Introduction to Helicopter Aerodynamics and Dynamics
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Lecture No. # 21

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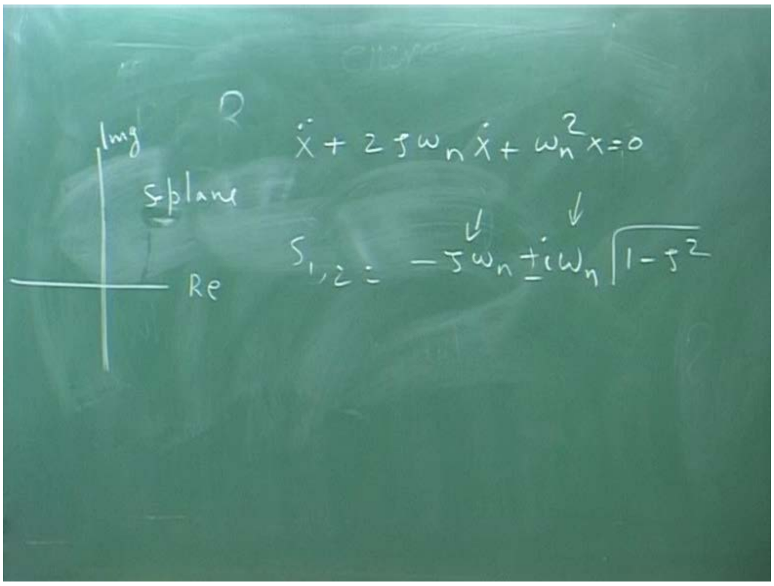

It is not necessary, you know, I will just tell the, see root locus is, you get the, suppose you have this equation, you get the homogeneous equation, roots, the roots are minus zeta omega and plus minus, what is that, I… this is the root. So, basically, root locus is, you take the real part, imaginary part, and real part put a point, imaginary, that will come. Now, you have some parameter in this system, which is basically the mu. For every mu, you will have a value and every gamma. That means, mu and gamma is one combination, I gave 4 sets for a given set of mu and gamma, sorry, not mu, that is, what is that omega R F.

For omega R F and gamma with varying mu, you will get this and this. So, you will plot those points, mu will be varying, but the other 2 quantities are fixed. Then, you go to the next set, next set, that is all. You plot this in this curve, real and imaginary, only thing is you have to estimate these 2 from your response curves that is all. Basically, root locus is, how the roots vary with any parameter because this is standard in control system or even in a flight dynamics because flight dynamics, they will always want to see if I vary some parameter. How my roots are and you know, that any root, real part, if it is on the right half, this is, they call it S plane, real and imaginary, the plus plane. So, if the root is on the right half, system is unstable, that is all. So, this is a standard root locus.

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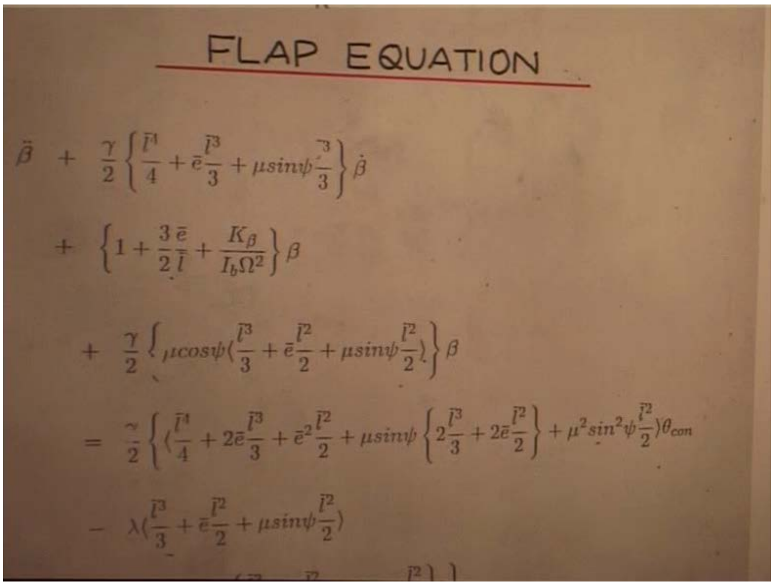

FLAP EQUATION

$$\ddot{\beta} + \frac{\gamma}{2}\left\{\frac{\bar{l}^4}{4} + \bar{e}\frac{\bar{l}^3}{3} + \mu sin\psi\frac{\bar{l}^3}{3}\right\}\dot{\beta}$$

$$+ \left\{1 + \frac{3}{2}\frac{\bar{e}}{\bar{l}} + \frac{K_\beta}{I_b\Omega^2}\right\}\beta$$

$$+ \frac{\gamma}{2}\left\{\mu cos\psi(\frac{\bar{l}^3}{3} + \bar{e}\frac{\bar{l}^2}{2} + \mu sin\psi\frac{\bar{l}^2}{2})\right\}\beta$$

$$= \frac{\gamma}{2}\left\{(\frac{\bar{l}^4}{4} + 2\bar{e}\frac{\bar{l}^3}{3} + \bar{e}^2\frac{\bar{l}^2}{2} + \mu sin\psi\left\{2\frac{\bar{l}^3}{3} + 2\bar{e}\frac{\bar{l}^2}{2}\right\} + \mu^2 sin^2\psi\frac{\bar{l}^2}{2})\theta_{con}\right.$$

$$- \lambda(\frac{\bar{l}^3}{3} + \bar{e}\frac{\bar{l}^2}{2} + \mu sin\psi\frac{\bar{l}^2}{2})$$

Now, we will start with the blade equations because you learnt about the, basically the natural frequency, how it gets influenced by the hinge offset and the root string, because hinge offset and root string, they increase the flap frequency. When the flap frequency goes up, rotating flap frequency, then the hub movements go up and you get a more control movement at the fuselage. So, that is how the flap frequency dominates your design; really, flap frequency is very, very important and you can have different types of rotor heads, like articulated or hingeless, etcetera. Now, this is without any, I introduce the word coupling, only flap motion and pitch angle is set, but there are certain cases where they introduce coupling.

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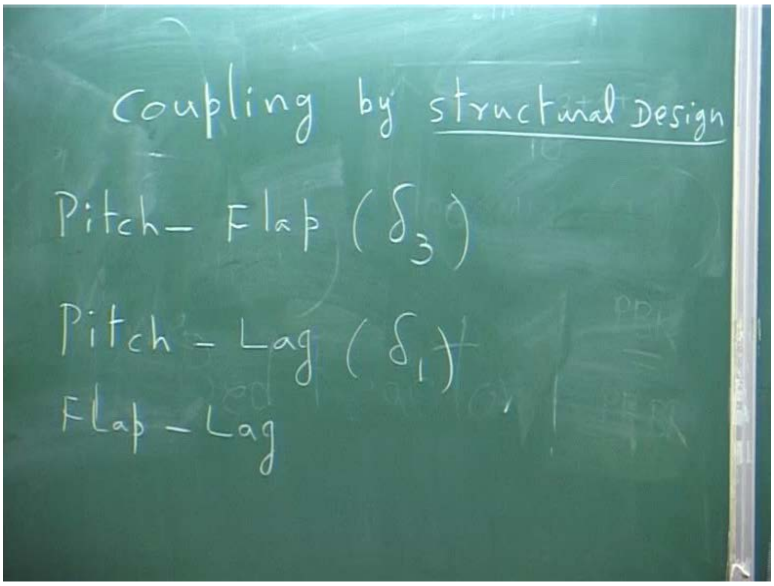

Coupling by structural Design

Pitch – Flap (δ_3)

Pitch – Lag (δ_1)

Flap – Lag

Coupling means, I will call it, what type of coupling you introduce, pitch-flap, then pitch-lag and flap-lag, that is, structurally coupling by some structural design, I will put a structural design, though we do not call it like this, you can introduce these couplings. When I say pitch-flap or flap-pitch, when the blade flaps up, the pitch angle will change. This coupling you call the, they call it delta 3, delta 3, alpha 3, something is there, they just call it delta 3.

