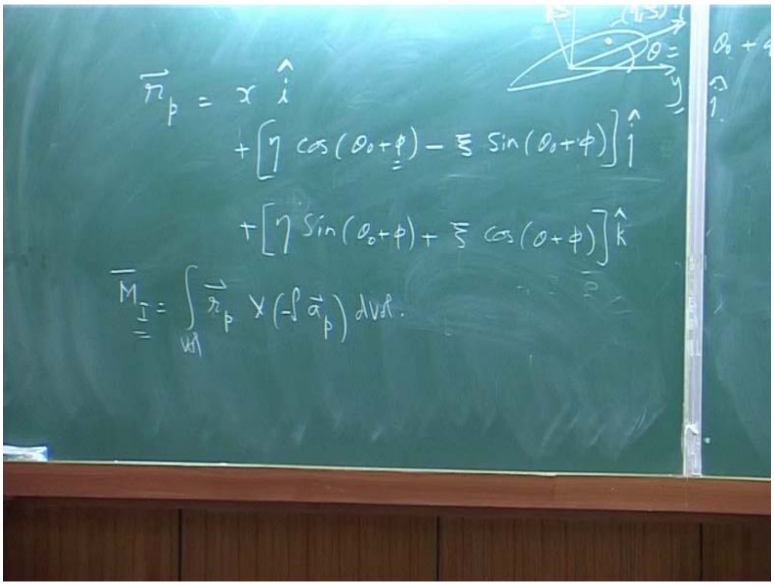

You take a position vector of any point which is x I because, why I said x i is from the center. You go to any line x i and then in the cross section, you are given a pitch angle theta naught and there is a elastic twist elastic twist because of the deformation of the spring. So, I have added a phi theta naught is what was originally said that is theta naught depends on where your that position is and then there is a torsional speed. So, you take any point on the cross section which is this is your theta this is xi and this is your y and this is your z and this angle theta naught plus phi.

So, take any point on the cross section and write the position vector position. Vector will be x i i is along the x direction then you will have y is this is j and this is k direction and this point is theta psi that is this point I am keeping the cross sectional coordinate system you will have plus theta cosine theta naught plus phi minus xi sine theta naught plus phi this is j and then plus theta sin is just a simple coordinate transformation plus xi cosine theta naught plus phi k.

Now, this is my position vector in the rotating frame of any point p in the cross section any point p this is a possession vector of any point on the blade. Now, you have to find out the velocity acceleration and then put the expression inertia load.

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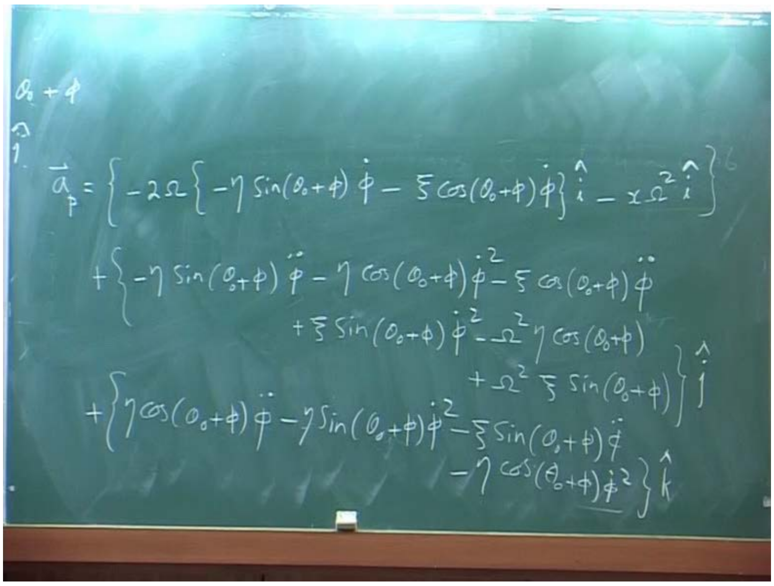

I will write the expression for the acceleration of the point p because, this is the rotating frame what are constants are theta x xi theta naught variable is phi we can vary. So, you differentiate you do everything because find out the acceleration that final expression will be like this

Minus 2 omega because you will have please note this you can your dynamic should be good then you will be able to you can try cosine theta naught plus phi e dot minus x omega square i then plus you can you can close this bracket minus eta sine theta naught plus phi phi double dot minus theta cosine theta naught plus phi phi dot square.