And similarly, when you, when the blade goes back and forth, lead-lag, again the pitch angle can change. So, this, they call it delta 1 and of course, flap-lag coupling is, there is no symbol attached to that, it is just a structural coupling and how that comes, I will explain. So, these are 3 and these couplings are very powerful couplings, in the sense, they alter the dynamics of the blade frequency-wise as well as stability wise. So, we will, I will just briefly, this is not my, I will not be deriving a lot in these things, just to give you, that these are very critical.

Our flap equation is this, beta double dot beta dot beta some beta with respect to lot number with the forward speed and then, there is a theta control, control input, which is independent of flap motion, a pilot gives that input. Suppose, that input itself is a function of flap motion, that is, theta control is proportional to some factor to flap angle.

Then what will happen? You may have pilot input plus another term, which is proportional to the flap deformation. Now, the flap deformation term, from here will be taken to the left hand side because that will come as a beta. So, you find, I can actually alter the stiffness term because this is related to flap beta, which is a stiffness term. I can alter the stiffness term by introducing, that pitch-flap coupling, but since we have not introduced the lag dynamics, you can also alter the pitch of the blade by introducing lag-pitch coupling. But how do you introduce the coupling is the question.

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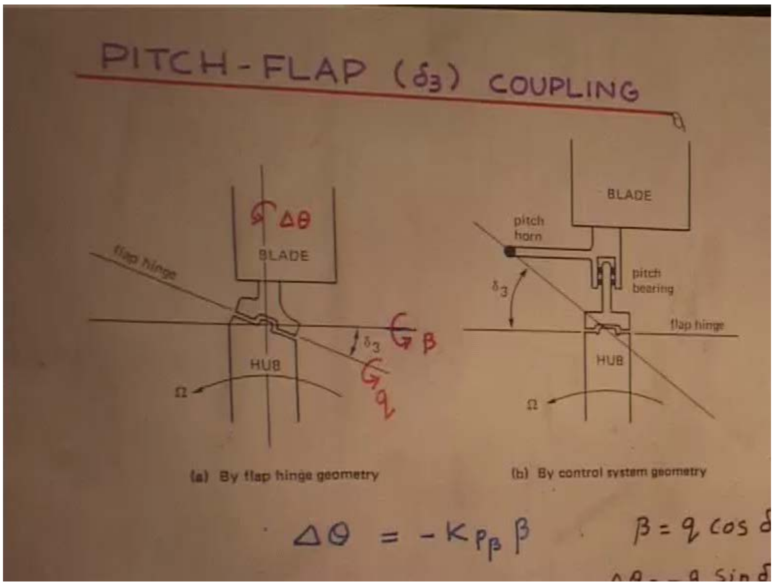

PITCH-FLAP (δ_3) COUPLING

(a) By flap hinge geometry

(b) By control system geometry

$\Delta\theta = -K_{P_\beta}\beta$

$\beta = q \cos \delta$

Mechanically or I would say structurally, which I showed there, I will show a diagram now, that will tell you how that coupling is… This is the pitch-flap coupling, what is done is, you have the hinge that is the flap hinge here. If the flap hinge is like this, look at only this diagram, this I will come to the later, flap hinge is like this when the, where flap motion means blade will come up and down because this is the hub and it is the direction of rotation omega.

But if the flap hinge is slightly inclined, now it can only flap about this hinge, the blade can only rotate about this hinge because it cannot rotate about.

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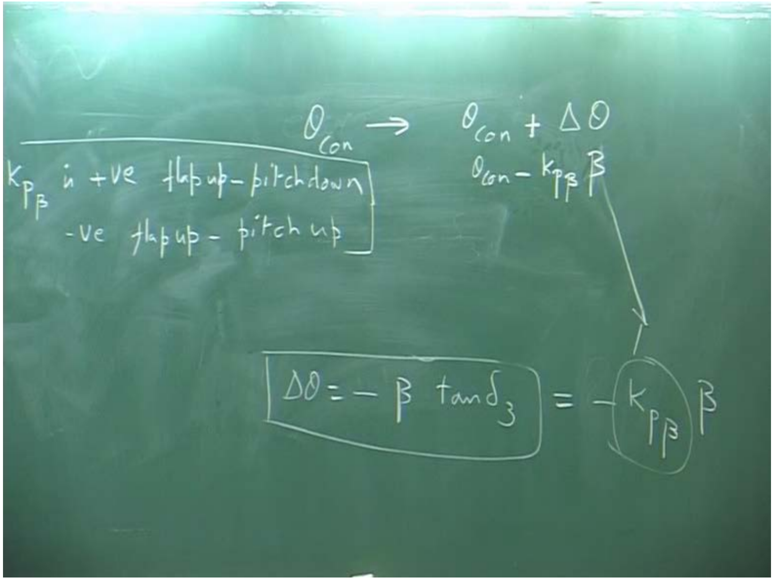

Now, what you would say is, I resolve this rotation q into 2 components, so q cos delta 3. So, I, so I call it beta as q cos delta 3, delta 3 is that angle. And then, the other component, which is pushing down, that is, change in pitch angle because the blade pitch angle is changing, that delta theta is q because I put a minus sign, because it is nose down because when it is flapping up, there is a nose down. So, your delta theta becomes minus, sorry, equal minus q sine delta 3. Now, you replace q by beta. So, you will have minus beta sine delta 3 by cosine delta 3, which will be...

So, please remember, whenever I call flap, flap deflection is only about the hub plane what it is coming out, that angle, not about what axis it is rotated. That is why, I put flap beta not in terms of q, q can be something else, I am replacing this q in terms of beta. Now, you see, whenever there is a blade is flapping, you have a change in the pitch angle. Now, depending on what direction this is, because it can be like this, it can be like this also. If it is like this, then when it flaps up, pitch angle will increase.

So, that is why this is written as minus k p beta beta; this quantity is your pitch-flap coupling, this quantity is coupling, but that minus sign is there, that is how you have arranged your, the hinge. And you may ask why should I put it, that we will come to that later, this is one type of arrangement.

Another arrangement, which is very obvious, is this. Suppose, I have a pitch horn, pitch horn means pilot input comes to this point, it will come out of plane and that means, my pitch angle changing.

But once pilot has kept a particular pitch, the spot light is held and assumed, the control linkage is stiff. Then, when it is flapping, flap hinge is here, but this point cannot come out because it is held, which means it has to rotate about this axis because 2 points are held and you introduce a delta 3 completely. So, this happens automatically when your pitch horn kept it someplace.

So, in the design, it is a little tricky. Suppose, if you take this horn, instead of putting it here, you bring it up to this point and then connect it here, like a 90 degree, then it flaps. No problem, that is why, if you see, in most of the design the pitch link, they will take it like this, bring it back to the axis and from there link is to go, then these 2 points are in the same axis, in the sense, 1 horizontal. So, it will do.

Now, why do you do it is a question? Mathematically, what we said is this is the change in the pitch angle. So, I have written here, in the hover case, from the flap equation just change my theta control to theta control minus, whatever is the change, delta theta because delta theta, because is the change in the angle. So, you just put it and this delta theta goes there, if it is flap up pitch down, this is flap up pitch is down, now what will happen? This term is flap up, when I put minus, what is, then this will become plus. So, I should put a plus sign here because flap up beta is positive. That means pitch angle is reduced. Now, you bring this term to the left hand side, add it with the flap term and then substitute for delta theta as minus k p beta beta.

So, when the minus, when it comes to the left hand side it will become plus because here this plus if I put this directly this will become minus k p beta beta. So, theta control become minus, I will take this minus to the left hand side, that will become plus along with the aerodynamic terms, that gamma because it will be theta as a coefficient, that coefficient term is basically aerodynamic term, that is, I will show that equation, say all these terms, all these terms, gamma over 2, entire term will come.

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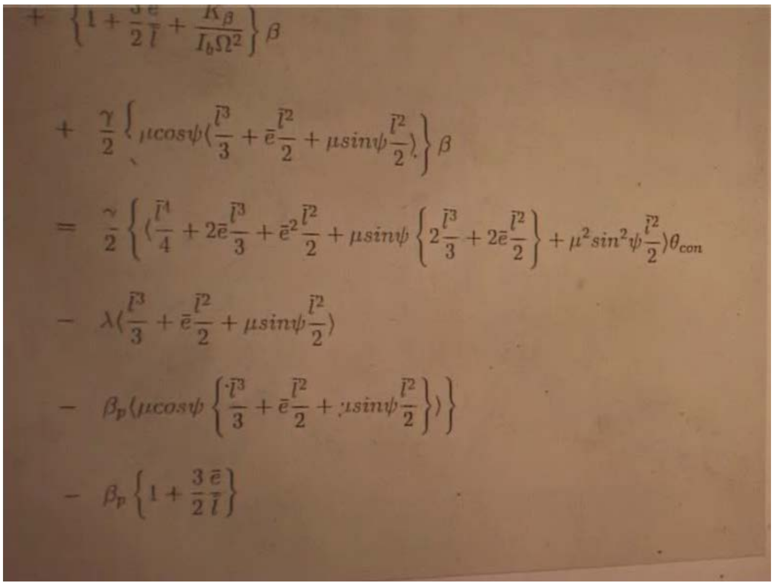

$$\begin{aligned}
+ & \left\{1+\frac{3}{2}\frac{\bar{e}}{\bar{l}}+\frac{K_\beta}{I_b\Omega^2}\right\}\beta \\
+ & \frac{\gamma}{2}\left\{\mu cos\psi\left(\frac{\bar{l}^3}{3}+\bar{e}\frac{\bar{l}^2}{2}+\mu sin\psi\frac{\bar{l}^2}{2}\right)\right\}\beta \\
= & \frac{\gamma}{2}\left\{\left(\frac{\bar{l}^4}{4}+2\bar{e}\frac{\bar{l}^3}{3}+\bar{e}^2\frac{\bar{l}^2}{2}+\mu sin\psi\left\{2\frac{\bar{l}^3}{3}+2\bar{e}\frac{\bar{l}^2}{2}\right\}+\mu^2 sin^2\psi\frac{\bar{l}^2}{2}\right)\theta_{con}\right. \\
- & \lambda\left(\frac{\bar{l}^3}{3}+\bar{e}\frac{\bar{l}^2}{2}+\mu sin\psi\frac{\bar{l}^2}{2}\right) \\
- & \left.\beta_p\left(\mu cos\psi\left\{\frac{\bar{l}^3}{3}+\bar{e}\frac{\bar{l}^2}{2}+\mu sin\psi\frac{\bar{l}^2}{2}\right\}\right)\right\} \\
- & \beta_p\left\{1+\frac{3}{2}\frac{\bar{e}}{\bar{l}}\right\}
\end{aligned}$$

In hover, mu will go off, only this term will remain and that will go and get added to the already existing stiffness term. And I have written for the case of hover with mu 0, that coupling will be this term. You see, it is o.k., I should put a plus sign here, this is some because repeated use, this is the additional stiffness term, which comes because of pitch-flap coupling. If this is positive, please understand, positive pitch, pitch-flap coupling means k p beta is positive; that means, flap up-pitch down.

This is the convention and then this is negative means flap-up pitch-up, pitch-up. So, you always do not take this negative into consideration, only the k p beta term as it is, if it flaps up, pitch angle will come down automatically, but it affects; please, the interesting part is, it affects the stiffness term.

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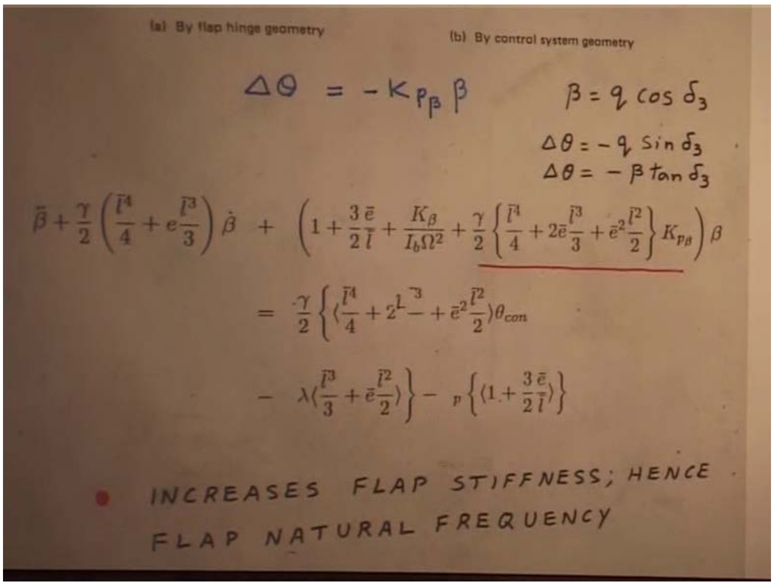

(a) By flap hinge geometry

(b) By control system geometry

$$\Delta\theta = -K_{P_\beta}\beta$$

$$\beta = q\cos\delta_3$$

$$\Delta\theta = -q\sin\delta_3$$

$$\Delta\theta = -\beta\tan\delta_3$$

$$\ddot{\beta} + \frac{\gamma}{2}\left(\frac{\bar{l}^4}{4} + e\frac{\bar{l}^3}{3}\right)\dot{\beta} + \left(1 + \frac{3}{2}\frac{\bar{e}}{\bar{l}} + \frac{K_\beta}{I_b\Omega^2} + \underline{\frac{\gamma}{2}\left\{\frac{\bar{l}^4}{4} + 2\bar{e}\frac{\bar{l}^3}{3} + \bar{e}^2\frac{\bar{l}^2}{2}\right\}K_{p_\beta}}\right)\beta$$

$$= \frac{\gamma}{2}\left\{\left(\frac{\bar{l}^4}{4} + 2^{L}\frac{{}^{3}}{} + \bar{e}^2\frac{\bar{l}^2}{2}\right)\theta_{con}\right.$$

$$\left. - \lambda\left(\frac{\bar{l}^3}{3} + \bar{e}\frac{\bar{l}^2}{2}\right)\right\} - p\left\{\left(1 + \frac{3}{2}\frac{\bar{e}}{\bar{l}}\right)\right\}$$

- INCREASES FLAP STIFFNESS; HENCE FLAP NATURAL FREQUENCY

In a simple (( )), I do not take the dynamics of the torsion here, I am just taking in a (( )) study change in pitch, if it gone up, change, pitch angle is changed.