Minus xi cosine theta naught plus phi phi double dot then minus, sorry, plus xi sin theta naught plus phi phi dot square then minus omega square eta cosine theta naught plus phi plus omega square xi sin theta naught plus phi this entire term is j then you add k eta cosine theta naught plus phi phi double dot minus eta sine because you can try this because phi phi dot square that is why phi double dot minus eta cosine theta naught plus phi phi dot square. This is k this is the acceleration of the any mass point on the cross section.

Now, you want to write the equation of the motion. So, inertia moment you write inertia moment is as a vector is integral volume r cross p cross rho rho is the mass per unit length sorry unit volume. I am sorry this is the mass per unit volume now what you do is you take this cross product.

I am not going to write all the things; you will have i j k components. What we need is because we are interested in only the torsional motion; that means, the inertia movement only above the x axis is what we need because after that you take the cross product then simplify. You get the inertia moment about only the x axis that I will write here and you will get cross sectional integrals.

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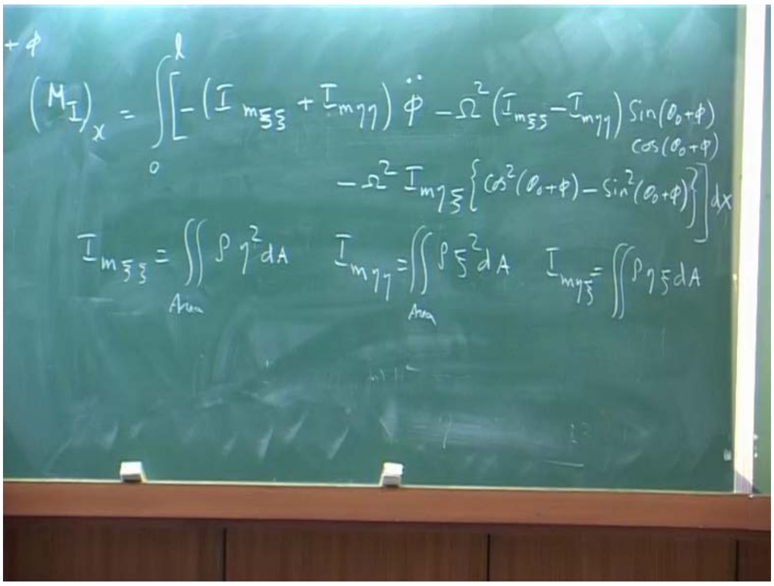

You will get the sorry M phi along x that is the inertia M phi i or (( )) equal to phi M I only along x axis and I am going to do this volume integral. What I will do is, I will split one along length another one cross sectional area now cross sectional area the rho d a I will write separately because for the entire cross section only thing variable theta and xi theta naught phi they do not change.

So, you do the cross sectional integral separately and then when you do it you will have 0 to l this is length of blade minus I m. This is the symbol I am using I will later I will tell you what is this eta eta phi double dot minus omega square I m I m eta eta sine theta naught plus phi cosine theta naught plus phi.

This is the product; then, minus omega square I m eta xi cosine square theta naught plus phi minus sine square theta naught plus phi into d x now where this I m xi is area rho eta square d A and then I m eta eta is rho xi square d A this is the area. And then this is the cross product I m eta xi is rho eta xi d A these are the cross products now you see this xi is eta square please understand do not this like your area inertia this is now mass moment of inertia per unit length that is all.

Now, this is my inertia moment and I have to get aerodynamic moment and then effect of the spring moment; when I add all of them that is my equation of motion. So, inertia moment I got this M i plus M aerodynamic moment and then spring moment due to the control system or all the combination the torsional spring. You put all of them equal to 0 that is your torsional equation. So, please understand I am having everything in cosine theta naught plus phi sin theta naught plus phi this not a very simple equation. So, what it done is yes this is about the because I have put that pitch axis.