Now, you see, this is an aerodynamic term. So, you have stiffness even in hover forward flight. Of course, mu sine psi etcetera is there, this gives additional stiffness to the blade in flap, but why do you want to have an additional stiffness to the blade? Make the flap frequency go very high, but earlier we said, if you make the flap frequency go very high your control moment will become very large.

So, you do not introduce this in main rotors, you put this in tail rotor because tail rotor loads are not much. Why you want to put in tail rotor? They do not want the blade to flap too much; if you make it stiff, the blade will not flap-up. But imagine if you have to put a stiffness that is going to become structurally very big weight, etcetera. This is nothing to do with structure, just pure structural design, that coupling.

So, you introduce a coupling, I have increased my stiffness of the blade tremendously and this, if you see in tail rotor blades in cheetah, chetak it is there. Cheetah, chetak, I do not know, whether you have seen it, you will go and see cheetah, chetak, that those (( )) helicopter, they have this, it will… That is because you do not want the blade to go too much, it is stiff and it will stay, but main rotor you do not introduce this intentionally in articulated blades. But if you introduce, yes you can. But now you see, by changing the sign of this, I can increase my flap frequency or I can decrease my flap frequency, it can be less than 1 also. That is why, I would say, flap frequency is normally greater than 1 or it can be less than 1, depending on what coupling you have put in. So, usually, people are a flap 1 point something, yes, it is it is true, but there can be situations where you can introduce this coupling and influence the flap. So, that is why, I said, it increases flap stiffness, hence flap natural frequency. But you have to rotate in air because gamma is there. Suppose, if there is no air in vacuuming, gamma is 0 because rho a c because it is a ratio of aerodynamics to inertia.

So, this is one coupling, but in hingeless blades and bearingless blade, this coupling always exist, you cannot avoid because of the elastic deformation of the blade. They come, but you cannot quantify it, is, it is a bit tricky, that is why they do not introduce this, but you solve a full elastic problem, elastic blade problem because depending on the deformation you can have.

It is like, suppose if this is the beam, the beam has, because it can have both flap and lag deformation, so what will happen is the blade will bend like this. Now, any lift will give lift force, is acting on the blade. The pitch link is here, pitch bearing; if the lift increases, this will give a pitching moment with respect to how far I am back.

Suppose, if I increase the lag load, then how much I have flapped up? That much, it will give the pitch. So, this is both flaps and lag will introduce this coupling. So, when they actually met the elastic blade, this was one of the very important things, they have to consider because that deformation you introduce intentionally, you do actually unintentionally I would put it and that depends on what is the elastic deformation of the blade, elastic deformation depends on what is the load.

So, it is very, very tricky to say, what exactly is the deformation? You will get coupling, but one has to know that that this effect occurs in those places because you may think, that this is what the blade angle etcetera, etcetera, but actually what you may get, this may be totally different in operation. That is why, they do lot of, you know world tower test, this test, several test and then check, match the data at least to a reasonable extent. Then, they say, you fly; that is why, these are all the pitch-flap and pitch-lag coupling.

They did research essentially by the, experimentally they made models and then studied the effect of this models, these parameters. What it does to the damping of the blade? This effects frequency, but this pitch-lag coupling, people do not use. Now, you see, pitch-lag also you can give delta 1, lag is what, blade is here, you have a hinge like this. Now, if this hinge is tilted like this, then when it, when the blade goes, it has to rotate, you will get a change in pitch, but it is not introduced in blade intentionally. But they studied in experiments it is very powerful, in the sense, in influencing the aero-elastic stability, even though these stills are, but how much angle.

Now, I will come to delta 3 in the, they use 45 degrees, that mean tan delta 3 is almost is 1, so tan 45 degree is 1. So, whatever, if it moves through 1 degree flap, 1 degree reduction in pitch, so that is 45 degrees they put. It is not a small 5 degrees, 10 degrees, you say substantially large, but the lag pitch coupling is very powerful. So, all these couplings are, they create, please understand additional problems, additional effect into the dynamics of the blade.

Now, I will, because pitch-lag is similar, because there is nothing I cannot show, because we do not put that in this equation, because lag dynamics is not part of this, because this is only flap dynamics, unless you introduce lag motion you cannot add that coupling. But just for information I am saying, there is another coupling, which is called the lag coupling. Then, you can have flap-lag coupling. How do you get that flap-lag coupling? See, one is from inertia that is the Coriolis. As the blade goes up like this, you will get Coriolis acceleration, that is, from inertia, that is very powerful; in addition to that, structurally can you have flap-lag coupling.

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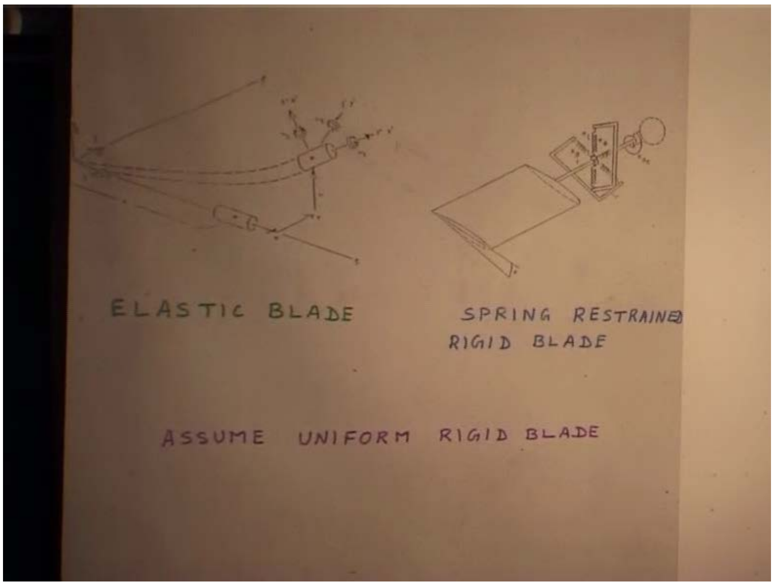

Show you, that one diagram, which you have seen earlier, this spring assembly, this is an approximation because I am only giving an idealization, because you have a soft spot on the blade about which you want the blade to flex in a hingeless blade.

Now, whether your pitch angle, whatever the pilot gives, does he gives inboard of this or outboard of this. If we change just a pitch angle outboard of that, that means, the spring assembly is not rotating because this is lag spring, this is flap spring. But suppose, you go and change the spring assembly, if your pitch angle of the blade is changed inboard, inboard means inside this, then what will happen? When you change the pitch angle, the spring also is going to rotate.

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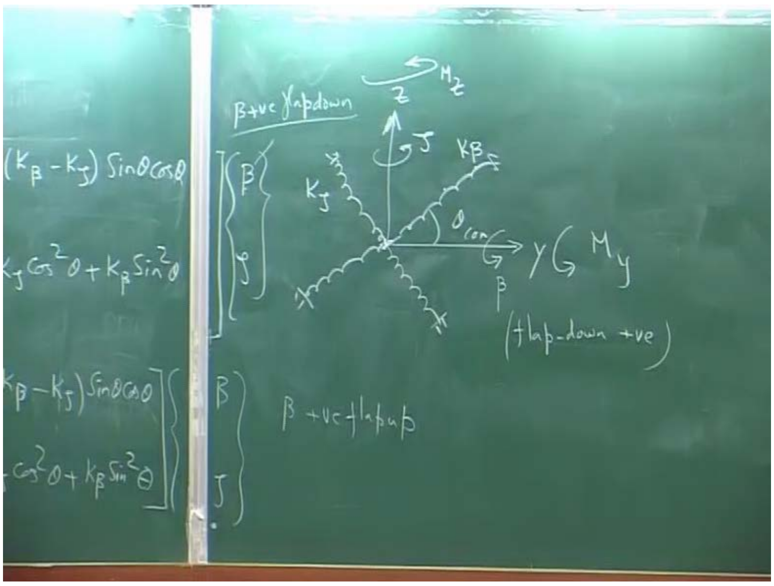

So, when you have a pitch, your spring assembly can come like this and this is my hub and this is my theta control or theta pitch angle, whatever it may be, spring is changed and my blade may be coming out like this. That means I am having my pitch bearing inboard of this.

So, what is flap? Flap is rotation about y-axis, lead-lag rotation about z-axis. Now, when I rotate, the spring assembly is now inclined. Because of this, if I make 1 rotation, you make, now flap and lag are small angles you take, whereas theta control can be 10, 15 degrees you can go. So, this you take it as slightly large, no problem, but flap you take small angle and assume, that it is a vector, small angle; you can treat it as a vector, not large angles. You give, if you say flap, that is, flap down, I call it beta or if I want flap up, I can put beta that side.

But sine convention will get a little (( )), in the sense, this is counterclockwise rotation. The blade is here, if I do flap, means this blade is coming down even though this is a plane of rotation. But if I want to use helicopter convention, flap up, speed positive, then this will be other way round and lead-lag counter clockwise rotation, this you may give zeta, this is the lead-lag. Now, if I have these 2, you have to find out, what is the flap moment, what is the lead-lag moment, that is, you can say applied external moment. Because the blade is going down, means you put some moment about this. So, you may call it M y and M z, you can have this as one convention, but you will find, because when I did this in the eighties, eighties means beginning eighties, we were discussing lot of this, how do you get that spring. Then, what different people had different, but this is one simple assembly, you have more complicated assembly also.

That means, you introduce hub stiffness also, not only blade stiffness put hub stiffness. That means, you will have 2 sets of assembly of springs, one hub, which it does not rotate; another blade, which is rotating like this. Now, you get an equivalent spring for that entire assembly, that is there in my (( )), I will not go into the all the formulation, but this one simple explain. But here please note, this is the flap down, I am taking positive counterclockwise rotation here, counterclockwise rotation. You know, there is lot of confusion because flap is up, you know what do I do, how I take it finally. If you take this way, you will get one set of equation, which is consistent, but then you want to introduce beta, I want beta up is positive, means you have to change the symbol sine, sine of beta.

When you do that, then you will get one set up equations. So, different people have used different, only thing is depending on how they define this beta. If you follow this as a conventional, counterclockwise rotation is positive, you will get one very systematically, absolutely no problem, that I will write it, I will write the stiffness. But you will find in different reports, it may be given in different format. So, you should not think, that this right or that is right, if you follow what assumption he has used, then you will find, that everything will be fine. So, I am just using this convention, applied moment M y, M z, because of that the blade is down and it is rotating. So, both flap, lag, this is purely structural coupling because my pitch bearing is inboard of the spring assembly.

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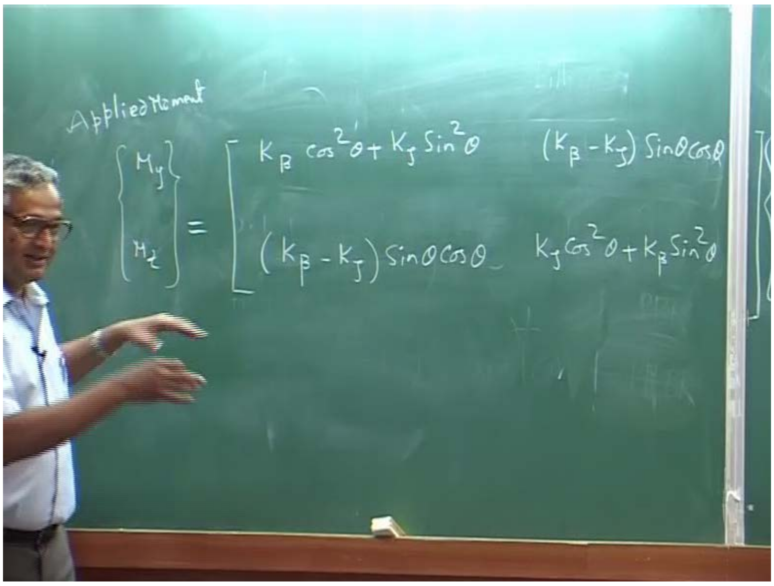

Now, if I have, I am putting in a matrix form, k, I call this as, this is the flap spring, k beta; this is the lead lag spring, which is k zeta. So, k beta cosine square theta plus… If you are interested, you try, you derive yourself, this is not very difficult because I want to discuss this, cosine theta k beta sine square theta multiplied by beta zeta, this equation corresponds to this, whatever this relation. This relation corresponds to this, it depends on how they, what they, this is the applied moment; applied moment M y, M z, flap moment, lag moment, but downward; please understand. So, the convention everything you have to be very careful.

Now, in NASA report, what I used was, I used minus beta and then I get the, not an applied moment, I used the restoring moment, so I put one more minus sign. So, you will find, that there will be some sign difference between this and there, and some other research paper you will find there be other sign, but you have to be careful, that what sign he has used for everything. Finally, all will come out to be correct equation, only thing is do not take it as though this is…

This is following that convention, if you modify the convention and apply restoring moment, this is the applied moment; restoring moment means what, the spring gives on the blade, this is what externally you apply on the spring, you can take what is the spring load, that comes on to the blade. Then, you will put a minus sign and then, if you say, that I want to define flap up is positive, means I put a minus beta. Then what will happen is some of the terms here change sign; that is just for information I am giving. If you want I can write the other expression also, restoring moment with the minus beta I will put, this is applied moment with that.

Now, I am saying restoring moment that means, the spring moment with flap up positive, with flap up positive; that means this is flap down is positive. If you want I can write flap down positive here. Because of my convention, here if I write this is M y, M z, this will become k beta cosine square theta plus, but you will, you have a minus sign here, k beta sine theta cos theta and this will be same, this is k beta sine theta cosine theta and this will be minus sign, minus of k zeta cosine square theta plus k beta beta zeta.

In this case, beta positive flap up; in this case, beta positive flap down. This is external moment; this is the restoring moment. Basically, what you do is if you want to apply moment to spring moment, you put a minus sign, everything minus, minus, minus, minus and then you change wherever beta is there, that sign will change. So, this minus, minus, minus, this will become plus plus, but these two will stay as minus, that is all.

This is just a sign convention, but what I want to discuss here is, this is very interesting discussion you have. If I introduce pitch bearing inboard of flap and lag hinges, I introduced flap-lag coupling and all the hingeless blades because hingeless blade you have here, pitch bearing is inboard. You change the pitch here, coupling is always there, you cannot avoid.