Now, my mass can be anywhere mass center can be anywhere and mass center see that comes in the in this where is the mass center in the inertia it will come this is about the x axis because this is the feathering axis. When a this is acceleration at the particular point see this is the what are the moment is force this is force per unit volume r cross f that is the inertia moment that is all now what you can do is you can combined this aerodynamic moment and, but aerodynamic moment when you say it is the oscillating aerofoil because the phi is not constant it is a time varying because phi is a function of time; that means, you are talking about moment on the aerofoil which is the function of time.

So, it immediately goes to unsteady aerodynamics when you talk about lead lag I never express any aerodynamic term for lead lag motion also you are having the blade moving like this.

The flow is coming; that means, it is the pulsating flow you want the drag force in a pulsating flow we do not have expressions for those things later we will make lot of approximation and that is why we say this is the aerodynamic load that is why I did not put any aerodynamic expression when I derived my lead lag motion flap yes we use the quasi steady here again if you want you use the same thing torsion also you can use again quasi steady motion

So, this is where your complexities start appearing from the aerodynamic modeling now I will just write the equation of motion and if I now I have to assume because this is an integral over l my cross section properties can vary along the span because they need not be constant. Now, I have to make an assumption that I am having these properties are constant along the span. Then it becomes easy for my integration.

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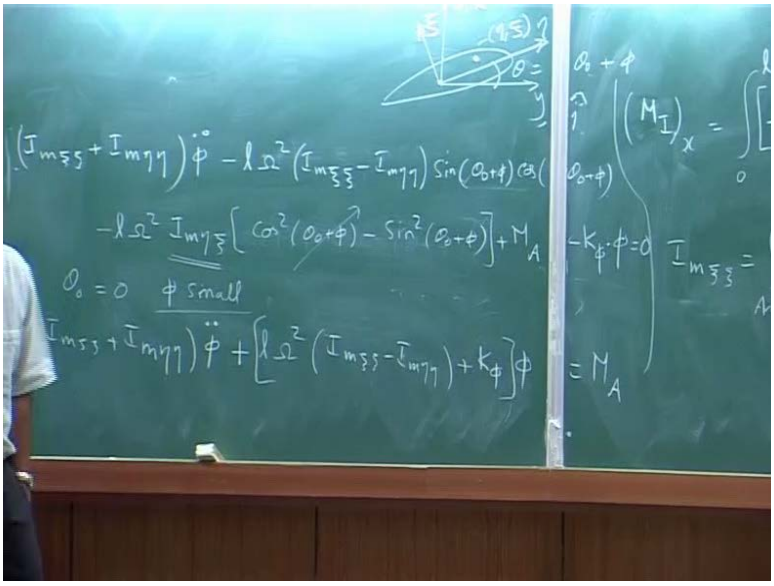

So, I assume that the properties are constant and my equation becomes minus l l is the length of the blade right from the root to tip you can take it I m xi xi plus I m eta eta phi double dot minus l omega square I m xi xi minus I m eta eta sine theta naught plus phi cosine theta naught plus phi minus l omega square same I m eta xi cosine square theta naught plus phi minus sine square theta naught plus phi then you put a aerodynamic load M A and because this other load will be k phi into phi equals 0 this is my equation because this is the moment due to the spring inertia moment this moment, but you will; now if you want you can take all the inertia and spring on one side keep the aerodynamics on one side may be, I will put now I do not think I you can remove this bracket itself (( )) moment that is all rho earlier that includes that is why I do not take it separately because acceleration at a point what I wrote the big expression it has everything coriolis term, but omega is constant.

So, there is the you know that whe-n you take a rotating being you will write the acceleration as four components relative acceleration coriolis acceleration angular acceleration centripetal acceleration that is for easy ease of thing, but if you this derivation follow the everything because, if you want to be systematic it better to start from scratch. So, that your fundamental principles are clear position vector then velocity then absolute acceleration half of point which is in a blade which is rotating then you will not make any mistake. Now, if you derive that this is my equation it will be sine and suppose, if you say my cross section is symmetric cross section then this term drops out because I m eta xi this will go to 0.

Even if you make this go to 0 this is non-linear sine. So, what you do is you expand; you do not say my theta naught suppose you say I am operating at 0 pitch angle then theta naught also is gone then my motion is very small about phi is small then this term you set it one you will have sine phi is phi.