Now, how much coupling comes, this is again, see lot of debate, see one is what they did was, whether, whatever pitch angle I give, whether the same pitch angle goes to the spring also I can have, if it is, if I rotate by theta, the spring assembly rotates by R theta where R is less than 1. That means, I give a pitch angle of theta, but the spring assembly does not rotate to same angle, but it rotates slightly less than that, so they, that is the coupling. What they did was, they put R parameter, R. So, theta is R times theta control, this R can be from 0 to 1; 0 means pitch bearing is outboard.

So, wherever zeta is there, you put theta. If R is 1, this is directly theta control; pitch bearing is inboard. But if it is in between some value because it does not it there could be a slight deformation because this is a structure, I give a rotation it need not go, faithfully full rotation need not go and there can be a slight increase.

So, this interesting thing, what happened is if you look at that coupling term, suppose I have only flap motion, only flap motion, I do not have lead lag automatically, I get a lead lag moment, I get a lead lag moment because of this.

Now, if I do not want coupling, one is you take the R is 0, that means, pitch bearing outboard of the spring assembly. That means, the spring will not, there is no coupling between flap and lag structurally. But if you introduce K beta equals K zeta, that means, the flap stiffness is same as lag stiffness, that is, structural stiffness, the spring assembly. Then, I do not have any coupling, no matter what, I can have a rotation, whether by pitch bearing is inboard, pitch bearing is outboard, does not matter.

Now, this, whether you can design a blade? This is called matched stiffness blade configuration that flap stiffness, but please understand, I am deriving all these things with this type of idealized model elastic blade. You have to see at the junction where the stiffness is there, where it is the flexure you have to see, whether you can get matched stiffness, usually it is difficult.

But please note, experiments were carried out to study the effect of these and I will bring that, that you will find, data will, results will just change the damp thing data because I am, I am not gone into aero-elastic stability part, but these parameters are tremendous influence on the damp.

Structural coupling, flap lag, but now you realize elastic blade, hingeless blade, if I take, I cannot avoid this coupling because by construction I am getting that and pitch bearing is inboard. Now, you say, I do not want stiffness, then can I make the blade at the point where it is both, the flap stiffness, lag stiffness must be same. That means it should be a square section with the same distribution of material at the point, where it is flexing. If I can design this, coupling is removed.

(( ))

Yeah, isotropic, no, dimension, see dimension also equally matter. See, suppose you take isotropic, you take straight edge roller, you can bend one way easily, other way you can bend it, that is the…

(( ))

Yeah, yeah, yeah, but the stiffness part, I am representing the flexural stiffness by a spring. See, the blade is not going to bend; you are putting at some place where you deliberately make it a little soft. Yesterday, he was mentioning that, so that you create, but the stress levels, the strain levels are below the limit, but still it is flexing at that location. You make these 2 stiffness same, then you can eliminate the flap lag coupling, but the flap lag coupling is not, most normally lag stiffness is high, lead lag stiffness is much higher than flap stiffness in the design of the blade, but you find, you try to reduce, but this coupling will affect.

Now, you know automatically, I cannot analyze only flap motion because the moment I have flap, I get a lag moment, lag moment will make a blade go lead lag and if I have any pitch associated with that, then that will create a problem. And then, lead lag motion will come and give me a flap deformation because these 2 are coupled lead lag. If I have, I will get a coupling to lag.

So, flap and lag motion cannot be decoupled all along. We were, the full course till now only flap motion we were concentrating, only flap, we never bothered about lead lag motion. Actually as a subject or the dynamics of the helicopter, if you look at it, flap is very important. But lead lag creates more problems, but you cannot avoid. So, you have to consider lead lag; lead lag will come into the picture, then of course, elastic torsion. So, that is why, flap lag torsion, that treatment is the full dynamics.

And all these things start around seventies; people try to understand what you really going on in the flap lag those days. It is a rigid blade with this type of spring assembly only. All research paper, if you look at it, even eighties, yes, but now elastic blade models have come.

Now, I will just, that is a matched stiffness configuration, which can eliminate the structural flap lag coupling.

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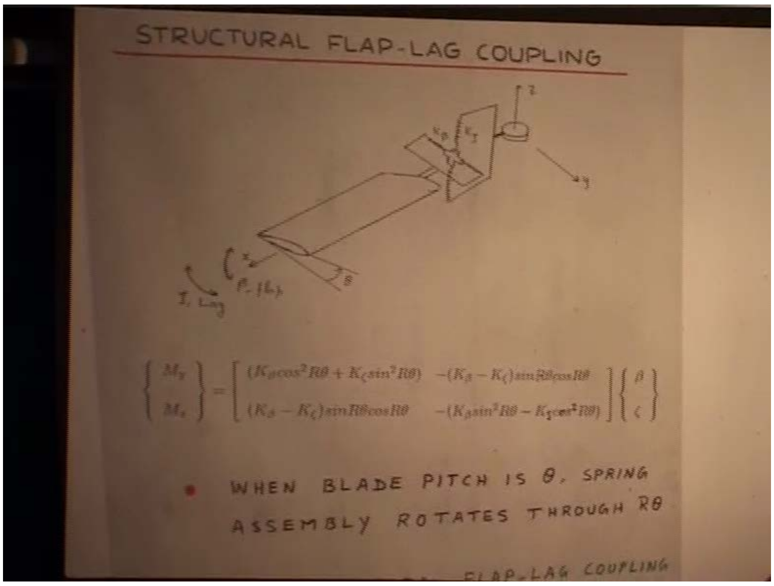

STRUCTURAL FLAP-LAG COUPLING

$$\begin{Bmatrix} M_y \\ M_z \end{Bmatrix} = \begin{bmatrix} (K_\beta\cos^2 R\theta + K_\zeta\sin^2 R\theta) & -(K_\beta - K_\zeta)\sin R\theta\cos R\theta \\ (K_\beta - K_\zeta)\sin R\theta\cos R\theta & -(K_\beta\sin^2 R\theta - K_\zeta\cos^2 R\theta) \end{bmatrix} \begin{Bmatrix} \beta \\ \zeta \end{Bmatrix}$$

- WHEN BLADE PITCH IS θ, SPRING ASSEMBLY ROTATES THROUGH $R\theta$

FLAP-LAG COUPLING

So, I just show here a diagram, whatever I have written there with the, because here I put a minus sign; you see, that is the restoring moment with the flap up positive, that is slightly modified.

When the blade pitch is theta, spring assembly rotates through R theta, so I put R. So, experimental set up was made coincident flap lag springs, so that the, and blade is made rigid and they rotated and then studied the, and then the data was given for correlation. So, those data I do not, whether I have for some problems, but not for all the cases, yeah.

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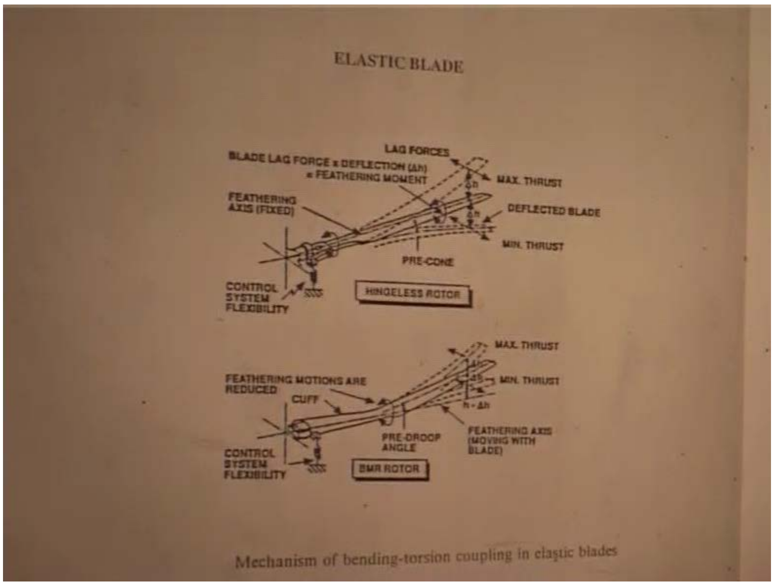

ELASTIC BLADE

Mechanism of bending-torsion coupling in elastic blades

Other case, which I want to tell you, how the flap lag coupling will come is because of this, this picture, I would know whether it… See, I will show this diagram, bit messier taken from some reference, from, to see this is the original blade, this is the deflection; this is the deflection, up, down, that dash line.

Now, lag forces will try to find a pitching moment lag, lead lag forces. Similarly, when the blade goes down, that lead lag force will give a pitching moment, that depends on how much it has flapped. So, how much you flap and that becomes a factor with the lead lag force, that is, the drag force in changing the pitch angle. Similarly, how much the blade has gone back and how much is the lift that will give a pitching? This is an elastic blade, that is why then it will give.

So, if you do not take these things properly, your control rod loads, everything will be wrong. So, I want to tell you, this is structural part, aerodynamics part is another.

So, usually predicting control rod load is always tougher, you really do not know because the elastic blade. Now, you will take more modes into according, whether this is really happening, get all the moments properly and then transfer them. And so, this is just a picture showing how one load will affect based on the deformation in another mode to something else, pitch angle. So, you see, flapped deformation lag load affecting pitch angle; similarly, lag deformation flap load influencing pitch angle.

Now, you see, if you want to analyze a problem, you cannot neglect anything, you have to have flap lag torsion problem, which is very complicated. So, we are not going to do that in this course, but this is just an introduction to you, to see the complexity in the factor of safety for what, for factor of safety. See, its all, see sometime I can only say what a little I know because factor safety is always it should have, infinite life that is the design usually. But how much load you are getting in the pitch link is very difficult to predict, sometimes the error can be a matter of 200, 300 percent, a particularly pitch link load.

So, you design by experience. So, you will learn now what is that load after you monitor pitch link load and then, you may replace it after so many hours. But once the flight, that is why, initial design you do, they have done theoretical calculations and theoretical calculations, if you are a beginner, nobody is assisting you, nobody has made anything, but you have to take a risk, you put a pitch, fly it, you measure it, you see it is now failing, then you make it bigger.

Then, you will find this is good, after that only you go by both, this much is a load, that is coming, where is it coming from, how it is coming from, but I do not know. Normally, pitch link may be, they will have safe life, I do not think they will have infinite life, I do not know, may be he will be able to tell you safe life means, you operated for so many hours, after that even if it looks good, remove it, put a new one, is that safe life. (( )) you do not take risk in those things because what you do not know, you have a, you know safe approach towards those problems.

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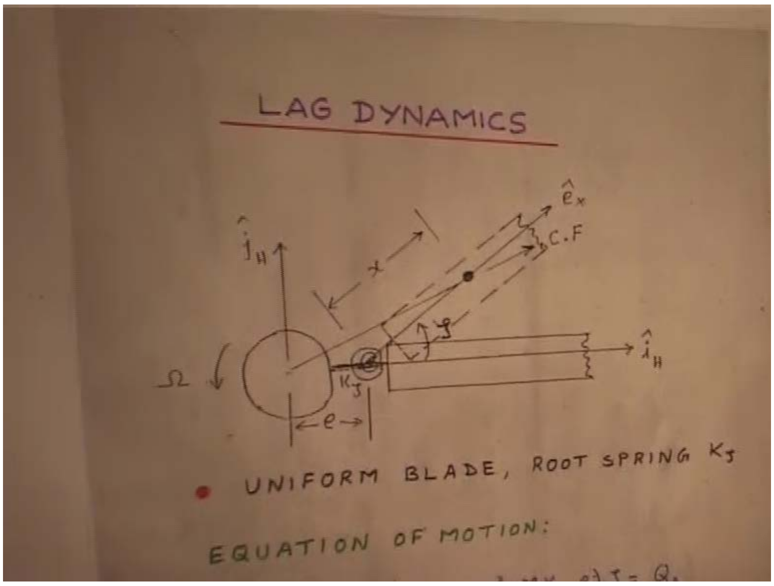
LAG DYNAMICS

- UNIFORM BLADE, ROOT SPRING K_ζ

EQUATION OF MOTION:

Now, I will introduce to you the lead lag dynamics. Now, please I, whether should I derive equations that you have to tell me. If you want to derive, that will go into full derivation, again position vector velocity acceleration, then inertia moment and write it, but I would not write any aerodynamic moment for this because that is a separate expression, which you have to because that is a drag load.

I will just briefly tell you that lead lag dynamics. So, this is isolated, isolated in the sense, only lag motion, I do not have any flap motion, please understand, like what all we derived earlier, it is only flap motion. Now, I am saying, either only lead lag motion. So, you put a spring with the hinge offset, alright. Now, like you have flap frequency, natural frequency, rotating flap natural frequency, you will have rotating lag natural frequency because this is a dynamic system, but what is that value.

If you derive the equation of motion for this, please understand, this is I am giving a deformation zeta, the blade is hinged about this point and there is a route spring, which is k sub zeta, which is, so the motion is in the plane of the diagram, if just.

So, when it goes, rotates, a blade can move back and forth, this is the equilibrium position. Now, if you write the dynamics for this, the equation will be like this, that is what you can derive it. If you want the derivation, I will start next class, not today, if you want the derivation.

This is my inertia, of course there is another term, which is inertia M X CG, you know the definition integral, m x d x, m is mass per unit length.

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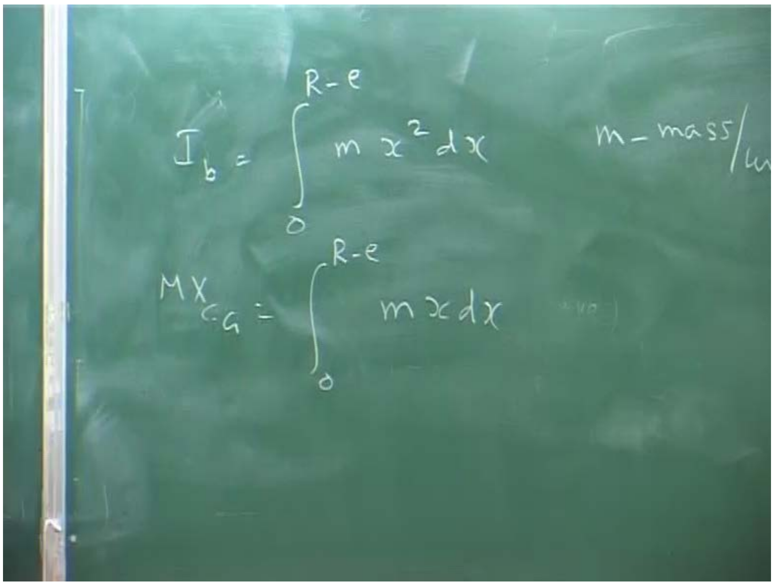

$$I_b = \int_0^{R-e} m x^2 dx \qquad m - \text{mass}/u$$

$$M X_{CG} = \int_0^{R-e} m x \, dx$$

So, here I b, we know that it is integral 0 to R minus e m x square d x and m x, m is mass per unit length.