Then you will have phi double dot there is the phi there is a phi term. So, you have I have my equations very simplified you follow; that means, you are making theta naught equal to 0 phi is small please understand if you make do not make these assumption then you have to retain this then you expand this by sin theta naught cosine phi plus cosine theta naught sin phi then phi is small you put that not that theta naught is small. Phi is small then you keep it, but if you make a very gross assumption theta naught is 0 phi is small then your equation will become l because this is just to give you the frequency, but the precise one is l then minus sorry plus I am changing the sign plus l omega square I m xi minus I m eta eta plus k phi phi equals aerodynamic moment i made it highly simplified with this assumption and I said I m eta xi is 0. So, I make all those assumptions it is a symmetric aerofoil theta naught is 0 phi is small now you know that this equation you can find out what is the natural frequency.

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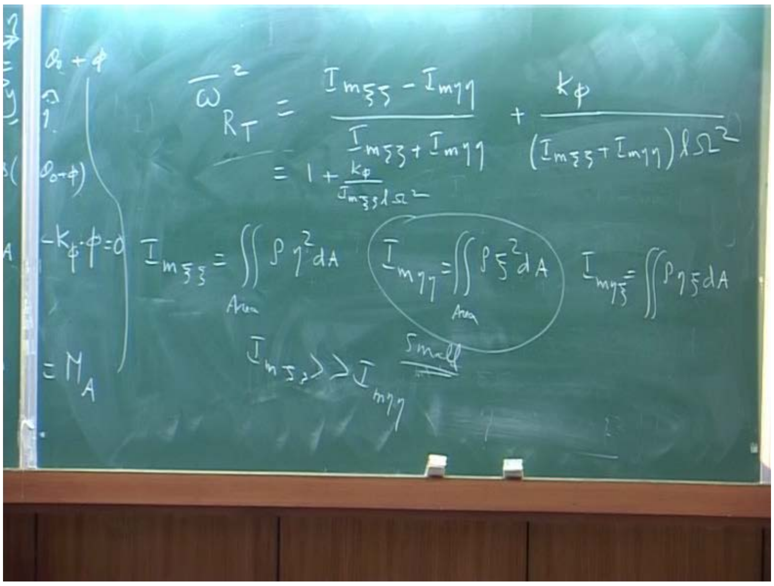

The frequency of this will be omega bar rotating torsion square this is if this is a dimensional non-dimensional you will get l omega square because this will be second derivative. So, this is d by d t. So, d omega will come. So, you put a non-dimensional then l omega square will cancel out.

you will have I m minus I m eta eta over I m plus I m plus k phi over I m xi xi plus i m eta eta l omega square location a feathering as a 25 percent that is all no in rotor blade now you see they do not want to keep the c g away from the elastic axis feathering axis feathering axis is if you keep the feathering axis separately aerodynamics center separately mass center separately then you will have lot of coupling flap pitch coupling everything will come. Because, when you flap up then inertia force will somewhere else that will give a twisting. So, you do not want do that and if your aerodynamics is ahead of the feathering axis it may twist lift will twist the pitch angle you can have some kind of divergence type of problem which is there in the fixed wing you will start having.

So, here the blade is designed such that mass center 25 percent chord elastic axis at 25 percent, but exactly 25. You know if it comes to 24.589, it is take, yes, it elastic axis of the cross section is also at 25 percent this is where the design goes very several iteration the box beam is kept only in the near trialing it does not contribute anything.

So, if you really want to know the design of the of rotor blade, you can spend your life 25 years, 30 years. Then, you would be an expert in how to design a rotor blade. Changing an aerofoil you may change the shape, but then the distribution of the structure in the mass you have to do. So, it is not that simple that I will go and then change as I want now you see this is my rotating torsional frequency usually this quantity this because eta if you look at the eta thickness is related to xi. So, this is usually small this is a small quantity.

Compare to this, but do not think that all these are for rotor blade they are of the order of 10 power minus 3 minus four something like that now if you say this quantity is small than this that is you write I m xi xi is much smaller than smaller I am making an assumption I m eta eta this becomes 1 1 plus k phi over I m l omega square sorry, I am sorry about that. Now, you see this particular one comes because of effect; this is due to your lead torsional spring if you have a non-rotating.