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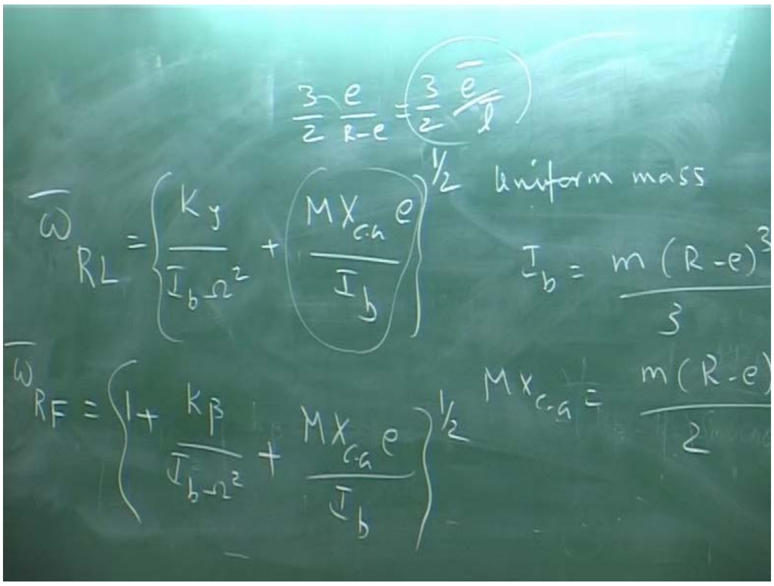

So, if you have uniform mass, uniform, uniform mass, you will have I b is m R minus 3 (( )) I am just writing, it is what I wrote last class and M X CG becomes m…, this for uniform mass. So, if I substitute for uniform mass case, I divide by I b converted into non-dimensional time, this is my lead lag rotating natural frequency.

So, I will put it omega bar RL is k zeta over I b omega square plus. If you want to put it as 3 by 2 e bar by l bar that is one form, that is uniform or else M X CG e over I b, both are fine and this is power half.

This is my rotating lag, if you, just for comparison if you take rotating, what we got, flap you add 1 plus. This is flap, you see this in flap frequency, you have 1 plus and of course, I do not have all the pitch, pitch flap coupling. If I put pitch flap coupling, that term will come. The one is missing here, that is not there, that is all.

Now, because of this, you find, that suppose k zeta is 0, but there is no spring, it is the articulated blade. Lead lag frequency rotating depends on what is the hinge offset at an M X CG RL, this whole term can be replaced as if it is a uniform mass, it will be 3 by 2 e over R minus e or you can write it as 3 by 2 e bar l bar e bar is e over r and 1 minus e bar is l bar and that is what is written there.

If this is 0, this is the way and the hinge offset, usually you will have about 4 percent 5 percent, but sometimes it may go to 10 percent hinge offset.

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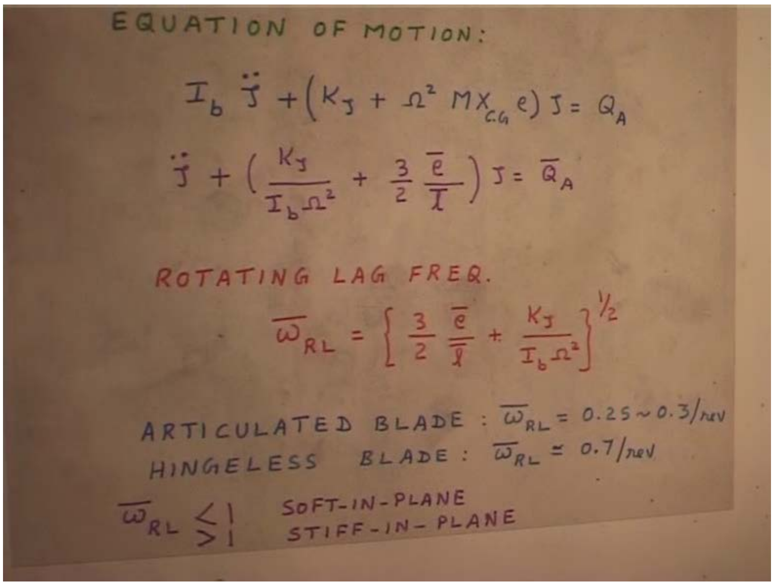

EQUATION OF MOTION:

$$I_b \ddot{\zeta} + (K_\zeta + \Omega^2 M X_{CG} e)\zeta = Q_A$$

$$\ddot{\zeta} + \left(\frac{K_\zeta}{I_b \Omega^2} + \frac{3}{2}\frac{\bar{e}}{I}\right)\zeta = \bar{Q}_A$$

ROTATING LAG FREQ.

$$\bar{\omega}_{RL} = \left\{\frac{3}{2}\frac{\bar{e}}{I} + \frac{K_\zeta}{I_b \Omega^2}\right\}^{1/2}$$

ARTICULATED BLADE : $\bar{\omega}_{RL} = 0.25 \sim 0.3/\text{rev}$

HINGELESS BLADE : $\bar{\omega}_{RL} \simeq 0.7/\text{rev}$

$\bar{\omega}_{RL} < 1$ SOFT-IN-PLANE

$\bar{\omega}_{RL} > 1$ STIFF-IN-PLANE

So, the lag frequency, non-dimensional, it is not very large, it is usually of the order of, I have given here, it is about for articulated blade, it is in the range 0.25 to 0.3, 0.25 to 0.3. Whereas, flap is one point, something and hingeless blade because articulated blade, there is no spring, it is just the hinge.

So, you get a hinge offset with this, that means, this term is 0, you can find out what is the corresponding hinge offset. So, immediately you can calculate, what is the rotating lag natural frequency, but if it is the hingeless blade, usually it goes to around 0.7.

Now, there is a one more terminology, which is there, there is a hingeless blade, I said 0.7, but can I have greater than 1? Yes, you can have greater than 1, provided you do this stiffness, you increase.

And suppose, another part, if this omega, if it is small that is, the rotor rpm, when you were starting, the blade rpm is low; when you go to the operational 100 percent rpm, it will have a much higher value. That means, my lead lag frequency, I am changing it as my omega changes the non-dimensional number itself is changing if I have stiffness, is it clear? Because as I changed my omega, lag frequency is changing, so this is a terminology.

If omega bar rotating lag is less than 1, this is called soft-in-plane blade, this is the terminology. If it is greater than 1, it is called stiff-in-plane blade; soft-in-plane, stiff-in-plane and at the operating rpm. Suppose, you operate that, I say, like our blade we operated 1500 rpm small one, a big rotor 300, 400, something like that.

If your lead lag frequency is greater than 1, then it will be called a stiff-in-plane; if it is less than 1 it is soft-in-plane. Now, how do you fix, which blade I will, which soft-in-plane blade or stiff-in-plane blade, but the same blade if I increase by rpm, what is originally stiff-in-plane will become a soft-in-plane.

So, it is really dependent on the operating rpm, but at the operating rpm which I should choose.